

Social Contagion

Principles of Complex Systems | @pocsvox
 CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
 Vermont Advanced Computing Core | University of Vermont



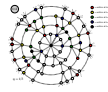
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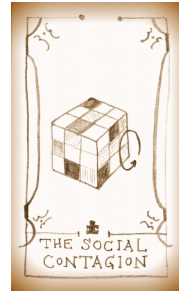
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



1 of 109

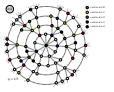


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Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



4 of 109

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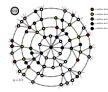


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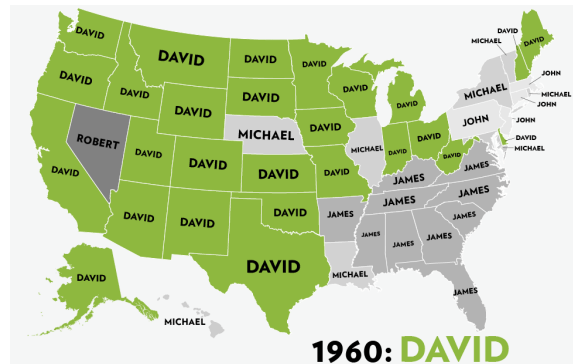
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



2 of 109



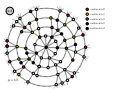
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Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



5 of 109

Outline

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

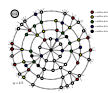
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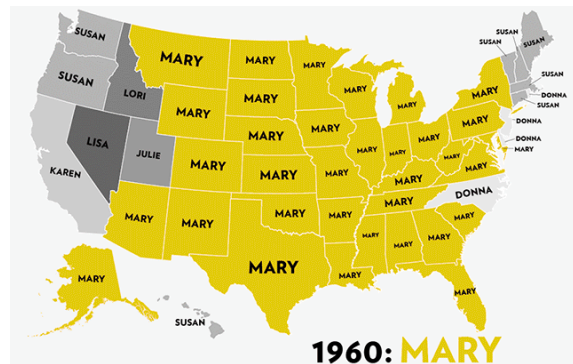
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



3 of 109



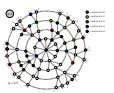
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Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



6 of 109

Things that spread well:

buzzfeed.com



Dangerously self aware: 11 Elements that make a perfect viral video.

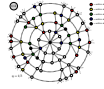
+ News ...

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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



9 of 109

Some things really stick:

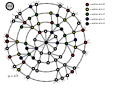


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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



12 of 109

LOL + cute + fail + wtf:

Oopsie!



BUZZFEED FELL DOWN AND WENT BOOM.

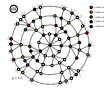
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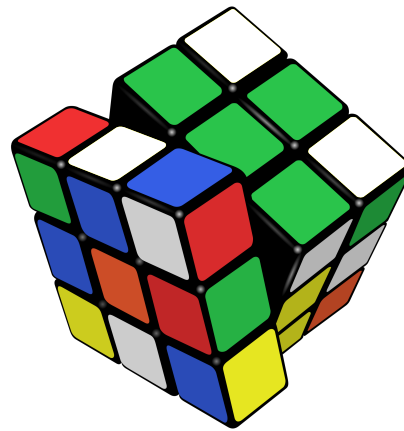
Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



10 of 109

wtf + geeky + omg:

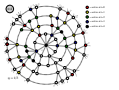


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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



13 of 109

The whole lolcats thing:

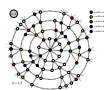


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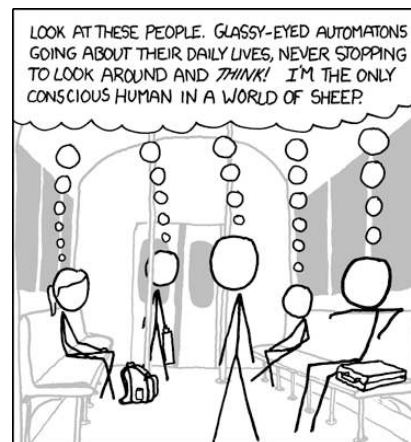
Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



11 of 109

Why social contagion works so well:



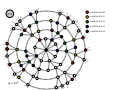
<http://xkcd.com/610/>

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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



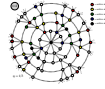
15 of 109

Social Contagion



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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16 of 109

Framingham heart study:

Evolving network stories (Christakis and Fowler):

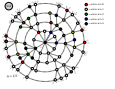
- The spread of [quitting smoking](#) [7]
- The spread of [spreading](#) [6]
- Also: [happiness](#) [11], [loneliness](#), ...
- The book: [Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives](#)

Controversy:

- [Are your friends making you fat?](#) (Clive Thomson, NY Times, September 10, 2009).
- [Everything is contagious](#)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

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Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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20 of 109

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Examples abound

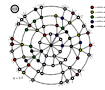
- fashion
- striking
- [smoking](#) [7]
- regional segregation [22]
- iPhones and iThings
- [obesity](#) [6]
- Harry Potter
- voting
- gossip
- Rubik's cube
- religious beliefs
- school shootings
- [leaving lectures](#)

SIR and SIRS type contagion possible

- Classes of behavior versus specific behavior : [dieting](#), [horror movies](#), [getting married](#), [invading countries](#), ...

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Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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17 of 109

Social Contagion

Two focuses for us

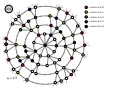
- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers? Highly popularized by Gladwell [12] as 'connectors'
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [19]

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



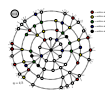
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Mixed messages: Please copy, but also, don't copy ...

- Cindy Harrell [appeared](#) in the (terrifying) music video for Ray Parker Jr.'s [Ghostbusters](#).
- Misframing: Appeals only to seed on exponential growth.

