Random Networks Nutshell

Complex Networks | @networksvox CSYS/MATH 303, Spring, 2016

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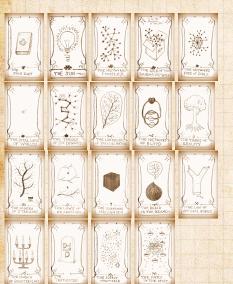
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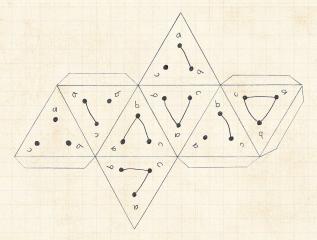








Random network generator for N=3:



- ▶ Get your own exciting generator here .
- \blacktriangleright As $N \nearrow$, polyhedral die rapidly becomes a ball...

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- Consider set of all networks with N labelled nodes and m edges.

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- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.

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- Sometimes equiprobability is a good assumption, but it is always an assumption.

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- Consider set of all networks with N labelled nodes and m edges.
- Standard random network = one randomly chosen network from this set.
- ▶ To be clear: each network is equally probable.
- Sometimes equiprobability is a good assumption, but it is always an assumption.
- Known as Erdős-Rényi random networks or ER graphs.

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Random networks—basic features:

Number of possible edges:

$$0 \leq m \leq {N \choose 2} = \frac{N(N-1)}{2}$$

- Limit of m=0: empty graph.
- Limit of $m = \binom{N}{2}$: complete or fully-connected graph.
- Number of possible networks with N labelled nodes:

$$2^{\binom{N}{2}} \sim e^{\frac{\ln 2}{2}N^2}$$

- ▶ Given m edges, there are $\binom{\binom{N}{2}}{m}$ different possible networks.
- ▶ Crazy factorial explosion for $1 \ll m \ll \binom{N}{2}$.
- Real worth links are usually costly so real networks are almost always sparse.

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- ightharpoonup Given N and m.

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How to build standard random networks:

- ightharpoonup Given N and m.
- Two probablistic methods (we'll see a third later on)
- 1. Connect each of the $\binom{N}{2}$ pairs with appropriate probability p.
- 2. Take N nodes and add exactly m links by selecting edges without replacement.
 - Algorithm: Randomly choose a pair of nodes i and j, $i \neq j$, and connect if unconnected; repeat until all m edges are allocated.
 - Best for adding relatively small numbers of links (most cases).
 - 1 and 2 are effectively equivalent for large N.

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A few more things:

For method 1, # links is probablistic:

$$\langle m \rangle = p {N \choose 2} = p \frac{1}{2} N(N-1)$$

➤ So the expected or average degree is

$$\langle k \rangle = rac{2 \langle m \rangle}{N}$$

- ▶ Which is what it should be...
- If we keep (k) constant then $p \propto 1/N \to 0$ as $N \to \infty$

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$$= \frac{2}{N}p\frac{1}{2}N(N+1) = \frac{2}{N}p\frac{1}{2}\mathcal{N}(N-1) = p(N-1)$$

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Random networks: examples

Next slides:

Example realizations of random networks

- N = 500
 - ▶ Vary m, the number of edges from 100 to 1000.
- Average degree (k) runs from 0.4 to 4
- ► Look at full network plus the largest component

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Example realizations of random networks

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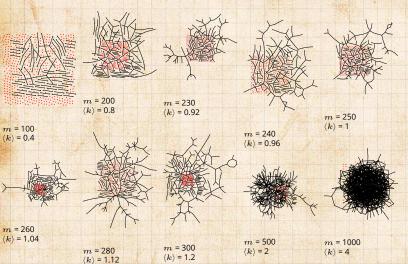
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Random networks: examples for N=500

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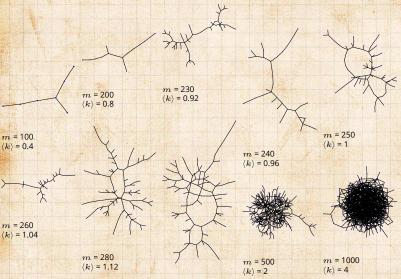






Random networks: largest components

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m = 300

 $\langle k \rangle = 1.2$

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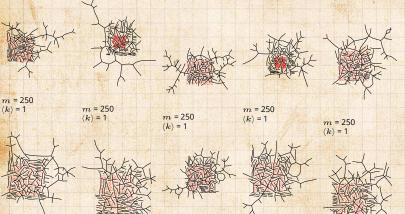






