Mixed, correlated random networks Complex Networks | @networksvox CSYS/MATH 303, Spring, 2016

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Directed random networks

Mixed random networks Definition Correlations

Mixed Random Network Contagion Spreading condition Full generalization Triggering probabilities

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So far, we've largely studied networks with undirected, unweighted edges. COcoNuTS

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🚳 Now consider directed, unweighted edges.

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- So far, we've largely studied networks with undirected, unweighted edges.
- 🚳 Now consider directed, unweighted edges.
- Nodes have k_i and k_o incoming and outgoing edges, otherwise random.

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- Network defined by joint in- and out-degree distribution: P_{k_i,k_0}

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3 Normalization: $\sum_{k_i=0}^{\infty}\sum_{k_o=0}^{\infty}P_{k_i,k_o}=1$

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Marginal in-degree and out-degree distributions:

$$P_{k_{i}} = \sum_{k_{o}=0}^{\infty} P_{k_{i},k_{o}} \text{ and } P_{k_{o}} = \sum_{k_{i}=0}^{\infty} P_{k_{i},k_{o}}$$

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Required balance:

$$\langle k_{\rm i} \rangle = \sum_{k_{\rm i}=0}^{\infty} \sum_{k_{\rm o}=0}^{\infty} k_{\rm i} P_{k_{\rm i},k_{\rm o}} = \sum_{k_{\rm i}=0}^{\infty} \sum_{k_{\rm o}=0}^{\infty} k_{\rm o} P_{k_{\rm i},k_{\rm o}} = \langle k_{\rm o} \rangle$$

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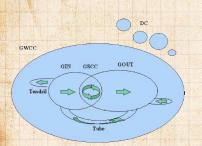
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Directed network structure:



From Boguñá and Serano.^[1]

GWCC = Giant Weakly Connected Component (directions removed);

GIN = Giant In-Component;

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GOUT = Giant Out-Component;

GSCC = Giant Strongly Connected Component;

DC = Disconnected Components (finite).

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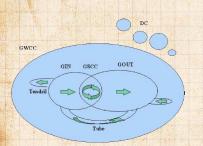
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When moving through a family of increasingly connected directed random networks, GWCC usually appears before GIN, GOUT, and GSCC which tend to appear together. ^[4, 1]

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Directed and undirected random networks are separate families ...

...and analyses are also disjoint. Need to examine a larger family of random network with mixed directed and undirected edges.

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Consider nodes with three types of edges:

- 1. $k_{\rm u}$ undirected edges,
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Consider nodes with three types of edges:

- 1. $k_{\rm u}$ undirected edges,
- 2. k_i incoming directed edges,
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Define a node by generalized degree:

$$\vec{k} = [k_{\rm u} \ k_{\rm i} \ k_{\rm o}]^{\rm T}.$$

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👶 Joint degree distribution:

$$P_{\vec{k}}$$
 where $\vec{k} = [k_{u} k_{i} k_{o}]^{\mathsf{T}}$.

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🚳 Joint degree distribution:

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As for directed networks, require in- and out-degree averages to match up:

$$\langle k_{\mathbf{i}} \rangle = \sum_{k_{\mathbf{u}}=0}^{\infty} \sum_{k_{\mathbf{i}}=0}^{\infty} \sum_{k_{\mathbf{o}}=0}^{\infty} k_{\mathbf{i}} P_{\vec{k}} = \sum_{k_{\mathbf{u}}=0}^{\infty} \sum_{k_{\mathbf{i}}=0}^{\infty} \sum_{k_{\mathbf{o}}=0}^{\infty} k_{\mathbf{o}} P_{\vec{k}} = \langle k_{\mathbf{o}} \rangle$$

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Otherwise, no other restrictions and connections are random.

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Otherwise, no other restrictions and connections are random.

Directed and undirected random networks are disjoint subfamilies:

Undirected: $P_{\vec{k}} = P_{k_u} \delta_{k_i,0} \delta_{k_o,0}$,

Directed:
$$P_{\vec{k}} = \delta_{k_0,0} P_{k_0,k_0}$$

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🙈 Now add correlations (two point or Markovian) 🗆:

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1 $P^{(0)}(k, k')$ must be related to $P^{(0)}(k', k)$. 2. $P^{(0)}(k, k')$ and $P^{(0)}(k, k')$ must be connected



Now add correlations (two point or Markovian) \Box : 1. $P^{(u)}(\vec{k} | \vec{k}')$ = probability that an undirected edge leaving a degree \vec{k}' nodes arrives at a degree \vec{k} node. COcoNuTS

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- 1. $P^{(u)}(\vec{k} | \vec{k}')$ = probability that an undirected edge leaving a degree \vec{k}' nodes arrives at a degree \vec{k} node.
- 2. $P^{(i)}(\vec{k} | \vec{k}')$ = probability that an edge leaving a degree \vec{k}' nodes arrives at a degree \vec{k} node is an in-directed edge relative to the destination node.



