Mechanisms for Generating Power-Law Size Distributions, Part 2

Principles of Complex Systems | @pocsvox
CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

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What's the Story?

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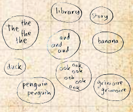
Outline



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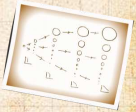

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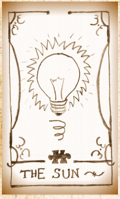

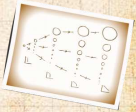

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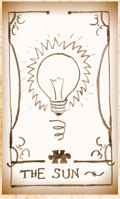

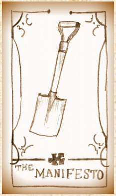

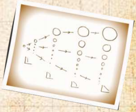

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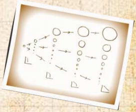

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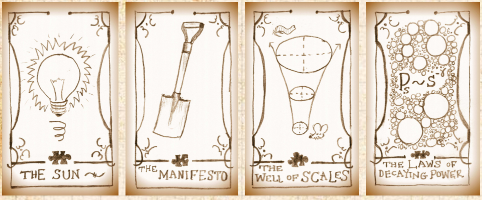
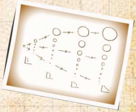

THE SUN
THE MANIFESTO
THE WELL OF SCALES
$P_s \sim s^{-\gamma}$
THE LAWS OF DECAYING POWER



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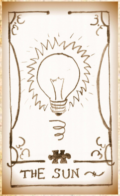
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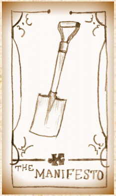
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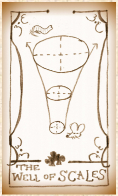
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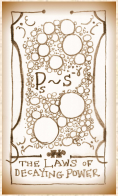
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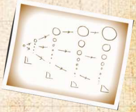
MINISTRY OF RANDOM WALKS



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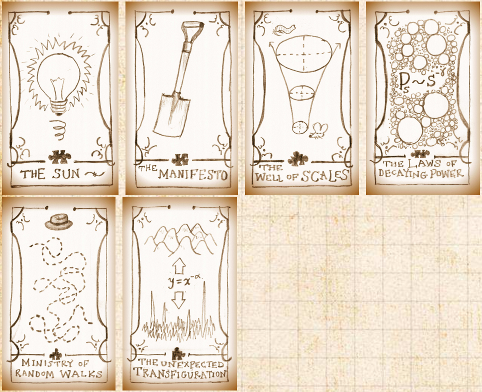
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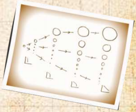

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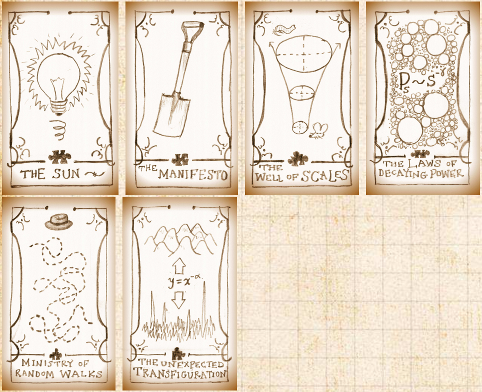
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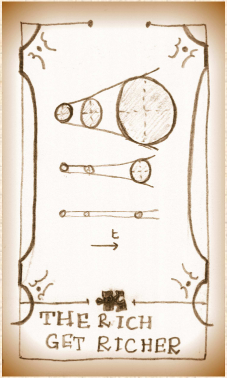
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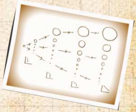
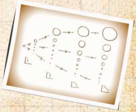


Aggregation:

- Random walks represent additive aggregation
- Mechanism: Random addition and subtraction
- Compare across realizations, no competition.
- Next: Random Additive/Copying Processes involving Competition.
- Widespread: Words, Cities, the Web, Wealth, Productivity (Lotka), Popularity (Books, People, ...)
- Competing mechanisms (trickiness)



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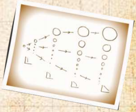

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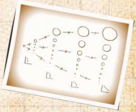

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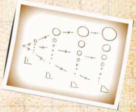

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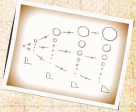

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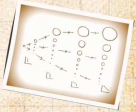
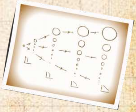


Pre-Zipf's law observations of Zipf's law

- 1910s: Word frequency examined re Stenography (or shorthand or brachygraphy or tachygraphy), Jean-Baptiste Estoup [11].
- 1910s: Felix Auerbach pointed out the Zipfitude of city sizes in "Das Gesetz der Bevölkerungskonzentration" ("The Law of Population Concentration").
- 1924: G. Udny Yule [28]: # Species per Genus
- 1926: Lotka [15]: # Scientific papers per author (Lotka's law)



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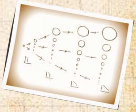

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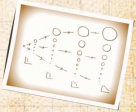

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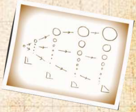

Theoretical Work of Yore:

- 1953: Mandelbrot [17]:
  Optimality argument for Zipf's law; focus on language.
- 1955: Herbert Simon [24, 30]:
  Zipf's law for word frequency, city size, income, publications, and species per genus.
- 1965/1976: Derek de Solla Price [7, 8]:
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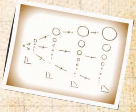
- 1999: Barabasi and Albert [2]:
  The World Wide Web, networks-at-large.



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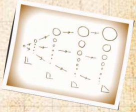

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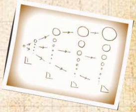

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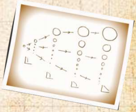
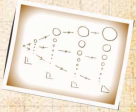


Herbert Simon (1916–2001):

- Political scientist (and much more)
- Involved in Cognitive Psychology, Computer Science, Public Administration, Economics, Management, Sociology
- Coined 'bounded rationality' and 'satisficing'
- Nearly 1000 publications (see Google Scholar)
- An early leader in Artificial Intelligence, Information Processing, Decision-Making, Problem-Solving, Attention Economics, Organization Theory, Complex Systems, And Computer Simulation Of Scientific Discovery.
- 1978 Nobel Laureate in Economics (his Nobel bio is here).