Random networks: examples for N=500





m = 250

 $\langle k \rangle = 1$

m = 250

 $\langle k \rangle = 1$

m = 250 $\langle k \rangle = 1$

m = 250

m = 250

 $\langle k \rangle = 1$

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Random networks: largest components

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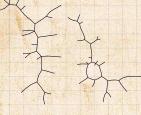
References











m = 250

 $\langle k \rangle = 1$

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 $\langle k \rangle = 1$

$$m = 250$$
 $\langle k \rangle = 1$

 $\langle k \rangle = 1$

$$m$$
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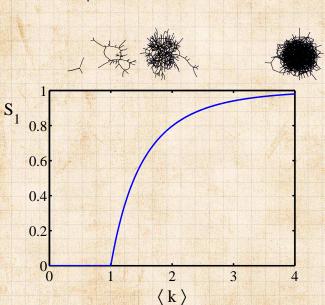
m = 250 $\langle k \rangle = 1$

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▶ For construction method 1, what is the clustering coefficient for a finite network?

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- For construction method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient: [5]

$$C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$$

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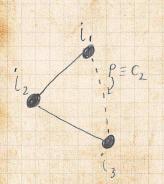






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ightharpoonup Recall: C_2 = probability that two friends of a node are also friends.

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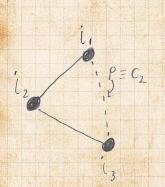






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$$C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$$



- ightharpoonup Recall: C_2 = probability that two friends of a node are also friends.
- ightharpoonup Or: C_2 = probability that a triple is part of a triangle.

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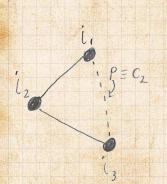






- For construction method 1, what is the clustering coefficient for a finite network?
- Consider triangle/triple clustering coefficient: [5]

$$C_2 = \frac{3 \times \text{\#triangles}}{\text{\#triples}}$$



- ▶ Recall: C₂ = probability that two friends of a node are also friends.
- ➤ Or: C₂ = probability that a triple is part of a triangle.
- For standard random networks, we have simply that

$$C_2 = p$$
.

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So for large random networks $(N \to \infty)$, clustering drops to zero. COCONUTS

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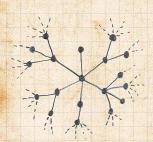
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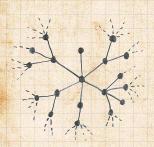
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- So for large random networks $(N \to \infty)$, clustering drops to zero.
- Key structural feature of random networks is that they locally look like pure branching networks
- ► No small loops.

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Recall P_k = probability that a randomly selected node has degree k.



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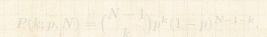
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- Recall P_k = probability that a randomly selected node has degree k.
- Consider method 1 for constructing random networks: each possible link is realized with probability p.



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- Recall P_k = probability that a randomly selected node has degree k.
- ► Consider method 1 for constructing random networks: each possible link is realized with probability p.
- Now consider one node: there are 'N-1 choose k' ways the node can be connected to k of the other N-1 nodes.
- Each connection occurs with probability p, each non-connection with probability (1-p).
- Therefore have a prhomial distri

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- ▶ Each connection occurs with probability p, each non-connection with probability (1-p).
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$$P(k;p,N) = {N-1 \choose k} p^k (1-p)^{N-1-k}.$$

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$$P(k; p, N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \to \frac{\langle k \rangle}{k!}$$

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- ▶ But we want to keep $\langle k \rangle$ fixed..
- So examine limit of P(k; p, N) when $p \to 0$ and $N \to \infty$ with $\langle k \rangle = |p(N-1)| = \text{constant.}$

$$P(k; p, N) \simeq \frac{\langle k \rangle^k}{k!} \left(1 - \frac{\langle k \rangle}{N-1} \right)^{N-1-k} \rightarrow 0$$

This is a graph distributible with mean $\langle k \rangle$.

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This is a property distribution \mathbb{Z} with mean $\langle k \rangle$.

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▶ This is a Poisson distribution \square with mean $\langle k \rangle$.