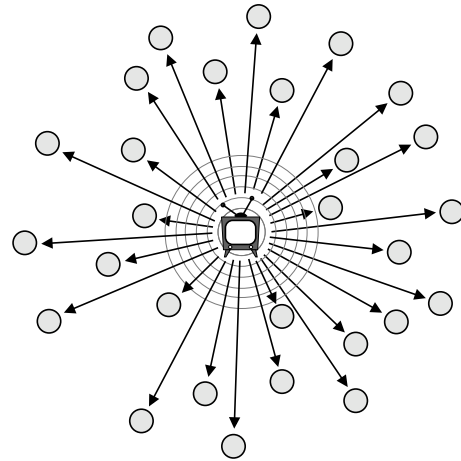
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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



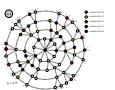
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The hypodermic model of influence



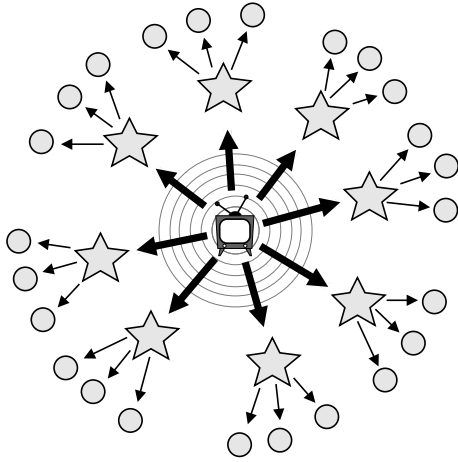
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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



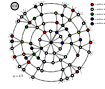
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The two step model of influence [19]



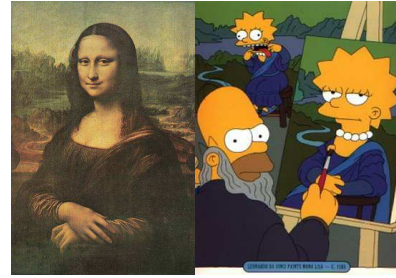
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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



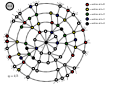
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23 of 109

The Mona Lisa



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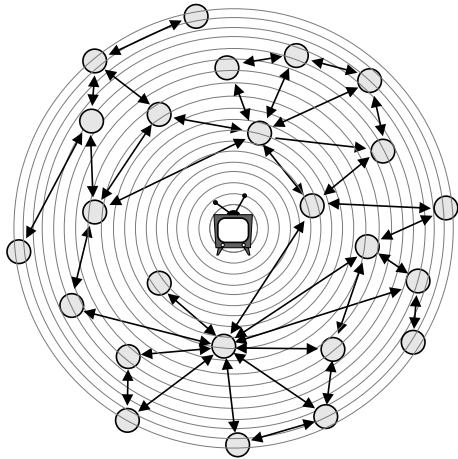
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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27 of 109

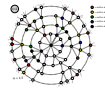
- “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon
- Not the world’s greatest painting from the start...
- Escalation through theft, vandalism, **parody**, ...

The general model of influence: the Social Wild



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



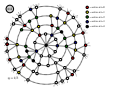
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24 of 109

‘Tattooed Guy’ Was Pivotal in Armstrong Case [nytimes]



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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28 of 109

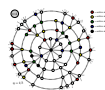
- “... Leogrande’s doping sparked a series of events ...”

Why do things spread socially?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are storytellers: *homo narrativus*.
- We like to think things happened for reasons ...
- Reasons for success are usually ascribed to intrinsic properties (examples next).
- Teleological stories of fame are often easy to generate and believe.
- System/group dynamics harder to understand because most of our stories are built around individuals.
- Always good to examine what is said before and after the fact ...

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



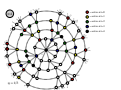
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26 of 109

The completely unpredicted fall of Eastern Europe



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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29 of 109

Timur Kuran: [20, 21] “Now Out of Never: The Element of Surprise in the East European Revolution of 1989”

The dismal predictive powers of editors...

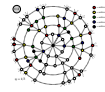


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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



30 of 109

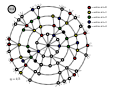
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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



33 of 109

From a 2013 Believer Magazine interview with Maurice Sendak

BLVR: Did the success of *Where the Wild Things Are* ever feel like an albatross?

MS: It's a nice book. It's perfectly nice. I can't complain about it. I remember Herman Melville said, "When I die no one is going to mention *Moby-Dick*. They're all going to talk about my first book, about f**ing maidens in Tahiti." He was right. No mention of *Moby-Dick* then. Everyone wanted another Tahitian book, a beach book. But then he kept writing deeper and deeper and then came *Moby-Dick* and people hated it. The only ones who liked it were Mr. and Mrs. Nathaniel Hawthorne. *Moby-Dick* didn't get famous until 1930.

Sendak named his dog Herman.

The essential Colbert interview: [Pt. 1](#) and [Pt. 2](#).

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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



31 of 109

Getting others to do things for you

A very good book: 'Influence' by Robert Cialdini

Six modes of influence:

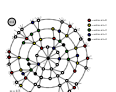
- Reciprocation:** *The Old Give and Take... and Take;* e.g., Free samples, Hare Krishnas.
- Commitment and Consistency:** *Hobgoblins of the Mind;* e.g., Hazing.
- Social Proof:** *Truths Are Us;* e.g., Jonestown, Kitty Genovese (contested).
- Liking:** *The Friendly Thief;* e.g., Separation into groups is enough to cause problems.
- Authority:** *Directed Deference;* e.g., Milgram's obedience to authority experiment.
- Scarcity:** *The Rule of the Few;* e.g., Prohibition.

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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

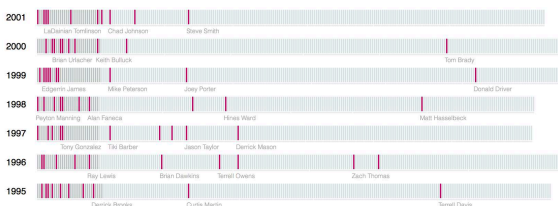
References



34 of 109

Drafting success in the NFL:

Top Players by Round, 1995-2012



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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References



32 of 109

Social contagion

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Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

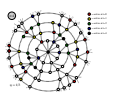
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Cialdini's modes are heuristics that help us get through life.

