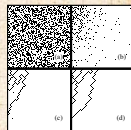


# System Robustness

Principles of Complex Systems | @pocsvox  
CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center  
Vermont Advanced Computing Core | University of Vermont



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Robustness

HOT theory

Narrative causality

Random forests

Self-Organized Criticality

COLD theory

Network robustness

References

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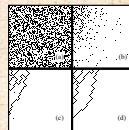
Sealie & Lambie  
Productions



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# Outline

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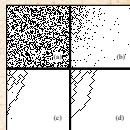
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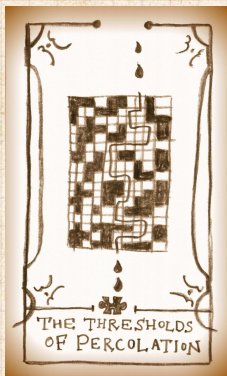
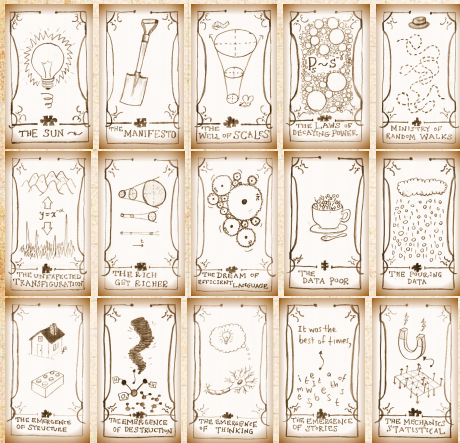
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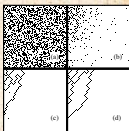
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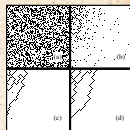
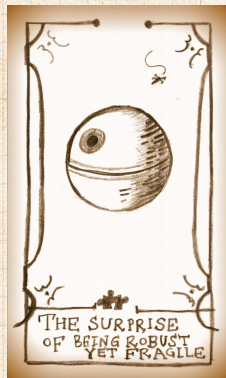
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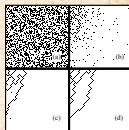
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 Many complex systems are prone to cascading catastrophic failure:

-  Blackouts
-  Disease outbreaks
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 But complex systems also show persistent **robustness**

 Robustness and Failure may be a power-law story...

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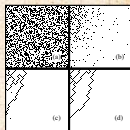
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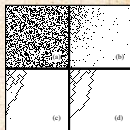
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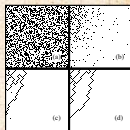
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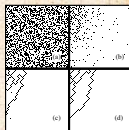
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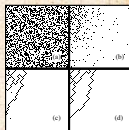
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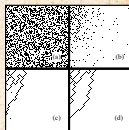
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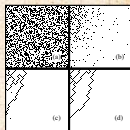
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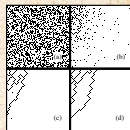
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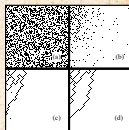
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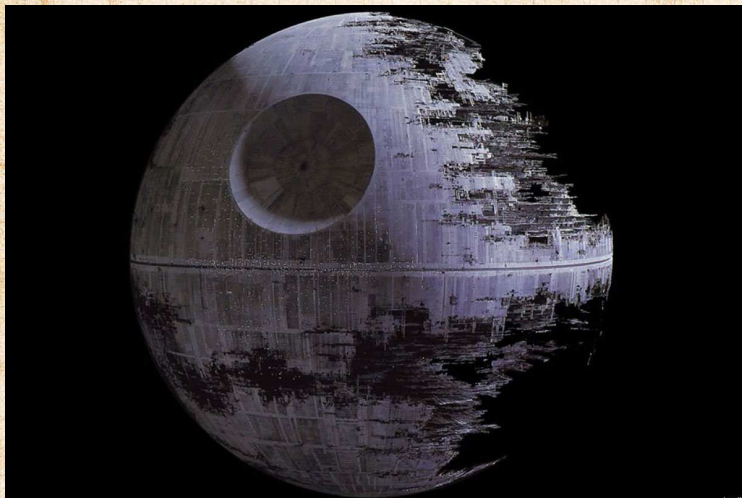
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# Our emblem of Robust-Yet-Fragile:

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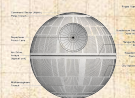
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“Trouble ...”

