

Overview of Complex Networks

Principles of Complex Systems | @pocsvox
 CSYS/MATH 300, Fall, 2016 | #FallPoCS2016

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 Vermont Advanced Computing Core | University of Vermont



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 Overview of Complex Networks

Complex Networks Basics

Etymology
 Popularity
 Graph theory?
 Basic definitions

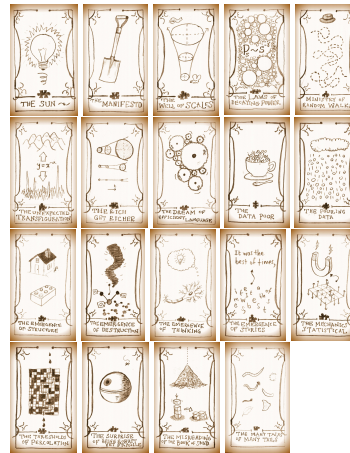
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net·work |'netwɜ:k|

noun

- an arrangement of intersecting horizontal and vertical lines.
 - a complex system of roads, railroads, or other transportation routes : *a network of railroads.*
- a group or system of interconnected people or things : *a trade network.*
 - a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
 - a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
 - a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
 - a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

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Outline

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Thesaurus deliciousness:

network

noun

- a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- a network of lanes* MAZE, labyrinth, warren, tangle.
- a network of friends* SYSTEM, complex, nexus, web, webwork.

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Ancestry:

From Keith Briggs's excellent [etymological investigation](#):

- Opus reticulatum:
- A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network/>]

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Key Observation:

- Many complex systems can be viewed as complex networks of physical or abstract interactions.
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mech flavor.
- Mindboggling amount of work published on complex networks since 1998 ...
- ...largely due to your typical theoretical physicist:



- Piranha physicus
- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)
- See also: <https://xkcd.com/793/>

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Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- 1658-: reticulate structures in animals
- 1839-: rivers and canals
- 1869-: railways
- 1883-: distribution network of electrical cables
- 1914-: wireless broadcasting networks

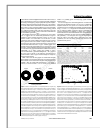
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Popularity (according to Google Scholar)



"Collective dynamics of 'small-world' networks"
 Watts and Strogatz,
 Nature, **393**, 440-442, 1998. [14]

Times cited: ~ 27,184 (as of October 8, 2015)



"Emergence of scaling in random networks"
 Barabási and Albert,
 Science, **286**, 509-511, 1999. [2]

Times cited: ~ 23,532 (as of October 8, 2015)

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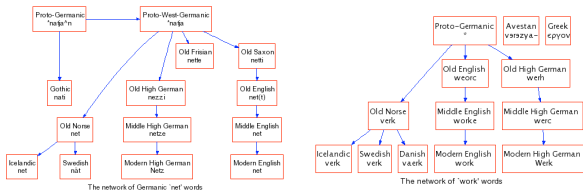


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Ancestry:

Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appear to have long meant purposeful action.



- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

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Review articles:



"Complex Networks: Structure and Dynamics"
 Boccaletti et al.,
 Physics Reports, **424**, 175-308, 2006. [3]

Times cited: ~ 5,791 (as of October 8, 2015)



"The structure and function of complex networks"
 M. E. J. Newman,
 SIAM Rev., **45**, 167-256, 2003. [10]

Times cited: ~ 13,156 (as of October 8, 2015)



"Statistical mechanics of complex networks"
 Albert and Barabási,
 Rev. Mod. Phys., **74**, 47-97, 2002. [1]

Times cited: ~ 15,676 (as of October 8, 2015)

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Popularity according to textbooks:

Textbooks:

- 🔗 Mark Newman (Physics, Michigan)
"Networks: An Introduction" [↗](#)
- 🔗 David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
"Networks, Crowds, and Markets: Reasoning About a Highly Connected World" [↗](#)

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Numerous others ...

- 🔗 Complex Social Networks—F. Vega-Redondo ^[12]
- 🔗 Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo ^[11]
- 🔗 Random Graph Dynamics—R. Durrett
- 🔗 Scale-Free Networks—Guido Caldarelli
- 🔗 Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- 🔗 Complex Graphs and Networks—Fan Chung
- 🔗 Social Network Analysis—Stanley Wasserman and Kathleen Faust
- 🔗 Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster ^[5]
- 🔗 Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes ^[6]

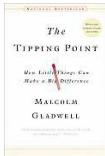
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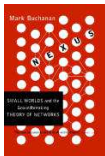


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Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell ^[7]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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More observations

- 🔗 But surely **networks aren't new** ...
- 🔗 Graph theory is well established ...
- 🔗 Study of social networks started in the 1930's ...
- 🔗 So why all this 'new' research on networks?
- 🔗 **Answer:** Oodles of Easily Accessible Data.
- 🔗 We can now inform (alas) our theories with a much more measurable reality.*
- 🔗 A worthy goal: establish **mechanistic explanations**.
**If this is upsetting, maybe string theory is for you ...*

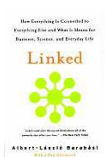
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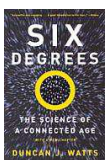


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Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts ^[13]

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More observations

- 🔗 Web-scale data sets can be overly **exciting**.

