Random Networks

Principles of Complex Systems | @pocsvox
CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

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Vermont Advanced Computing Core | University of Vermont

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What's the Story?

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These slides are brought to you by:

Outline

Models

Some important models:

1. Generalized random networks;
2. Small-world networks;
3. Generalized affiliation networks;
4. Scale-free networks;
5. Statistical generative models (p^*).


THE SUN
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER
MINISTRY OF RANDOM WALKS
THE UNEXPECTED TRANSFIGURATION
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE DATA POOR
THE POURING DATA
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
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THE THRESHOLDS OF PERCOLATION
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THE MISREADING OF THE BOOK OF SAND
THE MANY TALES OF MANY TAILS
THE NETWORKS COMPLEX


THE ELEMENTS OF WEBS


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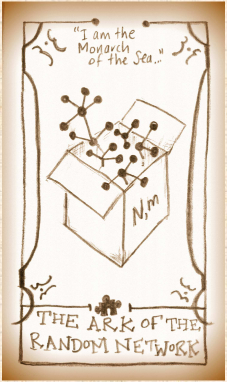
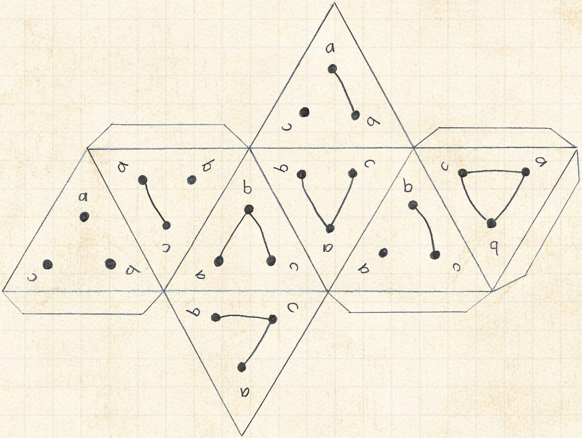

"I am the Monarch of the Sea."
N,m
THE ARK OF THE RANDOM NETWORK

Random network generator for $N = 3$:



- Get your own exciting generator here.
- As $N \nearrow$, polyhedral die rapidly becomes a ball...

Outline

Random networks

Pure, abstract random networks:

- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

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Random networks—basic features:

- Number of possible edges:

$$0 \leq m \leq \binom{N}{2} = \frac{N(N-1)}{2}$$

- Limit of $m = 0$: empty graph.
- Limit of $m = \binom{N}{2}$: complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2} N^2}.$$

- Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.
- Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$.
- Real world: links are usually costly so real networks are almost always sparse.

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Random networks

How to build standard random networks:

- Given N and m.
- Two probablistic methods (we'll see a third later on)

1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.
   - Useful for theoretical work.
2. Take N nodes and add exactly m links by selecting edges without replacement.
   - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
   - Best for adding relatively small numbers of links (most cases).
   - 1 and 2 are effectively equivalent for large N.

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A few more things:

- For method 1, # links is probablistic:

$$\langle m \rangle = p\binom{N}{2} = p\frac{1}{2}N(N-1)$$

- So the expected or average degree is

$$\langle k \rangle = \frac{2\,\langle m \rangle}{N}$$

$$= \frac{2}{N}p\frac{1}{2}N(N-1) = \frac{2}{N}p\frac{1}{2}N(N-1) = p(N-1).$$

- Which is what it should be...
- If we keep $\langle k \rangle$ constant then $p \propto 1/N \to 0$ as $N \to \infty$.

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The UNIVERSITY of VERMONT

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Random networks: examples

Next slides:

Example realizations of random networks

- $N = 500$
- Vary m, the number of edges from 100 to 1000.
- Average degree $\langle k \rangle$ runs from 0.4 to 4.
- Look at full network plus the largest component.

