

Properties of Complex Networks

Principles of Complex Systems | @pocsvox

CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

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Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont



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Properties of
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Sealie & Lambie
Productions



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Degree distributions
Assortativity
Clustering
Motifs
Concurrency
Branching ratios
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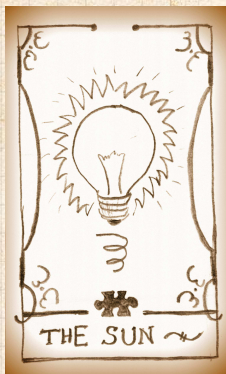
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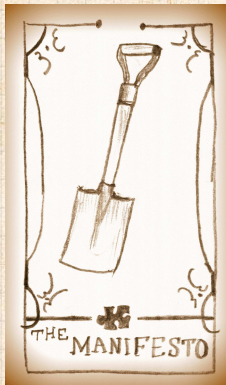
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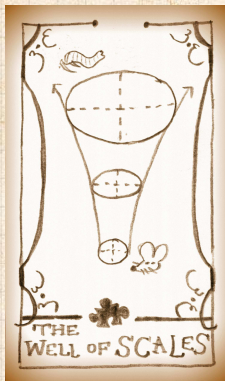
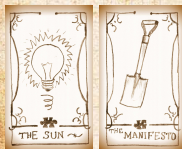
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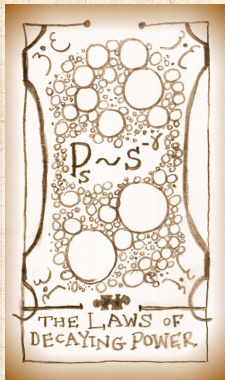
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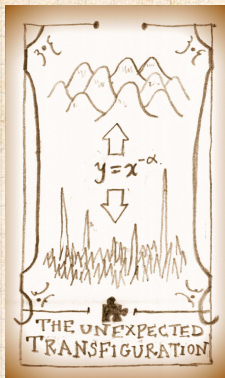
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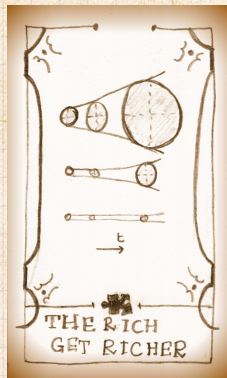
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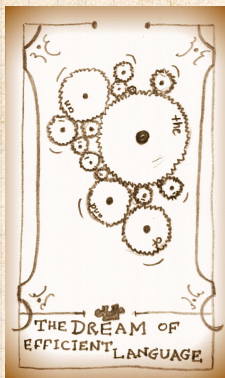
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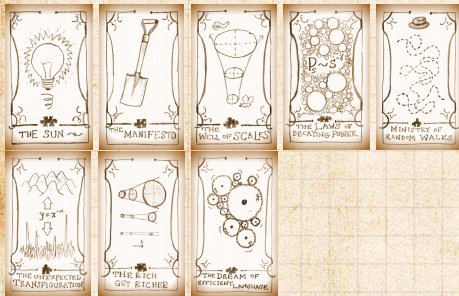
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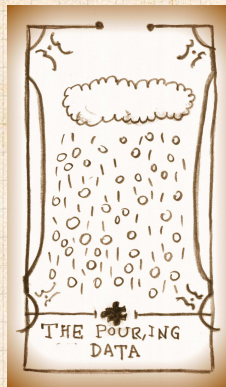