Witness:

- 🔗 The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) [↗](#)
- 🔗 "The Unreasonable Effectiveness of Data," Halevy et al. ^[8]
- 🔗 c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences" ^[15]

But:

- 🔗 For scientists, description is only part of the battle.
- 🔗 We still need to understand.

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Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

e.g., people, forks in rivers, proteins, webpages, organisms, ...

Links = Connections between nodes

Links may be directed or undirected.
Links may be binary or weighted.

Other spiffing words: vertices and edges.



Examples

So what passes for a complex network?

- Complex networks are **large** (in node number)
- Complex networks are **sparse** (low edge to node ratio)
- Complex networks are usually **dynamic** and **evolving**
- Complex networks can be social, economic, natural, informational, abstract, ...



Super Basic definitions

Node degree = Number of links per node

Notation: Node i 's degree = k_i .

$k_i = 0, 1, 2, \dots$

Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)

Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}$$

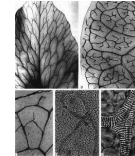
Defn: N_i = the set of i 's k_i neighbors



Examples

Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks
- The Internet
- Road networks
- Power grids



Distribution (branching) versus redistribution (cyclical)



Super Basic definitions

Adjacency matrix:

We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .

e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

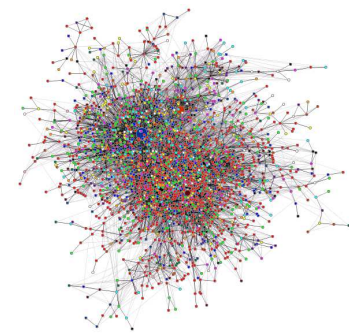
(n.b., for numerical work, we always use sparse matrices.)



Examples

Interaction networks

- The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs: who eats whom
- The World Wide Web (?)
- Airline networks
- Call networks (AT&T)
- The Media

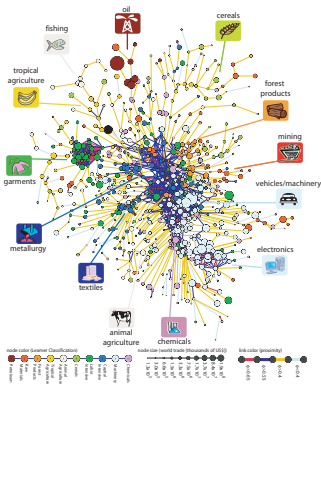


datamining.typepad.com



topics:

- Hidalgo et al.'s "The Product Space Conditions the Development of Nations" [9]
- How do products depend on each other, and how does this network evolve?
- How do countries depend on each other for water, energy, people (immigration), investments?



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Examples

Relational networks

- Consumer purchases (Wal-Mart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging: bit.ly [flickr](http://flickr.com)

common tags [cloud](#) | [list](#)

[community](#) [daily](#) [dictionary](#) [education](#) [encyclopedia](#)
[english](#) [free](#) [imported](#) [info](#) [information](#) [internet](#) [knowledge](#)
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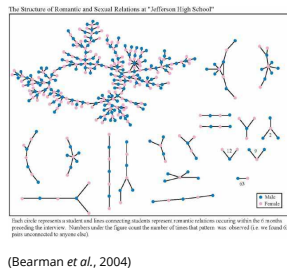


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Examples

Interaction networks: social networks

- Snogging
- Friendships
- Acquaintances
- Boards and directors
- Organizations
- [facebook](#) [twitter](#)



(Bearman et al., 2004)

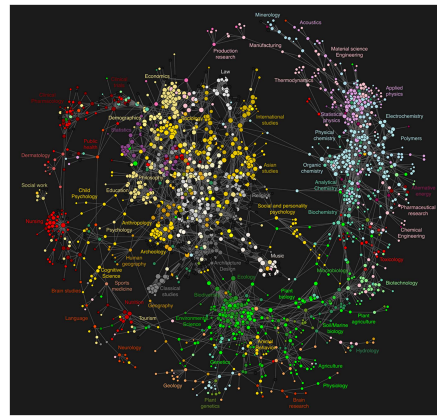
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Clickworthy Science:



"Clickstream Data Yields High-Resolution Maps of Science", Bollen et al. [4], 2009.

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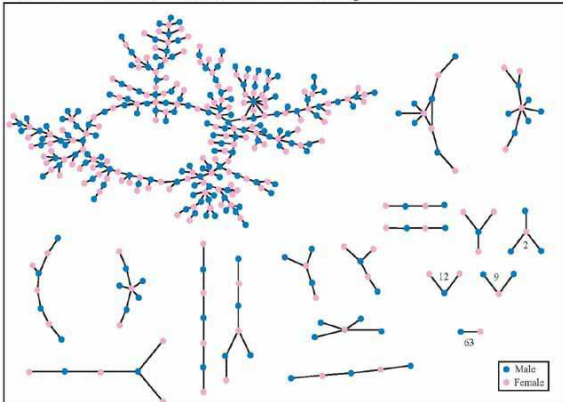
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Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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Handbook of Graphs and Networks.
Wiley-VCH, Berlin, 2003.
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Oxford University Press, Oxford, UK, 2003.
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[Communications on Pure and Applied Mathematics, 13:1-14, 1960. pdf](#)

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[Science, 317:482-487, 2007. pdf](#)
- [10] M. E. J. Newman.
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[SIAM Rev., 45\(2\):167-256, 2003. pdf](#)

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