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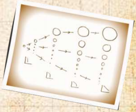

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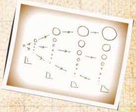

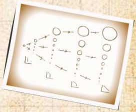

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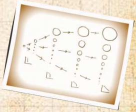

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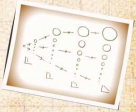
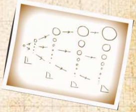


Essential Extract of a Growth Model:

Random Competitive Replication (RCR):

1. Start with 1 elephant (or element) of a particular flavor at $t = 1$
2. At time $t = 2, 3, 4, \ldots$, add a new elephant in one of two ways:
   - With probability ρ, create a new elephant with a new flavor
   - With probability $1 - \rho$, randomly choose from all existing elephants, and make a copy.
   - Elephants of the same flavor form a group



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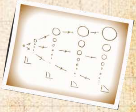

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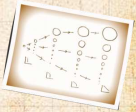

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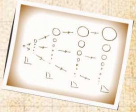

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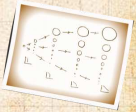

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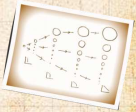
     = Replication/Imitation
   - Elephants of the same flavor form a group

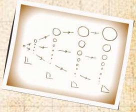


Random Competitive Replication:

Example: Words appearing in a language

- Consider words as they appear sequentially.
- With probability ρ, the next word has not previously appeared
- With probability $1 - \rho$, randomly choose one word from all words that have come before, and reuse this word

Note: This is a terrible way to write a novel.



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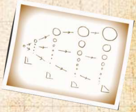

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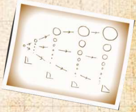

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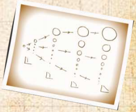

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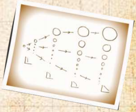

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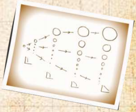

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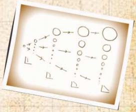

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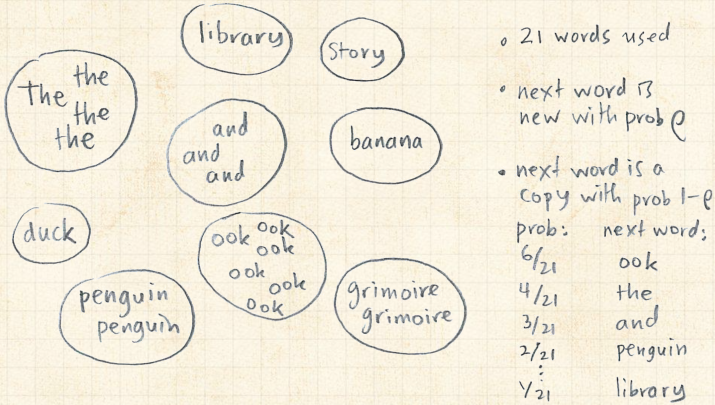
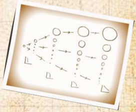

library
Story
the the the the
and and and
banana
duck
ook ook ook ook ook ook
penguin penguin
grimoire grimoire
• 21 words used
• next word is new with prob ρ
• next word is a copy with prob 1−ρ
prob: next word:
6/21 ook
4/21 the
3/21 and
2/21 penguin
1/21 library

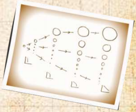


Some observations:

- Fundamental Rich-get-Richer story;
- Competition for replication between individual elephants is random;
- Competition for growth between groups of matching elephants is not random;
- Selection on groups is biased by size;
- Random selection sounds easy;
- Possible that no great knowledge of system needed (but more later ...).

Your free set of tofu knives:

- Related to Pólya's Urn Model, a special case of problems involving urns and colored balls
- Sampling with super-duper replacement and sneaky sneaking in of new colors.



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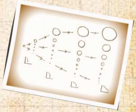

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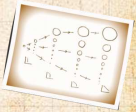

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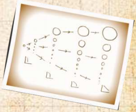

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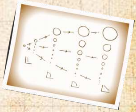

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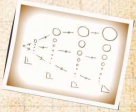

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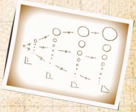

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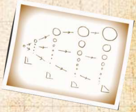

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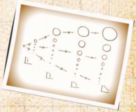

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Some observations:

- Steady growth of system: +1 elephant per unit time.
- Steady growth of distinct flavors at rate ρ
- We can incorporate
  1. Elephant elimination
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  3. Variable innovation rate ρ
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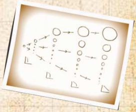

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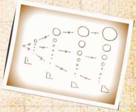

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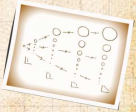

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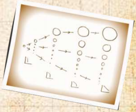

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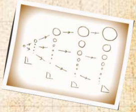

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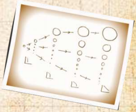

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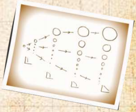

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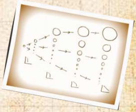

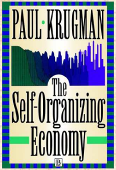

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Ch. 3: An Urban Mystery, p. 46

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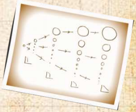

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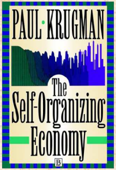

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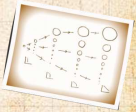

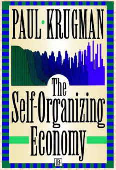

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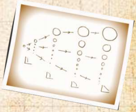

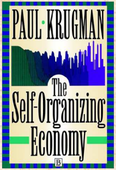

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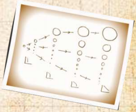
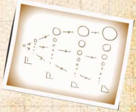


Outline



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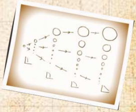
Definitions:

- k_i = size of a group i
- $N_{k,t}$ = # groups containing k elephants at time t.



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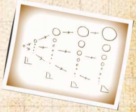

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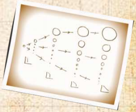

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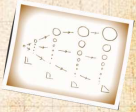

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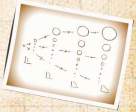
First: $\sum_k kN_{k,t} = t =$ number of elephants at time t

Random Competitive Replication:

$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:

- $N_{k,t}$ size k groups
- $\Rightarrow kN_{k,t}$ elephants in size k groups
- t elephants overall



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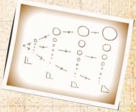

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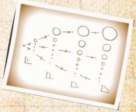

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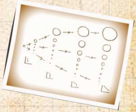
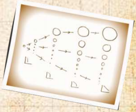


Random Competitive Replication:

$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:

- $N_{k,t}$ size k groups
- $\Rightarrow kN_{k,t}$ elephants in size k groups
- t elephants overall

$$P_k(t) = \frac{kN_{k,t}}{t}.$$

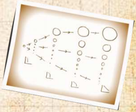


Random Competitive Replication:

$N_{k,t}$, the number of groups with k elephants, changes at time t if

1. An elephant belonging to a group with k elephants is replicated:

2. An elephant belonging to a group with $k-1$ elephants is replicated:



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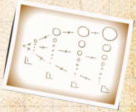

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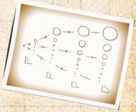

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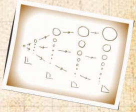

