# **Fundamentals**

Principles of Complex Systems | @pocsvox CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

#### Prof. Peter Dodds | @peterdodds

# Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont



These slides are brought to you by:

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# Outline

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Exponential growth:  $\sim$  60% per year.

2005 06 07 08 09 10 11

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Data, Data, Everywhere-the Economist, Feb 25, 2010

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2,000

1,750 1,500

1,250

1,000 750

500 lable storage 250

ed and available storage

FORECAST

1965

Internet estimate to reach 2/3 Zettabytes  $(1ZB = 10^{3}EB = 10^{6}PB =$ 10<sup>9</sup>TB)

TB/second.

🗞 2016—Large Synoptic Survey Telescope: 140 TB every 5 days.

photos (mid 2013)  $\clubsuit$  Twitter:  $\sim$  500 billion









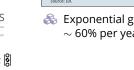
# **Big Data Science:**

🗞 2013: year traffic on

🗞 Large Hadron Collider: 40

 $\clubsuit$  Facebook:  $\sim$  250 billion

tweets (mid 2013)



Overload

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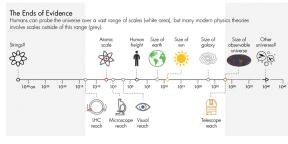
References

# No really, that's a lot of data

| Unit           | Size                              | What it means  |
|----------------|-----------------------------------|--|
| Bit (b)        | 1 or 0                            | Short for "binary digit", after the binary code (1 or 0)<br>computers use to store and process data                              |
| Byte (B)       | 8 bits                            | Enough information to create an English letter or number<br>in computer code. It is the basic unit of computing                  |
| Kilobyte (KB)  | 1,000, or 2 <sup>10</sup> , bytes | From "thousand" in Greek. One page of typed text is 2KB  |
| Megabyte (MB)  | 1,000KB; 2 <sup>20</sup> bytes    | From "large" in Greek. The complete works of Shakespeare total 5MB.<br>A typical pop song is about 4MB                           |
| Gigabyte (GB)  | 1,000MB; 2 <sup>30</sup> bytes    | From "giant" in Greek. A two-hour film can be compressed into 1-2GB  |
| Terabyte (TB)  | 1,000GB; 2 <sup>40</sup> bytes    | From "monster" in Greek. All the catalogued books<br>in America's Library of Congress total 15TB                                 |
| Petabyte (PB)  | 1,000TB; 2 <sup>50</sup> bytes    | All letters delivered by America's postal service this year will amount<br>to around 5PB. Google processes around 1PB every hour |
| Exabyte (EB)   | 1,000PB; 2 <sup>60</sup> bytes    | Equivalent to 10 billion copies of The Economist   |
| Zettabyte (ZB) | 1,000EB; 2 <sup>70</sup> bytes    | The total amount of information in existence<br>this year is forecast to be around 1.2ZB   |
| Yottabyte (YB) | 1,000ZB; 2 <sup>80</sup> bytes    | Currently too big to imagine   |

# Limits of testability and happiness in Science:

From A Fight for the soul of Science I in Quanta Magazine (2016/02):



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# **Big Data—Culturomics:**

#### "Quantitative analysis of culture using millions of digitized books" by Michel et al., Science, 2011<sup>[6]</sup>



🗞 http://www.culturomics.org/ 🗹 and Google Books ngram viewer 🗹

- Silan

### Barney Rubble:

| - |  |  |  |  |  |  |
|---|--|--|--|--|--|--|
|   |  |  |  |  |  |  |

"Characterizing the Google Books corpus: Strong limits to inferences of socio-cultural and linguistic evolution" Pechenick, Danforth, and Dodds, PLoS ONE, 10, e0137041, 2015. [7]



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Basic Science  $\simeq$  Describe + Explain:



### Lord Kelvin (possibly):

🚳 "To measure is to know." 🚳 "lf you cannot measure it, you cannot improve it."

#### Bonus:

- 🚳 "X-rays will prove to be a hoax."
- line and the set of th discovered in physics now, All that remains is more and more precise measurement."



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# The Newness of being a Scientist (1833 on):

Google books Ngram Viewer



## 🗞 Etymology here 🗹.

\delta "Scientists are the people who ask a question about a phenomenon and proceed to systematically go about answering the question themselves. They are by nature curious, creative and well organized."



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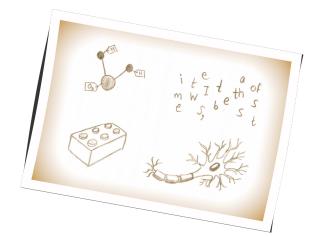
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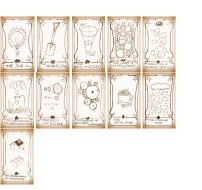
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#### Data Emergence







# Emergence

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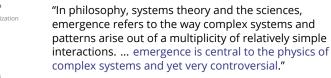
emergence is a process whereby larger entities arise through interactions among smaller or simpler entities such that the larger entities exhibit properties the smaller/simpler entities do not exhibit.

