Principles of Complex Systems | @pocsvox CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

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Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont















Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Largest component

References







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29 C 1 of 70

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References



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Outline

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 3 of 70



Models

Some important models:

- 1. Generalized random networks;
- 2. Small-world networks;
- 3. Generalized affiliation networks;
- 4. Scale-free networks;
- 5. Statistical generative models (p^*) .

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

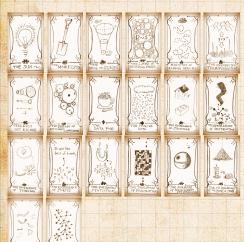
References

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990 6 of 70



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"I am the Monarch

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

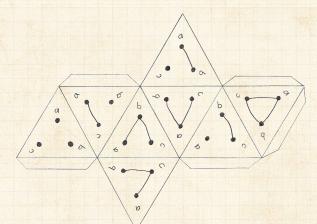




200 6 of 70



Random network generator for N = 3:



Set your own exciting generator here \mathbb{Z} . As $N \nearrow$, polyhedral die rapidly becomes a ball...

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 7 of 70

Outline

Pure random networks Definitions

Clastering Degree distributions renalized introductions Configuration model

How to build in practice Motifs Random mends are strang Largest component

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Random Networks

Pure random networks

Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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200 8 of 70

Pure, abstract random networks:

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Random Networks

Pure random networks

Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 9 of 70

Pure, abstract random networks:

- Solution Consider set of all networks with N labelled nodes and m edges.
 - Standard random network = one randomly chosen network from this set. To be clear: each network is equally probable. Sometimes equiprobability is a good assumptio but it is always an assumption.
 - Known as Erdős-Rényi random networks or graphs.

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Random Networks

Pure random networks

Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 9 of 70

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





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Pure random networks

Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 9 of 70

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larges component

References





200 9 of 70

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





Number of possible edges:

$$0 \le m \le {N \choose 2} = \frac{N(N-1)}{2}$$

Limit of m = 0: empty graph. Limit of $m = \binom{N}{2}$: complete or fully-connected graph. Number of possible networks with N labelled nodes:

Given *m* edges, there are $\binom{N}{m}$ different possib networks. Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$. Real world: links are usually costly so real networks are almost always so real

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References

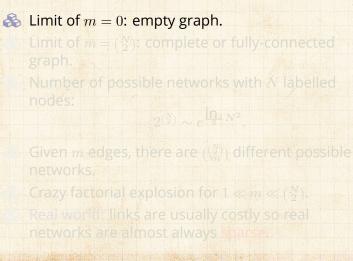




990 10 of 70

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Random Networks

Pure random networks

Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 10 of 70

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





200 10 of 70

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Given m edges, there are $\binom{N}{m}$ different possible networks. Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$. Real world links are usually costly so real networks are almost always sparse.

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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





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Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





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Definitions How to build theoretically Some visual examples Degree distributions

Generalized Networks Configuration model How to build in practice





Outline

Pure random networks

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Random Networks

Pure random networks

Definitions

How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





200 11 of 70

How to build standard random networks:

Two probablistic methods

Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.

Take N nodes and add exactly m links by selectin edges without replacement.

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How to build theoretically

Some visual examples

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Dac 12 of 70

How to build standard random networks:

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How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Largest component





Dac 12 of 70

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How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Largest component





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Degree distributions

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Algorithm: Randomly choose a pair of nodes *i* and *j*, $i \neq j$, and connect if unconnected; repeat until all *m* edges are allocated. Best for adding relatively small numbers of links (most cases).

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How to build theoretically Some visual examples Clustering

Degree distributions

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How to build theoretically Some visual examples Clustering

Degree distributions

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How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





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How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





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How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p {N \choose 2}$$

Which is what it should be... If we keep (k) constant then $p \propto$

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Definitions

How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





200 13 of 70

A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p {N \choose 2} = p \frac{1}{2} N(N-1)$$

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Definitions

How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





200 13 of 70

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$$\langle m \rangle = p {N \choose 2} = p \frac{1}{2} N(N-1)$$

So the expected or average degree is

$$\langle k \rangle = \frac{2 \langle m \rangle}{N}$$

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How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





990 13 of 70

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How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component





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How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





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$$= \frac{2}{N}p\frac{1}{2}N(N-1) = \frac{2}{N}p\frac{1}{2}N(N-1) = p(N-1)$$

Which is what it should be... If we keep (4) constant then $p \propto 1/N$

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How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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How to build theoretically Some visual examples

Clustering Degree distributions

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Which is what it should be...

