Principles of Complex Systems | @pocsvox CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

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Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont















Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Largest component

References







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Models

Some important models:

- 1. Generalized random networks;
- 2. Small-world networks;
- 3. Generalized affiliation networks;
- 4. Scale-free networks;
- 5. Statistical generative models (p^*) .

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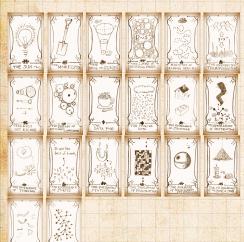
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"I am the Monarch

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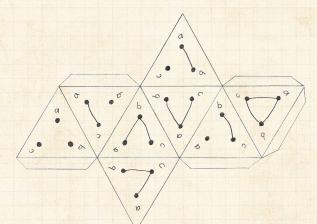




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Random network generator for N = 3:



Set your own exciting generator here \mathbb{Z} . As $N \nearrow$, polyhedral die rapidly becomes a ball...

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Pure, abstract random networks:

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Pure, abstract random networks:

- Solution Consider set of all networks with N labelled nodes and m edges.
 - Standard random network = one randomly chosen network from this set. To be clear: each network is equally probable. Sometimes equiprobability is a good assumptio but it is always an assumption.
 - Known as Erdős-Rényi random networks or graphs.

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Number of possible edges:

$$0 \le m \le {N \choose 2} = \frac{N(N-1)}{2}$$

Limit of m = 0: empty graph. Limit of $m = \binom{N}{2}$: complete or fully-connected graph. Number of possible networks with N labelled nodes:

Given *m* edges, there are $\binom{N}{m}$ different possib networks. Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$. Real world: links are usually costly so real networks are almost always so real

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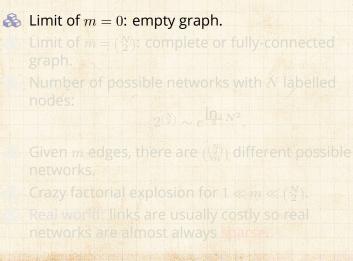




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Given m edges, there are $\binom{N}{m}$ different possible networks. Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$. Real world links are usually costly so real networks are almost always sparse.

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How to build standard random networks:

Two probablistic methods

Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.

Take N nodes and add exactly m links by selectin edges without replacement.

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How to build standard random networks:

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Algorithm: Randomly choose a pair of nodes *i* and *j*, $i \neq j$, and connect if unconnected; repeat until all *m* edges are allocated. Best for adding relatively small numbers of links (most cases).

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A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p {N \choose 2}$$

Which is what it should be... If we keep (k) constant then $p \propto$

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A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p {N \choose 2} = p \frac{1}{2} N(N-1)$$

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$$\langle m \rangle = p {N \choose 2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

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$$= \frac{2}{N}p\frac{1}{2}N(N-1) = \frac{2}{N}p\frac{1}{2}N(N-1) = p(N-1)$$

Which is what it should be... If we keep (4) constant then $p \propto 1/N$

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Which is what it should be...

 \bigotimes If we keep $\langle k \rangle$ constant then $p \propto 1/N \rightarrow 0$ as $N \to \infty$.

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Next slides: Example realizations of random networks

Vary *m*, the number of edges from 100 to 1000.
Average degree (*k*) runs from 0.4 to 4.
Look at full network plus the largest component.

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Next slides: Example realizations of random networks $\gg N = 500$

Vary *µ*, the number of edges from 100 to 1000
Average degree (*k*) runs from 0.4 to 4,
Look at full network plus the largest component

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Example realizations of random networks

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Example realizations of random networks

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Random networks: examples for N=500