Wikipedia, 2016:

The philosopher G. H. Lewes first used the word explicity in 1875.

In philosophy, systems theory, science, and art,

The Wikipedia on Emergence (2006):



#### **Emergence:**

Tornadoes, financial collapses, human emotion aren't found in water molecules, dollar bills, or carbon atoms.

#### Examples:

- $\clubsuit$  Fundamental particles  $\Rightarrow$  Life, the Universe, and Everything
- & Genes  $\Rightarrow$  Organisms
- $\clubsuit$  Neurons etc.  $\Rightarrow$  Brain  $\Rightarrow$  Thoughts
- $Rightarrow People \Rightarrow Religion, Collective behaviour$
- $\clubsuit$  People  $\Rightarrow$  The Web
- $\mathfrak{F}$  People  $\Rightarrow$  Language, and rules of language
- $\mathfrak{F}$  ?  $\Rightarrow$  time; ?  $\Rightarrow$  gravity; ?  $\Rightarrow$  reality.

"The whole is more than the sum of its parts" -Aristotle

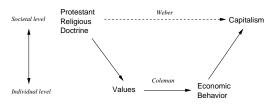
# **Emergence:**

#### Friedrich Hayek (Economist/Philospher/Nobelist):

- legal systems, political systems are emergent and not designed.
- 🚓 'Taxis' = made order (by God, Sovereign, Government, ...)
- 🚳 'Cosmos' = grown order
- Archetypal limits of hierarchical and decentralized structures.
- Hierarchies arise once problems are solved. [4]
- Decentralized structures help solve problems.
- lacktrian state and the second state of the se

# **Emergence:**

# James Coleman C in Foundations of Social Theory:



- 🚳 Understand macrophenomena arises from microbehavior which in turn depends on macrophenomena.<sup>[3]</sup>
- 🚳 More on Coleman here 🗹.

# Emergence:

## Thomas Schelling C (Economist/Nobelist):



🚳 "Micromotives and Macrobehavior"<sup>[10]</sup> Segregation<sup>[8, 11]</sup> Wearing hockey helmets<sup>[9]</sup> Seating choices

# The emergence of taste:

## $\clubsuit$ Molecules $\Rightarrow$ Ingredients $\Rightarrow$ Taste





# Reductionism

#### Reductionism and food:

- Pollan: "even the simplest food is a hopelessly complex thing to study, a virtual wilderness of chemical compounds, many of which exist in complex and dynamic relation to one another ... "
- 🗞 "So ... break the thing down into its component parts and study those one by one, even if that means ignoring complex interactions and contexts, as well as the fact that the whole may be more than, or just different from, the sum of its parts. This is what we mean by reductionist science."

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Vi Hart and

Nicky Case's

Polygon-

themed visualization 🗷:



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nytimes.com







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# Reductionism

- line and "people don't eat nutrients, they eat foods, and foods can behave very differently than the nutrients they contain."
- Studies suggest diets high in fruits and vegetables help prevent cancer.
- 🗞 So... find the nutrients responsible and eat more of them
- 🗞 But "in the case of beta carotene ingested as a supplement, scientists have discovered that it actually increases the risk of certain cancers. Oops."

| Modeling                 |
|--------------------------|
| Statistical<br>Mechanics |
| Nutshell                 |
| References               |
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[cnn.com]

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#### Thyme's known antioxidants:

4-Terpineol, alanine, anethole, apigenin, ascorbic acid, beta carotene, caffeic acid, camphene, carvacrol, chlorogenic acid, chrysoeriol, eriodictyol, eugenol, ferulic acid, gallic acid, gamma-terpinene isochlorogenic acid, isoeugenol, isothymonin, kaempferol, labiatic acid, lauric acid, linalyl acetate, luteolin, methionine, myrcene, myristic acid, naringenin, oleanolic acid, p-coumoric acid, p-hydroxy-benzoic acid, palmitic acid, rosmarinic acid, selenium, tannin, thymol, tryptophan, ursolic acid, vanillic acid.

## Reductionism

"It would be great to know how this all works, but in the meantime we can enjoy thyme in the knowledge that it probably doesn't do any harm (since people have been eating it forever) and that it may actually do some good (since people have been eating it forever) and that even if it does nothing, we like the way it tastes."

Gulf between theory and practice (see baseball and bumblebees).