 \bigotimes If we keep $\langle k \rangle$ constant then $p \propto 1/N \rightarrow 0$ as $N \to \infty$.

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How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice





Outline

Pure random networks

Some visual examples

Configuration model How to build in practice Motifs Random mends are strange Largest component

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Random Networks

Pure random networks Definitions How to build theoretically

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





200 14 of 70

Next slides: Example realizations of random networks

Vary *m*, the number of edges from 100 to 1000.
Average degree (*k*) runs from 0.4 to 4.
Look at full network plus the largest component.

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Pure random networks Definitions How to build theoretically

Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Next slides: Example realizations of random networks $\gg N = 500$

Vary *µ*, the number of edges from 100 to 1000
Average degree (*k*) runs from 0.4 to 4,
Look at full network plus the largest component

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Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

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Example realizations of random networks

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Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





Dac 15 of 70

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Example realizations of random networks

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Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Random networks: examples for N=500

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Pure random networks How to build theoretically Some visual examples

Degree distributions

Networks Configuration model How to build in practice Random friends are strange Largest component





Clustering Generalized Random







m = 260(k) = 1.04



m = 280

(k) = 1.12



m = 300

 $\langle k \rangle = 1.2$

m = 230

 $\langle k \rangle = 0.92$



m = 240

(k) = 0.96

m = 500 $\langle k \rangle = 2$

m = 1000 $\langle k \rangle = 4$

m = 250

 $\langle k \rangle = 1$

Random networks: largest components

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Degree distributions

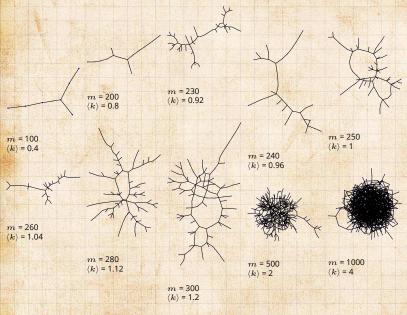
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References





200 17 of 70



Random networks: examples for N=500



m = 250

m = 250

 $\langle k \rangle = 1$

 $\langle k \rangle = 1$



m = 250 $\langle k \rangle = 1$



m = 250 $\langle k \rangle = 1$





m = 250 $\langle k \rangle = 1$



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Clustering Degree distributions

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References





m = 250 $\langle k \rangle = 1$

Random networks: largest components

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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

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m = 250

m = 250

 $\langle k \rangle = 1$



m = 250 $\langle k \rangle = 1$





m = 250

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m = 250

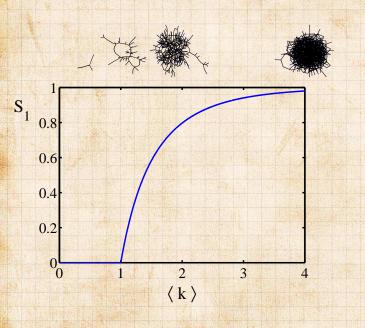
m = 250 $\langle k \rangle = 1$

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Giant component



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Pure random networks

Clustering

Configuration model How to build in practice Motifs Random friends are strange Largest component

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Clustering Degree distributions

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References





990 21 of 70

Clustering in random networks: For construction method 1, what is the clustering coefficient for a finite network?

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For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: ^[5]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$

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References





na @ 22 of 70

For construction method 1, what is the clustering coefficient for a finite network? S Consider triangle/triple clustering coefficient: [5]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$



 \mathbb{R} Recall: C_2 = probability that two friends of a node are also friends.

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For construction method 1, what is the clustering coefficient for a finite network?
 Consider triangle/triple clustering coefficient: ^[5]

 $C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$

Recall: C_2 = probability that two friends of a node are also friends.

Or: C_2 = probability that a triple is part of a triangle.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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na 22 of 70

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Recall: C_2 = probability that two friends of a node are also friends.

- Or: C_2 = probability that a triple is part of a triangle.
- For standard random networks, we have simply that

$$C_2 = p.$$

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Pure random networks Definitions How to build theoretically Some visual examples

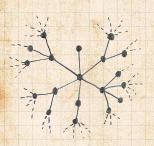
Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References







So for large random networks $(N \rightarrow \infty)$, clustering drops to zero.

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Pure random networks Definitions How to build theoretically Some visual examples

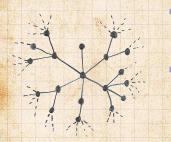
Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples

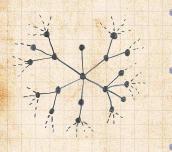
Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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So for large random networks (N → ∞), clustering drops to zero.
 Key structural feature of random networks is that they locally look like pure branching networks
 No small loops.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples

Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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Outline

Pure random networks

Some visual examples clustering Degree distributions

Configuration model How to build in practice Motifs Random friends are strange Largest domponent

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References







Recall P_k = probability that a randomly selected node has degree k.

