Generating Functions and Networks

Complex Networks | @networksvox
CSYS/MATH 303, Spring, 2016

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Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

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Outline


THE JOHN DORY
THE SUN
THE NETWORKS COMPLEX
"I am the Monarch of the Sea."
THE ARK OF THE RANDOM NETWORK
THE NETWORKS FREE OF SCALE
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE UNKNOWN MECHANISM
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
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THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
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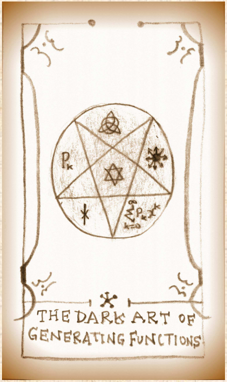
THE DARK ART OF GENERATING FUNCTIONS


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Outline

Generating Functions

- Definitions
- Basic Properties
- Giant Component Condition
- Component sizes
- Useful results
- Size of the Giant Component
- Average Component Size

References

Generatingfunctionology [1]

- Idea: Given a sequence $a_0, a_1, a_2, \ldots,$ associate each element with a distinct function or other mathematical object.
- Well-chosen functions allow us to manipulate sequences and retrieve sequence elements.

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- The generating function (g.f.) for a sequence $\{a_n\}$ is

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- Roughly: transforms a vector in R^∞ into a function defined on R^1.
- Related to Fourier, Laplace, Mellin, ...

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Simple examples:

Rolling dice and flipping coins:

- $p_k^{(⚂)} = \mathbf{Pr}(\text{throwing a } k) = 1/6$ where $k = 1, 2, \ldots, 6$.

$$F^{(⚂)}(x) = \sum_{k=1}^{6} p_k^{(⚂)} x^k = \frac{1}{6}(x + x^2 + x^3 + x^4 + x^5 + x^6).$$

- $p_0^{(\text{coin})} = \mathbf{Pr}(\text{head}) = 1/2$, $p_1^{(\text{coin})} = \mathbf{Pr}(\text{tail}) = 1/2$.

$$F^{(\text{coin})}(x) = p_0^{(\text{coin})} x^0 + p_1^{(\text{coin})} x^1 = \frac{1}{2}(1 + x).$$

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- Take a degree distribution with exponential decay:

$$P_k = ce^{-\lambda k}$$

where geometricsumfully, we have $c = 1 - e^{-\lambda}$

- The generating function for this distribution is

$$F(x) = \sum_{k=0}^{\infty} P_k x^k = \sum_{k=0}^{\infty} c e^{-\lambda k} x^k = \frac{c}{1 - xe^{-\lambda}}.$$

- Notice that $F(1) = c/(1 - e^{-\lambda}) = 1$.
- For probability distributions, we must always have $F(1) = 1$ since

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Properties:

- Average degree:

$$\langle k \rangle = \sum_{k=0}^{\infty} k P_k = \sum_{k=0}^{\infty} k P_k x^{k-1} \Big|_{x=1}$$

$$= \frac{\mathrm{d}}{\mathrm{d}x} F(x) \Big|_{x=1} = F'(1)$$

- In general, many calculations become simple, if a little abstract.
- For our exponential example:

$$F'(x) = \frac{(1-e^{-\lambda})e^{-\lambda}}{(1-xe^{-\lambda})^2}.$$

So: $\langle k \rangle = F'(1) = \frac{e^{-\lambda}}{(1-e^{-\lambda})}.$

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Useful pieces for probability distributions:

- Normalization:

$$F(1) = 1$$

- First moment:

$$\langle k \rangle = F'(1)$$

- Higher moments:

$$\langle k^n \rangle = \left(x \frac{\mathsf{d}}{\mathsf{d}x} \right)^n F(x) \bigg|_{x=1}$$

- kth element of sequence (general):

$$P_k = \frac{1}{k!} \frac{\mathsf{d}^k}{\mathsf{d}x^k} F(x) \bigg|_{x=0}$$

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A beautiful, fundamental thing:

- The generating function for the sum of two random variables

$$W = U + V$$

is

$$F_W(x) = F_U(x) F_V(x).$$

- Convolve yourself with Convolutions: Insert question from assignment 5 .
- Try with die and coin p.g.f.'s.
  1. Add two coins (tail=0, head=1).
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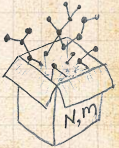


Edge-degree distribution

- Recall our condition for a giant component:

$$\langle k \rangle_R = \frac{\langle k^2 \rangle - \langle k \rangle}{\langle k \rangle} > 1.$$

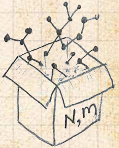
- Let's re-express our condition in terms of generating functions.
- We first need the g.f. for R_k.
- We'll now use this notation:

  $F_P(x)$ is the g.f. for P_k.
  $F_R(x)$ is the g.f. for R_k.

- Giant component condition in terms of g.f. is:

$$\langle k \rangle_R = F_R'(1) > 1.$$

- Now find how F_R is related to F_P ...