Random networks: examples

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Random networks: examples for N=500

m = 100
$\langle k \rangle$ = 0.4

m = 200
$\langle k \rangle$ = 0.8

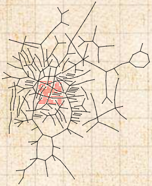
m = 230
$\langle k \rangle$ = 0.92

m = 240
$\langle k \rangle$ = 0.96

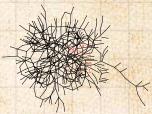
m = 250
$\langle k \rangle$ = 1

m = 260
$\langle k \rangle$ = 1.04

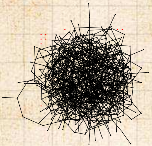
m = 280
$\langle k \rangle$ = 1.12



m = 300
$\langle k \rangle$ = 1.2

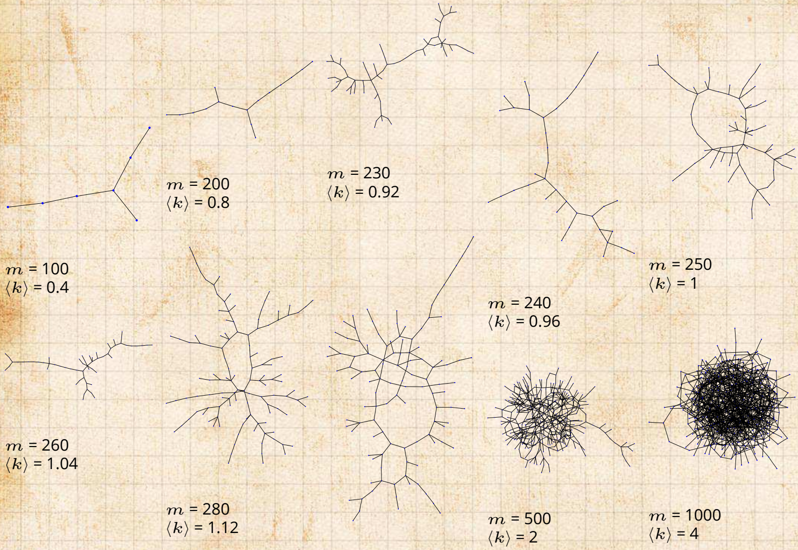


m = 500
$\langle k \rangle$ = 2



m = 1000
$\langle k \rangle$ = 4

Random networks: largest components



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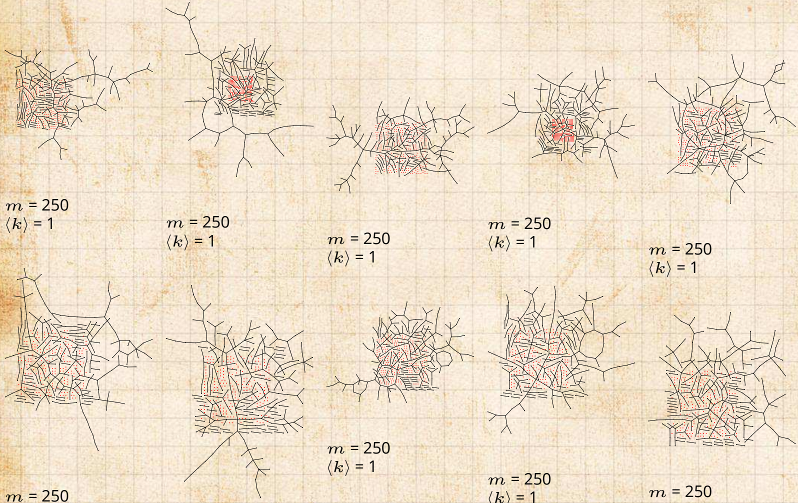