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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Largest component



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29 c 25 of 70

- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.
 - Now consider one node: there are N-1 choose ways the node can be connected to k of the othe N-1-nodes.
 - Each connection occurs with probability p, each non-connection with probability (1-p).
 - Therefore have a

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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- Recall P_k = probability that a randomly selected node has degree k.
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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





na 25 of 70

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- Now consider one node: there are N 1 choose k' ways the node can be connected to k of the other N 1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1-p).
- Therefore have a binomial distribution C:

$$P(k;p,N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larges component

References



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Our degree distribution: $P(k; p, N) = {\binom{N-1}{k}}p^k(1-p)^{N-1-k}$. What happens as $N \to \infty$? We must end up with the normal distribution right?

If p is fixed, then we would end up with a Gaussia with average degree $\langle k \rangle \simeq pN \to \infty$. But we want to keep $\langle k \rangle$ fixed...

So examine limit of P(k; p, N) when $p \rightarrow 0$ and $N \rightarrow \infty$ with $\langle k \rangle = p(N - 1) = \text{constant.}$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Solution: $P(k; p, N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$ What happens as $N \to \infty$?

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





na 26 of 70

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Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





Limiting form of P(k; p, N):

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$$P(k;p,N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \to \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle}$$

Random Networks

Pure random networks How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Networks Configuration model How to build in practice



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Limiting form of P(k; p, N):

- Our degree distribution: $P(k; p, N) = \binom{N-1}{k} p^k (1-p)^{N-1-k}.$
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 \mathfrak{S} This is a Poisson distribution \mathbb{C} with mean $\langle k \rangle$.

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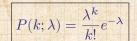
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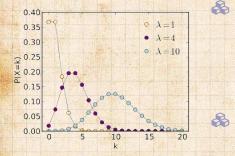
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λ > 0
k = 0, 1, 2, 3, ...
Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.
e.g.:

phone calls/minute, horse-kick deaths. 'Law of small numbers'

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Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

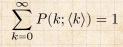
Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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🚳 Normalization: we must have



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Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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🚳 Normalization: we must have

$$\sum_{k=0}^{\infty} P(k; \langle k \rangle) = 1$$

🚳 Checking:

$$\sum_{k=0}^{\infty} P(k;\langle k\rangle) = \sum_{k=0}^{\infty} \frac{\langle k\rangle^k}{k!} e^{-\langle k\rangle}$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clusterine

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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$$= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!}$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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🚳 Mean degree: we must have

$$\langle k\rangle = \sum_{k=0}^\infty k P(k;\langle k\rangle).$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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DQC 29 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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In CocoNuTs, we get to a better and crazier way of doing this...

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





The variance of degree distributions for random networks turns out to be very important.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References



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20 0 30 of 70

The variance of degree distributions for random networks turns out to be very important.
 Using calculation similar to one for finding (k) we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





20 0 30 of 70

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 $\sigma^2 = \langle k^2 \rangle - \langle k \rangle^2$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





200 30 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



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So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larrest component

References





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So standard deviation *σ* is equal to √⟨k⟩.
 Note: This is a special property of Poisson distribution and can trip us up...

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Larges component

References





Neural reboot (NR):

Unrelated: Feline elevation

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clusterine

Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References





DQC 31 of 70

Outline

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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice

Largest component





2 9 0 32 of 70

Generalized Random Networks Configuration model

So... standard random networks have a Poisson degree distribution

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

Configuration model How to build in practice

Random friends are strange

Largest component

References



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うへで 33 of 70

So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

Configuration model How to build in practice

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strange Largest component

References



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na @ 33 of 70

- So... standard random networks have a Poisson degree distribution
- Seneralize to arbitrary degree distribution P_k .
- lso known as the configuration model. [5]

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

Configuration model How to build in practice

Motifs

Random friends are strange

Largest component

References



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So... standard random networks have a Poisson degree distribution
 Generalize to arbitrary degree distribution P_k.
 Also known as the configuration model. ^[5]
 Can generalize construction method from ER random networks.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

Configuration model How to build in practice

Motifs

Random friends are strange

Largest component

References



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 Assign each node a weight w from some

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 $P(\text{link between } i \text{ and } j) \propto w_i w_j.$

nodes with fixed degrees 2. Examining mechanisms that lead to networks certain degree distributions