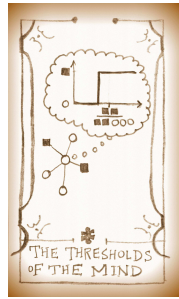
Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

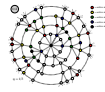


35 of 109



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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36 of 109

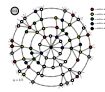
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Some important models:

- Tipping models—Schelling (1971) [22, 23, 24]
 - Simulation on checker boards
 - Idea of thresholds
 - Polygon-themed online visualization. (Includes optional diversity-seeking proclivity.)
 - Explore the Netlogo online implementation [29]
- Threshold models—Granovetter (1978) [15]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [2, 3]
 - Social learning theory, Informational cascades,...

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
37 of 109

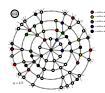
Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a **certain fraction of others** have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter... (**unrealistic**).
- Assumption: level of influence per person is uniform (**unrealistic**).

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
38 of 109

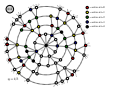
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Some possible origins of thresholds:

- Inherent, evolution-devised inclination to coordinate, to conform, to imitate. [1]
- Lack of information:** impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: **Network effects** or **network externalities**
 - Externalities = Effects on others not directly involved in a transaction
 - Examples: telephones, fax machine, Facebook, operating systems
 - An individual's utility increases with the adoption level among peers and the population in general

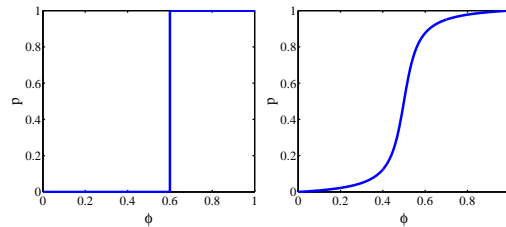
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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
39 of 109

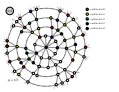
Threshold models—response functions



- Example threshold influence response functions: **deterministic** and **stochastic**
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Two states: S and I.

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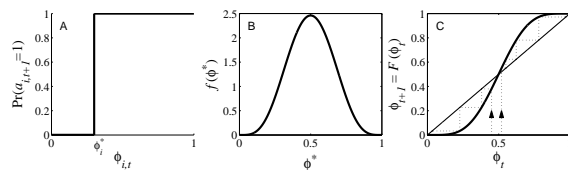
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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42 of 109

Threshold models

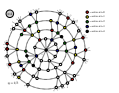
Action based on perceived behavior of others:



- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a **Critical mass model**

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

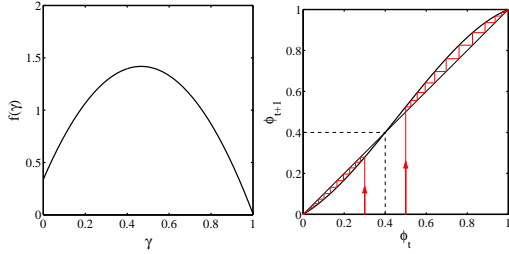


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43 of 109

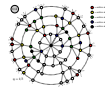
Threshold models

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Another example of critical mass model:



Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



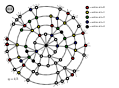
Threshold models—Nutshell

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Implications for collective action theory:

1. Collective uniformity \nRightarrow individual uniformity
2. Small individual changes \Rightarrow large global changes
3. The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
4. System stories live in left null space of our stories—we can't even see them.
5. But we happily impose simplistic, individual-centric stories—we can't help ourselves.

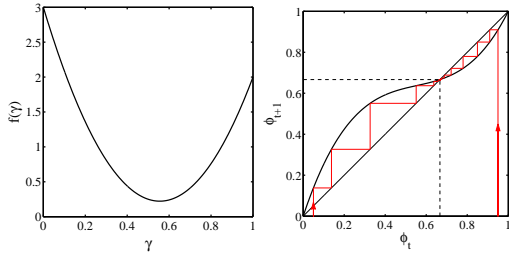
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



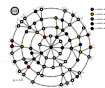
Threshold models

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Example of single stable state model:



Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

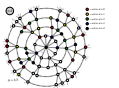


Many years after Granovetter and Soong's work:

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- "A simple model of global cascades on random networks"
D. J. Watts. Proc. Natl. Acad. Sci., 2002 [26]
- Mean field model \rightarrow network model
- Individuals now have a limited view of the world

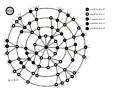
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



We'll also explore:

- "Seed size strongly affects cascades on random networks" [14]
Gleeson and Cahalane, Phys. Rev. E, 2007.
- "Direct, physically motivated derivation of the contagion condition for spreading processes on generalized random networks" [10] Dodds, Harris, and Payne, Phys. Rev. E, 2011
- "Influentials, Networks, and Public Opinion Formation" [27]
Watts and Dodds, J. Cons. Res., 2007.
- "Threshold models of Social Influence" [28]
Watts and Dodds, The Oxford Handbook of Analytical Sociology, 2009.

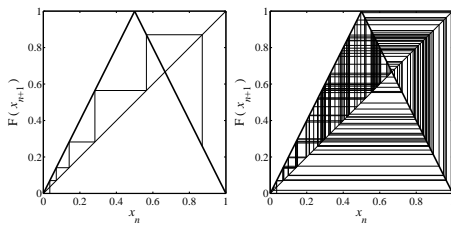
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Threshold models

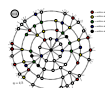
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Chaotic behavior possible [17, 16, 9, 18]



- Period doubling arises as map amplitude r is increased.
- Synchronous update assumption is crucial

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

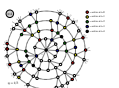


Threshold model on a network

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- Interactions between individuals now represented by a network.
- Network is **sparse**.
- Individual i has k_i contacts.
- Influence on each link is **reciprocal** and of **unit weight**.
- Each individual i has a fixed threshold ϕ_i .
- Individuals repeatedly poll contacts on network.
- Synchronous, discrete time updating.
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$.
- Individuals remain active when switched (no recovery = SI model).