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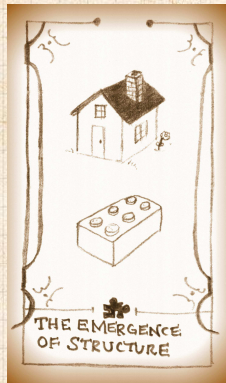
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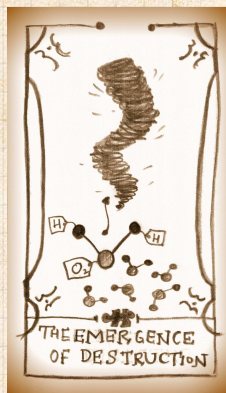
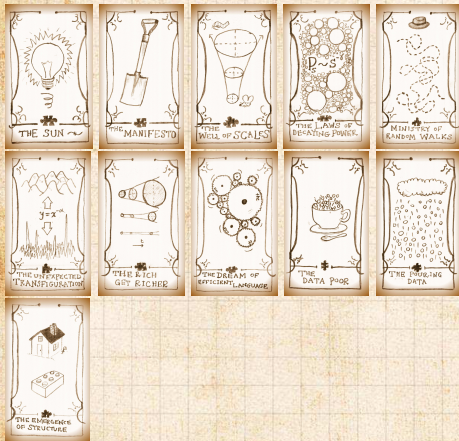
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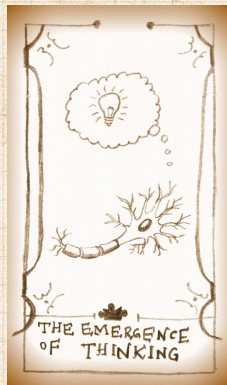
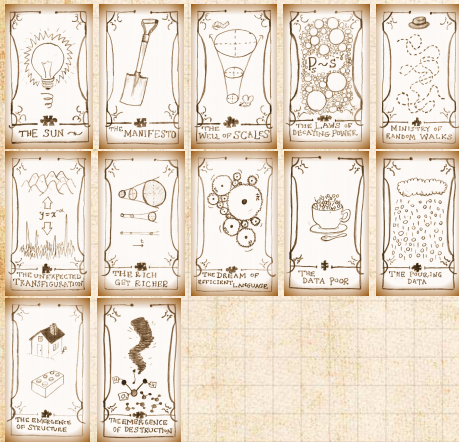
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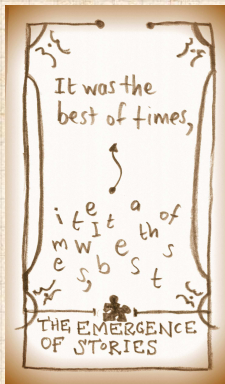
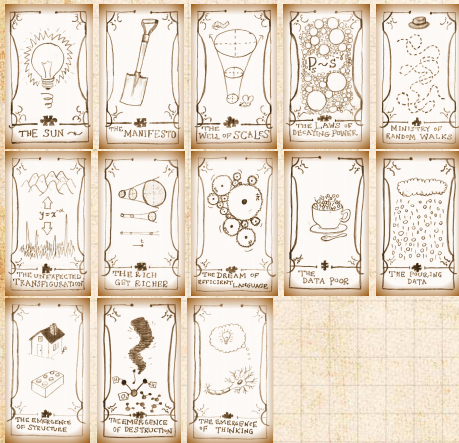
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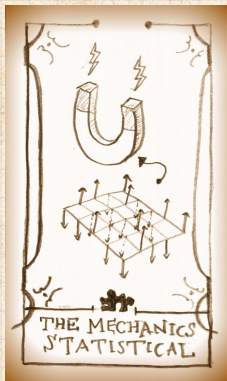
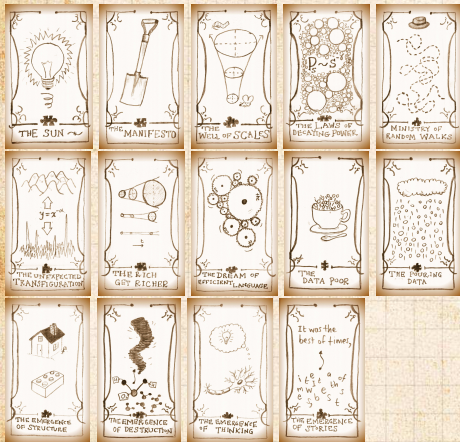
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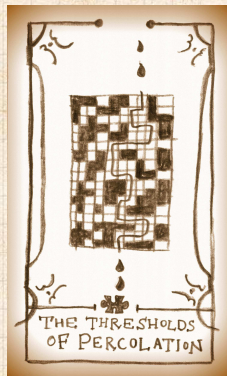
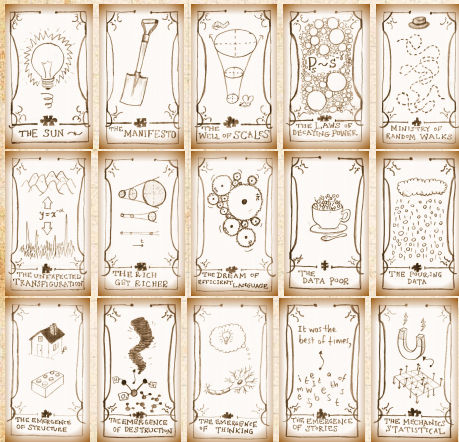
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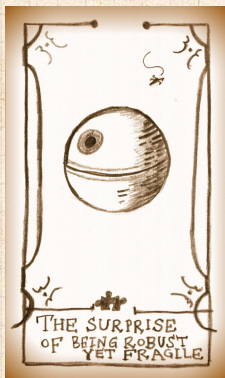
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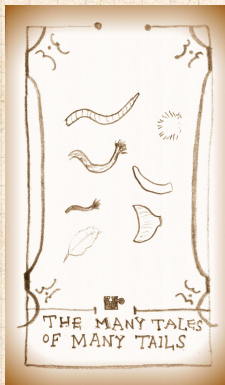
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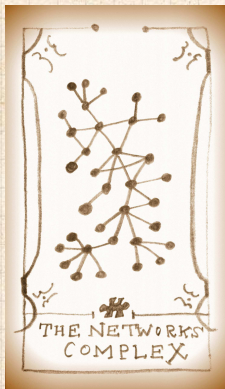
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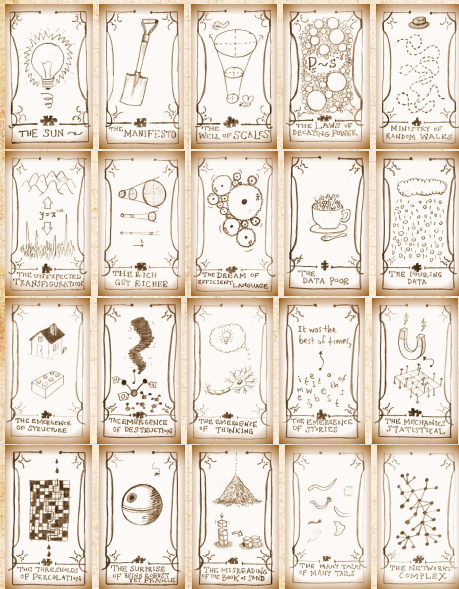
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A notable feature of large-scale networks:

Graphical renderings are often just a big mess.

And even when renderings somehow look good:

We need to extract **digestible, meaningful aspects**.

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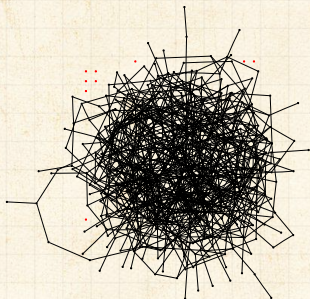
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A notable feature of large-scale networks:

 Graphical renderings are often just a big mess.



⇐ Typical hairball

-  number of nodes $N = 500$
-  number of edges $m = 1000$
-  average degree $\langle k \rangle = 4$

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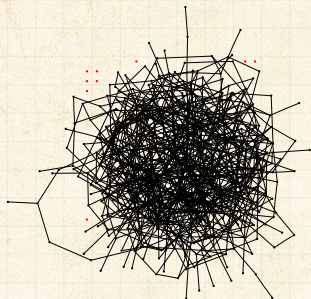
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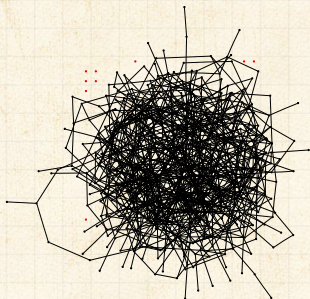
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said Ponder [Stibbons] —*Making Money*, T. Pratchett.

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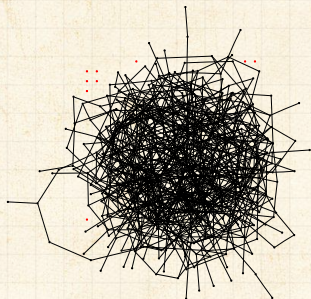
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Some key aspects of real complex networks:

-  degree distribution*
-  assortativity
-  homophily
-  clustering
-  motifs
-  modularity
-  concurrency
-  hierarchical scaling
-  network distances
-  centrality
-  efficiency
-  interconnectedness
-  robustness

 Plus coevolution of network structure and processes on networks.