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

Configuration model How to build in practice

Motifs

Random friends are strange

argest component

References





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But we'll be more interested in

Random Networks

Pure random networks How to build theoretically Some visual examples Degree distributions

Generalized Networks

Configuration model How to build in practice





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 Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

Configuration model How to build in practice

Random friends are strange

argest component

References





Random networks: examples

Coming up:

Example realizations of random networks with power law degree distributions:

 $P_k \propto k^{-\gamma}$ for $k \ge 1$. Set $P_0 = 0$ (no isolated nodes). Vary exponent γ between 2.10 and 2.91. Again, look at full network plus the largest component.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice

Random friends are strange

Largest component

References





200 34 of 70

Random networks: examples

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice

Random friends are strange

Largest component

References





200 34 of 70

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Apart from degree distribution, wiring is random

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice

Motifs Random friends ar

Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice

Motifs Random friend

Largest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice

Motifs Random friends ar

argest component

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice

Motifs

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice

Motifs

Random friends are strange

argest component

References

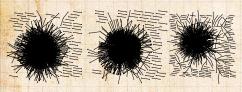


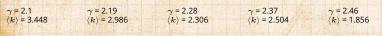


Random networks: examples for N=1000

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Random Networks







 $\gamma = 2.64$

 $\langle k \rangle = 1.6$

 $\gamma = 2.55$

(k) = 1.712



 $\gamma = 2.73$

(k) = 1.862



 $\gamma = 2.82$

(k) = 1.386





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Generalized Random Networks Configuration model

How to build in practice Motifs Random friends are strange

Largest component

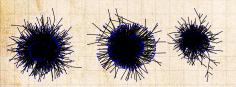
References





na @ 35 of 70

Random networks: largest components





 $\gamma = 2.55$

(k) = 1.712

 γ = 2.19 $\langle k \rangle$ = 2.986







 $\gamma = 2.46$ $\langle k \rangle = 1.856$



Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





2 C 36 of 70





 $\gamma = 2.64$ $\langle k \rangle = 1.6$

 $\gamma = 2.73$ $\langle k \rangle = 1.862$

 $\gamma = 2.82$ $\langle k \rangle = 1.386$

 γ = 2.91 $\langle k \rangle$ = 1.49

Outline

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References



VERMONT

Da @ 37 of 70

Some visual examples Clustering Degree distributions

Generalized Random Networks

How to build in practice

Random friends are strans

PoCS | @pocsvox

Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References



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Generalized random networks:

Arbitrary degree distribution P_k . Create (unconnected) nodes with degrees sampled from P_k . Wire nodes together randomly.

randomness.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References



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Generalized random networks:

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Wire nodes together randomly. Create ensemble to test deviations from randomness.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References





うへで 38 of 70

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- Solution Create (unconnected) nodes with degrees sampled from P_k .
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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References





200 38 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References

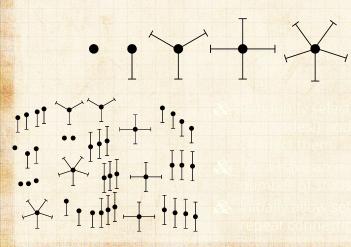




200 38 of 70

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References

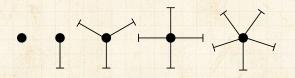


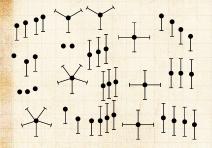
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うへで 39 of 70

Phase 1:

Idea: start with a soup of unconnected nodes with stubs (half-edges):





Randomly select stuk (not nodes!) and connect them. Must have an even number of stubs. Initially allow self- an connections.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References

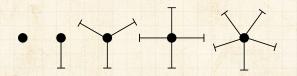


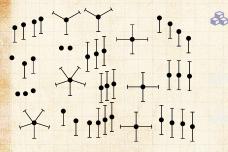
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200 39 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

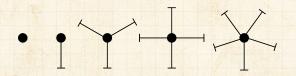
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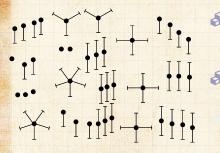




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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

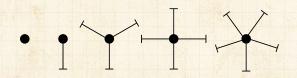
References

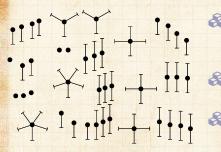


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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References



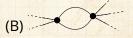
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Building random networks: First rewiring

Phase 2:

Now find any (A) self-loops and (B) repeat edges and randomly rewire them.