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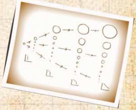

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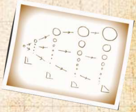

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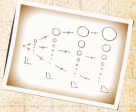

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Special case for $N_{1,t}$:

1. The new elephant is a new flavor:

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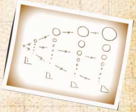

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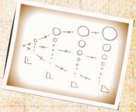

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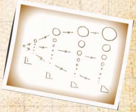

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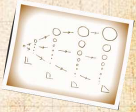

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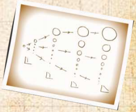

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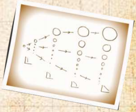

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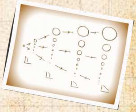
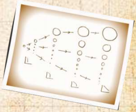


Random Competitive Replication:

Putting everything together:

For $k > 1$:

$$\langle N_{k,t+1} - N_{k,t} \rangle = (1-\rho)\left((+1)(k-1)\frac{N_{k-1,t}}{t} + (-1)k\frac{N_{k,t}}{t} \right)$$



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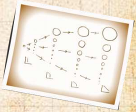


Random Competitive Replication:

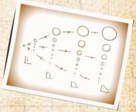
Assume distribution stabilizes: $N_{k,t} = n_k t$

(Reasonable for t large)

- Drop expectations
- Numbers of elephants now fractional
- Okay over large time scales

- For later: the fraction of groups that have size k is n_k/ρ since

$$\frac{N_{k,t}}{\rho t} = \frac{n_k t}{\rho t} = \frac{n_k}{\rho}.$$



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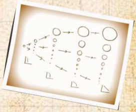

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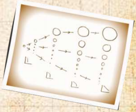

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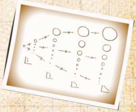

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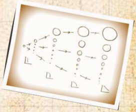

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Stochastic difference equation:

$$\left\langle N_{k,t+1} - N_{k,t} \right\rangle = (1-\rho)\left((k-1)\frac{N_{k-1,t}}{t} - k\frac{N_{k,t}}{t}\right)$$

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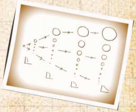

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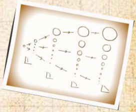

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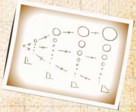

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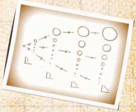

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We have a simple recursion:

$$\frac{n_k}{n_{k-1}} = \frac{(k-1)(1-\rho)}{1+(1-\rho)k}$$

- Interested in k large (the tail of the distribution)
- Can be solved exactly.

Insert question from assignment 4

- For just the tail: Expand as a series of powers of $1/k$

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We (okay, you) find

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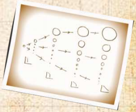

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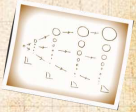

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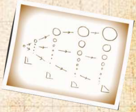

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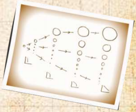

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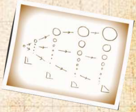

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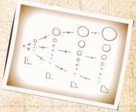

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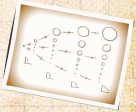

Micro-to-Macro story with ρ and γ measurable.

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$

- Observe $2 < \gamma < \infty$ for $0 < \rho < 1$.
- For $\rho \simeq 0$ (low innovation rate):

$$\gamma \simeq 2$$

- 'Wild' power-law size distribution of group sizes, bordering on 'infinite' mean.
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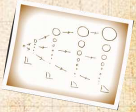
- All elephants have different flavors.
- Upshot: Tunable mechanism producing a family of universality classes.



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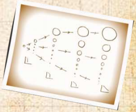

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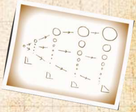

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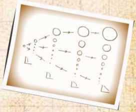

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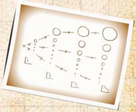

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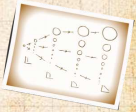

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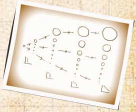

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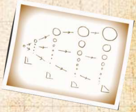

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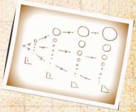

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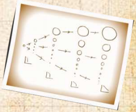

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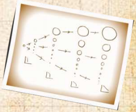

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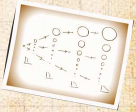

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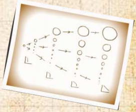

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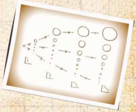

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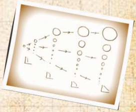

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We had one other equation:

$$\langle N_{1,t+1} - N_{1,t} \rangle = \rho - (1-\rho)1 \cdot \frac{N_{1,t}}{t}$$

As before, set $N_{1,t} = n_1 t$ and drop expectations

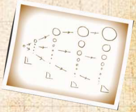
$$n_1(t+1) - n_1 t = \rho - (1-\rho)1 \cdot \frac{n_1 t}{t}$$

$$n_1 = \rho - (1-\rho)n_1$$

Rearrange:

$$n_1 + (1-\rho)n_1 = \rho$$

$$\boxed{n_1 = \frac{\rho}{2-\rho}}$$



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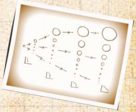

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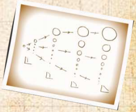

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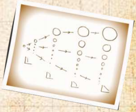

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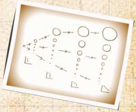

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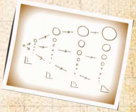
- For ρ small, fraction of unique elephants $\sim 1/2$
- Roughly observed for real distributions
- ρ increases, fraction increases
- Can show fraction of groups with two elephants $\sim 1/6$
- Model works well for large and small λ #awesome



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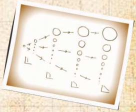

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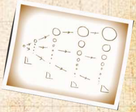

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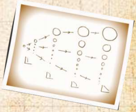

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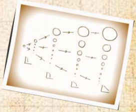

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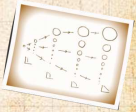

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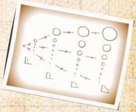

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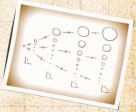

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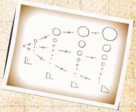
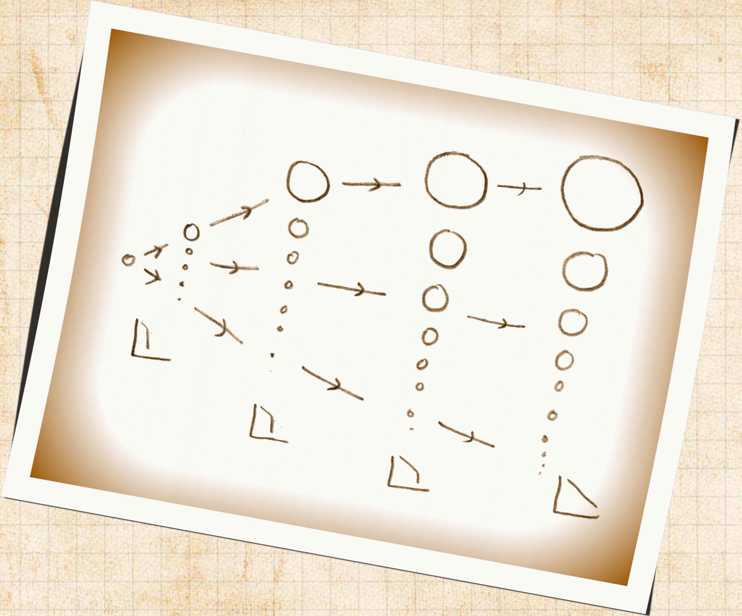
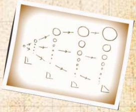