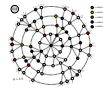
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Threshold model on a network

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Social Contagion

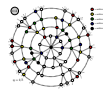
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
51 of 109

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Social Contagion

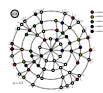
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



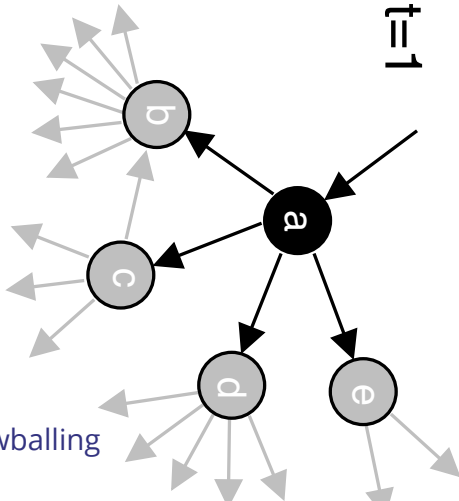
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52 of 109

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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53 of 109



Snowballing

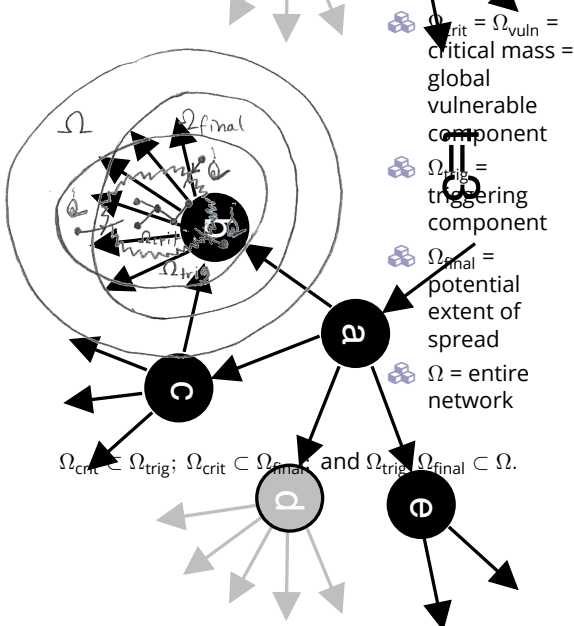
First study random networks:

- Start with N nodes with a degree distribution P_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a **cascade condition**

The Cascade Condition:

- If one individual is initially activated, what is the probability that an activation will spread over a network?
- What features of a network determine whether a cascade will occur or not?

Example random network structure:

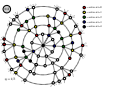


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53 of 109

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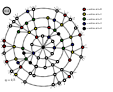
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
54 of 109

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Social Contagion

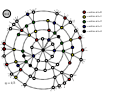
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
55 of 109

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
56 of 109

Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to **at least 1 more infected link**, then **activation spreads**.
- We need to understand which nodes can be activated when only one of their neighbors becomes active.

The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active **vulnerables**
- The vulnerability condition for node i :

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a **global cluster of vulnerables** [26]
- Cluster of vulnerables = critical mass**
- Network story: 1 node \rightarrow critical mass \rightarrow everyone.

Cascade condition

Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k .
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

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53 of 109

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56 of 109

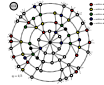
Cascade condition

Next: Vulnerability of linked node

Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_k=0}^{1/k} f(\phi'_k) d\phi'_k$$

- If linked node is **vulnerable**, it produces $k - 1$ new outgoing active links
- If linked node is **not vulnerable**, it produces **no** active links.



Cascade condition

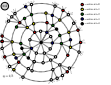
Two special cases:

(1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

(2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$



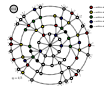
Cascade condition

Putting things together:

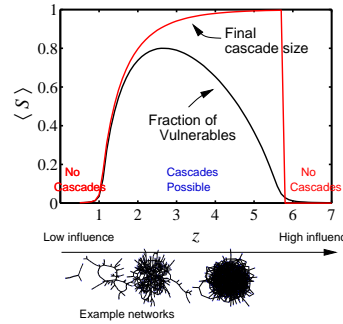
Expected number of active edges produced by an active edge:

$$R = \left[\underbrace{\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1 - \beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}} \right]$$

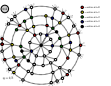
$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$



Cascades on random networks



- Cascades occur only if size of max vulnerable cluster > 0.
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

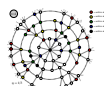


Cascade condition

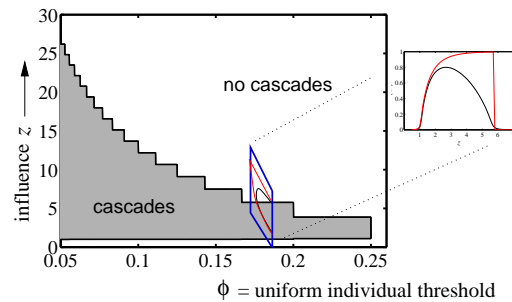
So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

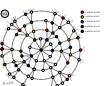
- β_k = probability a degree k node is vulnerable.
- P_k = probability a node has degree k .



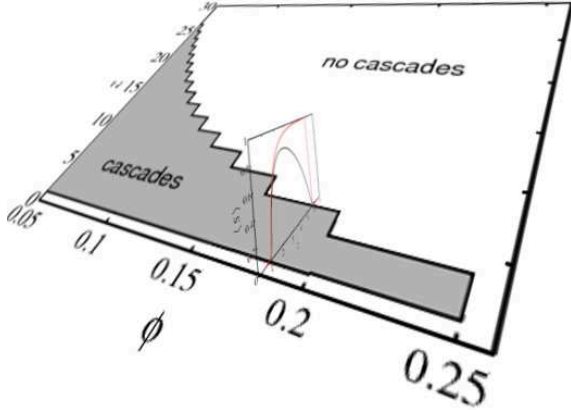
Cascade window for random networks



- 'Cascade window' widens as threshold ϕ decreases.
- Lower thresholds enable spreading.