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Clustering Generalized Random







m = 260(k) = 1.04



m = 280

(k) = 1.12



m = 300

 $\langle k \rangle = 1.2$

m = 230

 $\langle k \rangle = 0.92$



m = 240

(k) = 0.96

m = 500 $\langle k \rangle = 2$

m = 1000 $\langle k \rangle = 4$

m = 250

 $\langle k \rangle = 1$

Random networks: largest components

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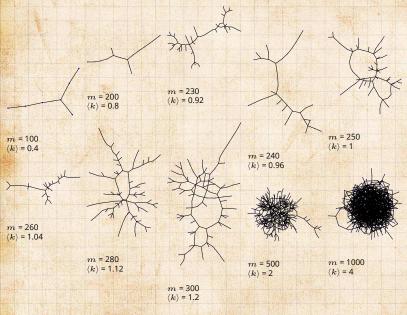
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Random networks: examples for N=500



m = 250

m = 250

 $\langle k \rangle = 1$

 $\langle k \rangle = 1$



m = 250 $\langle k \rangle = 1$



m = 250 $\langle k \rangle = 1$





m = 250 $\langle k \rangle = 1$



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m = 250 $\langle k \rangle = 1$

Random networks: largest components

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m = 250

m = 250

 $\langle k \rangle = 1$



m = 250 $\langle k \rangle = 1$





m = 250

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m = 250

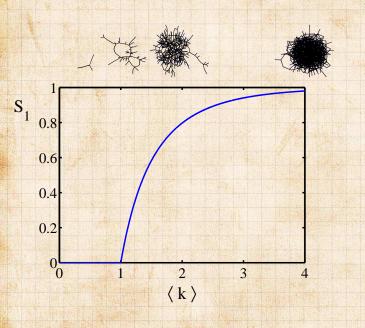
m = 250 $\langle k \rangle = 1$

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Giant component



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Clustering in random networks: For construction method 1, what is the clustering coefficient for a finite network?

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For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: ^[5]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$

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For construction method 1, what is the clustering coefficient for a finite network? S Consider triangle/triple clustering coefficient: [5]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$



 \mathbb{R} Recall: C_2 = probability that two friends of a node are also friends.

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For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: ^[5]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$

Recall: C_2 = probability that two friends of a node are also friends.

Or: C_2 = probability that a triple is part of a triangle.

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Recall: C_2 = probability that two friends of a node are also friends.

- Or: C_2 = probability that a triple is part of a triangle.
- For standard random networks, we have simply that

$$C_2 = p.$$

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So for large random networks $(N \rightarrow \infty)$, clustering drops to zero.

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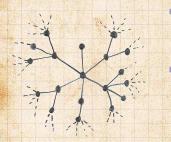
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So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks

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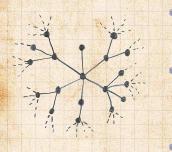
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So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks
 No small loops.

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Recall P_k = probability that a randomly selected node has degree k.

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- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
 - Now consider one node: there are N-1 choose ways the node can be connected to k of the othe N-1-nodes.
 - Each connection occurs with probability p, each non-connection with probability (1-p).
 - Therefore have a

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- Recall P_k = probability that a randomly selected node has degree k.
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- Now consider one node: there are N 1 choose k' ways the node can be connected to k of the other N 1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1-p).
- Therefore have a binomial distribution C:

$$P(k;p,N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$$

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Our degree distribution: $P(k; p, N) = {\binom{N-1}{k}}p^k(1-p)^{N-1-k}$. What happens as $N \to \infty$? We must end up with the normal distribution right?

If p is fixed, then we would end up with a Gaussia with average degree $\langle k \rangle \simeq pN \to \infty$. But we want to keep $\langle k \rangle$ fixed...

So examine limit of P(k; p, N) when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k \rangle = p(N - 1) = \text{constant.}$

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Solution: $P(k; p, N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$ What happens as $N \to \infty$?

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Limiting form of P(k; p, N):

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$$P(k;p,N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \to \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

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Limiting form of P(k; p, N):

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 \mathfrak{S} This is a Poisson distribution \mathbb{C} with mean $\langle k \rangle$.