- * Degree distribution is the elephant in the room that we are now all very aware of...

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1. degree distribution P_k

- P_k is the probability that a randomly selected node has degree k .
- k = node degree = number of connections.
- ex 1: Erdős-Rényi random networks have Poisson degree distributions:
insert question from assignment 7 ↗


$$P_k = e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$$

- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'.
- link cost controls skew.
- hubs may facilitate or impede contagion.



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Note:

🧩 Erdős-Rényi random networks are a *mathematical construct*.

🧩 'Scale-free' networks are *growing networks* that form according to a *plausible mechanism*.

🧩 Randomness is out there, just not to the degree of a completely random network.

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2. Assortativity/3. Homophily:

 Social networks: Homophily  = birds of a feather

 e.g., degree is standard property for sorting:
measure degree-degree correlations.



 **Assortative** network: ^[5] similar degree nodes
connecting to each other.


 **Disassortative** network: high degree nodes
connecting to low degree nodes.

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*Often technological or biological: Internet, WWW,
protein interactions, neural networks, food webs.*

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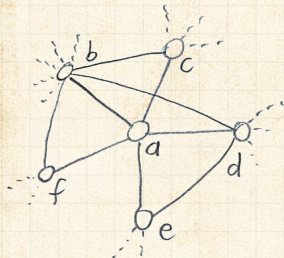
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Local socialness:

4. Clustering:

- ☞ Your friends tend to know each other.
- ☞ Two measures (explained on following slides):



2. Newman [6]

Clustering
#triples

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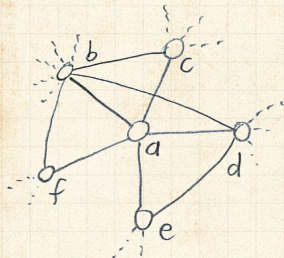
4. Clustering:



Your friends tend to know each other.



Two measures (explained on following slides):



1. Newman [6]

2. Clauset & Shalizi
#triples

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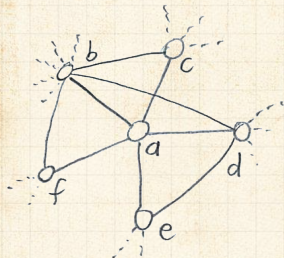
Two measures (explained on following slides):

1. Watts & Strogatz [8]

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

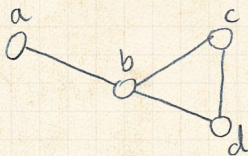
2. Newman [6]

$$C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$$

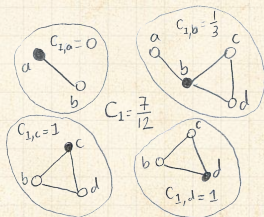




Example network:



Calculation of C_1 :



C_1 is the average fraction of pairs of neighbors who are connected.

Fraction of pairs of neighbors who are connected is

$$\frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$

where k_i is node i 's degree, and N_i is the set of i 's neighbors.

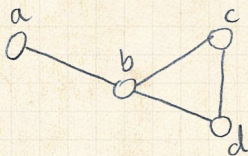
Averaging over all nodes, we have:

$$C_1 = \frac{1}{n} \sum_{i=1}^n \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2}$$



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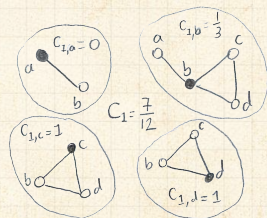
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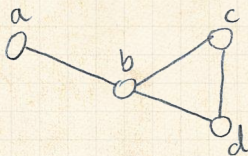