Outline



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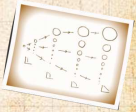

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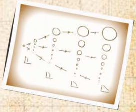

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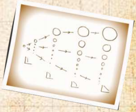
N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870

Outline



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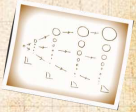
Its appearance is so frequent, and the phenomena so diverse, that one is led to conjecture that if these phenomena have any property in common it can only be a similarity in the structure of the underlying probability mechanisms.



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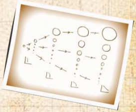

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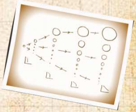

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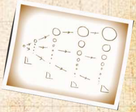

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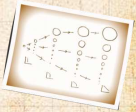

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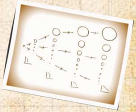

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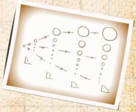

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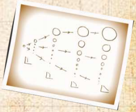

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- First to study network evolution with these kinds of models.
- Citation network of scientific papers
- Price's term: Cumulative Advantage
- Idea: papers receive new citations with probability proportional to their existing # of citations
- Directed network
- Two (surmountable) problems:
  1. New papers have no citations
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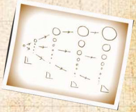

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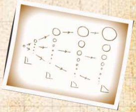

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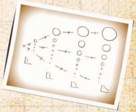

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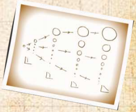

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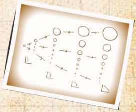

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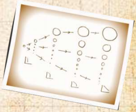

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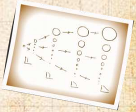

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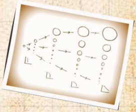

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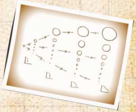

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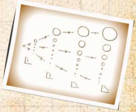

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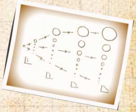

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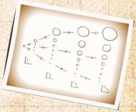

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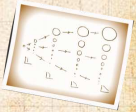

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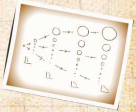

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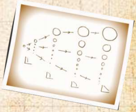

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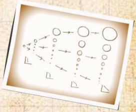

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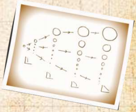

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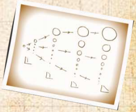

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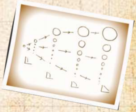

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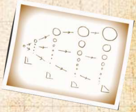

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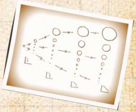
- "Scale-free networks" = food on the table for physicists



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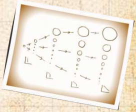

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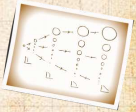

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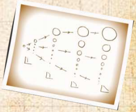

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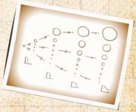

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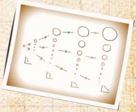

Evolution of catch phrases:

- Barabasi and Albert [2]—thinking about the Web
- Independent reinvention of a version of Simon and Price's theory for networks
- Another term: "Preferential Attachment"
- Considered undirected networks (not realistic but avoids 0 citation problem)
- Still have selection problem based on size (non-random)
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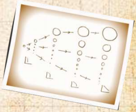

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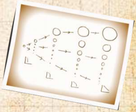
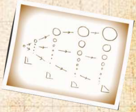


Another analytic approach: [9]

- Focus on how the nth arriving group typically grows.
- Analysis gives:

$$S_{n,t} \sim \begin{cases} \frac{1}{\Gamma(2-\rho)}\left[\frac{1}{t}\right]^{-(1-\rho)} & \text{for } n = 1, \\ \rho^{1-\rho}\left[\frac{n-1}{t}\right]^{-(1-\rho)} & \text{for } n \geq 2. \end{cases}$$

- First mover is a factor $1/\rho$ greater than expected.
- Because ρ is usually close to 0, the first element is truly an elephant in the room.
- Appears that this has been missed for 60 years ...



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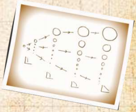

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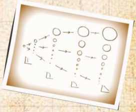
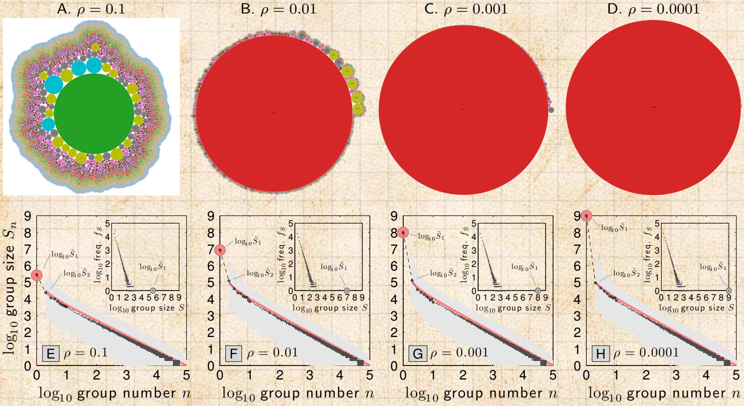
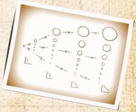


"Simon's fundamental rich-gets-richer model entails a dominant first-mover advantage"
Dodds et al.,
Available online at
http://arxiv.org/abs/0909.1104, 2016. [9]



See visualization at paper's online app-endices



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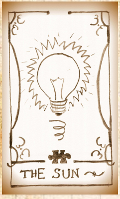

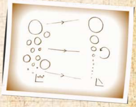

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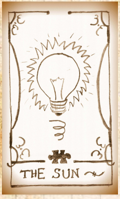

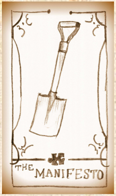

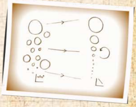

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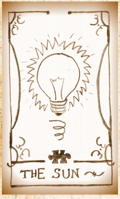
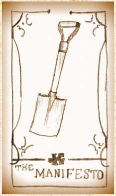
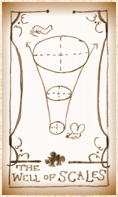
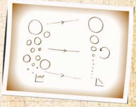
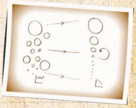