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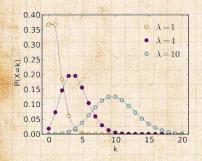
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Poisson basics:

$$\boxed{P(k;\lambda) = \frac{\lambda^k}{k!} e^{-\lambda}}$$





- $\lambda > 0$
- k = 0, 1, 2, 3, ...
- Classic use: probability that an event occurs k times in a given time period, given an average rate of occurrence.
- e.g.: phone calls/minute, horse-kick deaths.
- 'Law of small numbers'

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The variance of degree distributions for random networks turns out to be very important.

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- ▶ The variance of degree distributions for random networks turns out to be very important.
- ▶ Using calculation similar to one for finding $\langle k \rangle$ we find the second moment to be:

$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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▶ So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.

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- ▶ So standard deviation σ is equal to $\sqrt{\langle k \rangle}$.
- Note: This is a special property of Poisson distribution and can trip us up...

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So... standard random networks have a Poisson degree distribution

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- So... standard random networks have a Poisson degree distribution
- \triangleright Generalize to arbitrary degree distribution P_k .

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 $P(\text{link between } i \text{ and } j) \propto w_i w_j$.

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 - 1. Randomly wiring up (and rewiring) already existing nodes with fixed degrees.

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 - Randomly wiring up (and rewiring) already existing nodes with fixed degrees.
 - 2. Examining mechanisms that lead to networks with certain degree distributions.

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Example realizations of random networks with power law degree distributions:

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Example realizations of random networks with power law degree distributions:

- N = 1000.

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Example realizations of random networks with power law degree distributions:

- N = 1000.
- $P_k \propto k^{-\gamma}$ for $k \geq 1$.
- ► Set $P_0 = 0$ (no isolated nodes).
- Vary exponent γ between 2.10 and 2.9
- Again, look at full network plus the largest component.
- Apart from degree distribution, wiring is random.

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Random networks: examples for N=1000













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 $\gamma = 2.19$ $\langle k \rangle = 2.986$

 $\gamma = 2.28$ $\langle k \rangle = 2.306$

 $\gamma = 2.37$ $\langle k \rangle = 2.504$

 $\gamma = 2.46$ $\langle k \rangle = 1.856$







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 $\gamma = 2.55$ $\langle k \rangle = 1.712$

 $\gamma = 2.64$ $\langle k \rangle = 1.6$



 $\gamma = 2.82$ $\langle k \rangle = 1.386$

 $\gamma = 2.91$ $\langle k \rangle = 1.49$





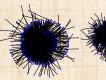


Random networks: largest components

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 $\gamma = 2.64$ $\langle k \rangle = 1.6$

 $\gamma = 2.73$ $\langle k \rangle = 1.862$



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- \blacktriangleright Arbitrary degree distribution P_k .
- Create (unconnected) nodes with degrees sampled from P_b.
- ➤ Wire nodes together randomly.
- Create ensemble to test deviations from randomness.

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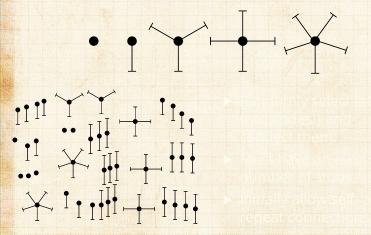
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► Idea: start with a soup of unconnected nodes with stubs (half-edges):



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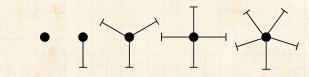
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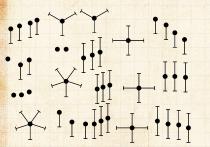
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► Idea: start with a soup of unconnected nodes with stubs (half-edges):





- Randomly select stubs (not nodes!) and connect them.
 - Must have an even number of stubs.
 - Initially allow self- and rebeat connections.

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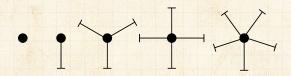
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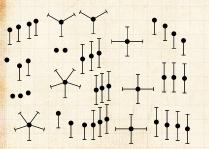
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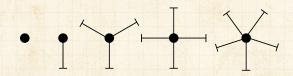
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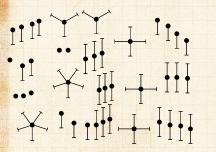
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► Idea: start with a soup of unconnected nodes with stubs (half-edges):





- Randomly select stubs (not nodes!) and connect them.
- Must have an even number of stubs.

Initially allow self- and repeat connections.