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## System robustness may result from

1. Evolutionary processes
2. Engineering/Design

🔍 Idea: Explore systems optimized to perform under **uncertain conditions**.

🔍 The handle:  
'Highly Optimized Tolerance' (HOT) [4, 5, 6, 10]

🔍 The catchphrase: Robust yet Fragile

🔍 The people: Jean Carlson and John Doyle [7]

🔍 Great abstracts of the world #73: "There aren't any." [11]

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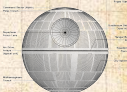
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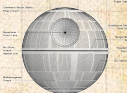
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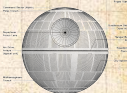
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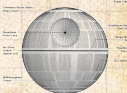
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## Features of HOT systems: [5, 6]

- High performance and robustness
- Designed/evolved to handle known stochastic environmental variability
- Fragile** in the face of unpredicted environmental signals
- Highly specialized, low entropy configurations
- Power-law distributions appear (of course...)

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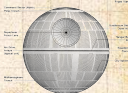
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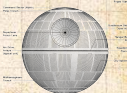
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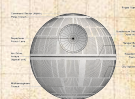
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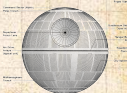
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HOT combines things we've seen:



Variable transformation



Constrained optimization



Need power law transformation between variables:  $(Y = X^{-b})$



Recall PLIPLD is bad...



MIWO is good



$X$  has a characteristic size but  $Y$  does not

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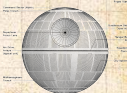
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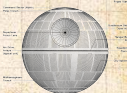
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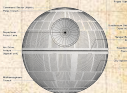
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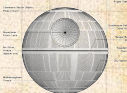
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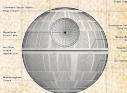
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## Forest fire example: [5]

- 1 Square  $N \times N$  grid
- 2 Sites contain a tree with probability  $\rho =$  density
- 3 Sites are empty with probability  $1 - \rho$
- 4 Fires start at location  $(i, j)$  according to some distribution  $P_{ij}$
- 5 Fires spread from tree to tree (nearest neighbor only)
- 6 Connected clusters of trees burn completely
- 7 Empty sites block fire
- 8 **Best case scenario:**  
Build firebreaks to maximize average # trees left intact given one spark

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- ☒ Fires start at location  $(i, j)$  according to some distribution  $P_{ij}$
- ☒ Fires spread from tree to tree (nearest neighbor only)
- ☒ Connected clusters of trees burn completely
- ☒ Empty sites block fire
- ☒ **Best case scenario:**  
Build firebreaks to maximize average # trees left intact given one spark

## Robustness

### HOT theory

- Narrative causality
- Random forests
- Self-Organized Criticality
- COLD theory
- Network robustness

## References



## Forest fire example: [5]

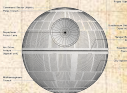
- 🧱 Square  $N \times N$  grid
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Narrative causality

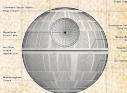
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## Forest fire example: [5]

- Build a forest by adding one tree at a time
- Test  $D$  ways of adding one tree
- $D =$  design parameter
- Average over  $P_{i,j}$  = spark probability
- $D = 1$ : random addition
- $D = N^2$ : test all possibilities

## Measure average area of forest left untouched

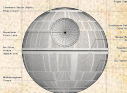
- $f(r)$  = distribution of fire sizes  $r$  (= cost)
- Yield =  $\sum_r = \int_0^\infty f(r) dr$

### Robustness


#### HOT theory

- Narrative causality
- Random forests
- Self-Organized Criticality
- COLD theory
- Network robustness

### References




## Forest fire example: [5]

 Build a forest by adding one tree at a time

 Test  $D$  ways of adding one tree

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 Average over  $P_{i,j}$  = spark probability

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## Measure average area of forest left untouched

  $f(c)$  = distribution of fire sizes  $c$  (= cost)

 Yield =  $\sum_c p_c \cdot c$

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Network robustness

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Self-Organized Criticality

COLD theory

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### References





## Forest fire example: [5]

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🧱 Test  $D$  ways of adding one tree

🧱  $D =$  **design parameter**

🧱 Average over  $P_{i,j}$  = spark probability

🧱  $D = 1$ : random addition

🧱  $D = N^2$ : test all possibilities

## Measure average area of forest left untouched

🧱  $f(c)$  = distribution of fire sizes  $c$  (= cost)