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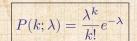
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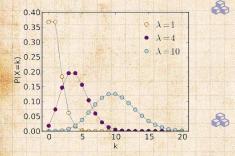
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λ > 0
k = 0, 1, 2, 3, ...
Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.
e.g.:

phone calls/minute, horse-kick deaths. 'Law of small numbers'

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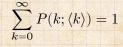
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🚳 Normalization: we must have



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🚳 Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

🚳 Checking:

$$\sum_{k=0}^{\infty} P(k;\langle k\rangle) = \sum_{k=0}^{\infty} \frac{\langle k\rangle^k}{k!} e^{-\langle k\rangle}$$

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$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!}$$

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🚳 Mean degree: we must have

$$\langle k\rangle = \sum_{k=0}^\infty k P(k;\langle k\rangle).$$

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In CocoNuTs, we get to a better and crazier way of doing this...

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The variance of degree distributions for random networks turns out to be very important.

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The variance of degree distributions for random networks turns out to be very important.
 Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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 $\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$

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So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.

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So standard deviation *σ* is equal to √⟨k⟩.
 Note: This is a special property of Poisson distribution and can trip us up...

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Neural reboot (NR):

Unrelated: Feline elevation

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So... standard random networks have a Poisson degree distribution

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So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.

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- So... standard random networks have a Poisson degree distribution
- Seneralize to arbitrary degree distribution P_k .
- lso known as the configuration model. [5]

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So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.
 Also known as the configuration model. ^[5]
 Can generalize construction method from ER random networks.

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 Assign each node a weight w from some

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 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$

nodes with fixed degrees 2. Examining mechanisms that lead to networks certain degree distributions

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But we'll be more interested in

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 Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

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Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:

 $P_k \propto k^{-\gamma}$ for $k \ge 1$. Set $P_0 = 0$ (no isolated nodes). Vary exponent γ between 2.10 and 2.91. Again, look at full network plus the largest component.

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Random networks: examples

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Apart from degree distribution, wiring is random

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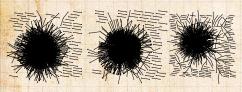


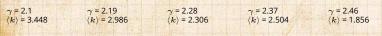


Random networks: examples for N=1000

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Random Networks







 $\gamma = 2.64$

 $\langle k \rangle = 1.6$

 $\gamma = 2.55$

(k) = 1.712



 $\gamma = 2.73$

(k) = 1.862



 $\gamma = 2.82$

(k) = 1.386





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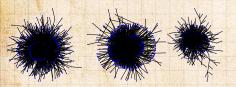
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Random networks: largest components





 $\gamma = 2.55$

(k) = 1.712

 γ = 2.19 $\langle k \rangle$ = 2.986







 $\gamma = 2.46$ $\langle k \rangle = 1.856$



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 $\gamma = 2.73$ $\langle k \rangle = 1.862$

 $\gamma = 2.82$ $\langle k \rangle = 1.386$

 γ = 2.91 $\langle k \rangle$ = 1.49

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Generalized random networks:

Arbitrary degree distribution P_k . Create (unconnected) nodes with degrees sampled from P_k . Wire nodes together randomly.

randomness.

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Generalized random networks:

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- Solution Create (unconnected) nodes with degrees sampled from P_k .
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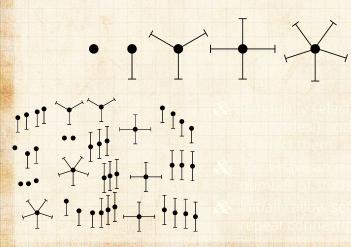




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Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



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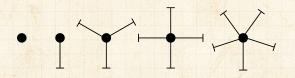


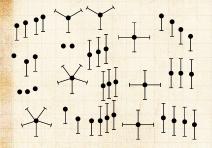
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Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stuk (not nodes!) and connect them. Must have an even number of stubs. Initially allow self- an connections.