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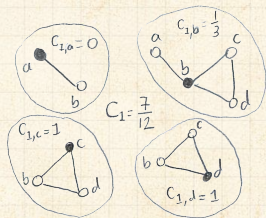
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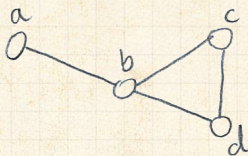
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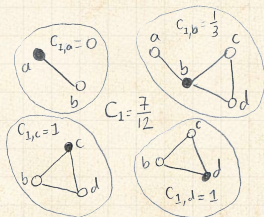
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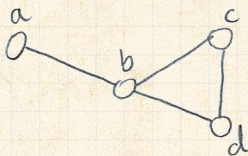
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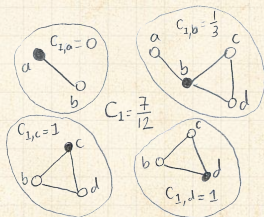
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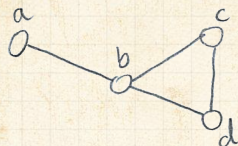


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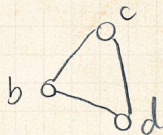
$$C_1 = \frac{1}{n} \sum_{i=1}^n \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} = \left\langle \frac{\sum_{j_1 j_2 \in N_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i$$

Triples and triangles

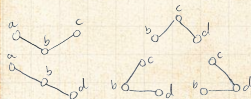
Example network:





Triangles:





Triples:



 Nodes i_1 , i_2 , and i_3 form a **triple** around i_1 if i_1 is connected to i_2 and i_3 .

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 The definition $C_2 = \frac{3 \times \# \text{triangles}}{\# \text{triples}}$ measures the fraction of **closed triples**

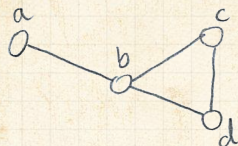
 The '3' appears because for each triangle, we have 3 closed triples.

 Social Network Analysis (SNA): fraction of **transitive triples**.

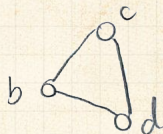


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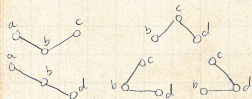
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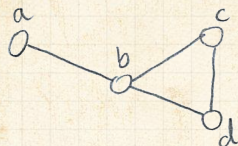
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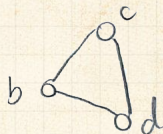


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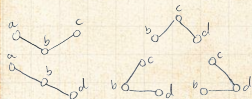
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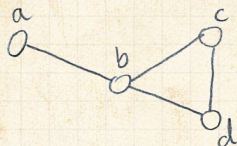
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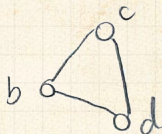


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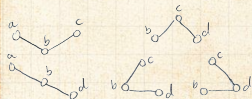
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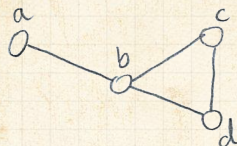
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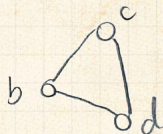


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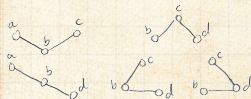
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Clustering:

Sneaky counting for undirected, unweighted networks:

• If the path $i-j-l$ exists then $a_{ij}a_{jl} = 1$.

• Otherwise, $a_{ij}a_{jl} = 0$.

• We want $i \neq l$ for good triples.

• In general, a path of n edges between nodes i_1 and i_n travelling through nodes i_2, i_3, \dots, i_{n-1} exists

$$\iff a_{i_1 i_2} a_{i_2 i_3} a_{i_3 i_4} \cdots a_{i_{n-2} i_{n-1}} a_{i_{n-1} i_n} = 1.$$

$$\# \text{triples} = \frac{1}{2} \left(\sum_{i=1}^N \sum_{\ell=1}^N [A^2]_{i\ell} - \text{Tr} A^2 \right)$$

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 For sparse networks, C_1 tends to discount highly connected nodes.

 C_2 is a useful and often preferred variant

 In general, $C_1 \neq C_2$.

 C_1 is a global average of a local ratio.

 C_2 is a ratio of two global quantities.