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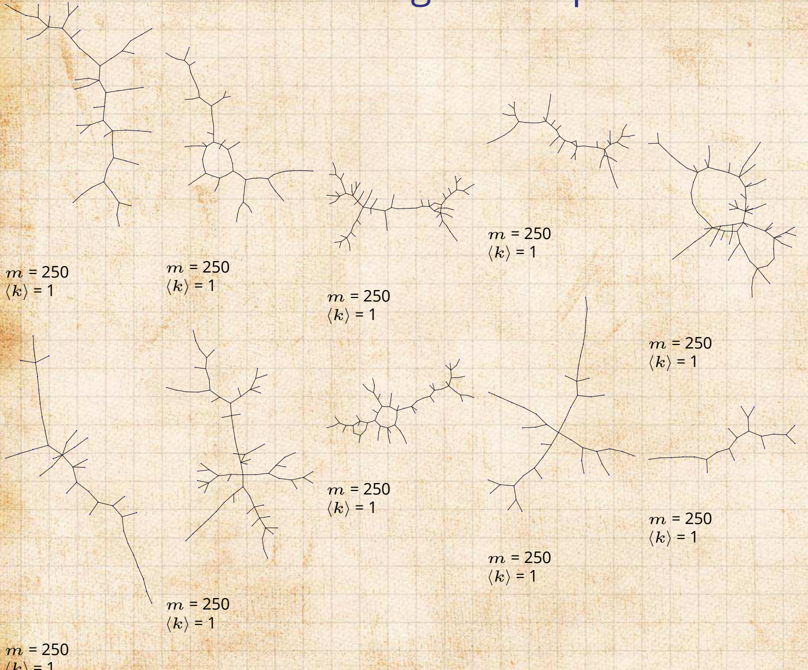
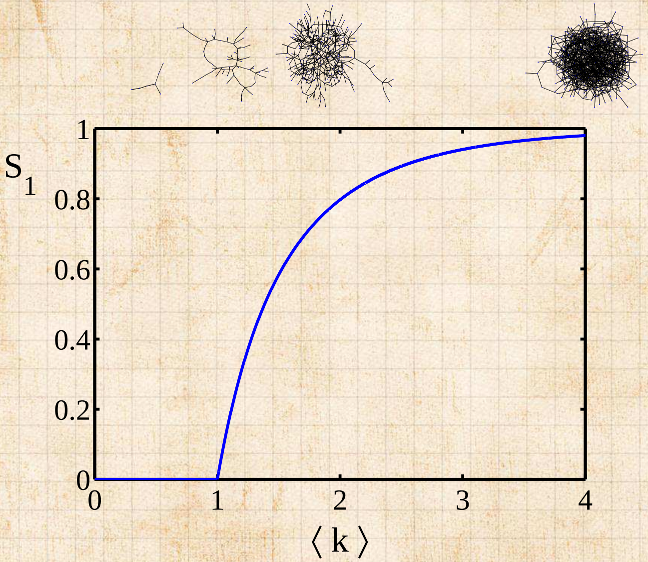


Giant component



Outline

Clustering in random networks:

- For construction method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient: [5]

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}}$$

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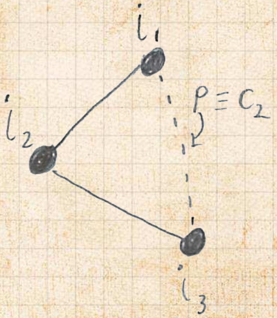

- Recall: C_2 = probability that two friends of a node are also friends.
- Or: C_2 = probability that a triple is part of a triangle.
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$$C_2 = p.$$

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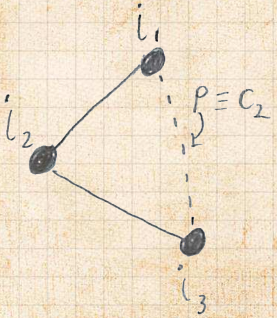

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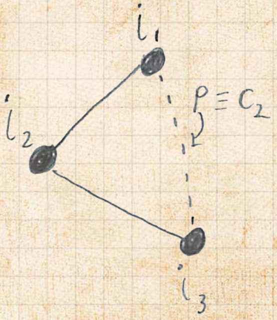

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Clustering in random networks:

- So for large random networks ($N \rightarrow \infty$), clustering drops to zero.
- Key structural feature of random networks is that they locally look like pure branching networks
- No small loops.