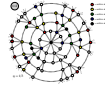


Cascade window for random networks



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



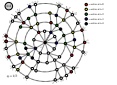
UNIVERSITY OF VERMONT
63 of 109

Threshold contagion on random networks

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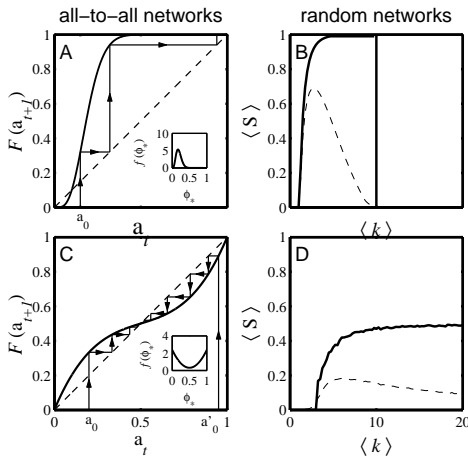
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

- Next: Find expected fractional size of spread.
- Not obvious even for uniform threshold problem.
- Difficulty is in figuring out if and when nodes that need ≥ 2 hits switch on.
- Problem beautifully solved for infinite seed case by Gleeson and Cahalane: "Seed size strongly affects cascades on random networks," Phys. Rev. E, 2007. [14]
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008. [13]



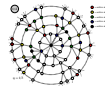
UNIVERSITY OF VERMONT
67 of 109

All-to-all versus random networks



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



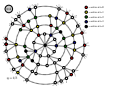
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64 of 109

Determining expected size of spread:

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

- Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$
- Capitalize on local branching network structure of random networks (again)
- Now think about what must happen for a specific node i to become active at time t :
 - $t = 0$: i is one of the seeds (prob = ϕ_0)
 - $t = 1$: i was not a seed but enough of i 's friends switched on at time $t = 0$ so that i 's threshold is now exceeded.
 - $t = 2$: enough of i 's friends and friends-of-friends switched on at time $t = 0$ so that i 's threshold is now exceeded.
 - $t = n$: enough nodes within n hops of i switched on at $t = 0$ and their effects have propagated to reach i .



UNIVERSITY OF VERMONT
68 of 109

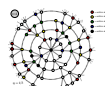
Cascade window—summary

For our simple model of a uniform threshold:

1. Low $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.
2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in "Cascade window."

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

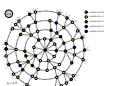
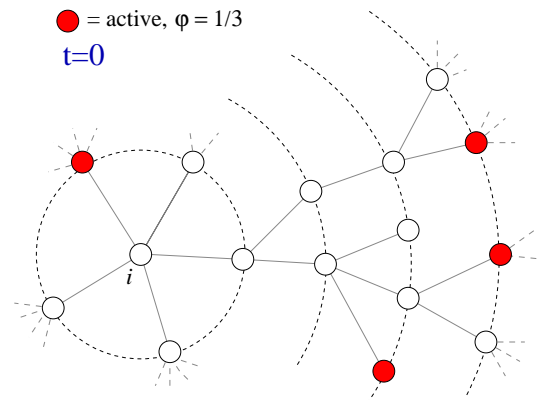


UNIVERSITY OF VERMONT
65 of 109

Expected size of spread

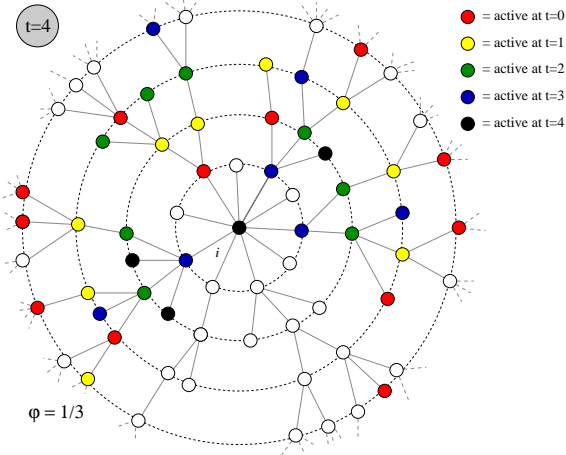
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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



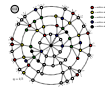
UNIVERSITY OF VERMONT
69 of 109

Expected size of spread



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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
70 of 109

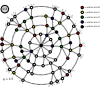
Expected size of spread

- Notation:** $\phi_{k,t} = \Pr(\text{a degree } k \text{ node is active at time } t).$
- Notation:** $B_{kj} = \Pr(\text{a degree } k \text{ node becomes active if } j \text{ neighbors are active}).$
- Our starting point: $\phi_{k,0} = \phi_0.$
- $\binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} = \Pr(j \text{ of a degree } k \text{ node's neighbors were seeded at time } t = 0).$
- Probability a degree k node was a seed at $t = 0$ is ϕ_0 (as above).
- Probability a degree k node was not a seed at $t = 0$ is $(1 - \phi_0).$
- Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^k \binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} B_{kj}.$$

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
73 of 109

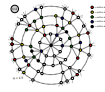
Expected size of spread

Notes:

- Calculations are possible if nodes do not become inactive (strong restriction).
- Not just for threshold model—works for a wide range of contagion processes.
- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determine $\Pr(\text{node of degree } k \text{ switching on at time } t).$
- Asynchronous updating can be handled too.

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
71 of 109

- For general t , we need to know the probability an edge coming into a degree k node at time t is active.

- Notation:** call this probability $\theta_t.$
- We already know $\theta_0 = \phi_0.$
- Story analogous to $t = 1$ case. For node i :

$$\phi_{i,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{k_i} \binom{k_i}{j} \theta_t^j (1 - \theta_t)^{k_i-j} B_{k_i j}.$$

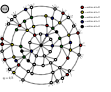
- Average over all nodes to obtain expression for ϕ_{t+1} :

$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}.$$

- So we need to compute $\theta_t \dots$ massive excitement...