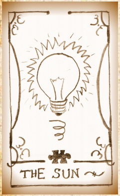




The deal:


THE SUN
THE MANIFESTO
THE WELL OF SCALES
THE LAWS OF DECAYING POWER

The deal: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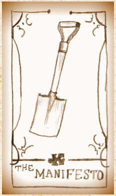
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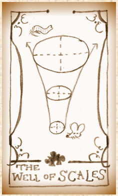
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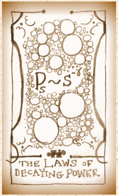
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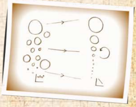
MINISTRY OF RANDOM WALKS



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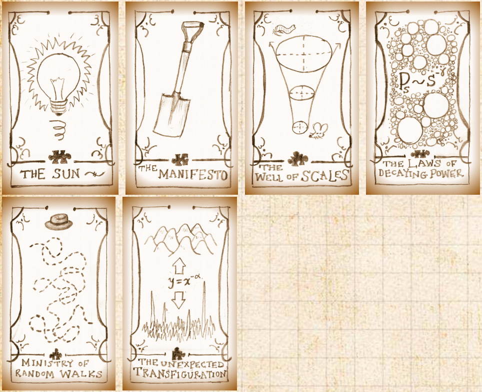

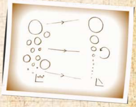

The UNIVERSITY of VERMONT

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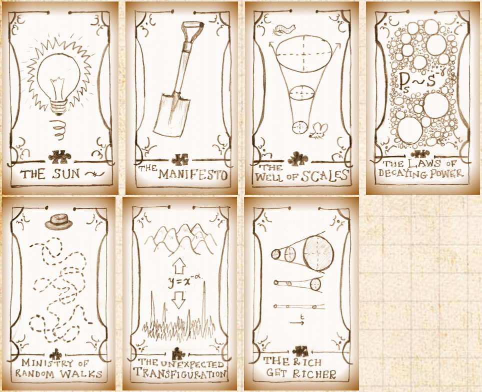
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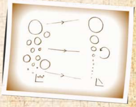

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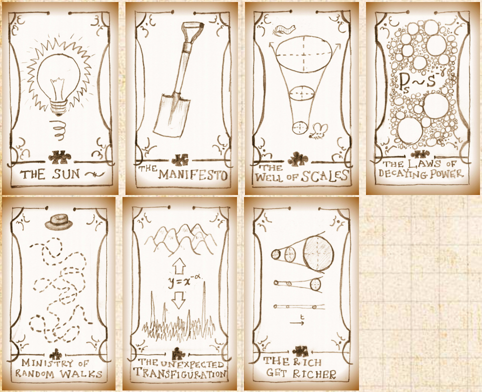
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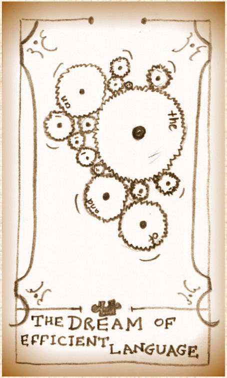
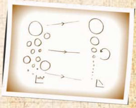

THE DREAM OF EFFICIENT LANGUAGE



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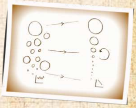
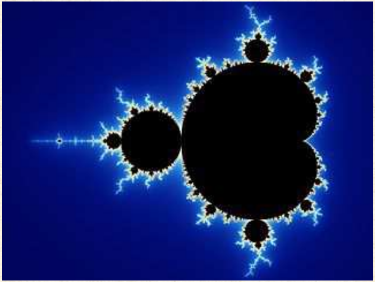
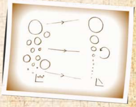


Benoît Mandelbrot



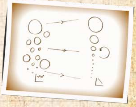
- Mandelbrot = father of fractals
- Mandelbrot = almond bread
- Bonus Mandelbrot set action: here.



Another approach:

Benoît Mandelbrot

- Derived Zipf's law through optimization [17]
- Idea: Language is efficient
- Communicate as much information as possible for as little cost
- Need measures of information (H) and average cost (C)...
- Language evolves to maximize H/C, the amount of information per average cost.
- Equivalently: minimize C/H.
- Recurring theme: what role does optimization play in complex systems?



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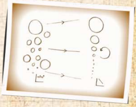

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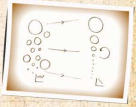

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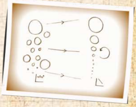

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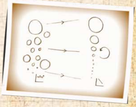

The University of Vermont

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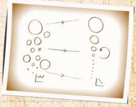

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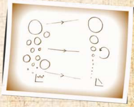

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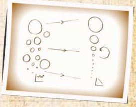

The Quickening—Mandelbrot v. Simon:

There Can Be Only One:

- Things there should be only one of: Theory, Highlander Films.
- Feel free to play Queen's It's a Kind of Magic in your head (funding remains tight).

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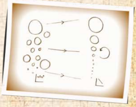
Now let us enjoy the Trailer for Highlander:



We were born to be Princes of the Universe

Mandelbrot vs. Simon:

- Mandelbrot (1953): "An Informational Theory of the Statistical Structure of Languages" [17]
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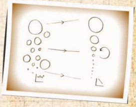

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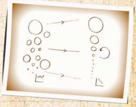

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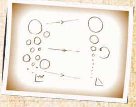

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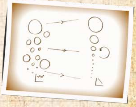

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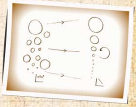
I have no rival, No man can be my equal

Mandelbrot vs. Simon:

- Mandelbrot (1961): "Final note on a class of skew distribution functions: analysis and critique of a model due to H.A. Simon" [20]
- Simon (1961): "Reply to 'final note' by Benoit Mandelbrot" [27]
- Mandelbrot (1961): "Post scriptum to 'final note'" [20]
- Simon (1961): "Reply to Dr. Mandelbrot's post scriptum" [28]



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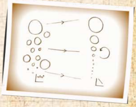

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Mandelbrot vs. Simon:

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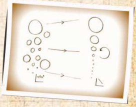

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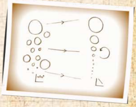

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"We shall restate in detail our 1959 objections to Simon's 1955 model for the Pareto-Yule-Zipf distribution. Our objections are valid quite irrespectively of the sign of p-1, so that most of Simon's (1960) reply was irrelevant." [19]

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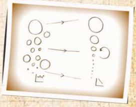

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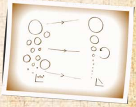

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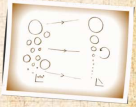