Clustering in random networks:

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- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are '$N-1$ choose k' ways the node can be connected to k of the other $N-1$ nodes.
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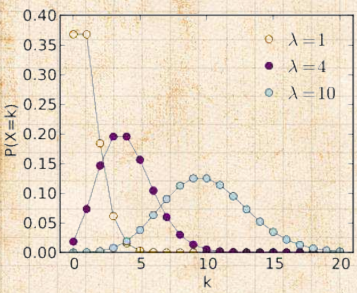
Poisson basics:

$$P(k;\lambda) = \frac{\lambda^k}{k!}e^{-\lambda}$$



- $\lambda > 0$
- $k = 0, 1, 2, 3, \ldots$
- Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.
- e.g.:
  phone calls/minute,
  horse-kick deaths.
- 'Law of small numbers'

Poisson basics:

- Normalization: we must have

$$\sum_{k=0}^{\infty} P(k;\langle k \rangle) = 1$$

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- The variance of degree distributions for random networks turns out to be very important.
- Using calculation similar to one for finding $\langle k \rangle$ we find the second moment to be:

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$$\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2 = \langle k \rangle^2 + \langle k \rangle - \langle k \rangle^2 = \langle k \rangle.$$

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Neural reboot (NR):

Unrelated: Feline elevation

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Pure random networks
Definitions
How to build theoretically
Some visual examples
Clustering
Degree distributions
Generalized Random Networks
Configuration model
How to build in practice
Motifs
Random friends are strange
Largest component
References

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- Also known as the configuration model.[5]
- Can generalize construction method from ER random networks.
- Assign each node a weight w from some distribution P_w and form links with probability

$$P(\text{link between } i \text{ and } j) \propto w_i w_j.$$

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General random networks

- So... standard random networks have a Poisson degree distribution
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Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:

- $N = 1000$.
- $P_k \propto k^{-\gamma}$ for $k \geq 1$.
- Set $P_0 = 0$ (no isolated nodes).
- Vary exponent γ between 2.10 and 2.91.
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random.

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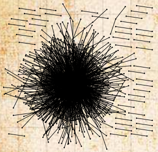
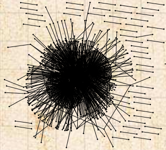


γ = 2.1
$\langle k \rangle$ = 3.448



γ = 2.19
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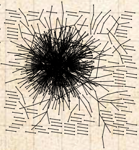

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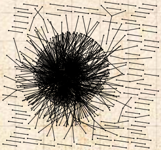

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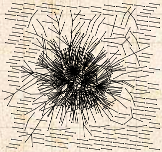

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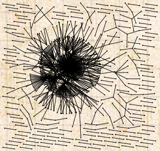

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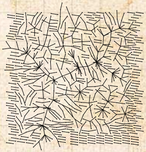

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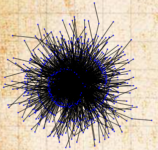
Random networks: largest components



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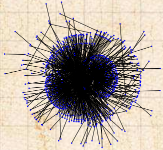

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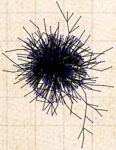

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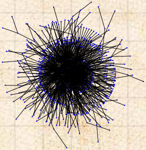

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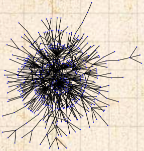

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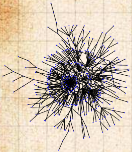

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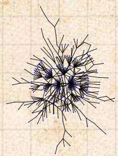

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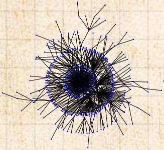

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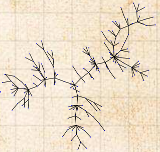

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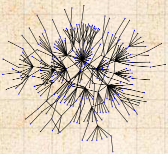

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Outline

Models

Generalized random networks:

- Arbitrary degree distribution P_k.
- Create (unconnected) nodes with degrees sampled from P_k.
- Wire nodes together randomly.
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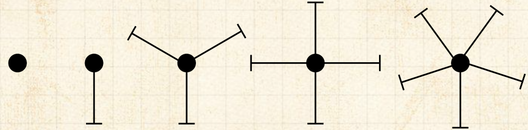
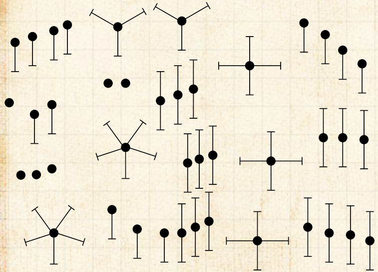
Phase 1:

- Idea: start with a soup of unconnected nodes with stubs (half-edges):





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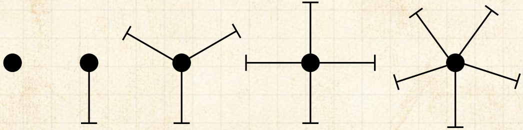

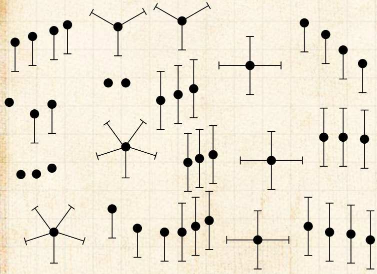

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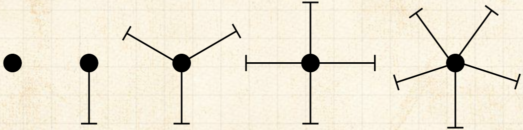

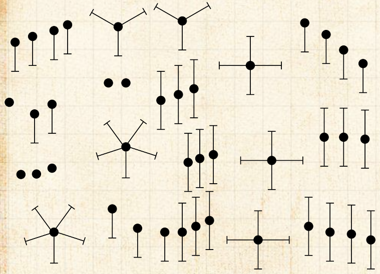

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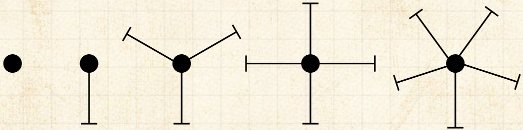

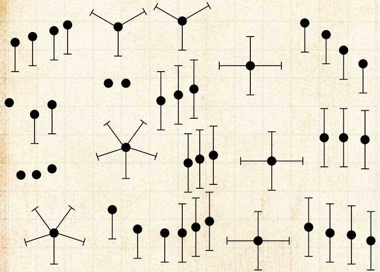

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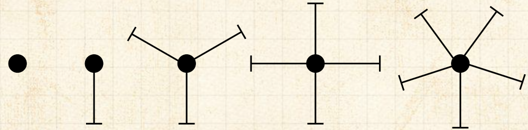

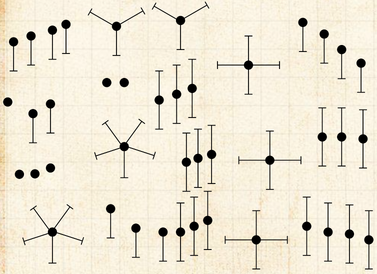

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Building random networks: First rewiring

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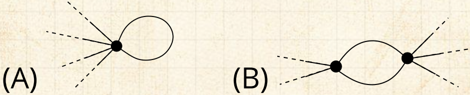
- Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



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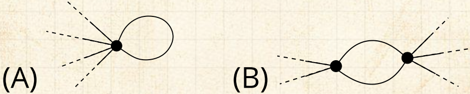

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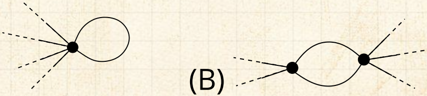

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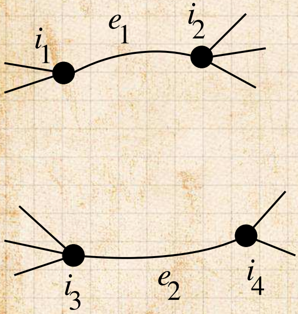

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- Check to make sure edges are disjoint.
- Rewire one end of each edge.
- Node degrees do not change.
- Works if e_1 is a self-loop or repeated edge.
- Same as finding on/off/on/off 4-cycles, and rotating them.