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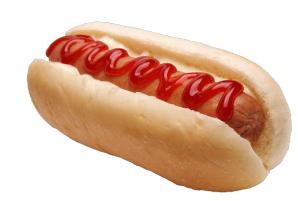
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# This is a Collateralized Debt Obligation:



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🗞 From "The Speed of Darkness" (1968) by Muriel Rukeyser 🗹

\*The Universe is made of stories, not of atoms.

🚳 Quoted by Metatron in Supernatural, Meta Fiction, S9F18.

🍪 "The most common element on

the disc, although not included in

the list of the standard five: earth,

fire, air, water and surprise. It

ensures that everything runs

little narrativium goes a long

way: the simpler the story, the better you understand it.

Storytelling is the opposite of

reductionism: 26 letters and some rules of grammar are no

properly as a story."

story at all."



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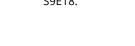
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(Sir Terry) Pratchett's 🖸 Narrativium 🖸:











## **Emergence:**

#### Higher complexity:

- Many system scales (or levels) that interact with each other.
- Potentially much harder to explain/understand.

#### Even mathematics: <sup>[5]</sup>



Gödel's Theorem **∠**: we can't prove every theorem that's true ...

Suggests a strong form of emergence: Some phenomena cannot be analytically deduced from elementary aspects of a system.

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# **Emergence:**

Roughly speaking, there are two types of emergence:

#### I. Weak emergence:

System-level phenomena is different from that of its constituent parts yet can be connected theoretically.

#### II. Strong emergence:

System-level phenomena fundamentally cannot be deduced from how parts interact.





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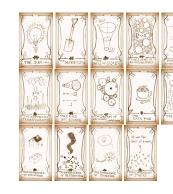
🗞 Reductionist techniques can explain weak emergence.

- & Magic explains strong emergence.<sup>[2]</sup>
- But: maybe magic should be interpreted as an inscrutable yet real mechanism that cannot ever be simply described.
- 🚳 Gulp.

**Emergence:** 













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Definitions

"Self-organization 🗷 is a process in which the internal organization of a system, normally an open system, increases in complexity without being guided or managed by an outside source." (also: Self-assembly)

 $\clubsuit$  Molecules/Atoms liking each other  $\rightarrow$ Gases, liquids, and solids.

- $\clubsuit$  Spin alignment  $\rightarrow$  Magnetization.
- 🗞 Protein folding.
- $\ll$  Imitation  $\rightarrow$  Herding, flocking, mobs, ...

Fundamental question: how likely is 'complexification'?



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Listen to Steve Strogatz, Hod Lipson, and Michael Schmidt (Cornell) in the last piece C (11:16) on Radiolab's show 'Limits' 🕝 (April 5, 2010).

Limits of Science | Radiolab



Dr. Steve Strogatz wonders if we've reached the limits of human scientific understanding, and should soon turn the reins of research over to robots. Cold, calculating robots. Then, Dr. Hod Lipson and Michael Schmidt walk us through the workings of a revolutionary computer program that they developed -- a program that can deduce mathematical relationships in nature, through simple observation. The catch? As Dr. Gurol Suel explains, the program gives answers to complex biological questions that we humans have yet to ask,

TAGS: mind bending



# Pair with some slow tv 🖸



or even to understand.



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Examples:

#### Tools and techniques:

- linear Differential equations, difference equations, linear algebra, stochastic models.
- line and statistical techniques for comparisons and descriptions.
- 🗞 Methods from statistical mechanics and computer science.
- \lambda Machine learning (but beware the black box).
- 🗞 Computer modeling, everything from Artisanal toy models
  - to kitchen sink models.

### Key advance (more soon):

- Representation of complex interaction patterns as complex networks.
- The driver: Massive amounts of Data

# Rather silly but great example of real science:

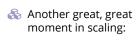
"How Cats Lap: Water Uptake by Felis catus" Reis et al., Science, 2010.





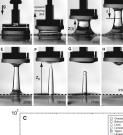


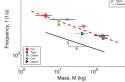
Amusing interview here



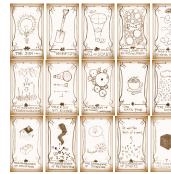
 $f\sim M^{-1/6}$ 

The balance of inertia and gravity yields a prediction for the lapping frequency of other feines. Assuming isometry within the Felidae family (i.e., that lapping height *H* scales inerdy with longe width *R* and animal mass *M* scales as *R'*), the finding that  $\Gamma^{ab}_{c}$  is of order one translates to the prediction  $f - R^{-2} - M^{-1}$ . Summerly or marginally positive allometry wanong the Felidae has been demonstrated for skull (*Q*, 21) and linh boses (22). Althoogh variability by function can lead to departures from isometry in interspecific scalings (23), reported variations within the Felidae (23, 24) only minimally affect the predicted scaling  $f - M^{-16}$ . We tested this -16 power-law dependence by measuring the lapping frequency for eight species of felines, from videos equirited at the 250 wer England on variable on Var Tube (16). The lapping frequency was observed to decrease with animal mass sa  $f - 46 M = ^{411+100} (16^{-1}, M$  in kg) (Fig. 4C), close to the predicted  $M^{-16}$ . This close agreement suggests that the domestic car's incritis- and gravity-controlled lapping mechanism is conserved mong felines. lapping mechanism is conserved among felines.

