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5. motifs:

- small, recurring functional subnetworks
- e.g., Feed Forward Loop:

Shen-Orr, Uri Alon, *et al.* ^[74]

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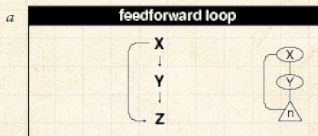
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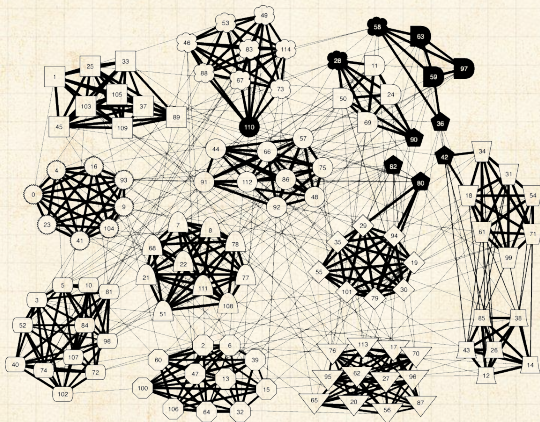
- A problem
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6. modularity and structure/community detection:



Clauset *et al.*, 2006 ^[2]: NCAA football

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- transmission of a contagious element only occurs during contact
- rather obvious but easily missed in a simple model
- dynamic property—static networks are not enough
- knowledge of previous contacts crucial
- beware cumulated network data
- Kretzschmar and Morris, 1996 [4]
- “Temporal networks” become a concrete area of study for Piranha Physicists in 2013.

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8. Horton-Strahler ratios:



Metrics for branching networks:

- Method for ordering streams hierarchically
- Number: $R_n = N_\omega / N_{\omega+1}$
- Segment length: $R_l = \langle l_{\omega+1} \rangle / \langle l_\omega \rangle$
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9. network distances:

(a) shortest path length d_{ij} :

(b) average path length $\langle d_{ij} \rangle$:

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(a) shortest path length d_{ij} :


- 1 Fewest number of steps between nodes i and j .
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network diameter d_{\max} :

Maximum shortest path length between any two nodes.



closeness $d_{cl} = [\sum_{i,j} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:

Average 'distance' between any two nodes.



Closeness handles disconnected networks
($d_{ij} = \infty$)



$d_{cl} = \infty$ only when all nodes are isolated.



Closeness perhaps compresses too much into one number



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10. centrality:

Many such measures of a node's 'importance.'

ex 1: Degree centrality: k_i .

ex 2: Node i 's betweenness
= fraction of shortest paths that pass through i .

ex 3: Edge ℓ 's betweenness
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ex 4: Recursive centrality: Hubs and Authorities
(Jon Kleinberg ^[3])

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Interconnected networks and robustness (two for one deal):

“Catastrophic cascade of failures in interdependent networks” [1]. Buldyrev et al., Nature 2010.

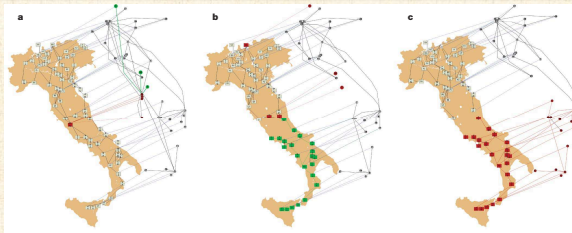


Figure 1 | Modelling a blackout in Italy. Illustration of an iterative process of a cascade of failures using real-world data from a power network (located on the map of Italy) and an Internet network (shifted above the map) that were implicated in an electrical blackout that occurred in Italy in September 2003³⁹. The networks are drawn using the real geographical locations and every Internet server is connected to the geographically nearest power station. **a.** One power station is removed (red node on map) from the power network and as a result the Internet nodes depending on it are removed from the Internet network (red nodes above the map). The nodes that will be disconnected from the giant cluster (a cluster that spans the entire network)

at the next step are marked in green. **b.** Additional nodes that were disconnected from the Internet communication network giant component are removed (red nodes above map). As a result the power stations depending on them are removed from the power network (red nodes on map). Again, the nodes that will be disconnected from the giant cluster at the next step are marked in green. **c.** Additional nodes that were disconnected from the giant component of the power network are removed (red nodes on map) as well as the nodes in the Internet network that depend on them (red nodes above map).

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Overview Key Points:

- 🧩 The field of complex networks came into existence in the late 1990s.
- 🧩 Explosion of papers and interest since 1998/99.
- 🧩 Hardened up much thinking about complex systems.
- 🧩 Specific focus on networks that are **large-scale**, sparse, **natural** or **man-made**, evolving and dynamic, and (crucially) **measurable**.
- 🧩 Three main (blurred) categories:
 1. **Physical** (e.g., river networks),
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