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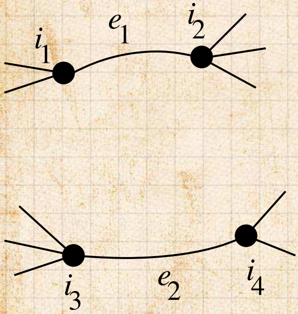

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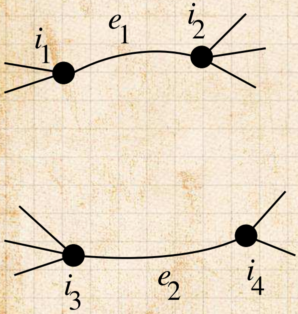

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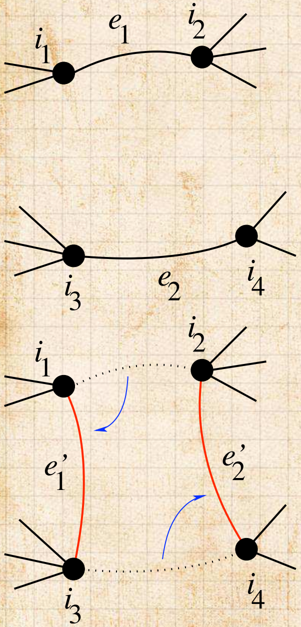

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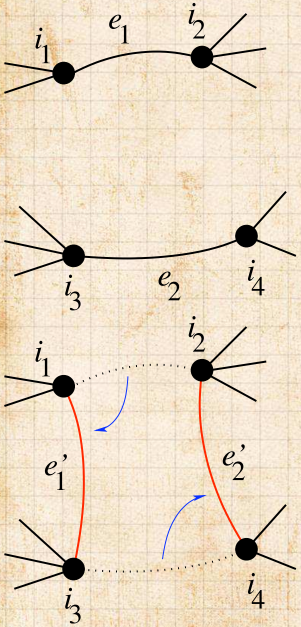

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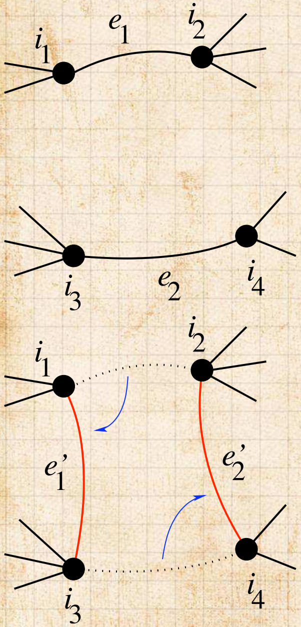

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Phase 2:

- Use rewiring algorithm to remove all self and repeat loops.

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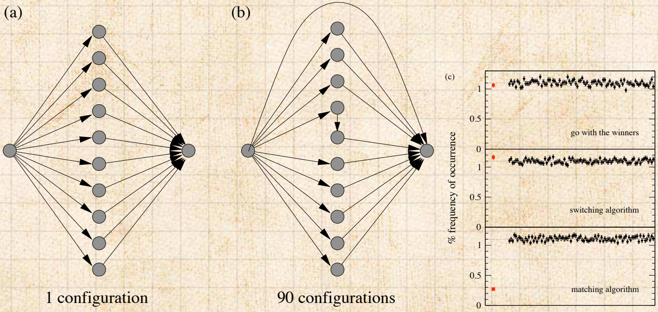

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- Must now create nodes before start of the construction algorithm.
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- Idea of motifs [6] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Used network randomization to produce ensemble of alternate networks with same degree frequency N_k.
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

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- Looked for certain subnetworks (motifs) that appeared more or less often than expected

Network motifs

- Idea of motifs [6] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- Specific example of Escherichia coli.
- Directed network with 577 interactions (edges) and 424 operons (nodes).
- Used network randomization to produce ensemble of alternate networks with same degree frequency N_k.
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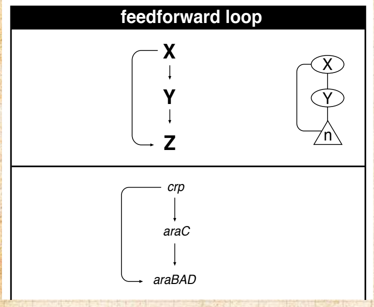
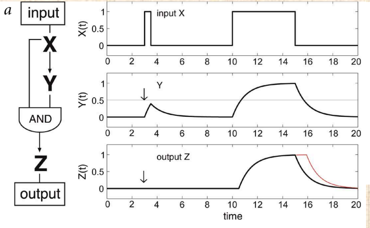
Network motifs