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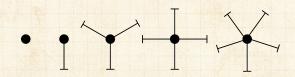


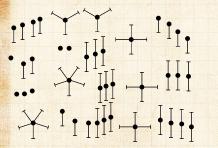


Building random networks: Stubs

Phase 1:

► Idea: start with a soup of unconnected nodes with stubs (half-edges):





- Randomly select stubs (not nodes!) and connect them.
- Must have an even number of stubs.
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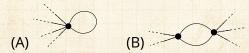
References





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Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



- Being careful: we can't change the degree of any node, so we can't simply move links around.
- Simplest solution: randomly rewire two edges at a time

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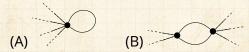
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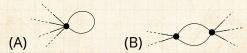
Random friends are







Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



- ▶ Being careful: we can't change the degree of any node, so we can't simply move links around.
- ➤ Simplest solution: randomly rewire two edges at a time.

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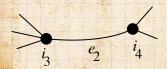
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- Randomly choose two edges.
 (Or choose problem edge and a random edge)
 - Check to make sure edges are disjoint.

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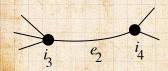
Random friends are strange











- Randomly choose two edges.
 (Or choose problem edge and a random edge)
- Check to make sure edges are disjoint.

- Rewire one end of each edge.
- ▶ Node degree:
- ► Works if e₁ is a self-loop or repeated edge.
- Same as finding on/off/on/off
 4-cycles, and rotating them.

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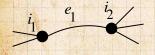
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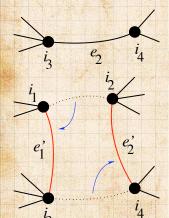
Random friends are strange











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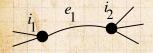
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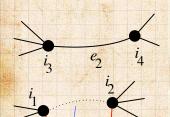
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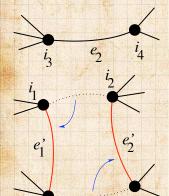
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Sampling random networks

Phase 2:

Use rewiring algorithm to remove all self and repeat loops.

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Use rewiring algorithm to remove all self and repeat loops.

Phase 3:

- Randomize network wiring by applying rewiring algorithm liberally.
- Rule of thumb: # Rewirings ≈ 10 × # edges

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Use rewiring algorithm to remove all self and repeat loops.

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Problem with only joining up stubs is failure to randomly sample from all possible networks. Pure random networks

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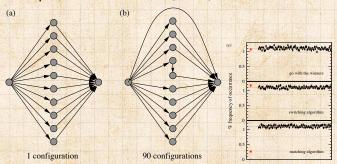
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- Problem with only joining up stubs is failure to randomly sample from all possible networks.
- ► Example from Milo et al. (2003) [3]:



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Sampling random networks

▶ What if we have P_k instead of N_k ?

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Sampling random networks

- ▶ What if we have P_k instead of N_k ?
- Must now create nodes before start of the construction algorithm.

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How to build in practice







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- Must now create nodes before start of the construction algorithm.
- Generate N nodes by sampling from degree distribution P_k .
- ▶ Easy to do exactly numerically since *k* is discrete.
- Note: not all P_k will always give nodes that can be wired together.

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▶ Idea of motifs [6] introduced by Shen-Orr, Alon et al. in 2002.

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Motifs Random friends are







- ▶ Idea of motifs [6] introduced by Shen-Orr, Alon et al. in 2002.
- Looked at gene expression within full context of transcriptional regulation networks.

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Motifs Random friends are









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Motifs Random friends are







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Network motifs

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- Directed network with 577 interactions (edges) and 424 operons (nodes).
- ▶ Used network randomization to produce ensemble of alternate networks with same degree frequency N_k .
- ► Looked for certain subnetworks (motifs) that appeared more or less often than expected

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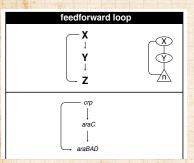
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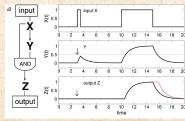
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▶ Z only turns on in response to sustained activity in X.

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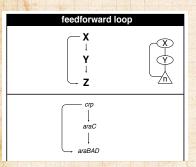
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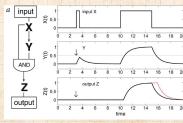
Motifs Random friends are











- ▶ Z only turns on in response to sustained activity in X.
- ightharpoonup Turning off X rapidly turns off Z.

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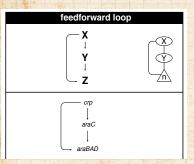
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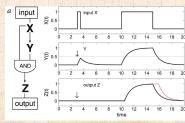
Motifs Random friends are











- ▶ Z only turns on in response to sustained activity in X.
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- Analogy to elevator doors.