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

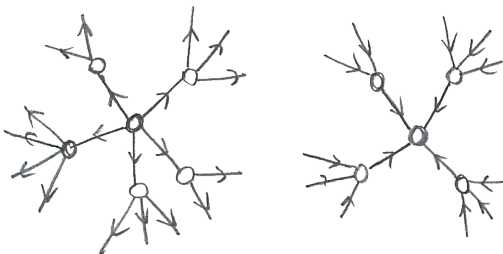


UNIVERSITY OF VERMONT
74 of 109

Expected size of spread

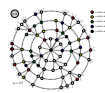
Pleasantness:

- Taking off from a single seed story is about **expansion** away from a node.
- Extent of spreading story is about **contraction** at a node.



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
72 of 109

Expected size of spread

First connect θ_0 to θ_1 :

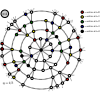
- $\theta_1 = \phi_0 +$

$$(1 - \phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_0^j (1 - \theta_0)^{k-1-j} B_{kj}$$

- $\frac{k P_k}{\langle k \rangle} = R_k = \Pr(\text{edge connects to a degree } k \text{ node}).$
- $\sum_{j=0}^{k-1}$ piece gives $\Pr(\text{degree } k \text{ node activates})$ of its neighbors $k - 1$ incoming neighbors are active.
- θ_0 and $(1 - \theta_0)$ terms account for state of node at time $t = 0.$
- See this all generalizes to give θ_{t+1} in terms of $\theta_t \dots$

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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
75 of 109

Expected size of spread

Two pieces: edges first, and then nodes

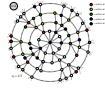
$$1. \theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{k,j}}_{\text{social effects}}$$

with $\theta_0 = \phi_0$.

$$2. \phi_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{k,j}}_{\text{social effects}}$$

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
76 of 109

Expected size of spread:

In words:

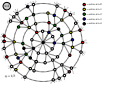
- If $G(0; \phi_0) > 0$, spreading must occur because some nodes turn on for free.
- If G has an **unstable fixed point** at $\theta = 0$, then cascades are also always possible.

Non-vanishing seed case:

- Cascade condition is more complicated for $\phi_0 > 0$.
- If G has a **stable fixed point** at $\theta = 0$, and an **unstable fixed point** for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
- Tricky point: G depends on ϕ_0 , so as we change ϕ_0 , we also change G .
- A version of a critical mass model again.

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
79 of 109

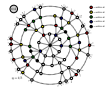
Expected size of spread

Iterative map for θ_t is key:

$$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1 - \phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{k,j}}_{\text{social effects}} = G(\theta_t; \phi_0)$$

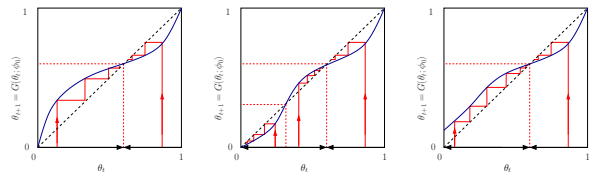
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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
77 of 109

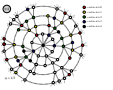
General fixed point story:



- Given $\theta_0 (= \phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.
- n.b., adjacent fixed points must have opposite stability types.
- Important:** Actual form of G depends on ϕ_0 .
- So choice of ϕ_0 dictates both G and starting point—can't start anywhere for a given G .

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
80 of 109

Expected size of spread:

- Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \rightarrow 0$.
- Depends on map $\theta_{t+1} = G(\theta_t; \phi_0)$.
- First: if self-starters are present, some activation is assured:

$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet B_{k,0} > 0.$$

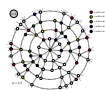
meaning $B_{k,0} > 0$ for at least one value of $k \geq 1$.

- If $\theta = 0$ is a fixed point of G (i.e., $G(0; \phi_0) = 0$) then spreading occurs if

$$G'(0; \phi_0) = \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet (k - 1) \bullet B_{k,1} > 1.$$

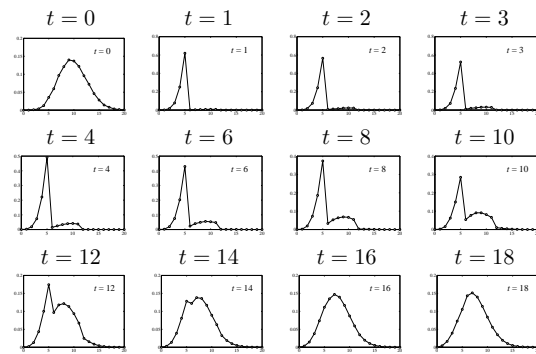
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
78 of 109

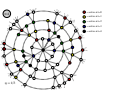
Early adopters—degree distributions



$P_{k,t}$ versus k

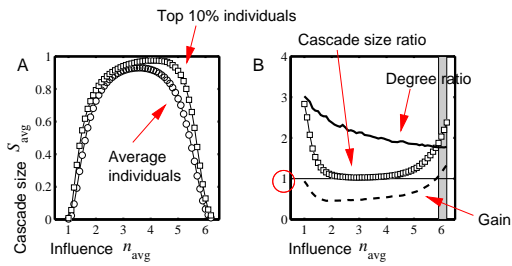
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
82 of 109

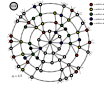
The multiplier effect:



- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.

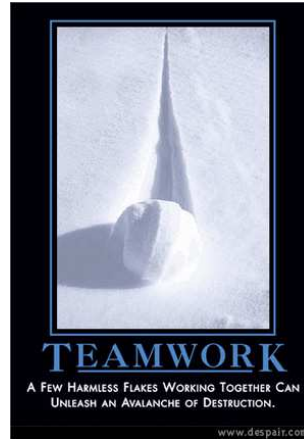
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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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83 of 109

The power of groups...

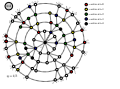


"A few harmless flakes working together can unleash an avalanche of destruction."

despair.com

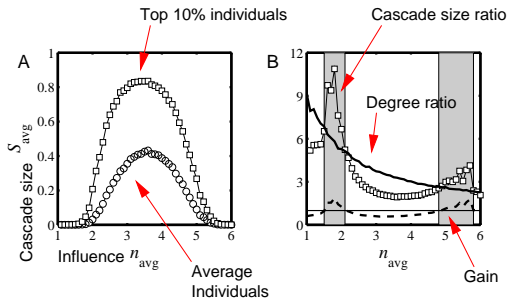
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
87 of 109

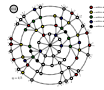
The multiplier effect:



- Skewed influence distribution example.