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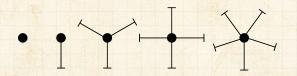


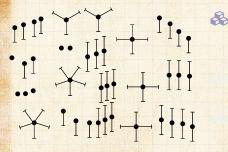
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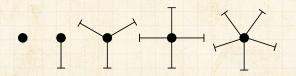
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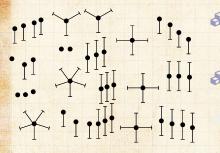




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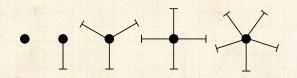
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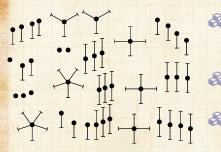


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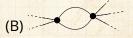
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Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.





Being careful, we can't change the degree of an node, so we can't simply move links around. Simplest solution: randomly rewire two edges time. PoCS | @pocsvox

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Building random networks: First rewiring

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Randomly choose two edges. (Or choose problem edge and a random edge)

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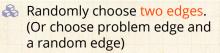
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Check to make sure edges are disjoint.

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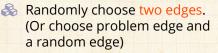
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Check to make sure edges are 3 disjoint.



Rewire one end of each edge.

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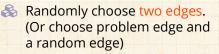
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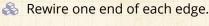


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e1



Check to make sure edges are disjoint.



Node degrees do not change.

Works if e₁ is a self-loop or repeated edge. Same as finding on/off/on/of PoCS | @pocsvox

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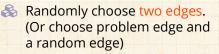
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e1



Check to make sure edges are disjoint.

- Rewire one end of each edge.
 - Node degrees do not change.
- Works if e_1 is a self-loop or repeated edge.
 - Same as finding on/off/on/off 4-cycles. and rotating them.

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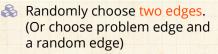
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e1



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Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

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Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

Rule of thumb: # Rewirings $\simeq 10 \times #$ edge

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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.

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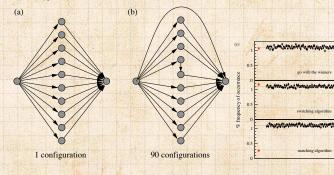


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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.
 Example from Milo et al. (2003) ^[3]:



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\mathbb{R} What if we have P_k instead of N_k ?

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 \mathbb{R} What if we have P_k instead of N_k ? Must now create nodes before start of the construction algorithm.

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What if we have P_k instead of N_k?
 Must now create nodes before start of the construction algorithm.
 Construct N nodes by campling from degree

Solution P_k . Generate N nodes by sampling from degree distribution P_k .

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What if we have P_k instead of N_k?
Must now create nodes before start of the construction algorithm.
Generate N nodes by sampling from degree distribution P_k.

Easy to do exactly numerically since k is discrete.

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What if we have P_k instead of N_k?
Must now create nodes before start of the construction algorithm.
Generate N nodes by sampling from degree distribution P_k.
Easy to do exactly numerically since k is discrete.
Note: not all P_k will always give nodes that can be wired together.

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Motifs

Idea of motifs^[6] introduced by Shen-Orr, Alon et al. in 2002.

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 - Directed network with 577 interactions (edges) and 424 operons (nodes).
- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

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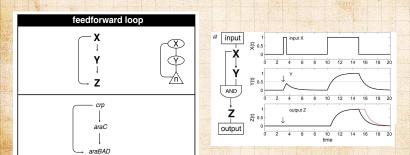
Configuration model

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 \mathbb{R} Z only turns on in response to sustained activity in X.

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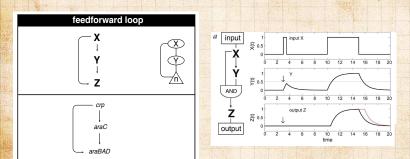
Generalized

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Z only turns on in response to sustained activity in X.
 Transition off X respirit to the set of Z.

 $\underset{\text{Constraints}}{\underset{\text{Constr$





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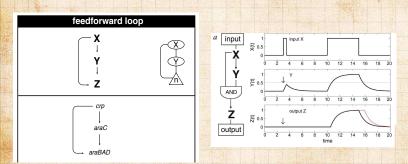
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Configuration model

Largest component

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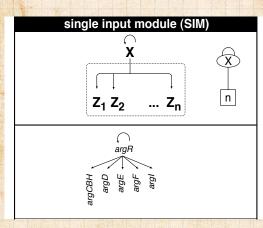


 \mathbb{R} Z only turns on in response to sustained activity in X.