🧱 Yield =  $\sum_c = p = \int f(c)$

### Robustness

#### HOT theory

Narrative causality

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Self-Organized Criticality


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Network robustness

### References




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 Build a forest by adding one tree at a time

 Test  $D$  ways of adding one tree

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 Average over  $P_{i,j}$  = spark probability

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### Robustness

#### HOT theory

Narrative causality

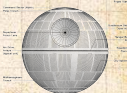
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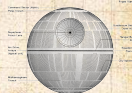
- $f(c) =$  distribution of fire sizes  $c$  (= cost)
- Yield =  $Y = \rho - \langle c \rangle$

### Robustness

#### HOT theory

- Narrative causality
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- Network robustness

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## Specifics:



$$P_{ij} = P_{i;a_x,b_x} P_{j;a_y,b_y}$$

where

$$P_{i;a,b} \propto e^{-[(i+a)/b]^2}$$



In the original work,  $b_y > b_x$



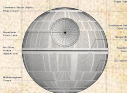
Distribution has more width in  $y$  direction.

## Robustness

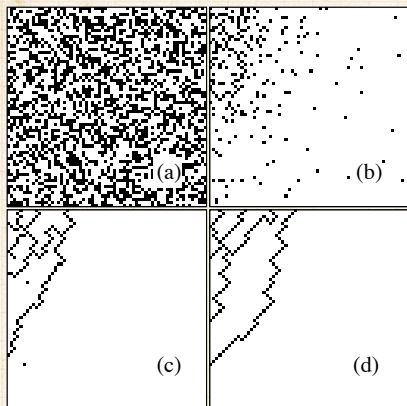
### HOT theory

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- Network robustness

## References



## HOT Forests



$$N = 64$$

$$(a) D = 1$$

$$(b) D = 2$$

$$(c) D = N$$

$$(d) D = N^2$$

$P_{ij}$  has a  
Gaussian decay

[5]

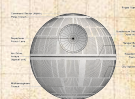
- Optimized forests do well on average
- But rare extreme events occur

## Robustness

HOT theory

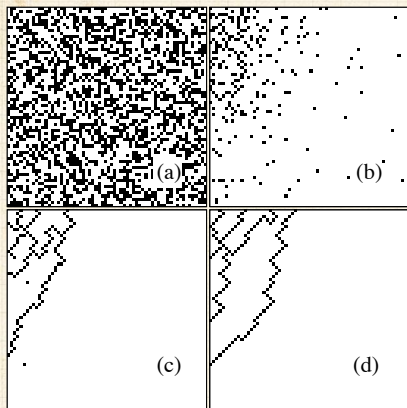
- Narrative causality
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Narrative causality

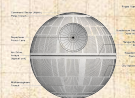
Random forests

Self-Organized Criticality

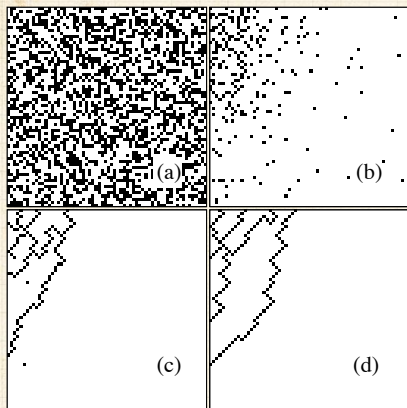
COLD theory

Network robustness

## References



## HOT Forests



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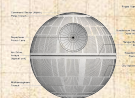
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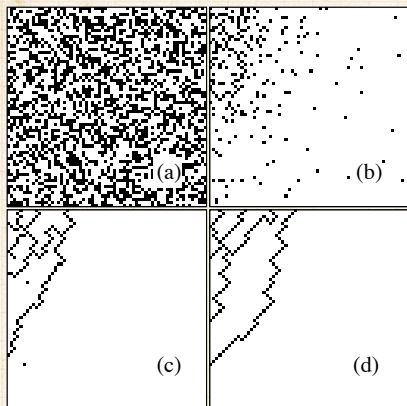
HOT theory

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- Random forests
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## References



## HOT Forests



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$P_{ij}$  has a  
Gaussian decay

-  Optimized forests do well on average (**robustness**)
-  But rare extreme events occur