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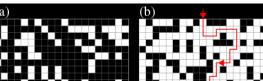
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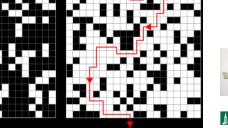
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Snared from Michael Gastner's page on percolation [no longer online]



















Statistical References















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- Statistical Mechanics is "a science of collective behavior.'
- Simple rules give rise to collective phenomena.

# Percolation:

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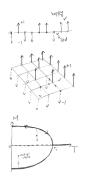
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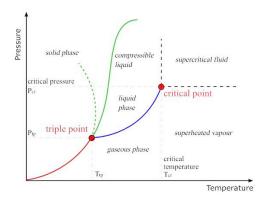


- 🚳 Each atom is assumed to have a local spin that can be up or down:  $S_i = \pm 1.$
- Spins are assumed to be arranged on a lattice.
- ln isolation, spins like to align with each other.
- Increasing temperature breaks these alignments.
- 🗞 The drosophila 🗹 of statistical mechanics.
- 🗞 Criticality: Power-law distributions at critical points.

# Example 2-d Ising model simulation:

http://dtjohnson.net/projects/ising

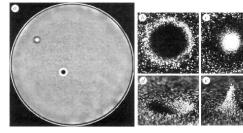
# Phase diagrams



Qualitatively distinct macro states.

# Phase diagrams

Oscillons, bacteria, traffic, snowflakes, ...



Umbanhowar et al., Nature, 1996<sup>[12]</sup>



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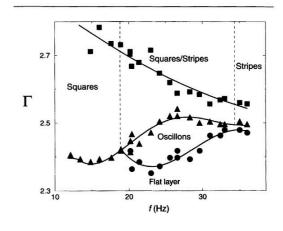
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# Phase diagrams

| S <sub>0</sub> |              |               |   |          |   |          |            |      |          |        |
|----------------|--------------|---------------|---|----------|---|----------|------------|------|----------|--------|
| 1.2            | $\bullet$    | (6)           |   |          |   | $\odot$  | t =<br>◀   | 400  | $\odot$  |        |
| 1.0            |              |               |   |          |   |          | t =<br>◀   | 400  | <b>G</b> |        |
| 0.8            | ۲            | ۲             |   | (e)      |   |          | t =<br>◀   | 400  |          |        |
| 0.6            | *            | ۲             |   |          |   |          | t =<br>◀   | 600  |          |        |
| 0.4            | t − 400<br>• | *             |   | *        |   |          | t = ]<br>← | 1000 | *        |        |
| 0.2            | t = 1000     | t = 2000<br>• |   | t - 3000 |   | t = 3840 |            |      | t = 2000 |        |
| +              | 2            | 3             | 4 | 5        | 6 | 7        | 8          | 9    | 10       | •<br>W |
|                |              |               |   |          |   |          |            |      |          |        |

 $W_0$  = initial wetness,  $S_0$  = initial nutrient supply http://math.arizona.edu/~lega/HydroBact.html

# Ising model

### Analytic issues:

- 1-d: simple (Ising & Lenz, 1925)
- 🗞 2-d: hard (Onsager, 1944)
- 🗞 3-d: extremely hard...
- 🗞 4-d and up: simple.



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# **Statistics**

#### Historical surprise:

- Origins of Statistical Mechanics are in the studies of people... (Maxwell and co.)
- Now physicists are using their techniques to study everything else including people...
- A See Philip Ball's "Critical Mass" [1]

#### **Beyond Statistical Mechanics:**

- limits, especially in Analytic approaches have their limits, especially in evolutionary, algorithm-rich systems.
- Algorithmic methods and simulation techniques will continue to rise in importance.

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# Nutshell

- The central concepts Complexity and Emergence are reasonably well defined.
- There is no general theory of Complex Systems.
- 🚳 But the problems exist... Complex (Adaptive) Systems abound...
- And the observation of Universality C of dynamical systems, statistical mechanics, and other quantitative areas means not everything is special and different.
- 🗞 Framing from the Manifesto: Science's focus is moving to Complex Systems because it finally can.
- 🛞 We use whatever tools we need.
- Science ~ Describe + Explain.

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