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- 1. $P^{(u)}(\vec{k} | \vec{k}')$ = probability that an undirected edge leaving a degree \vec{k}' nodes arrives at a degree \vec{k} node.
- P⁽ⁱ⁾(*k* | *k*') = probability that an edge leaving a degree *k*' nodes arrives at a degree *k* node is an in-directed edge relative to the destination node.
 P^(o)(*k* | *k*') = probability that an edge leaving a degree *k*' nodes arrives at a degree *k* node is an out-directed edge relative to the destination node.



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🚳 Now require more refined (detailed) balance.

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Randomly choose an edge, and randomly choose one end.

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- Randomly choose an edge, and randomly choose one end.
- Say we find a degree \vec{k} node at this end, and a degree $\vec{k'}$ node at the other end.

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- Randomly choose an edge, and randomly choose one end.
- Say we find a degree k node at this end, and a degree k' node at the other end.
 Define probability this happens as P^(u)(k, k').

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- Randomly choose an edge, and randomly choose one end.
- Say we find a degree \vec{k} node at this end, and a degree \vec{k}' node at the other end.
- \bigotimes Define probability this happens as $P^{(u)}(\vec{k}, \vec{k}')$.
- Solution Observe we must have $P^{(u)}(\vec{k}, \vec{k}') = P^{(u)}(\vec{k}', \vec{k}).$

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 $\begin{array}{l} \bigotimes \\ \text{Conditional probability} \\ \text{connection:} \\ P^{(\mathsf{u})}(\vec{k},\vec{k}') &= P^{(\mathsf{u})}(\vec{k} \mid \vec{k}') \frac{k'_{\mathsf{u}} P(\vec{k}')}{\langle k'_{\mathsf{u}} \rangle} \end{array} \end{array}$

$$P^{(\mathsf{u})}(\vec{k}',\vec{k}) = P^{(\mathsf{u})}(\vec{k}' \mid \vec{k}) \frac{k_{\mathsf{u}}P(\vec{k})}{\langle k_{\mathsf{u}} \rangle}$$

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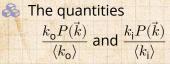
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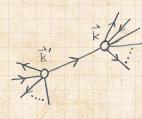
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Correlations—Directed edge balance:



give the probabilities that in starting at a random end of a randomly selected edge, we begin at a degree \vec{k} node and then find ourselves travelling:

- 1. along an outgoing edge, or
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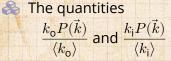
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🚳 We therefore have

$$P^{(\text{dir})}(\vec{k},\vec{k}') = P^{(i)}(\vec{k}\,|\,\vec{k}')\frac{k'_{\text{o}}P(\vec{k}')}{\langle k'_{\text{o}} \rangle} = P^{(o)}(\vec{k}'\,|\,\vec{k})\frac{k_{\text{i}}P(\vec{k})}{\langle k_{\text{i}} \rangle}$$

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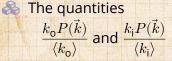
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$$P^{(\mathsf{dir})}(\vec{k},\vec{k}') = P^{(i)}(\vec{k}\,|\,\vec{k}')\frac{k_{\mathsf{o}}'P(\vec{k}')}{\langle k_{\mathsf{o}}' \rangle} = P^{(\mathsf{o})}(\vec{k}'\,|\,\vec{k})\frac{k_{\mathsf{i}}P(\vec{k})}{\langle k_{\mathsf{i}} \rangle}$$

Note that $P^{(\text{dir})}(\vec{k}, \vec{k}')$ and $P^{(\text{dir})}(\vec{k}', \vec{k})$ are in general not related if $\vec{k} \neq \vec{k}'$.

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Consider uncorrelated mixed networks first. Recall our first result for undirected random networks, that edge gain ratio must exceed 1

 $\mathbf{R} = \sum_{u=1}^{\infty} \frac{k_{u} P_{k_{u}}}{\langle k_{u} \rangle} \bullet (k_{u} - 1) \bullet B_{k_{d}, 1} > 1.$

Similar form for purely directed networks:

Both are composed of (1) probability of connection to a node of a given type; (2) numb of newly infected edges if successful; and (3) probability of infection.

 $\mathbf{R} = \sum_{k=0}^{\infty} \sum_{k_i=0}^{\infty} \frac{k_i P_{k_i, k_o}}{\langle k_i \rangle} \bullet k_o \bullet B_{k_i, 1}$

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Global spreading condition: ^[2] When are cascades possible?: Consider uncorrelated mixed networks first. $\mathbf{R} = \sum_{u=1}^{\infty} \frac{k_{u} P_{k_{u}}}{\langle k_{u} \rangle} \bullet (k_{u} - 1) \bullet B_{k_{d}, 1} > 1.$ $\mathbf{R} = \sum_{k=0}^{\infty} \sum_{k_i=0}^{\infty} \frac{k_i P_{k_i, k_o}}{\langle k_i \rangle} \bullet k_o \bullet B_{k_i, 1}$

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$$\mathbf{R} = \sum_{k_{\mathrm{u}}=0}^{\infty} \frac{k_{\mathrm{u}} P_{k_{\mathrm{u}}}}{\langle k_{\mathrm{u}} \rangle} \bullet (k_{\mathrm{u}}-1) \bullet B_{k_{\mathrm{u}},1} > 1.$$

🚳 Similar form for purely directed networks:

$$\mathbf{R} = \sum_{k_{\mathrm{i}}=0}^{\infty} \sum_{k_{\mathrm{o}}=0}^{\infty} \frac{k_{\mathrm{i}} P_{k_{\mathrm{i}},k_{\mathrm{o}}}}{\langle k_{\mathrm{i}} \rangle} \bullet k_{\mathrm{o}} \bullet B_{k_{\mathrm{i}},1} > 1.$$

Both are composed of (1) probability of connection to a node of a given type; (2) number of newly infected edges if successful; and (3) probability of infection.