Being careful, we can't change the degree of an node, so we can't simply move links around. Simplest solution: randomly rewire two edges time. PoCS | @pocsvox

Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References





うへへ 40 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

Largest component

References





うへへ 40 of 70

Building random networks: First rewiring

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Being careful: we can't change the degree of any node, so we can't simply move links around.
 Simplest solution: randomly rewire two edges at a time.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

Largest component

References



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うへ ~ 40 of 70

3

Randomly choose two edges. (Or choose problem edge and a random edge)

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

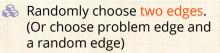
Largest component

References



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うへで 41 of 70



Check to make sure edges are disjoint.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

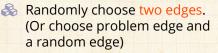
Largest component

References





うへで 41 of 70



Check to make sure edges are 3 disjoint.



Rewire one end of each edge.

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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice

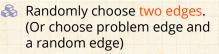
Largest component



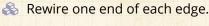


29 C 41 of 70

e1



Check to make sure edges are disjoint.



Node degrees do not change.

Works if e₁ is a self-loop or repeated edge. Same as finding on/off/on/of PoCS | @pocsvox

Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

Largest component

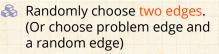
References





うへで 41 of 70

e1



Check to make sure edges are disjoint.

- Rewire one end of each edge.
 - Node degrees do not change.
- Works if e_1 is a self-loop or repeated edge.
 - Same as finding on/off/on/off 4-cycles. and rotating them.

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Random Networks

- Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions
- Generalized Random Networks
- How to build in practice Motifs
- Random friends are strange
- Largest component

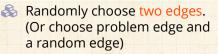
References





うへへ 41 of 70

e1



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References





Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

Largest component

References





200 42 of 70

Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

Randomize network wiring by applying rewiring algorithm liberally.

Rule of thumb: # Rewirings $\simeq 10 \times #$ edge

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

Largest component

References





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Solution Relation Re

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

argest component

References



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Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

Largest component

References

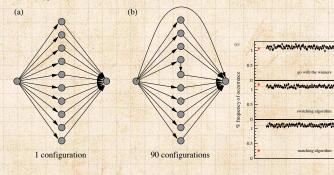


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20 43 of 70

Random sampling

Problem with only joining up stubs is failure to randomly sample from all possible networks.
 Example from Milo et al. (2003) ^[3]:



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References



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\mathbb{R} What if we have P_k instead of N_k ?

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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice

Largest component



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Dac 44 of 70



 \mathbb{R} What if we have P_k instead of N_k ? Must now create nodes before start of the construction algorithm.

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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model

How to build in practice

Largest component





Dac 44 of 70

What if we have P_k instead of N_k?
 Must now create nodes before start of the construction algorithm.
 Construct N nodes by campling from degree

Solution P_k . Generate N nodes by sampling from degree distribution P_k .

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References





200 44 of 70

What if we have P_k instead of N_k?
Must now create nodes before start of the construction algorithm.
Generate N nodes by sampling from degree distribution P_k.

Easy to do exactly numerically since k is discrete.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model

How to build in practice Motifs

Random friends are strange

Largest component

References





What if we have P_k instead of N_k?
Must now create nodes before start of the construction algorithm.
Generate N nodes by sampling from degree distribution P_k.
Easy to do exactly numerically since k is discrete.
Note: not all P_k will always give nodes that can be wired together.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks

How to build in practice Motifs

Random friends are strange

Largest component

References





200 44 of 70

Outline

PoCS | @pocsvox

Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Random Networks Configuration model How to build in practice

Random friends are





Generalized

Motifs

Largest component



29 C 45 of 70

Generalized Random Networks

Motifs

Idea of motifs^[6] introduced by Shen-Orr, Alon et al. in 2002.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice

Motifs Random friends are strange Largest component

References





- Idea of motifs^[6] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice

Motifs Random friends are strange Largest component







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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Motifs Random friends are strange Lareest component

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Motifs Random friends are strange

References





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- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.
 - Directed network with 577 interactions (edges) and 424 operons (nodes).
- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Motifs Random friends are strange





- Idea of motifs^[6] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.
- 🚳 Specific example of Escherichia coli.
 - Directed network with 577 interactions (edges) and 424 operons (nodes).
- Solution Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .
- Looked for certain subnetworks (motifs) that appeared more or less often than expected

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Motifs Random friends are strange

References





うへ ~ 46 of 70

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Random Networks

Pure random

Clustering Degree distributions

Random

Networks

Motifs

Generalized

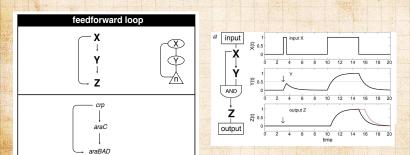
Configuration model

Random friends are

Largest component

How to build in practice

How to build theoretically Some visual examples



 \mathbb{R} Z only turns on in response to sustained activity in X.