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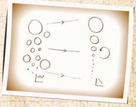

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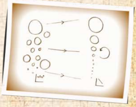

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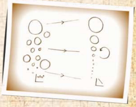

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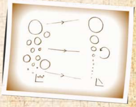

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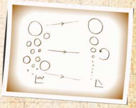

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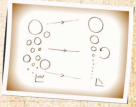
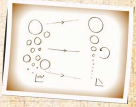


Zipfarama via Optimization:

Mandelbrot's Assumptions:

- Language contains n words: $w_1, w_2, \ldots, w_n$.
- ith word appears with probability p_i
- Words appear randomly according to this distribution (obviously not true...)
- Words = composition of letters is important
- Alphabet contains m letters
- Words are ordered by length (shortest first)



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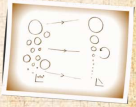

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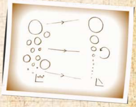

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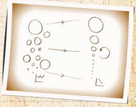

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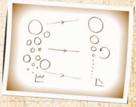

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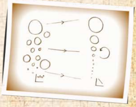

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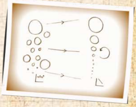


Zipfarama via Optimization:

Word Cost

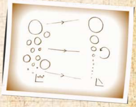
- Length of word (plus a space)
- Word length was irrelevant for Simon's method

Objection

- Real words don't use all letter sequences

Objections to Objection

- Maybe real words roughly follow this pattern (?)
- Words can be encoded this way
- Na na na-na naaaaa...



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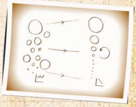

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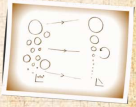

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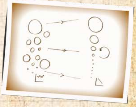

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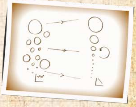

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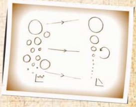

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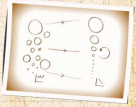

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Binary alphabet plus a space symbol

i	1	2	3	4	5	6	7	8
word	1	10	11	100	101	110	111	1000
length	1	2	2	3	3	3	3	4
$1 + \log_2 i$	1	2	2.58	3	3.32	3.58	3.81	4

- Word length of 2^kth word: $= k + 1$
- Word length of ith word $\simeq 1 + \log_2 i$
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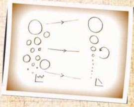

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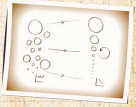

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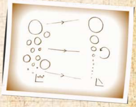

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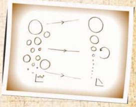

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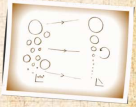

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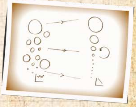

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Total Cost C

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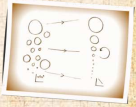
- Cost of the ith word plus space: $C_i \simeq 1 + \log_m (i+1)$
- Subtract fixed cost: $C'_i = C_i - 1 \simeq \log_m (i+1)$
- Simplify base of logarithm:

$$C'_i \simeq \log_m (i+1) = \frac{\log_e (i+1)}{\log_e m}$$

- Total Cost:

$$C \sim \sum_{i=1}^{n} p_i C'_i \propto \sum_{i=1}^{n} p_i \text{□□}(i+1)$$



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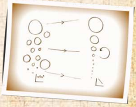

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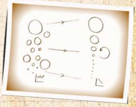

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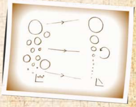

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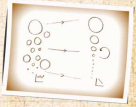
$$C_i' \simeq \log_m (i+1) = \frac{\log_e (i+1)}{\log_e m} \propto □□(i+1)$$

- Total Cost:

$$C \sim \sum_{i=1}^{n} p_i C_i' \propto \sum_{i=1}^{n} p_i □□(i+1)$$



The University of Vermont

Zipfarama via Optimization:

Total Cost C

- Cost of the ith word: $C_i \simeq 1 + \log_m i$
- Cost of the ith word plus space: $C_i \simeq 1 + \log_m (i+1)$
- Subtract fixed cost: $C'_i = C_i - 1 \simeq \log_m (i+1)$
- Simplify base of logarithm:

$$C'_i \simeq \log_m (i+1) = \frac{\log_e (i+1)}{\log_e m} \propto \square\square(i+1)$$

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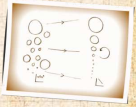


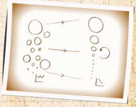
Zipfarama via Optimization:

Information Measure

- Use Shannon's Entropy (or Uncertainty):

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i$$

- (allegedly) von Neumann suggested 'entropy'...
- Proportional to average number of bits needed to encode each 'word' based on frequency of occurrence
- $-\log_2 p_i = \log_2 1/p_i$ = minimum number of bits needed to distinguish event i from all others
- If $p_i = 1/2$, need only 1 bit ($\log_2 1/p_i = 1$)
- If $p_i = 1/64$, need 6 bits ($\log_2 1/p_i = 6$)



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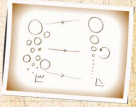

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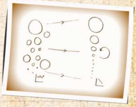

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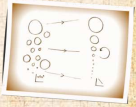

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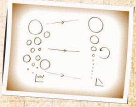

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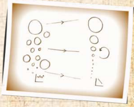

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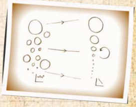
- Use a slightly simpler form:

$$H = -\sum_{i=1}^{n} p_i \log_e p_i / \log_e 2 = -g \sum_{i=1}^{n} p_i \ln p_i$$

where $g = 1/\ln 2$



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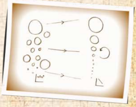


Zipfarama via Optimization:

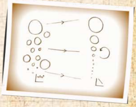
- Minimize

$$F(p_1, p_2, \ldots, p_n) = C/H$$

subject to constraint

$$\sum_{i=1}^{n} p_i = 1$$

- Tension:
(1) Shorter words are cheaper



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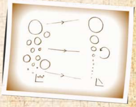

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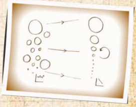
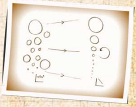
- Tension:
(1) Shorter words are cheaper
(2) Longer words are more informative (rarer)



Outline



Zipfarama via Optimization:

Time for Lagrange Multipliers:

- Minimize

$$\Psi(p_1, p_2, \dots, p_n) =$$

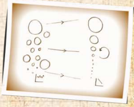
$$F(p_1, p_2, \dots, p_n) + \lambda G(p_1, p_2, \dots, p_n)$$

where

$$F(p_1, p_2, \dots, p_n) = \frac{C}{H} = \frac{\sum_{i=1}^{n} p_i \ln(i+1)}{-g \sum_{i=1}^{n} p_i \ln p_i}$$

and the constraint function is

$$G(p_1, p_2, \dots, p_n) = \sum_{i=1}^{n} p_i - 1 (= 0)$$



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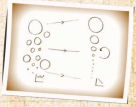