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Local growth equation:

Solution Define number of infected edges leading to nodes a distance d away from the original seed as f(d).

Applies for discrete time and continuous time contagion processes.

Now see $B_{k_{u},1}$ is the probability that an infected edge eventually infects a node. Also allows for recovery of nodes (SIR). COcoNuTS

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Local growth equation:

Define number of infected edges leading to nodes a distance *d* away from the original seed as *f*(*d*).
 Infected edge growth equation:

 $f(d+1) = \mathbf{R}f(d).$

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Mixed, uncorrelated random netwoks:

- Now have two types of edges spreading infection: directed and undirected.
 - Infected directed edges can lead to infected directed or undirected edges.
 Infected undirected edges can lead to infected directed or undirected edges.
 - Define $f^{(u)}(d)$ and $f^{(o)}(d)$ as the expected number of infected undirected and directed edges leading to nodes a distance *d* from seed.

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Mixed, uncorrelated random netwoks:

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- 🚳 Gain ratio now more complicated:
 - Infected directed edges can lead to infected directed or undirected edges.
 - 2. Infected undirected edges can lead to infected directed or undirected edges.

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$$\left[\begin{array}{c}f^{(\mathsf{u})}(d+1)\\f^{(\mathsf{o})}(d+1)\end{array}\right] = \mathbf{R}\left[\begin{array}{c}f^{(\mathsf{u})}(d)\\f^{(\mathsf{o})}(d)\end{array}\right]$$

$$\begin{bmatrix} f^{(\mathsf{u})}(d+1) \\ f^{(\mathsf{o})}(d+1) \end{bmatrix} = \mathbf{R} \begin{bmatrix} f^{(\mathsf{u})}(d) \\ f^{(\mathsf{o})}(d) \end{bmatrix}$$

🚳 Two separate gain equations:

$$f^{(\mathrm{u})}(d+1) = \sum_{\vec{k}} \left[\frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet (k_{\mathrm{u}} - 1) \bullet B_{k_{\mathrm{u}} + k_{\mathrm{i}}, 1} f^{(\mathrm{u})}(d) + \frac{k_{\mathrm{i}} P_{\vec{k}}}{\langle k_{\mathrm{i}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{i}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{i}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{o})}(d) + \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{u})}(d) + \frac{k_{\mathrm{u}} P_{\underline{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{u})}(d) + \frac{k_{\mathrm{u}} P_{\underline{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{u}} \bullet B_{k_{\mathrm{u}} + k_{\mathrm{u}}, 1} f^{(\mathrm{u})}(d) + \frac{k_{\mathrm{u}} P_{\underline{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet h^{(\mathrm{u})}(d) + \frac{k_{\mathrm{u}} P_{$$

$$\begin{bmatrix} f^{(\mathsf{u})}(d+1) \\ f^{(\mathsf{o})}(d+1) \end{bmatrix} = \mathbf{R} \begin{bmatrix} f^{(\mathsf{u})}(d) \\ f^{(\mathsf{o})}(d) \end{bmatrix}$$

🚳 Two separate gain equations:

$$\begin{split} f^{(\mathsf{u})}(d+1) &= \sum_{\vec{k}} \left[\frac{k_{\mathsf{u}} P_{\vec{k}}}{\langle k_{\mathsf{u}} \rangle} \bullet (k_{\mathsf{u}} - 1) \bullet B_{k_{\mathsf{u}}+k_{\mathsf{l}},1} f^{(\mathsf{u})}(d) + \frac{k_{\mathsf{i}} P_{\vec{k}}}{\langle k_{\mathsf{i}} \rangle} \bullet k_{\mathsf{u}} \bullet B_{k_{\mathsf{u}}+k_{\mathsf{i}},1} f^{(\mathsf{o})}(d) \right] \\ f^{(\mathsf{o})}(d+1) &= \sum_{\vec{k}} \left[\frac{k_{\mathsf{u}} P_{\vec{k}}}{\langle k_{\mathsf{u}} \rangle} \bullet k_{\mathsf{o}} B_{k_{\mathsf{u}}+k_{\mathsf{i}},1} f^{(\mathsf{u})}(d) + \frac{k_{\mathsf{i}} P_{\vec{k}}}{\langle k_{\mathsf{i}} \rangle} \bullet k_{\mathsf{o}} \bullet B_{k_{\mathsf{u}}+k_{\mathsf{i}},1} f^{(\mathsf{o})}(d) \right] \end{split}$$

$$\begin{bmatrix} f^{(\mathsf{u})}(d+1)\\ f^{(\mathsf{o})}(d+1) \end{bmatrix} = \mathbf{R} \begin{bmatrix} f^{(\mathsf{u})}(d)\\ f^{(\mathsf{o})}(d) \end{bmatrix}$$