- $\underset{\text{Comparison}}{\underset{\text{$
- logy to elevator doors.







🚳 Master switch.

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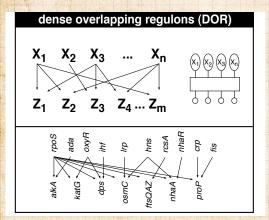
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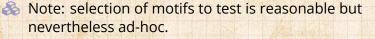




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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.
 For more, see work carried out by Wiggins *et al.* at Columbia.



Outline

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Generalized Random Networks

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The degree distribution P_k is fundamental for our description of many complex networks

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The degree distribution P_k is fundamental for our description of many complex networks

Solution Again: P_k is the degree of randomly chosen node.

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- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.

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- Solution Again: P_k is the degree of randomly chosen node.
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- Now choosing nodes based on their degree (i.e., size):

 $Q_k \propto k P_k$

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Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}}$$

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Big deal: Rich-get-richer mechanism is built into this selection process.

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Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7.$

Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16$, $Q_2 = 4/16$, $Q_3 = 3/16$, $Q_6 = 6/16$,

Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

 $R_0 = 3/16 R_1 = 4/10$ $R_2 = 3/16, R_5 = 6/1$

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Probability of finding #

outgoing edges = & after randomly selecting an edge and then randomly choosing one direction to travel: PoCS | @pocsvox

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Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

$$\begin{split} R_0 &= 3/16 \; R_1 = 4/16, \\ R_2 &= 3/16, \; R_5 = 6/16. \end{split}$$

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For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

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For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

 \bigotimes Useful variant on Q_k :

 R_k = probability that a friend of a random node has k other friends.

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 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

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$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Solution Equivalent to friend having degree k + 1.

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For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

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 R_k = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Equivalent to friend having degree k + 1.
 Natural question: what's the expected number of other friends that one friend has?

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Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k \right\rangle_R = \sum_{k=0}^{\infty} k R_k$$

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Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

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Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left< k \right>_R = \sum_{k=0}^\infty k R_k = \sum_{k=0}^\infty k \frac{(k+1) P_{k+1}}{\left< k \right>}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^\infty k(k+1) P_{k+1}$$

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$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{1} \left((k+1)^2 - (k+1) \right) P_{k+1}$$

(where we have sneakily matched up indices)

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$$= \frac{1}{\langle k \rangle} \sum_{j=0}^{\infty} (j^2 - j) P_j$$
 (using j = k+1)

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$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1} \left((k+1)^2 - (k+1) \right) P_{k+1}$$

(where we have sneakily matched up indices)

$$=rac{1}{\langle k
angle}\sum_{j=0}^{\infty}(j^2-j)P_j$$
 (using j = k+1)

$$=rac{1}{\langle k
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Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

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Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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A Therefore:

$$\left< k \right>_R = \frac{1}{\left< k \right>} \left(\left< k \right>^2 + \left< k \right> - \left< k \right> \right)$$

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Again, neatness of results is a special property of the Poisson distribution.

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$$\langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k \rangle^2 + \langle k \rangle - \langle k \rangle \right) = \langle k \rangle$$

Again, neatness of results is a special property of the Poisson distribution.

So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total friends...

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The edge-degree distribution: In fact, R_k is rather special for pure random networks ...

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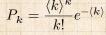


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In fact, R_k is rather special for pure random networks ...
 Substituting

into



 $R_{k}=\frac{(k+1)P_{k+1}}{\langle k\rangle}$

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In fact, R_k is rather special for pure random networks ...
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$$P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k | k!}$$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_k = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle}$$

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$$=\frac{\langle k\rangle^k}{k!}e^{-\langle k\rangle}$$

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$$= \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \equiv P_k.$$

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In fact, R_k is rather special for pure random networks ...
 Substituting

$$P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k}$$

into

$$R_{k} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_{k} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)k!} e^{-\langle k \rangle}$$

$$= \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \equiv P_k.$$

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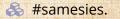
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Reason #1:

Average # friends of friends per node is

Key: Average depends on the 1st and 2nd moment: P_k and not just the 1st moment. Three peculiarities:

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Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R$$

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Average # friends of friends per node is

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Your friends really are different from you... See also: class size paradoxes (nod to: Gelm

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Two reasons why this matters More on peculiarity #3: A node's average # of friends: $\langle k \rangle$

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More on peculiarity #3:

A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$

So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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"Generalized friendship paradox in complex networks: The case of scientific collaboration" Eom and Jo, Nature Scientific Reports, **4**, 4603, 2014.^[1]

Your friends really are monsters #winners:¹ Go on the real Friends have more coauthors, citations, and publications. Other homitic studies: your connections on Twitter have more followers than you, your sex partners more partners than you, ... The hope: Maybe they have more enemies and diseases too.

¹Some press here C [MIT Tech Review].

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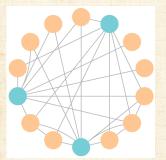
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¹https://www.washingtonpost.com/graphics/business/ wonkblog/majority-illusion/

Related disappointment:



Nodes see their friends' color choices.

Vhich color is more popular?¹

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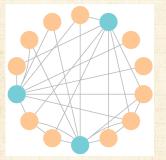
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🚳 Again: thinking in edge space changes everything.

Related disappointment:



Two reasons why this matters (Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.
 - e.g., we'd like to know what's the size of the larg component within a network. As $N \rightarrow \infty$, does our network have a giant component?
 - Defor Component = connected subnetwork of nodes such that = path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
 - Defn: Giant component = component that comprises a non-zero fraction of a network as
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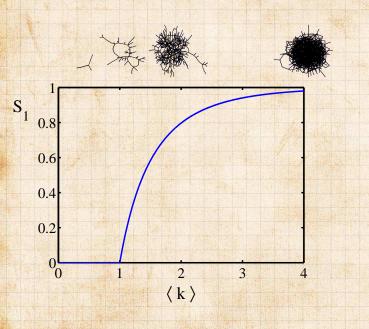


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Giant component



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Structure of random networks Giant component:

A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.

Equivalently, expect exponential growth in nod number as we move out from a random node. All of this is the same as requiring $\langle k \rangle_R > 1$. Giant-component condition (or percolation condition):

Again, see that the second moment is an essenti part of the story. Equivalent statement: $\langle k^2
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- All of this is the same as requiring $\langle k \rangle_R > 1$.

Giant component condition (or percolation condition):

$$\left< k \right>_R = \frac{\left< k^2 \right> - \left< k \right>}{\left< k \right>} > 1$$

Again, see that the second moment is an essential part of the story.

Equivalent statement: $\langle k^2 \rangle > 2 \langle k \rangle$

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Giant component for standard random networks: Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.

Determine condition for giant component:

Therefore when $\langle k \rangle > 1$, standard random networks have a giant component. When $\langle k \rangle < 1$, all components are finite. Fine example of a continuous phase where the say $\langle k \rangle = 1$ marks the critical point of the system.

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Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

6

Determine condition for giant component:

$$\left\langle k \right\rangle_R = rac{\left\langle k^2 \right\rangle - \left\langle k \right\rangle}{\left\langle k \right\rangle}$$

Therefore when $\langle k \rangle > 1$, standard random networks have a giant component. When $\langle k \rangle < 1$, all components are finite. Fine example of a continuous phase We say $\langle k \rangle = 1$ marks the critical point of th system.

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$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

4

Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle}$$

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Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
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Solution
 Solution

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Solution 3. Therefore when ⟨k⟩ > 1, standard random networks have a giant component.
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Solution State State

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 \mathfrak{R} e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \ge 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^\infty k^2 k^{-\gamma}$$

So giant component always exists for these kinds of networks. Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to lool harder at $\langle k \rangle_R$. How about $P_k = \delta_{kk_0}$? CocoNuTs: We figure out the final size and complete dynamics.

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