## Robustness

HOT theory

Narrative causality

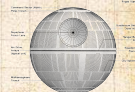
Random forests

Self-Organized Criticality

COLD theory

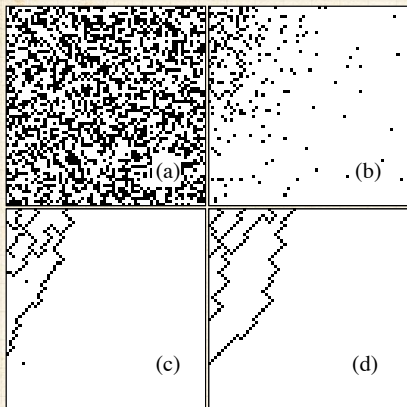
Network robustness

## References





## HOT Forests



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$$(c) D = N$$

$$(d) D = N^2$$

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[5]

 Optimized forests do well on average (**robustness**)

 But rare extreme events occur (**fragility**)

## Robustness

HOT theory

Narrative causality

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## References



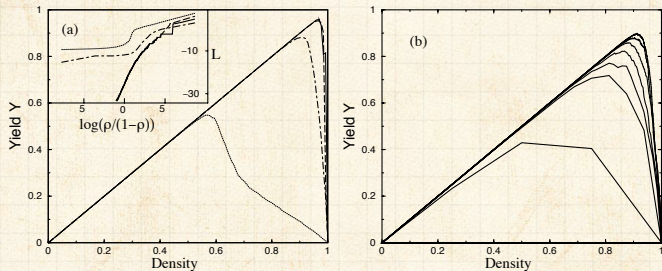


FIG. 2. Yield vs density  $Y(\rho)$ : (a) for design parameters  $D = 1$  (dotted curve),  $2$  (dot-dashed),  $N$  (long dashed), and  $N^2$  (solid) with  $N = 64$ , and (b) for  $D = 2$  and  $N = 2, 2^2, \dots, 2^7$  running from the bottom to top curve. The results have been averaged over 100 runs. The inset to (a) illustrates corresponding loss functions  $L = \log[\langle f \rangle / (1 - \langle f \rangle)]$ , on a scale which more clearly differentiates between the curves.

[5]


## Robustness

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## References



  $Y$  = 'the average density of trees left unburned in a configuration after a single spark hits.' [5]

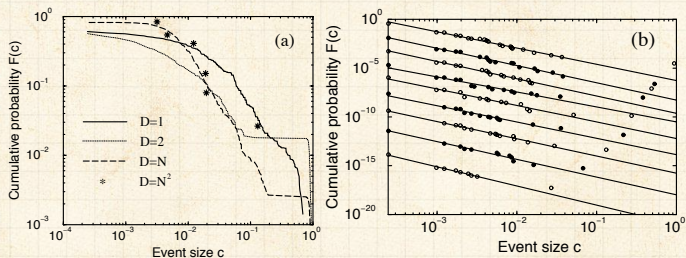


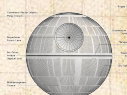
FIG. 3. Cumulative distributions of events  $F(c)$ : (a) at peak yield for  $D = 1, 2, N$ , and  $N^2$  with  $N = 64$ , and (b) for  $D = N^2$ , and  $N = 64$  at equal density increments of 0.1, ranging at  $\rho = 0.1$  (bottom curve) to  $\rho = 0.9$  (top curve).

## Robustness

### HOT theory

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- Random forests
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# Outline

PoCS | @pocsvox

System  
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## Robustness

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Random forests

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## Narrative causality:

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## References





## References





$D = 1$ : Random forests = Percolation [11]

 Randomly add trees.

-  Below critical density  $\rho_c$ , no fires take off.
-  Above critical density  $\rho_c$ , percolating cluster of trees burns.
-  Only at  $\rho_c$ , the critical density, is there a power-law distribution of tree cluster sizes.
-  Forest is random and featureless.