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$$f^{(\mathbf{u})}(d+1) = \sum_{\vec{k}} \left[\frac{k_{\mathbf{u}} P_{\vec{k}}}{\langle k_{\mathbf{u}} \rangle} \bullet (k_{\mathbf{u}} - 1) \bullet B_{k_{\mathbf{u}} + k_{\mathbf{i}}, 1} f^{(\mathbf{u})}(d) + \frac{k_{\mathbf{i}} P_{\vec{k}}}{\langle k_{\mathbf{i}} \rangle} \bullet k_{\mathbf{u}} \bullet B_{k_{\mathbf{u}} + k_{\mathbf{i}}, 1} f^{(\mathbf{o})}(d) \right]$$
$$f^{(\mathbf{o})}(d+1) = \sum_{\vec{k}} \left[\frac{k_{\mathbf{u}} P_{\vec{k}}}{\langle k_{\mathbf{u}} \rangle} \bullet k_{\mathbf{o}} B_{k_{\mathbf{u}} + k_{\mathbf{i}}, 1} f^{(\mathbf{u})}(d) + \frac{k_{\mathbf{i}} P_{\vec{k}}}{\langle k_{\mathbf{i}} \rangle} \bullet k_{\mathbf{o}} \bullet B_{k_{\mathbf{u}} + k_{\mathbf{i}}, 1} f^{(\mathbf{o})}(d) \right]$$

🚳 Gain ratio matrix:

$$\mathbf{R} = \sum_{\vec{k}} \left[\begin{array}{c} \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet (k_{\mathrm{u}} - 1) & \frac{k_{\mathrm{i}} P_{\vec{k}}}{\langle k_{\mathrm{i}} \rangle} \bullet k_{\mathrm{u}} \\ \frac{k_{\mathrm{u}} P_{\vec{k}}}{\langle k_{\mathrm{u}} \rangle} \bullet k_{\mathrm{o}} & \frac{k_{\mathrm{i}} P_{\vec{k}}}{\langle k_{\mathrm{i}} \rangle} \bullet k_{\mathrm{o}} \end{array} \right] \bullet B_{k_{\mathrm{u}} + k_{\mathrm{i}}, 1}$$

$$\begin{bmatrix} f^{(\mathsf{u})}(d+1)\\ f^{(\mathsf{o})}(d+1) \end{bmatrix} = \mathbf{R} \begin{bmatrix} f^{(\mathsf{u})}(d)\\ f^{(\mathsf{o})}(d) \end{bmatrix}$$

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$$f^{(\mathbf{o})}(d+1) = \sum_{\vec{k}} \left[\frac{k_{\mathbf{u}} P_{\vec{k}}}{\langle k_{\mathbf{u}} \rangle} \bullet k_{\mathbf{o}} B_{k_{\mathbf{u}} + k_{\mathbf{i}}, 1} f^{(\mathbf{u})}(d) + \frac{k_{\mathbf{i}} P_{\vec{k}}}{\langle k_{\mathbf{i}} \rangle} \bullet k_{\mathbf{o}} \bullet B_{k_{\mathbf{u}} + k_{\mathbf{i}}, 1} f^{(\mathbf{o})}(d) \right]$$

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Spreading condition: max eigenvalue of $\mathbf{R} > 1$.

Useful change of notation for making results more general: write $P^{(u)}(\vec{k} \mid *) = \frac{k_u P_{\vec{k}}}{\langle k_u \rangle}$ and $P^{(i)}(\vec{k} \mid *) = \frac{k_i P_{\vec{k}}}{\langle k_i \rangle}$ where * indicates the starting node's degree is irrelevant (no correlations).

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Useful change of notation for making results more general: write P^(U)(k | *) = k_uP_k/(k_u) and P⁽ⁱ⁾(k | *) = k_iP_k/(k_i) where * indicates the starting node's degree is irrelevant (no correlations).
 Also write B_{kuki,*} to indicate a more general infection probability, but one that does not depend on the edge's origin.
 Now have, for the example of mixed, uncorrelated random networks:

 $\mathbf{R} = \sum_{\vec{k}} \begin{bmatrix} P^{(u)}(\vec{k} \mid *) \bullet (k_{u} - 1) & P^{(i)}(\vec{k} \mid *) \bullet k_{u} \\ P^{(u)}(\vec{k} \mid *) \bullet k_{o} & P^{(i)}(\vec{k} \mid *) \bullet k_{o} \end{bmatrix} \bullet B_{k_{u}k_{i},*}$

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Summary of contagion conditions for uncorrelated networks:

 \mathfrak{R} I. Undirected, Uncorrelated— $f(d+1) = \mathbf{f}(d)$:

$$\mathbf{R} = \sum_{k_{\mathrm{u}}} P^{(\mathrm{u})}(k_{\mathrm{u}} \,|\, *) \bullet (k_{\mathrm{u}} - 1) \bullet B_{k_{\mathrm{u}}, *}$$

 $\mathbf{R} = \sum_{k_i, k_o} P^{(i)}(k_i, k_o | *) \bullet k_o \bullet B_{k_i, *}$