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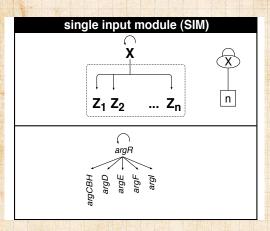
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Master switch.

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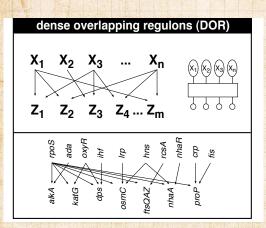
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Network motifs



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Note: selection of motifs to test is reasonable but nevertheless ad-hoc.

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- Note: selection of motifs to test is reasonable but nevertheless ad-hoc.
- For more, see work carried out by Wiggins *et al.* at Columbia.

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 \blacktriangleright The degree distribution P_k is fundamental for our description of many complex networks

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The edge-degree distribution:

- The degree distribution P_k is fundamental for our description of many complex networks
- \blacktriangleright Again: P_k is the degree of randomly chosen node.

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- The degree distribution P_k is fundamental for our description of many complex networks
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- ➤ A second very important distribution arises from choosing randomly on edges rather than on nodes.

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$$Q_k \propto k P_k$$

Normalized form:

$$Q_k = \frac{kP_k}{\sum_{k'=0}^{\infty} k' P_{k'}}$$

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▶ Big deal: Rich-get-richer mechanism is built into this selection process.

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- Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7$, $P_2 = 2/7$, $P_3 = 1/7$, $P_6 = 1/7$.

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- Probability of randomly selecting a node of degree k by choosing from nodes: $P_1 = 3/7, P_2 = 2/7, P_3 = 1/7,$ $P_6 = 1/7$.
- Probability of landing on a node of degree k after randomly selecting an edge and then randomly choosing one direction to travel:

$$Q_1 = 3/16, Q_2 = 4/16, Q_3 = 3/16, Q_6 = 6/16.$$

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- Probability of randomly selecting a node of degree k by choosing from nodes: $P_1=3/7,\,P_2=2/7,\,P_3=1/7,\,P_6=1/7.$
- ▶ Probability of landing on a node of degree *k* after randomly selecting an edge and then randomly choosing one direction to travel:

$$Q_1 = 3/16, Q_2 = 4/16, \ Q_3 = 3/16, Q_6 = 6/16.$$

▶ Probability of finding # outgoing edges = k after randomly selecting an edge and then randomly choosing one direction to travel:

$$R_0 = 3/16 \ R_1 = 4/16,$$

 $R_2 = 3/16, R_5 = 6/16.$

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 \triangleright For random networks, Q_k is also the probability that a friend (neighbor) of a random node has k friends.

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 R_k = probability that a friend of a random node has k other friends.

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 $R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}}$

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▶ Equivalent to friend having degree k + 1.

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$$R_k = \frac{(k+1)P_{k+1}}{\sum_{k'=0}(k'+1)P_{k'+1}} = \frac{(k+1)P_{k+1}}{\langle k \rangle}$$

- ▶ Equivalent to friend having degree k + 1.
- Natural question: what's the expected number of other friends that one friend has?

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• Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}$$

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• Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

$$\left\langle k\right\rangle _{R}=\sum_{k=0}^{\infty}kR_{k}=\sum_{k=0}^{\infty}k\frac{(k+1)P_{k+1}}{\left\langle k\right\rangle }$$

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$$\begin{split} \left\langle k \right\rangle_R &= \sum_{k=0}^\infty k R_k = \sum_{k=0}^\infty k \frac{(k+1)P_{k+1}}{\left\langle k \right\rangle} \\ &= \frac{1}{\left\langle k \right\rangle} \sum_{k=1}^\infty k (k+1)P_{k+1} \end{split}$$

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(where we have sneakily matched up indices)

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▶ Given R_k is the probability that a friend has k other friends, then the average number of friends' other friends is

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(where we have sneakily matched up indices)

$$= \frac{1}{\langle k \rangle} \sum_{j=0}^{\infty} (j^2 - j) P_j \quad \text{(using j = k+1)}$$

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(where we have sneakily matched up indices)

$$=\frac{1}{\langle k\rangle}\sum_{j=0}^{\infty}(j^2-j)P_j\quad \text{(using j = k+1)}$$

$$=\frac{1}{\langle k\rangle}\left(\langle k^2\rangle-\langle k\rangle\right)$$

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Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle - \langle k \rangle \right)$, is true for all random networks, independent of degree distribution.