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# HOT forests nutshell:



## Highly structured

Power law distribution of tree cluster sizes for

$$\rho > \rho_c$$

No specialness of  $\rho_c$

Forest states are **tolerant**

Uncertainty is okay if well characterized

If  $P_{i,j}$  is characterized poorly, failure becomes **highly likely**

## Robustness

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# HOT forests nutshell:



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# HOT forests nutshell:



Highly structured



Power law distribution of tree cluster sizes for

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HOT theory  
Narrative causality  
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Network robustness

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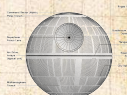
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- Highly structured
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- No specialness of  $\rho_c$
- Forest states are **tolerant**
- Uncertainty is okay if well characterized
- If  $P_{ij}$  is characterized **poorly**, failure becomes **highly likely**

## Robustness

HOT theory  
Narrative causality  
Random forests  
Self-Organized Criticality  
COLD theory  
Network robustness

## References



## “Complexity and Robustness,” Carlson & Dolye [6]

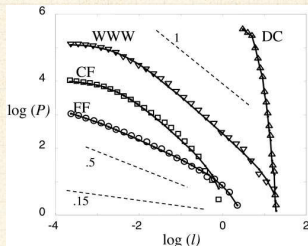


Fig. 1. Log-log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for  $\beta = 0, 0.9, 0.9, 1.85$ , or  $\alpha = 1/\beta = \infty, 1.1, 1.1, 1.0, 0.54$ , respectively) and the SOC FF model ( $\alpha = 0.15$ , dashed). Reference lines of  $\alpha = 0.5, 1$  (dashed) are included. The cumulative distributions of frequencies  $\mathcal{P}(l \geq l_i)$  vs.  $l_i$  describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the >10,000 largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km<sup>2</sup> (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.



PLR = probability-loss-resource.



Minimize cost subject to resource (barrier) constraints:

$$C = \sum_i p_i l_i$$

given

$$l_i = f(r_i) \text{ and } \sum r_i \leq R.$$



DC = Data Compression.



Horror: log. Screaming: “The base! What is the base!? You monsters!”

Robustness

HOT theory

Narrative causality

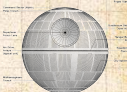
Random forests

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References





## “Complexity and Robustness,” Carlson & Dolye<sup>[6]</sup>

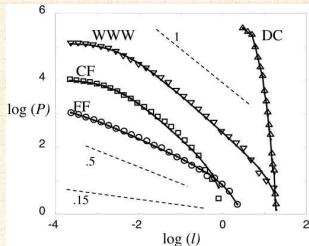


Fig. 1. Log-log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for  $\beta = 0, 0.9, 0.9, 1.85$ , or  $\alpha = 1/\beta = \infty, 1.1, 1.1, 0.054$ , respectively) and the SOC FF model ( $\alpha = 0.15$ , dashed). Reference lines of  $\alpha = 0.5, 1$  (dashed) are included. The cumulative distributions of frequencies  $\mathcal{P}(l \geq l)$  vs.  $l$  describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the >10,000 largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km<sup>2</sup> (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.



PLR = probability-loss-resource.



Minimize cost subject to resource (barrier) constraints:

$$C = \sum_i p_i l_i$$

given

$$l_i = f(r_i) \text{ and } \sum r_i \leq R.$$



DC = Data Compression.



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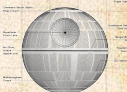
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## The abstract story, using figurative forest fires:

- Given some measure of failure size  $y_i$  and correlated resource size  $x_i$  with relationship  $y_i = x_i^{-\alpha}, i = 1, \dots, N_{\text{sites}}$ .
- Design system to minimize  $\langle y \rangle$  subject to a constraint on the  $x_i$ .
- Minimize cost:

$$C = \sum_{i=1}^{N_{\text{sites}}} Pr(y_i) y_i$$

- Subject to  $\sum_{i=1}^{N_{\text{sites}}} x_i = \text{constant}$ .

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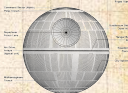
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## 1. Cost: Expected size of fire:

$$C_{\text{fire}} \propto \sum_{i=1}^{N_{\text{sites}}} p_i a_i.$$

$a_i$  = area of  $i$ th site's region, and  $p_i$  = avg. prob. of fire at  $i$ th site over some time frame.