II. Directed, Uncorrelated -f(d+1) = f(d):

III. Mixed Directed and Undirected, Uncorrelate

 $\mathbf{R} = \sum_{i} \begin{bmatrix} P^{(u)}(\vec{k} \mid *) \bullet (k_{u} \mid -1) & P^{(i)}(\vec{k} \mid *) \bullet k_{u} \\ P^{(u)}(\vec{k} \mid *) \bullet k_{0} & P^{(i)}(\vec{k} \mid *) \bullet k_{0} \end{bmatrix}$

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 $\bullet B_k$

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 \mathfrak{R} II. Directed, Uncorrelated— $f(d+1) = \mathbf{f}(d)$:

$$\mathbf{R} = \sum_{k_{i},k_{o}} P^{(\mathbf{i})}(k_{i},k_{o} \,|\, *) \bullet k_{o} \bullet B_{k_{i},*}$$

III. Mixed Directed and Undirected, Uncorrelate

 $\mathbf{R} = \sum_{i=1}^{n} \begin{bmatrix} P^{(0)}(\vec{k} \mid *) \bullet (k_{0} - 1) - P^{(0)}(\vec{k} \mid *) \bullet k_{0} \\ P^{(0)}(\vec{k} \mid *) \bullet k_{0} \end{bmatrix} \bullet B_{k_{0}} \bullet B_{k_{0}}$

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Summary of contagion conditions for uncorrelated networks:

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 \mathfrak{R} II. Directed, Uncorrelated— $f(d+1) = \mathbf{f}(d)$:

$$\mathbf{R} = \sum_{k_{\mathrm{i}}, k_{\mathrm{o}}} P^{(\mathrm{i})}(k_{\mathrm{i}}, k_{\mathrm{o}} \,|\, *) \bullet k_{\mathrm{o}} \bullet B_{k_{\mathrm{i}}, *}$$

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🚳 III. Mixed Directed and Undirected, Uncorrelated—

$$\begin{bmatrix} f^{(\mathbf{u})}(d+1)\\ f^{(\mathbf{o})}(d+1) \end{bmatrix} = \mathbf{R} \begin{bmatrix} f^{(\mathbf{u})}(d)\\ f^{(\mathbf{o})}(d) \end{bmatrix}$$
$$\mathbf{R} = \sum_{\vec{k}} \begin{bmatrix} P^{(\mathbf{u})}(\vec{k} \mid *) \bullet (k_{\mathbf{u}} - 1) & P^{(\mathbf{i})}(\vec{k} \mid *) \bullet k_{\mathbf{u}}\\ P^{(\mathbf{u})}(\vec{k} \mid *) \bullet k_{\mathbf{o}} & P^{(\mathbf{i})}(\vec{k} \mid *) \bullet k_{\mathbf{o}} \end{bmatrix} \bullet B_{k_{\mathbf{u}}k_{\mathbf{i}},*}$$

Now have to think of transfer of infection from edges emanating from degree \vec{k}' nodes to edges emanating from degree \vec{k} nodes.

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Now have to think of transfer of infection from edges emanating from degree \vec{k}' nodes to edges emanating from degree \vec{k} nodes. Replace $P^{(i)}(\vec{k} | *)$ with $P^{(i)}(\vec{k} | \vec{k}')$ and so on.

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Now have to think of transfer of infection from edges emanating from degree \vec{k}' nodes to edges emanating from degree \vec{k} nodes. Replace $P^{(i)}(\vec{k} \mid *)$ with $P^{(i)}(\vec{k} \mid \vec{k}')$ and so on. Edge types are now more diverse beyond directed and undirected as originating node type matters.

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Now have to think of transfer of infection from edges emanating from degree *k*' nodes to edges emanating from degree *k* nodes.
Replace P⁽ⁱ⁾(*k* | *) with P⁽ⁱ⁾(*k* | *k*') and so on.
Edge types are now more diverse beyond directed and undirected as originating node type matters.
Sums are now over *k*'.

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Summary of contagion conditions for correlated networks:

$$R_{k_{\mathsf{u}}k_{\mathsf{u}}'} = P^{(\mathsf{u})}(k_{\mathsf{u}} \,|\, k_{\mathsf{u}}') \bullet (k_{\mathsf{u}}-1) \bullet B_{k_{\mathsf{u}}k_{\mathsf{u}}'}$$

 $R_{k_{1}k_{2}k'_{1}k'_{0}} = P^{(i)}(k_{i},k_{0} \mid k'_{1},k'_{0}) \bullet k_{0} \bullet B_{k_{1}k_{2}k'_{1}k'_{0}}$

 $P^{(\mathrm{u})}(\vec{k} \mid \vec{k}') \bullet (k_{\mathrm{u}} + 1) \mid P^{(\mathrm{l})}(\vec{k} \mid \vec{k}') \bullet k_{\mathrm{u}}$

 $P^{(0)}(k|k') \bullet k_0 \qquad P^{(0)}(k|k') \bullet k_0$

VI. Mixed Directed and Undirected, Correlated

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Summary of contagion conditions for correlated networks:

 $\begin{array}{l} \bigotimes \quad \text{IV. Undirected,} \\ \text{Correlated} - f_{k_{\mathsf{u}}}(d+1) = \sum_{k'_{\mathsf{u}}} R_{k_{\mathsf{u}}k'_{\mathsf{u}}} f_{k'_{\mathsf{u}}}(d) \end{array}$

$$R_{k_{\mathrm{u}}k_{\mathrm{u}}'} = P^{(\mathrm{u})}(k_{\mathrm{u}} \,|\, k_{\mathrm{u}}') \bullet (k_{\mathrm{u}} - 1) \bullet B_{k_{\mathrm{u}}k_{\mathrm{u}}'}$$

 $\mathbf{R}_{\vec{k}\vec{k}'} = \begin{bmatrix} P^{(0)}(\vec{k} \mid \vec{k}') \bullet (k_0 - 1) & P^{(0)}(\vec{k} \mid \vec{k}') \bullet k_0 \\ P^{(0)}(\vec{k} \mid \vec{k}') \bullet k_0 & P^{(0)}(\vec{k} \mid \vec{k}') \bullet k_0 \end{bmatrix}$

 $R_{k_{\mathrm{i}}k_{\mathrm{o}}k_{\mathrm{i}}'k_{\mathrm{o}}'} = P^{(\mathrm{i})}(k_{\mathrm{i}},k_{\mathrm{o}}\,|\,k_{\mathrm{i}}',k_{\mathrm{o}}') \bullet k_{\mathrm{o}} \bullet B_{k_{\mathrm{i}}k_{\mathrm{o}}k_{\mathrm{i}}'k_{\mathrm{o}}'}$

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• $B_{\vec{k}\vec{k}'}$

The the

Summary of contagion conditions for correlated networks:

$$R_{k_{\mathrm{u}}k_{\mathrm{u}}'} = P^{(\mathrm{u})}(k_{\mathrm{u}} \,|\, k_{\mathrm{u}}') \bullet (k_{\mathrm{u}} - 1) \bullet B_{k_{\mathrm{u}}k_{\mathrm{u}}'}$$

$$R_{k_{\mathrm{i}}k_{\mathrm{o}}k_{\mathrm{i}}'k_{\mathrm{o}}'} = P^{(\mathrm{i})}(k_{\mathrm{i}},k_{\mathrm{o}}\,|\,k_{\mathrm{i}}',k_{\mathrm{o}}') \bullet k_{\mathrm{o}} \bullet B_{k_{\mathrm{i}}k_{\mathrm{o}}k_{\mathrm{i}}'k_{\mathrm{o}}'}$$

🚳 VI. Mixed Directed and Undirected, Correlated—

$$\begin{bmatrix} f_{\vec{k}}^{(\mathrm{U})}(d+1) \\ f_{\vec{k}}^{(\mathrm{O})}(d+1) \end{bmatrix} = \sum_{k'} \mathbf{R}_{\vec{k}\vec{k}'} \begin{bmatrix} f_{\vec{k}'}^{(\mathrm{U})}(d) \\ f_{\vec{k}'}^{(\mathrm{O})}(d) \end{bmatrix}$$
$$\mathbf{R}_{\vec{k}\vec{k}'} = \begin{bmatrix} P^{(\mathrm{U})}(\vec{k} \mid \vec{k}') \bullet (k_{\mathrm{U}} - 1) & P^{(\mathrm{i})}(\vec{k} \mid \vec{k}') \bullet k_{\mathrm{U}} \\ P^{(\mathrm{U})}(\vec{k} \mid \vec{k}') \bullet k_{\mathrm{O}} & P^{(\mathrm{i})}(\vec{k} \mid \vec{k}') \bullet k_{\mathrm{O}} \end{bmatrix} \bullet B_{\vec{k}\vec{k}'}$$

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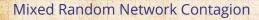
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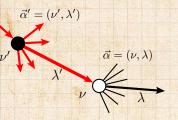
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Full generalization





$$f_{\vec{\alpha}}(d+1) = \sum_{\vec{\alpha}'} R_{\vec{\alpha}\vec{\alpha}'} f_{\vec{\alpha}'}(d)$$

 $R_{\vec{\alpha}\vec{\alpha}'}$ is the gain ratio matrix and has the form:

$$R_{\vec{\alpha}\vec{\alpha}'} = P_{\vec{\alpha}\vec{\alpha}'} \bullet k_{\vec{\alpha}\vec{\alpha}'} \bullet B_{\vec{\alpha}\vec{\alpha}'}$$

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 $\vec{\alpha}' = (\nu', \lambda')$

$$f_{\vec{\alpha}}(d+1) = \sum_{\vec{\alpha}'} R_{\vec{\alpha}\vec{\alpha}'} f_{\vec{\alpha}'}(d)$$

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 $k_{\vec{\alpha}\vec{\alpha}'} = \text{potential number of newly infected edges} \\ \text{of type } \lambda \text{ emanating from nodes of type } \nu.$

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\$\lambda_{\vec{a}\vec{a}\vec{a}'}\$ = potential number of newly infected edges of type \$\lambda\$ emanating from nodes of type \$\nu\$.
 \$\begin{aligned} B_{\vec{a}\vec{a}'}\$ = probability that a type \$\nu\$ node is eventually infected by a single infected type \$\lambda'\$ link arriving from a neighboring node of type \$\nu'\$.