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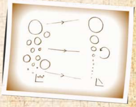
Insert question from assignment 3

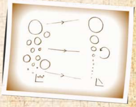


Zipfarama via Optimization:

Some mild suffering leads to:

- $$p_j = e^{-1-\lambda H^2/gC}(j+1)^{-H/gC} \propto (j+1)^{-H/gC}$$
- A power law appears [applause]: $\alpha = H/gC$
- Next: sneakily deduce λ in terms of g, C, and H.
- Find

$$p_j = (j+1)^{-H/gC}$$



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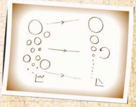

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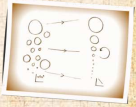

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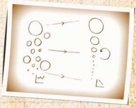

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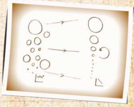


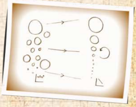
Zipfarama via Optimization:

Finding the exponent

- Now use the normalization constraint:

$$1 = \sum_{j=1}^{n} p_j = \sum_{j=1}^{n} (j+1)^{-H/gC} = \sum_{j=1}^{n} (j+1)^{-\alpha}$$

- As $n \to \infty$, we end up with $\zeta(H/gC) = 2$ where ζ is the Riemann Zeta Function
- Gives $\alpha \simeq 1.73$ (> 1, too high) or $\gamma = 1 + \frac{1}{\alpha} \simeq 1.58$ (very wild)
- If cost function changes ($j + 1 \to j + a$) then exponent is tunable
- Increase a, decrease α



The UNIVERSITY of VERMONT

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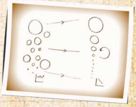

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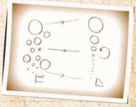

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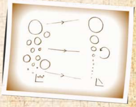

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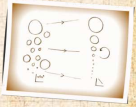

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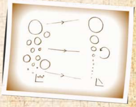

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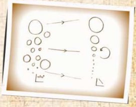
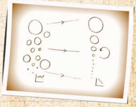


Zipfarama via Optimization:

All told:

- Reasonable approach: Optimization is at work in evolutionary processes
- But optimization can involve many incommensurate elephants: monetary cost, robustness, happiness,...
- Mandelbrot's argument is not super convincing
- Exponent depends too much on a loose definition of cost



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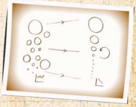

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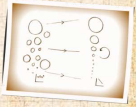

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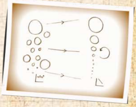
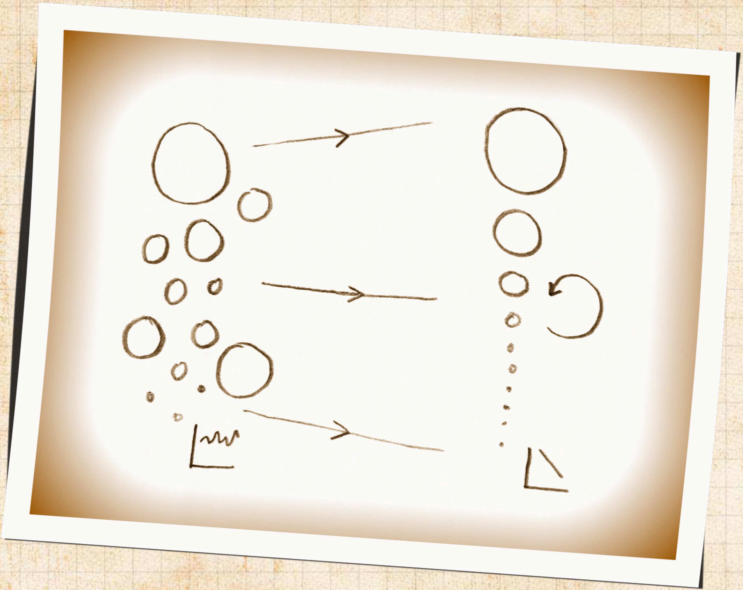
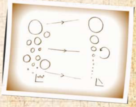




From the discussion at the end of Mandelbrot's paper:

- A. S. C. Ross: "M. Mandelbrot states that 'the actual direction of evolution (sc. of language) is, in fact, towards fuller and fuller utilization of places'. We are, in fact, completely without evidence as to the existence of any 'direction of evolution' in language, and it is axiomatic that we shall remain so. Many philologists would deny that a 'direction of evolution' could be theoretically possible; thus I myself take the view that a language develops in what is essentially a purely random manner."
- Mandelbrot: "As to the 'fundamental linguistic units being the least possible differences between pairs of utterances' this is a logical consequence of the fact that two is the least integer greater than one."



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Reconciling Mandelbrot and Simon

- Mixture of local optimization and randomness
- Numerous efforts...

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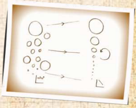

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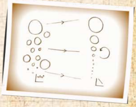

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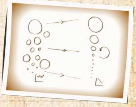

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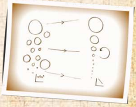

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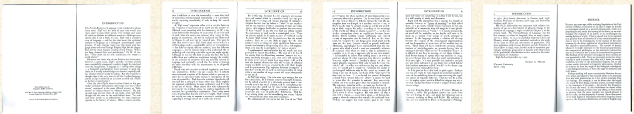

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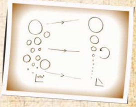

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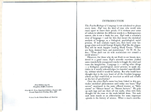
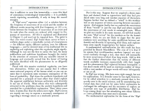
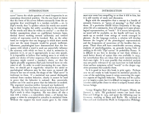
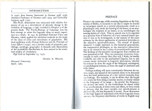

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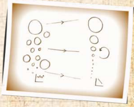

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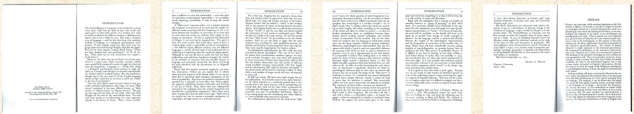

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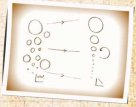

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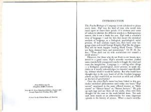
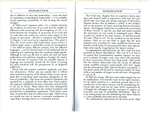
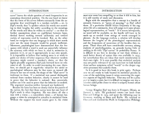
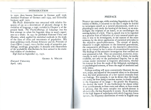

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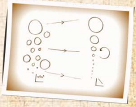

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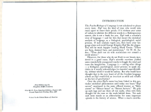
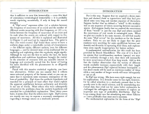
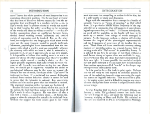
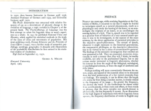

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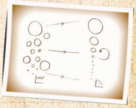
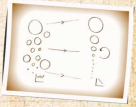
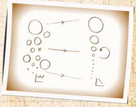


What Shannon said about meaning in his 1948 paper “A mathematical theory of communication”: [23]

The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have *meaning*; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one *selected from a set* of possible messages. The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design.