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20 A 47 of 70

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Random Networks

Pure random

Clustering Degree distributions

Random

Networks

Motifs

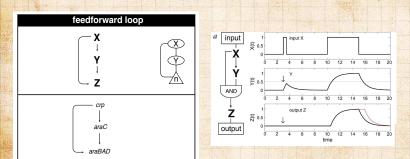
Generalized

Configuration model

Largest component

How to build in practice

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Z only turns on in response to sustained activity in X.
 Transition off X respirit to the set of Z.

 $\underset{\text{Constraints}}{\underset{\text{Constr$





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Random Networks

Pure random

Clustering Degree distributions

Random

Motifs

Networks

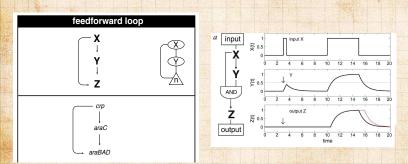
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Configuration model

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How to build in practice

How to build theoretically Some visual examples

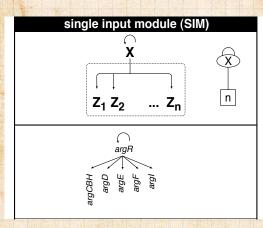


 \mathbb{R} Z only turns on in response to sustained activity in X.

- $\underset{\text{Comparison}}{\underset{\text{$
- logy to elevator doors.







🚳 Master switch.

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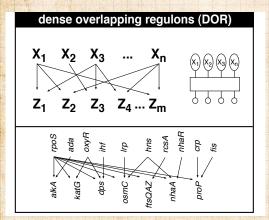
Motifs Random friends are strange Largest component

References



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200 48 of 70



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Motifs Random friends are strange Largest component

References



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200 49 of 70

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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Configuration model

Random friends are strange

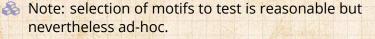




Networks How to build in practice Motifs

Largest component

2 C 50 of 70



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Random Networks

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Motifs Random friends are strange Largest component

References



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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.
 For more, see work carried out by Wiggins *et al.* at Columbia.



Outline

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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Random Networks Configuration model How to build in practice

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Generalized

Random friends are Largest component



Generalized Random Networks

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The degree distribution P_k is fundamental for our description of many complex networks

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References



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うへで 52 of 70

The degree distribution P_k is fundamental for our description of many complex networks

Solution Again: P_k is the degree of randomly chosen node.

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References



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na ~ 52 of 70

- The degree distribution P_k is fundamental for our description of many complex networks
- Solution Again: P_k is the degree of randomly chosen node.
- A second very important distribution arises from choosing randomly on edges rather than on nodes.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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- Solution Again: P_k is the degree of randomly chosen node.
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Random Networks

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Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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 $Q_k \propto k P_k$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k'P_{k'}}$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





na ~ 52 of 70

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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References



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Big deal: Rich-get-richer mechanism is built into this selection process.

うへで 52 of 70



Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7.$

Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel: $Q_1 = 3/16$, $Q_2 = 4/16$, $Q_3 = 3/16$, $Q_6 = 6/16$,

Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

 $R_0 = 3/16 R_1 = 4/10$ $R_2 = 3/16, R_5 = 6/1$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References



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うへで 53 of 70



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Probability of finding #

outgoing edges = & after randomly selecting an edge and then randomly choosing one direction to travel: PoCS | @pocsvox

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References



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うへで 53 of 70



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Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

$$\begin{split} R_0 &= 3/16 \; R_1 = 4/16, \\ R_2 &= 3/16, \; R_5 = 6/16. \end{split}$$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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References



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For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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References





200 54 of 70

For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

 \bigotimes Useful variant on Q_k :

 R_k = probability that a friend of a random node has k other friends.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





200 54 of 70

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 R_k = probability that a friend of a random node has k other friends.

$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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Solution Equivalent to friend having degree k + 1.

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Generalized Random Networks Configuration model How to build in practice Motifs

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For random networks, Q_k is also the probability that a friend (neighbor) of a random node has $\frac{k}{k}$ friends.

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 R_k = probability that a friend of a random node has k other friends.

$$R_{k} = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

Equivalent to friend having degree k + 1.
 Natural question: what's the expected number of other friends that one friend has?