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& k_{α̃α̃'} = potential number of newly infected edges of type λ emanating from nodes of type ν.
 & B_{α̃α̃'} = probability that a type ν node is eventually infected by a single infected type λ' link arriving from a neighboring node of type ν'.
 & Generalized contagion condition:

 $\max |\mu| : \mu \in \sigma(\mathbf{R}) > 1$

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 $Q_{\text{trig}} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet B_{k1} \bullet \left[1 - \left(1 - Q_{\text{trig}} \right)^{k-1} \right]$

 $P_{\mathrm{trig}} = S_{\mathrm{trig}} = \sum P_k \bullet \left[1 - (1 - Q_{\mathrm{trig}})^k\right],$

Equivalent to result found via the eldritch route of generating functions.

Generating functions arguably make some kinds of calculations easier (but perhaps we don't care abou component sizes that much).

On the other hand, a plainspoken physical argumen helps us generalize to correlated networks more eas

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Two good things:

$$\begin{split} Q_{\mathrm{trig}} &= \sum_{k=0}^{\infty} \frac{k P_k}{\langle k \rangle} \bullet B_{k1} \bullet \left[1 - \left(1 - Q_{\mathrm{trig}} \right)^{k-1} \right], \\ P_{\mathrm{trig}} &= S_{\mathrm{trig}} = \sum_k P_k \bullet \left[1 - (1 - Q_{\mathrm{trig}})^k \right]. \end{split}$$

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🗞 Two good things:

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generating functions.

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Summary of triggering probabilities for uncorrelated networks: ^[3]

 $\begin{array}{l} \diamondsuit \\ \textbf{I. Undirected, Uncorrelated} \\ Q_{\mathsf{trig}} = \sum_{k'_{\mathsf{u}}} P^{(\mathsf{u})}(k'_{\mathsf{u}} \mid \cdot) B_{k'_{\mathsf{u}}1} \left[1 - (1 - Q_{\mathsf{trig}})^{k'_{\mathsf{u}}-1} \right] \end{array}$

$$P_{\mathrm{trig}} = S_{\mathrm{trig}} = \sum_{k_{\mathrm{u}}'} P(k_{\mathrm{u}}') \left[1 - (1 - Q_{\mathrm{trig}})^{k_{\mathrm{u}}'}\right]$$

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Summary of triggering probabilities for uncorrelated networks: ^[3]

 $\begin{array}{l} \diamondsuit \\ \textbf{I. Undirected, Uncorrelated} \\ Q_{\mathsf{trig}} = \sum_{k'_{\mathsf{u}}} P^{(\mathsf{u})}(k'_{\mathsf{u}} \mid \cdot) B_{k'_{\mathsf{u}} \mathsf{1}} \left[1 - (1 - Q_{\mathsf{trig}})^{k'_{\mathsf{u}} - 1} \right] \end{array}$

$$P_{\mathrm{trig}} = S_{\mathrm{trig}} = \sum_{k_{\mathrm{u}}'} P(k_{\mathrm{u}}') \left[1 - (1 - Q_{\mathrm{trig}})^{k_{\mathrm{u}}'}\right]$$

 $\begin{cases} \text{II. Directed, Uncorrelated} \\ Q_{\text{trig}} = \sum_{k'_{i},k'_{o}} P^{(\text{u})}(k'_{i},k'_{o}|\cdot)B_{k'_{i}1}\left[1 - (1 - Q_{\text{trig}})^{k'_{o}}\right] \end{cases}$

$$S_{\mathsf{trig}} = \sum_{k_{\mathsf{i}}', k_{\mathsf{o}}'} P(k_{\mathsf{i}}', k_{\mathsf{o}}') \left[1 - (1 - Q_{\mathsf{trig}})^{k_{\mathsf{o}}'} \right]$$

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Summary of triggering probabilities for uncorrelated networks:

🚳 III. Mixed Directed and Undirected, Uncorrelated—

$$Q_{\text{trig}}^{(\text{u})} = \sum_{\vec{k}'} P^{(\text{u})}(\vec{k}'|\cdot) B_{\vec{k}'1} \left[1 - (1 - Q_{\text{trig}}^{(\text{u})})^{k'_{\text{u}}-1} (1 - Q_{\text{trig}}^{(\text{o})})^{k'_{\text{o}}} \right]$$

$$Q_{\rm trig}^{\rm (o)} = \sum_{\vec{k}'} P^{({\rm i})}(\vec{k}'|\cdot) B_{\vec{k}'1} \left[1 - (1 - Q_{\rm trig}^{\rm (u)})^{k'_{\rm u}} (1 - Q_{\rm trig}^{\rm (o)})^{k'_{\rm o}} \right]$$

$$S_{\rm trig} = \sum_{\vec{k}'} P(\vec{k}') \left[1 - (1 - Q_{\rm trig}^{\rm (u)})^{k'_{\rm u}} (1 - Q_{\rm trig}^{\rm (o)})^{k'_{\rm o}} \right]$$