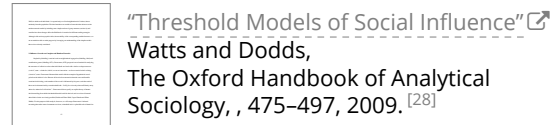
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
84 of 109

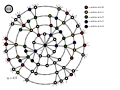
Extensions



- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: **group structure**

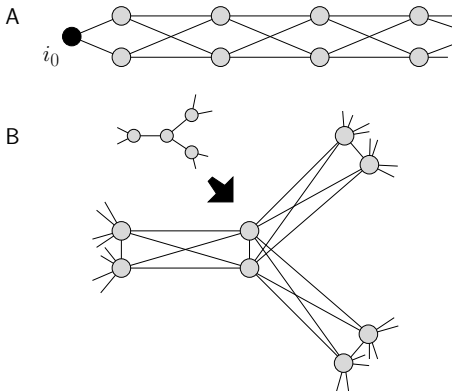
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
88 of 109

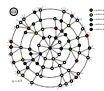
Special subnetworks can act as triggers



- $\phi = 1/3$ for all nodes

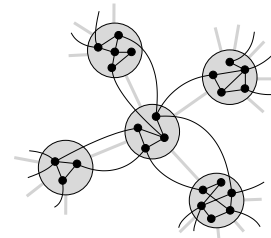
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



UNIVERSITY OF VERMONT
85 of 109

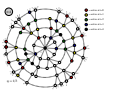
Group structure—Ramified random networks



p = intergroup connection probability
 q = intragroup connection probability.

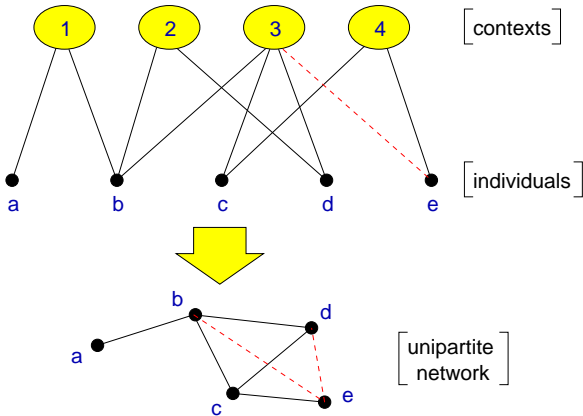
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



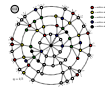
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89 of 109

Bipartite networks



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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



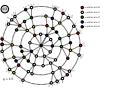
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90 of 109

Generalized affiliation model networks with triadic closure

- Connect nodes with probability $\propto \exp^{-\alpha d}$ where
 - α = homophily parameter
 - and d = distance between nodes (height of lowest common ancestor)
- τ_1 = intergroup probability of friend-of-friend connection
- τ_2 = intragroup probability of friend-of-friend connection

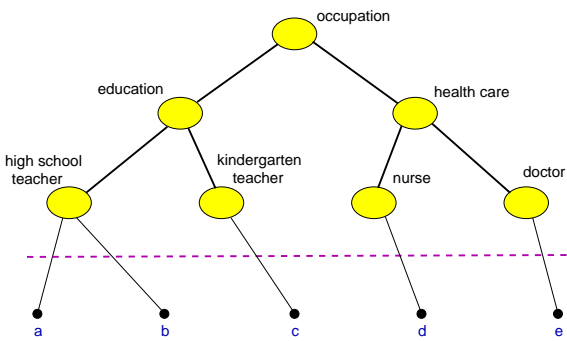
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



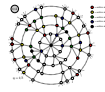
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93 of 109

Context distance



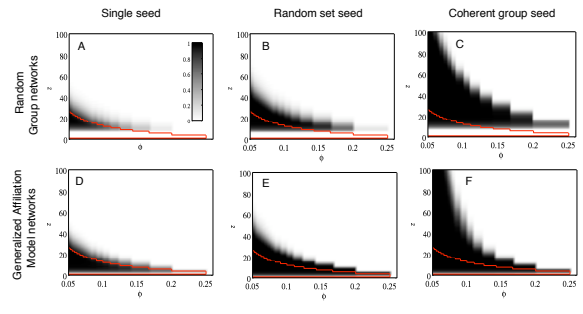
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



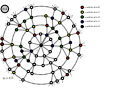
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91 of 109

Cascade windows for group-based networks



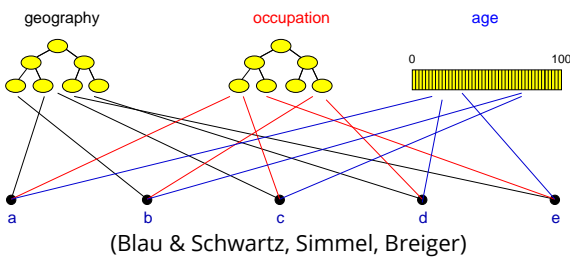
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



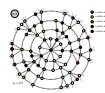
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94 of 109

Generalized affiliation model



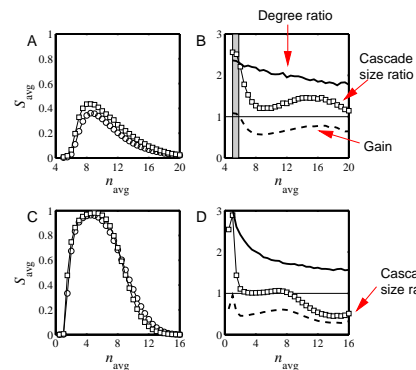
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



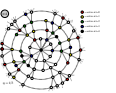
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92 of 109

Multiplier effect for group-based networks:



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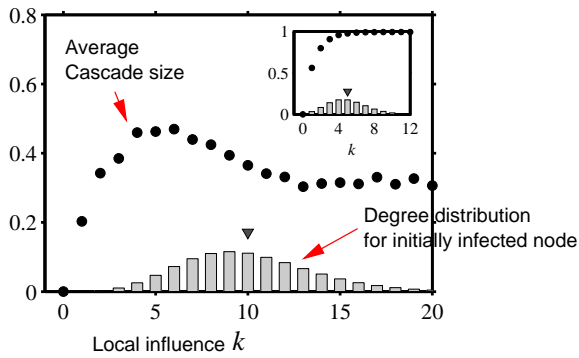
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



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95 of 109

- Multiplier almost always below 1.