INTRODUCTION

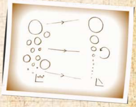
The Psycho-Biology of Language is not calculated to please every taste. Zipf was the kind of man who would take roses apart to count their petals; if it violates your sense of values to tabulate the different words in a Shakespearean sonnet, this is not a book for you. Zipf took a scientist's view of language — and for him that meant the statistical analysis of language as a biological, psychological, social process. If such analysis repels you, then leave your language alone and avoid George Kingsley Zipf like the plague. You will be much happier reading Mark Twain: "There are liars, damned liars, and statisticians." Or W. H. Auden: "Thou shalt not sit with statisticians nor commit a social science."

However, for those who do not flinch to see beauty murdered in a good cause, Zipf's scientific exertions yielded some wonderfully unexpected results to boggle the mind and tease the imagination. Language *is* — among other things — a biological, psychological, social process; to apply statistics to it merely acknowledges its essential unpredictability, without which it would be useless. But who would have thought that in the very heart of all the freedom language allows us Zipf would find an invariant as solid and reliable as the law of gravitation?

Put it this way. Suppose that we acquired a dozen monkeys and chained them to typewriters until they had produced some very long and random sequence of characters. Suppose further that we defined a "word" in this monkey-text as any sequence of letters occurring between successive spaces. And suppose finally that we counted the occurrences of these "words" in just the way Zipf and others counted the occurrences of real words in meaningful texts. When we plot our results in the same manner, we will find exactly the same "Zipf curves" for the monkeys as for the human authors. Since we are not likely to argue that the poor monkeys were searching for some equilibrium between uniformity and diversity in expressing their ideas, such explanations seem equally inappropriate for human authors.

A mathematical rationalization for this result has been provided by Benoit Mandelbrot. The crux of it is that if we assume that word-boundary markers (spaces) are scattered randomly through a text, then there will necessarily be more occurrences of short than long words. Add to this fact the further observation that the variety of different words available increases exponentially with their length and the phenomenon Zipf reported becomes inescapable: a few short words will be used an enormous number of times while a vast number of longer words will occur infrequently or not at all.

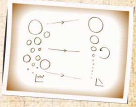
So Zipf was wrong. His facts were right enough, but not his explanations. In a broader sense he was right, however, for he called attention to a stochastic process that is frequently seen in the social sciences, and by accumulating statistical data that cried out for some better explanation he challenged his colleagues and his successors to explore an important new type of probability distribution. Zipf belongs among those rare but stimulating men whose failures are more profitable than most men's successes.



So who's right?

Bornholdt and Ebel (PRE), 2001:
"World Wide Web scaling exponent from Simon's 1955 model"[4].

- Show Simon's model fares well.
- Recall ρ = probability new flavor appears.
- Alta Vista crawls in approximately 6 month period in 1999 give $\rho \simeq 0.10$
- Leads to $\gamma = 1 + \frac{1}{1-\rho} \simeq 2.1$ for in-link distribution.
- Cite direct measurement of γ at the time: 2.1 ± 0.1 and 2.09 in two studies.



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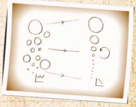

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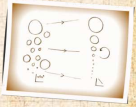

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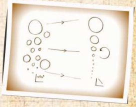

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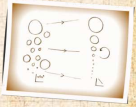
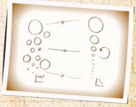


Outline



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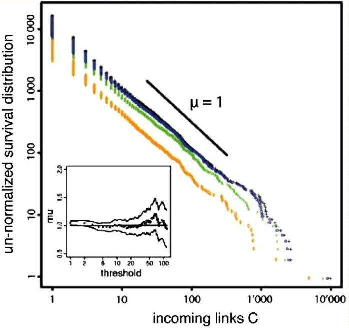
Recent evidence for Zipf's law...



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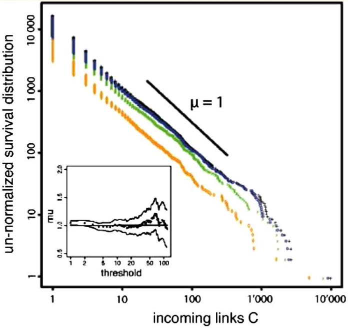

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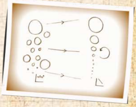

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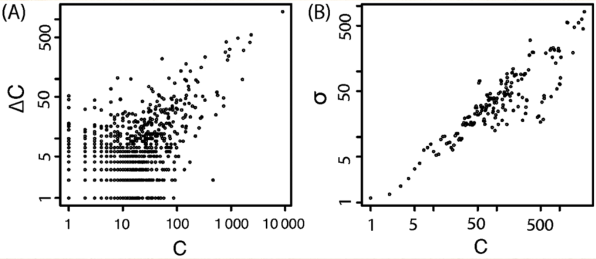
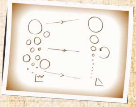


FIG. 2. Left panel: Plots of ΔC versus C from the Etch release (15.08.2007) to the latest Lenny version (05.05.2008) in double logarithmic scale. Only positive values are displayed. The linear regression $\Delta C = R \times C + C_0$ is significant at the 95% confidence level, with a small value $C_0 = 0.3$ at the origin and $R = 0.09$. Right panel: same as left panel for the standard deviation of ΔC.

- Rough, approximately linear relationship between C number of in-links and ΔC.



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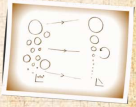

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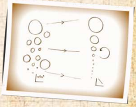
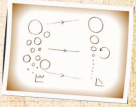


The University of Vermont

So who's right?

Nutshell:

- Simonish random 'rich-get-richer' models agree in detail with empirical observations.
- Power-lawfulness: Mandelbrot's optimality is still apparent.
- Optimality arises for free in Random Competitive Replication models.



Neural reboot (NR):

Walking with a baby robin:

https://www.youtube.com/v/CxiDTwvsLbA?rel=0

Others are also not happy:

Krugman and Simon

- "The Self-Organizing Economy" (Paul Krugman, 1996) [14]
- Krugman touts Zipf's law for cities, Simon's model
- "Déjà vu, Mr. Krugman" (Berry, 1999)
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Who needs a hug?

From Berry [3]

- Déjà vu, Mr. Krugman. Been there, done that. The Simon-Ijiri model was introduced to geographers in 1958 as an explanation of city size distributions, the first of many such contributions dealing with the steady states of random growth processes, ...
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