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Generalized Random Networks Configuration model How to build in practice Motifs

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Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k \right\rangle_R = \sum_{k=0}^{\infty} k R_k$$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





na ~ 55 of 70

Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References



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200 55 of 70

Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left< k \right>_R = \sum_{k=0}^\infty k R_k = \sum_{k=0}^\infty k \frac{(k+1) P_{k+1}}{\left< k \right>}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^\infty k(k+1) P_{k+1}$$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References



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$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{\infty} k(k+1) P_{k+1}$$

$$= \frac{1}{\langle k \rangle} \sum_{k=1}^{1} \left((k+1)^2 - (k+1) \right) P_{k+1}$$

(where we have sneakily matched up indices)

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





200 55 of 70

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$$= \frac{1}{\langle k \rangle} \sum_{j=0}^{\infty} (j^2 - j) P_j$$
 (using j = k+1)

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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$$=rac{1}{\langle k
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$$=rac{1}{\langle k
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Random Networks

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Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References



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200 55 of 70

8

Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

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Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





200 56 of 70

Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} (\langle k^2 \rangle - \langle k \rangle)$, is true for all random networks, independent of degree distribution.

🚳 For standard random networks, recall

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 56 of 70

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A Therefore:

$$\left< k \right>_R = \frac{1}{\left< k \right>} \left(\left< k \right>^2 + \left< k \right> - \left< k \right> \right)$$

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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Random friends are strange





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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Random friends are strange





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Again, neatness of results is a special property of the Poisson distribution.

Random Networks

Pure random networks How to build theoretically Some visual examples Degree distributions

Generalized Random Networks Configuration model How to build in practice

Random friends are strange





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Again, neatness of results is a special property of the Poisson distribution.

So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total friends...

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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The edge-degree distribution: In fact, R_k is rather special for pure random networks ...

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References

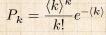


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200 57 of 70

In fact, R_k is rather special for pure random networks ...
 Substituting

into



 $R_{k}=\frac{(k+1)P_{k+1}}{\langle k\rangle}$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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References



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200 57 of 70

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$$P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k | k!}$$

into

$$R_k = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

we have

$$R_k = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle}$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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$$=\frac{\langle k\rangle^k}{k!}e^{-\langle k\rangle}$$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 57 of 70

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$$R_k = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)!} e^{-\langle k \rangle} = \frac{(k+1)}{\langle k \rangle} \frac{\langle k \rangle^{(k+1)}}{(k+1)k!} e^{-\langle k \rangle}$$

$$= \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \equiv P_k.$$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 57 of 70

In fact, R_k is rather special for pure random networks ...
 Substituting

$$P_k = \frac{\langle k \rangle^k}{k!} e^{-\langle k}$$

into

$$R_{k} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

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Random Networks

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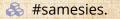
Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References







DQ @ 57 of 70

Reason #1:

Average # friends of friends per node is

Key: Average depends on the 1st and 2nd moment: P_k and not just the 1st moment. Three peculiarities:

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





200 58 of 70

Reason #1:

Average # friends of friends per node is

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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Average # friends of friends per node is

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





nac 58 of 70

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Your friends really are different from you... See also: class size paradoxes (nod to: Gelm

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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(e.g., in the case of a power-law distribution Your friends really are different from you...

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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Random Networks

Pure random networks How to build theoretically Some visual examples Degree distributions

Generalized Networks Configuration model How to build in practice Random friends are

strange





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Random Networks

Pure random networks How to build theoretically Some visual examples Degree distributions

Generalized Networks Configuration model How to build in practice Random friends are

strange





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Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

strange Largest component





Two reasons why this matters More on peculiarity #3: A node's average # of friends: $\langle k \rangle$

chosen as a friend

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange

References





うへで 59 of 70

More on peculiarity #3:

A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$

So only if everyone has the same degree (variance = $\sigma^2 = 0$) can a node be the same as friends.

Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





うへで 59 of 70

More on peculiarity #3:

- A node's average # of friends: $\langle k \rangle$ Friend's average # of friends: $\frac{\langle k^2 \rangle}{\langle k \rangle}$ Comparison:
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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





na c 59 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





na c 59 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





na c 59 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are

strange Largest component

References





うへで 59 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

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"Generalized friendship paradox in complex networks: The case of scientific collaboration" Eom and Jo, Nature Scientific Reports, **4**, 4603, 2014.^[1]

Your friends really are monsters #winners:¹ Go on the real Friends have more coauthors, citations, and publications. Other homitic studies: your connections on Twitter have more followers than you, your sex partners more partners than you, ... The hope: Maybe they have more enemies and diseases too.

¹Some press here C [MIT Tech Review].