## 2. Constraint: building and maintaining firewalls.

Per unit area, and over same time frame:

$$C_{\text{firewalls}} \propto \sum_{i=1}^{N_{\text{sites}}} a_i^{1/2} a_i^{-1}.$$



fire is a finite geometry

multidimensional, 1/2 is replaced by  $\gamma$

3. Insert question from assignment 6  to find:

$$\Pr(a_i) \propto a_i^{-\gamma}.$$

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### 1. Cost function:

$$\langle C \rangle = \int C(\vec{x})p(\vec{x})d\vec{x}$$

where  $C$  is some cost to be evaluated at each point in space  $\vec{x}$  (e.g.,  $V(\vec{x})^\alpha$ ), and  $p(\vec{x})$  is the probability an Ewok jabs position  $\vec{x}$  with a sharpened stick (or equivalent).

### 2. Constraint:

$$\int R(\vec{x})d(\vec{x}) = c$$

where  $c$  is a constant.

Claim/observation is that typically [4]

$$A(\vec{x}) \sim R^{-\beta}(\vec{x})$$

For spatial systems with barriers:  $\beta = d$ .





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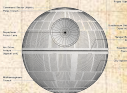
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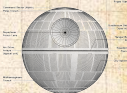
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# The Emperor's Robust-Yet-Fragileness:

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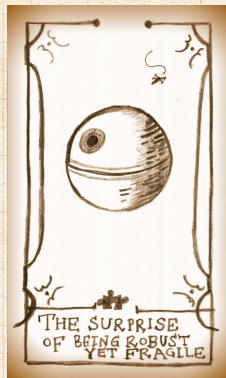
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- 🧱 Analogy: Ising model with temperature somehow self-tuning;
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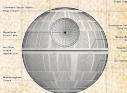
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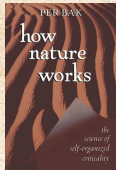
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“How Nature Works: the Science of  
Self-Organized Criticality” [a](#) [↗](#)  
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## Avalanches of Sand and Rice ...



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## HOT versus SOC



Both produce power laws



Optimization versus self-tuning



HOT systems viable over a wide range of high densities



SOC systems have one special density



HOT systems produce specialized structures



SOC systems produce generic structures





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# HOT theory—Summary of designed tolerance <sup>[6]</sup>

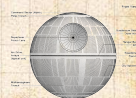
**Table 1. Characteristics of SOC, HOT, and data**

	Property	SOC	HOT and Data
1	Internal configuration	Generic, homogeneous, self-similar	Structured, heterogeneous, self-dissimilar
2	Robustness	Generic	Robust, yet fragile
3	Density and yield	Low	High
4	Max event size	Infinitesimal	Large
5	Large event shape	Fractal	Compact
6	Mechanism for power laws	Critical internal fluctuations	Robust performance
7	Exponent $\alpha$	Small	Large
8	$\alpha$ vs. dimension $d$	$\alpha \approx (d - 1)/10$	$\alpha \approx 1/d$
9	DDOFs	Small (1)	Large ( $\infty$ )
10	Increase model resolution	No change	New structures, new sensitivities
11	Response to forcing	Homogeneous	Variable

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## Avoidance of large-scale failures



Constrained Optimization with Limited Deviations<sup>[9]</sup>



Weight cost of larges losses more strongly



Increases average cluster size of burned trees...



... but reduces chances of catastrophe



Power law distribution of fire sizes is truncated

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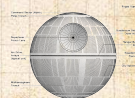
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- Constrained Optimization with Limited Deviations <sup>[9]</sup>
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe
- Power law distribution of fire sizes is truncated

### Robustness

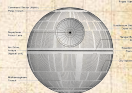
HOT theory  
Narrative causality  
Random forests  
Self-Organized Criticality  
COLD theory  
Network robustness

### References



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## Observed:

- Power law distributions often have an exponential cutoff

$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where  $x_c$  is the approximate cutoff scale.

- May be Weibull distributions:

$$P(x) \sim x^{-\gamma} e^{-ax^{-\gamma+1}}$$

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# Outline

PoCS | @pocsvox

System  
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## Robustness

HOT theory

Narrative causality

Random forests

Self-Organized Criticality

COLD theory

**Network robustness**

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## References



We'll return to this later on:

- 🧱 Network robustness.
- 🧱 Albert et al., Nature, 2000:  
"Error and attack tolerance of complex networks" [1]
- 🧱 General contagion processes acting on complex networks. [13, 12]
- 🧱 Similar robust-yet-fragile stories ...

Robustness

HOT theory

Narrative causality

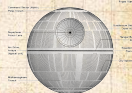
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# The Emperor's Robust-Yet-Fragileness:


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