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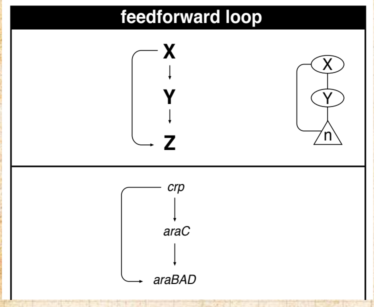

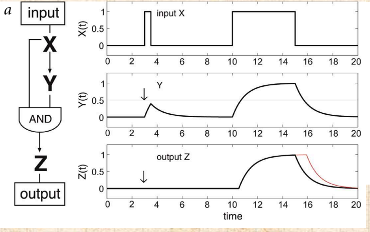

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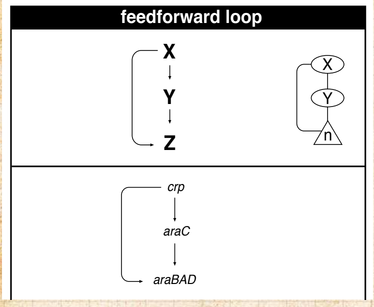

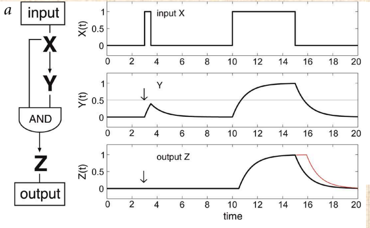

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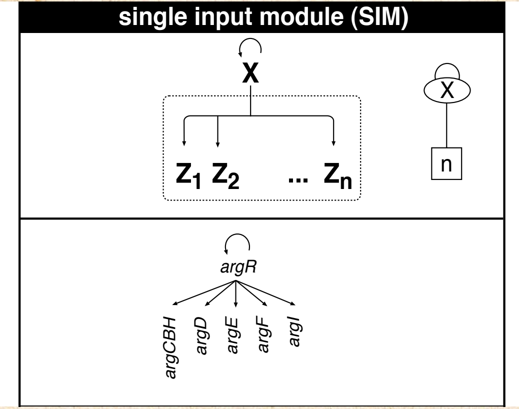
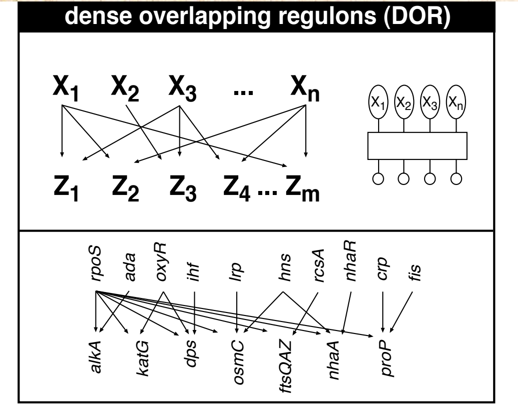


- Master switch.

Network motifs



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Outline

The edge-degree distribution:

- The degree distribution P_k is fundamental for our description of many complex networks
- Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.
- Define Q_k to be the probability the node at a random end of a randomly chosen edge has degree k.
- Now choosing nodes based on their degree (i.e., size):

$$Q_k \propto kP_k$$

- Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}} = \frac{kP_k}{\langle k \rangle}.$$

- Big deal: Rich-get-richer mechanism is built into this selection process.

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- Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7$, $P_2 = 2/7$, $P_3 = 1/7$, $P_6 = 1/7$.
- Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16$, $Q_2 = 4/16$, $Q_3 = 3/16$, $Q_6 = 6/16$.
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The edge-degree distribution:

- For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.
- Useful variant on Q_k:

  R_k = probability that a friend of a random node has k other friends.