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





200 60 of 70



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





990 60 of 70



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





20 60 of 70



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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





20 60 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

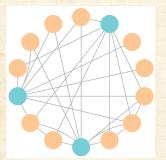
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¹https://www.washingtonpost.com/graphics/business/ wonkblog/majority-illusion/

Related disappointment:



Nodes see their friends' color choices.

Vhich color is more popular?¹

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Random friends are strange Largest component





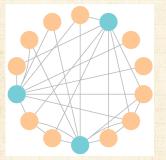
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Which color is more popular?¹

🚳 Again: thinking in edge space changes everything.

Related disappointment:



Two reasons why this matters (Big) Reason #2:

- $\langle k \rangle_R$ is key to understanding how well random networks are connected together.
 - e.g., we'd like to know what's the size of the larg component within a network. As $N \rightarrow \infty$, does our network have a giant component?
 - Defor Component = connected subnetwork of nodes such that = path between each pair of nodes in the subnetwork, and no node outside of the subnetwork is connected to it.
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Random Networks

- Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions
- Generalized Random Networks Configuration model How to build in practice Motifs
- Random friends are strange Largest component
- References





na c 62 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component

References





na c 62 of 70

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs

Random friends are strange Largest component





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Random Networks

Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Networks Configuration model How to build in practice

Random friends are strange





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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References



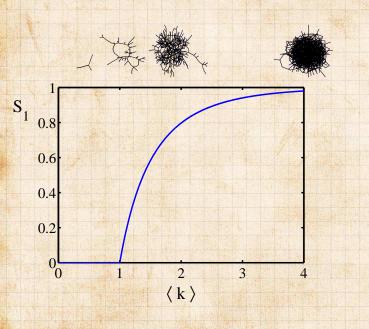


Generalized Random Networks

How to build in practice Motifs Random friends are strang Largest component

20 0 63 of 70

Giant component



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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References



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200 64 of 70

Structure of random networks Giant component:

A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.

Equivalently, expect exponential growth in nod number as we move out from a random node. All of this is the same as requiring $\langle k \rangle_R > 1$. Giant-component condition (or percolation condition):

Again, see that the second moment is an essenti part of the story. Equivalent statement: $\langle k^2
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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





20 65 of 70

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

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Random Networks

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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





Structure of random networks

Giant component:

- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- All of this is the same as requiring $\langle k \rangle_R > 1$.

Giant component condition (or percolation condition):

$$\left< k \right>_R = \frac{\left< k^2 \right> - \left< k \right>}{\left< k \right>} > 1$$

Again, see that the second moment is an essential part of the story.

Equivalent statement: $\langle k^2 \rangle > 2 \langle k \rangle$

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





Giant component for standard random networks: Recall $\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$.

Determine condition for giant component:

Therefore when $\langle k \rangle > 1$, standard random networks have a giant component. When $\langle k \rangle < 1$, all components are finite. Fine example of a continuous phase where the say $\langle k \rangle = 1$ marks the critical point of the system.

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





Recall
$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle$$
.

6

Determine condition for giant component:

$$\left\langle k \right\rangle_R = rac{\left\langle k^2 \right\rangle - \left\langle k \right\rangle}{\left\langle k \right\rangle}$$

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





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Determine condition for giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} = \frac{\langle k \rangle^2 + \langle k \rangle - \langle k \rangle}{\langle k \rangle}$$

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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





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Pure random networks How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice

Largest component





29 C 66 of 70

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Solution
 Solution

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





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Solution 3. Therefore when ⟨k⟩ > 1, standard random networks have a giant component.
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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





20 06 of 70

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Solution State State

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





 \mathfrak{R} e.g, if $P_k = ck^{-\gamma}$ with $2 < \gamma < 3$, $k \ge 1$, then

$$\langle k^2 \rangle = c \sum_{k=1}^\infty k^2 k^{-\gamma}$$

So giant component always exists for these kinds of networks. Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to lool harder at $\langle k \rangle_R$. How about $P_k = \delta_{kk_0}$? CocoNuTs: We figure out the final size and complete dynamics.

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





200 67 of 70

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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





200 67 of 70

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References





200 67 of 70

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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





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Pure random networks Definitions How to build theoretically Some visual examples Clustering Deeree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





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Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component





Neural reboot (NR):

Falling maple leaf

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange

Largest component

References



VERMONT

200 68 of 70

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Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component





200 69 of 70

References II

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Random Networks

Pure random networks Definitions How to build theoretically Some visual examples Clustering Degree distributions

Generalized Random Networks Configuration model How to build in practice Motifs Random friends are strange Largest component

References



VERMONT 8

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