Assortativity in group-based networks

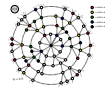


The most connected nodes aren't always the most 'influential.'

Degree assortativity is the reason.

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



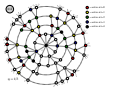
96 of 109

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



100 of 109

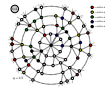
Social contagion

Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



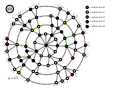
97 of 109

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



101 of 109

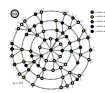
Social contagion

Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.

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Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



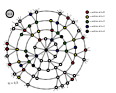
98 of 109

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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



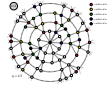
102 of 109

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Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.
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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
[References](#)



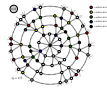
103 of 109

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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
[References](#)



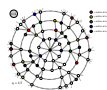
104 of 109

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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
[References](#)



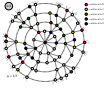
105 of 109

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PoCS | @pocsvox
Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
[References](#)



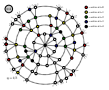
106 of 109

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Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
[References](#)



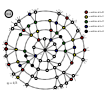
107 of 109

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PoCS | @pocsvox
Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
[References](#)



108 of 109

References X

PoCS | @pocsvox
Social Contagion

Social Contagion
Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115 116 117 118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133 134 135 136 137 138 139 140 141 142 143 144 145 146 147 148 149 150 151 152 153 154 155 156 157 158 159 160 161 162 163 164 165 166 167 168 169 170 171 172 173 174 175 176 177 178 179 180 181 182 183 184 185 186 187 188 189 190 191 192 193 194 195 196 197 198 199 200 201 202 203 204 205 206 207 208 209 210 211 212 213 214 215 216 217 218 219 220 221 222 223 224 225 226 227 228 229 230 231 232 233 234 235 236 237 238 239 240 241 242 243 244 245 246 247 248 249 250 251 252 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290 291 292 293 294 295 296 297 298 299 300 301 302 303 304 305 306 307 308 309 310 311 312 313 314 315 316 317 318 319 320 321 322 323 324 325 326 327 328 329 330 331 332 333 334 335 336 337 338 339 340 341 342 343 344 345 346 347 348 349 350 351 352 353 354 355 356 357 358 359 360 361 362 363 364 365 366 367 368 369 370 371 372 373 374 375 376 377 378 379 380 381 382 383 384 385 386 387 388 389 390 391 392 393 394 395 396 397 398 399 400 401 402 403 404 405 406 407 408 409 410 411 412 413 414 415 416 417 418 419 420 421 422 423 424 425 426 427 428 429 430 431 432 433 434 435 436 437 438 439 440 441 442 443 444 445 446 447 448 449 450 451 452 453 454 455 456 457 458 459 460 461 462 463 464 465 466 467 468 469 470 471 472 473 474 475 476 477 478 479 480 481 482 483 484 485 486 487 488 489 490 491 492 493 494 495 496 497 498 499 500 501 502 503 504 505 506 507 508 509 510 511 512 513 514 515 516 517 518 519 520 521 522 523 524 525 526 527 528 529 530 531 532 533 534 535 536 537 538 539 540 541 542 543 544 545 546 547 548 549 550 551 552 553 554 555 556 557 558 559 560 561 562 563 564 565 566 567 568 569 570 571 572 573 574 575 576 577 578 579 580 581 582 583 584 585 586 587 588 589 590 591 592 593 594 595 596 597 598 599 600 601 602 603 604 605 606 607 608 609 610 611 612 613 614 615 616 617 618 619 620 621 622 623 624 625 626 627 628 629 630 631 632 633 634 635 636 637 638 639 640 641 642 643 644 645 646 647 648 649 650 651 652 653 654 655 656 657 658 659 660 661 662 663 664 665 666 667 668 669 670 671 672 673 674 675 676 677 678 679 680 681 682 683 684 685 686 687 688 689 690 691 692 693 694 695 696 697 698 699 700 701 702 703 704 705 706 707 708 709 710 711 712 713 714 715 716 717 718 719 720 721 722 723 724 725 726 727 728 729 730 731 732 733 734 735 736 737 738 739 740 741 742 743 744 745 746 747 748 749 750 751 752 753 754 755 756 757 758 759 760 761 762 763 764 765 766 767 768 769 770 771 772 773 774 775 776 777 778 779 780 781 782 783 784 785 786 787 788 789 790 791 792 793 794 795 796 797 798 799 800 801 802 803 804 805 806 807 808 809 810 811 812 813 814 815 816 817 818 819 820 821 822 823 824 825 826 827 828 829 830 831 832 833 834 835 836 837 838 839 840 841 842 843 844 845 846 847 848 849 850 851 852 853 854 855 856 857 858 859 860 861 862 863 864 865 866 867 868 869 870 871 872 873 874 875 876 877 878 879 880 881 882 883 884 885 886 887 888 889 890 891 892 893 894 895 896 897 898 899 900 901 902 903 904 905 906 907 908 909 910 911 912 913 914 915 916 917 918 919 920 921 922 923 924 925 926 927 928 929 930 931 932 933 934 935 936 937 938 939 940 941 942 943 944 945 946 947 948 949 950 951 952 953 954 955 956 957 958 959 960 961 962 963 964 965 966 967 968 969 970 971 972 973 974 975 976 977 978 979 980 981 982 983 984 985 986 987 988 989 990 991 992 993 994 995 996 997 998 999 1000

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