- $$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

- Equivalent to friend having degree $k+1$.
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$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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- Again, neatness of results is a special property of the Poisson distribution.
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$$= \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \equiv P_k .$$

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The edge-degree distribution:

- In fact, R_k is rather special for pure random networks ...
- Substituting

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- So only if everyone has the same degree (variance$= \sigma^2 = 0$) can a node be the same as its friends.
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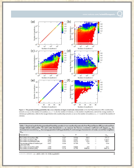

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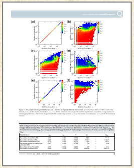

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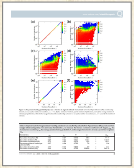

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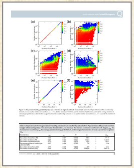

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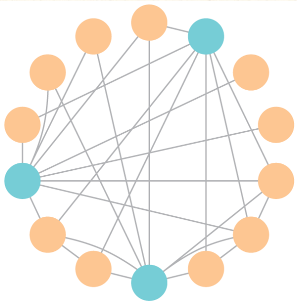

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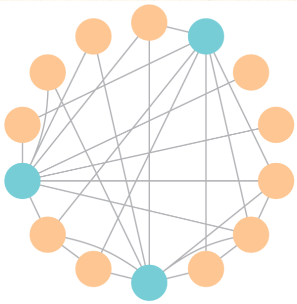

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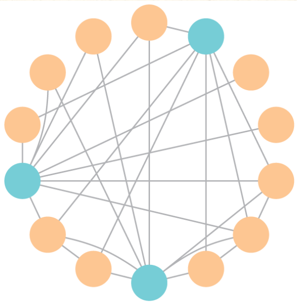

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- Again: thinking in edge space changes everything.

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Two reasons why this matters

(Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.
- As $N \to \infty$, does our network have a giant component?
- Defn: Component = connected subnetwork of nodes such that $\exists$ path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
- Defn: Giant component = component that comprises a non-zero fraction of a network as $N \to \infty$.
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Outline

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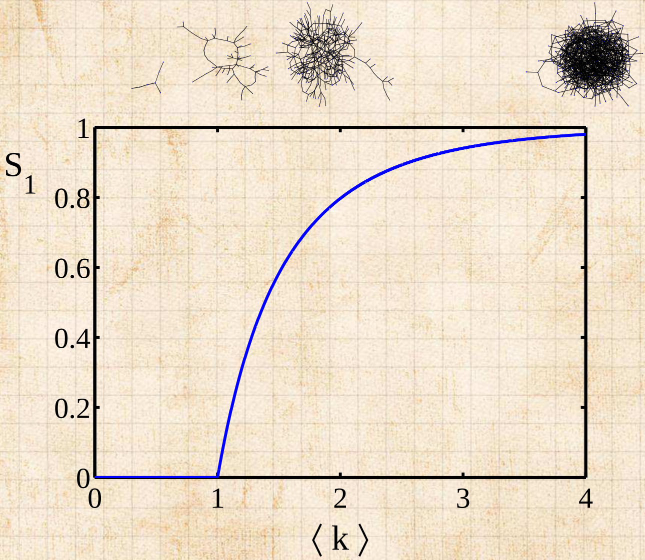

Structure of random networks

Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring $\langle k \rangle_R > 1$.
- Giant component condition (or percolation condition):

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

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- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring $\langle k \rangle_R > 1$.
- Giant component condition (or percolation condition):

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1$$

- Again, see that the second moment is an essential part of the story.
- Equivalent statement: $\langle k^2 \rangle > 2\langle k \rangle$

Giant component for standard random networks:

- Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.
- Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle}$$

- Therefore when $\langle k \rangle > 1$, standard random networks have a giant component.
- When $\langle k \rangle < 1$, all components are finite.
- Fine example of a continuous phase transition.
- We say $\langle k \rangle = 1$ marks the critical point of the system.

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Random networks with skewed P_k:

- e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \geq 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\sim \int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

$$\propto x^{3-\gamma} \Big|_{x=1}^{\infty} = \infty \quad (\gg \langle k \rangle).$$

- So giant component always exists for these kinds of networks.
- Cutoff scaling is k^{-3}: if $\gamma > 3$ then we have to look harder at $\langle k \rangle_R$.
- How about $P_k = \delta_{k k_0}$?
- CocoNuTs: We figure out the final size and complete dynamics.

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Neural reboot (NR):

Falling maple leaf

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