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Summary of triggering probabilities for correlated networks:

 $\begin{array}{l} & \fbox{IV. Undirected, Correlated} - Q_{\mathrm{trig}}(k_{\mathrm{u}}) = \\ & \sum_{k'_{\mathrm{u}}} P^{(\mathrm{u})}(k'_{\mathrm{u}} \mid k_{\mathrm{u}}) B_{k'_{\mathrm{u}}1} \left[1 - (1 - Q_{\mathrm{trig}}(k'_{\mathrm{u}}))^{k'_{\mathrm{u}}-1} \right] \end{array}$

$$S_{\mathrm{trig}} = \sum_{k_{\mathrm{u}}'} P(k_{\mathrm{u}}') \left[1 - (1 - Q_{\mathrm{trig}}(k_{\mathrm{u}}'))^{k_{\mathrm{u}}'} \right]$$

V. Directed, Correlated – $Q_{\text{trig}}(k_i, k_o) =$ $\sum_{i=0}^{n(0)} (k'_i k'_i k_i k_i) B_{i,i} [1 - (1 - O_{i,i})]$ Directed random networks

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Summary of triggering probabilities for correlated networks:

 $\begin{array}{l} & \fbox{IV. Undirected, Correlated} - Q_{\mathrm{trig}}(k_{\mathrm{u}}) = \\ & \sum_{k'_{\mathrm{u}}} P^{(\mathrm{u})}(k'_{\mathrm{u}} \mid k_{\mathrm{u}}) B_{k'_{\mathrm{u}}1} \left[1 - (1 - Q_{\mathrm{trig}}(k'_{\mathrm{u}}))^{k'_{\mathrm{u}}-1} \right] \end{array}$

$$S_{\mathrm{trig}} = \sum_{k_{\mathrm{u}}'} P(k_{\mathrm{u}}') \left[1 - (1 - Q_{\mathrm{trig}}(k_{\mathrm{u}}'))^{k_{\mathrm{u}}'} \right]$$

 $\begin{array}{l} & \& \\ & \bigvee \text{Directed, Correlated} - Q_{\text{trig}}(k_{\text{i}}, k_{\text{o}}) = \\ & \sum_{k_{i}', k_{\text{o}}'} P^{(\text{u})}(k_{\text{i}}', k_{\text{o}}'|\,k_{\text{i}}, k_{\text{o}}) B_{k_{i}'1} \left[1 - (1 - Q_{\text{trig}}(k_{\text{i}}', k_{\text{o}}'))^{k_{\text{o}}'} \right] \end{array}$

$$S_{\mathrm{trig}} = \sum_{k_{\mathrm{i}}^{\prime}, k_{\mathrm{o}}^{\prime}} P(k_{\mathrm{i}}^{\prime}, k_{\mathrm{o}}^{\prime}) \left[1 - (1 - Q_{\mathrm{trig}}(k_{\mathrm{i}}^{\prime}, k_{\mathrm{o}}^{\prime}))^{k_{\mathrm{o}}^{\prime}} \right]$$

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Summary of triggering probabilities for correlated networks:

🗞 VI. Mixed Directed and Undirected, Correlated—

$$Q_{\rm trig}^{\rm (u)}(\vec{k}) = \sum_{\vec{k}'} P^{\rm (u)}(\vec{k}'|\,\vec{k}) B_{\vec{k}'1} \left[1 - (1 - Q_{\rm trig}^{\rm (u)}(\vec{k}'))^{k'_{\rm u}-1} (1 - Q_{\rm trig}^{\rm (d)}(\vec{k}'))^{k'_{\rm u}-1} (1 - Q_{\rm trig}^{\rm (d)}(\vec{k}'))$$

$$Q_{\rm trig}^{\rm (o)}(\vec{k}) = \sum_{\vec{k}'} P^{\rm (i)}(\vec{k}'|\vec{k}) B_{\vec{k}'1} \left[1 - (1 - Q_{\rm trig}^{\rm (u)}(\vec{k}'))^{k'_{\rm u}} (1 - Q_{\rm trig}^{\rm (o)}(\vec{k}'))^{k'_{\rm u}} R^{\rm (o)}(\vec{k}') \right]^{k'_{\rm u}}$$

$$S_{\rm trig} = \sum_{\vec{k}\,'} P(\vec{k}') \left[1 - (1 - Q^{\rm (u)}_{\rm trig}(\vec{k}'))^{k'_{\rm u}} (1 - Q^{\rm (o)}_{\rm trig}(\vec{k}'))^{k'_{\rm o}} \right]$$

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Mixed, correlated random networks with undirected and directed edges form natural inclusive generalization of purely undirected and purely directed random networks.

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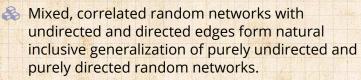
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Spreading conditions and triggering probabilities of contagion processes can be determined using a direct, physical approach.



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Mixed, correlated random networks with undirected and directed edges form natural inclusive generalization of purely undirected and purely directed random networks.

- Spreading conditions and triggering probabilities of contagion processes can be determined using a direct, physical approach.
- These conditions can be generalized to arbitrary random networks with arbitrary node and edge types.



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