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- Note: our result, $\langle k \rangle_R = \frac{1}{\langle k \rangle} \left(\langle k^2 \rangle \langle k \rangle \right)$, is true for all random networks, independent of degree distribution.
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$$\langle k^2 \rangle = \langle k \rangle^2 + \langle k \rangle.$$

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Again, neatness of results is a special property of the Poisson distribution.

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- Again, neatness of results is a special property of the Poisson distribution.
- So friends on average have $\langle k \rangle$ other friends, and $\langle k \rangle + 1$ total friends...

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In fact, R_k is rather special for pure random networks ...

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Two reasons why this matters

Reason #1:

$$\langle k_2 \rangle = \langle k \rangle \times \langle k \rangle_R$$

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Reason #1:

Average # friends of friends per node is

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 - 2. If P_k has a large second moment then $\langle k_2 \rangle$ will be big.
 - 3. Your friends really are different from you...
 - 4. See also; class size paradoxes (nod to: Gelman)

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- ▶ A node's average # of friends: $\langle k \rangle$

$$\frac{\langle k^2 \rangle}{\langle k \rangle} = 0$$

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More on peculiarity #3:

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- ▶ Intuition: for random networks, the more connected a node, the more likely it is to be chosen as a friend.

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Eom and Jo, Nature Scientific Reports, **4**, 4603, 2014. [1]

Your friends really are monsters #winners:1

- Go on that mer Friends have more coauthors, citations, and publications.
- ► Other horrific studies: your connections on Twitter have more followers than you, your sexua partners more partners than you, ...
- ▶ The hope. Maybe they have more enemies and diseases too.

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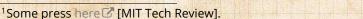
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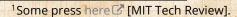
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- ► Other horrific studies: your connections on Twitter have more followers than you, your sexual partners more partners than you, ...
- The hope. Maybe they have more enemies and

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Eom and Jo, Nature Scientific Reports, **4**, 4603, 2014. [1]

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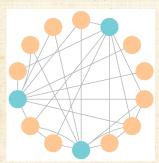
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Related disappointment:



- Nodes see their friends' color choices.

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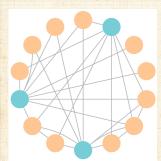






¹https://www.washingtonpost.com/graphics/business/ wonkblog/majority-illusion/

Related disappointment:



- Nodes see their friends' color choices.
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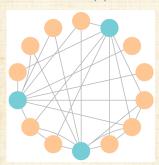






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Related disappointment:



- Nodes see their friends' color choices.
- Which color is more popular?1
- Again: thinking in edge space changes everything.

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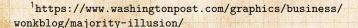
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- $\triangleright \langle k \rangle_B$ is key to understanding how well random networks are connected together.

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(Big) Reason #2:

- $\triangleright \langle k \rangle_B$ is key to understanding how well random networks are connected together.
- e.g., we'd like to know what's the size of the largest component within a network.

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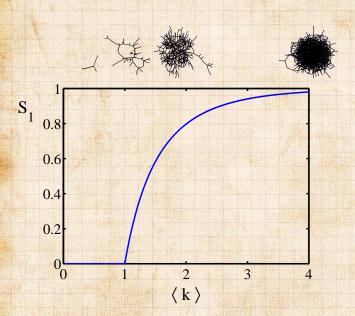






Giant component





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- A giant component exists if when we follow a random edge, we are likely to hit a node with at least 1 other outgoing edge.
- Equivalently, expect exponential growth in node number as we move out from a random node.
- lacktriangle All of this is the same as requiring $\langle k \rangle_R$
- ▶ Giant-component condition (or percolation condition):

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} >$$

- Again, see that the second moment is an essential part of the story.
- Equivalent statement: $\langle k^2 \rangle > 1$

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Structure of random networks

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Giant component for standard random networks:

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- ▶ When $\langle k \rangle$ < 1, all components are finite.
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- We say $\langle k \rangle = 1$ marks the critical point of the system.

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 $lackbox{ e.g, if } P_k = ck^{-\gamma} ext{ with } 2 < \gamma < 3 ext{, } k \geq 1 ext{, then}$

$$\langle k^2 \rangle = c \sum_{k=1}^{\infty} k^2 k^{-\gamma}$$

$$\int_{x=1}^{\infty} x^{2-\gamma} \mathrm{d}x$$

 $\propto x^{3-\gamma}$

- So giant component always exists for these kinds of networks
- Cutoff scaling is k^{-3} : if $\gamma > 3$ then we have to look harder at $\langle k \rangle_B$.
- How about $P_k = \delta_{kk_0}$?

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$$\begin{split} \langle k^2 \rangle &= c \sum_{k=1}^\infty k^2 k^{-\gamma} \\ &\sim \int_{x=1}^\infty x^{2-\gamma} \mathrm{d}x \\ &\propto x^{3-\gamma} \big|_{x=1}^\infty = \infty \quad (\gg \langle k \rangle). \end{split}$$

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And how big is the largest component?

- \blacktriangleright Define S_1 as the size of the largest component.

$$\delta = \sum_{k=0}^{\infty} P_k \delta^k$$

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Substitute in Poisson distribution...

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Carrying on:

$$\frac{\delta}{\delta} = \sum_{k=0}^{\infty} P_k \delta^k$$

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$$\frac{\delta}{\delta} = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k$$

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angle^k}{k!} e^{-\langle k
angle} \delta^k \ &= e^{-\langle k
angle} \sum_{k=0}^{\infty} rac{(\langle k
angle \delta)^k}{k!} \end{aligned}$$

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$$\begin{split} & \delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k \\ & = e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!} \\ & = e^{-\langle k \rangle} e^{\langle k \rangle \delta} \end{split}$$

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$$\begin{split} & \delta = \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k \\ & = e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!} \\ & = e^{-\langle k \rangle} e^{\langle k \rangle \delta} = e^{-\langle k \rangle (1 - \delta)}. \end{split}$$

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Carrying on:

$$\begin{split} \frac{\delta}{\delta} &= \sum_{k=0}^{\infty} P_k \delta^k = \sum_{k=0}^{\infty} \frac{\langle k \rangle^k}{k!} e^{-\langle k \rangle} \delta^k \\ &= e^{-\langle k \rangle} \sum_{k=0}^{\infty} \frac{(\langle k \rangle \delta)^k}{k!} \\ &= e^{-\langle k \rangle} e^{\langle k \rangle \delta} = e^{-\langle k \rangle (1 - \delta)}. \end{split}$$

Now substitute in $\delta = 1 - S_1$ and rearrange to obtain:

$$S_1 = 1 - e^{-\langle k \rangle S_1}.$$

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We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$.

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- ▶ We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}.$
- First, we can write $\langle k \rangle$ in terms of S_1 :

$$\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1-S_1}.$$

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- ▶ We can figure out some limits and details for $S_1 = 1 - e^{-\langle k \rangle S_1}$.
- \blacktriangleright First, we can write $\langle k \rangle$ in terms of S_1 :

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 \blacktriangleright As $\langle k \rangle \rightarrow 0$, $S_1 \rightarrow 0$.

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- ightharpoonup As $\langle k \rangle \to \infty$, $S_1 \to 1$.

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- Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.

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- ▶ Only solvable for $S_1 > 0$ when $\langle k \rangle > 1$.

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- We can figure out some limits and details for $S_1 = 1 e^{-\langle k \rangle S_1}$.
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- Notice that at $\langle k \rangle = 1$, the critical point, $S_1 = 0$.
- ▶ Only solvable for $S_1 > 0$ when $\langle k \rangle > 1$.
- ▶ Really a transcritical bifurcation. [7]

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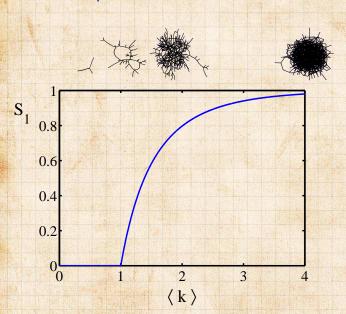
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Turns out we were lucky...

- Our dirty trick only works for ER random networks.

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Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- ▶ The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.

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Turns out we were lucky...

- Our dirty trick only works for ER random networks.
- ▶ The problem: We assumed that neighbors have the same probability δ of belonging to the largest component.
- But we know our friends are different from us...
- Works for ER random networks because $\langle k \rangle = \langle k \rangle_R$.
- We need a separate probability 8' for the chance that an edge leads to the giant (infinite) component.
- We can sort many things out with sensible probabilistic arguments...
- More detailed investigations will profit from a spot of Generating Unctionology.

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