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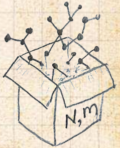

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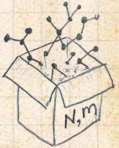

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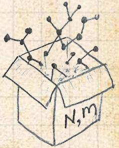

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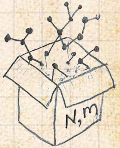

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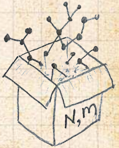

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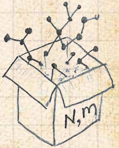

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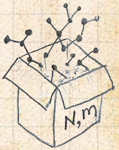

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$$F_R(x) = \sum_{k=0}^{\infty} R_k x^k = \sum_{k=0}^{\infty} \frac{(k+1)P_{k+1}}{\langle k \rangle} x^k.$$

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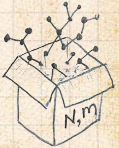

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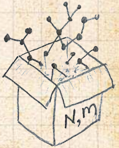

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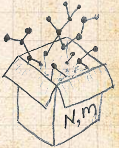

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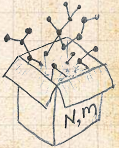

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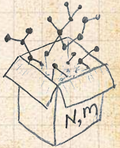

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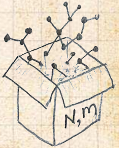

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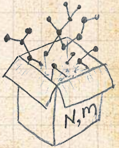

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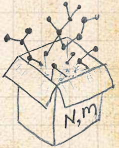

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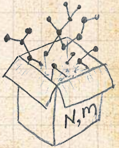

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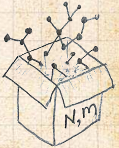

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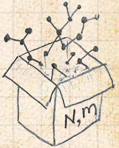

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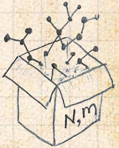

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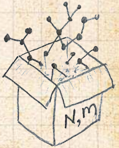
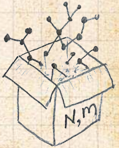


Outline



Size distributions

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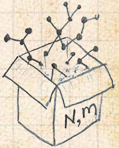
Definitions:

- π_n = probability that a random node belongs to a finite component of size $n < \infty$.
- ρ_n = probability that a random end of a random link leads to a finite subcomponent of size $n < \infty$.

Local-global connection:

$$P_k, R_k \Leftrightarrow \pi_n, \rho_n$$

neighbors ⇔ components



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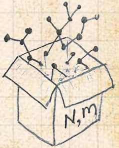

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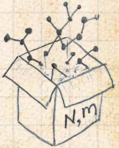

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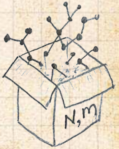

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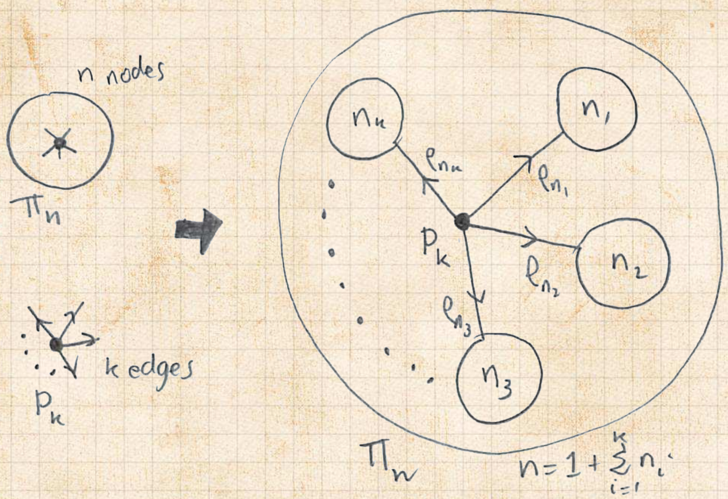

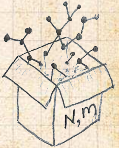

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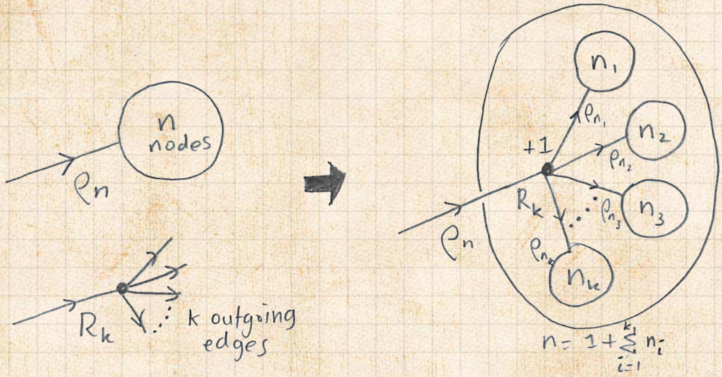

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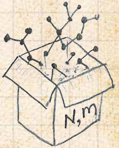

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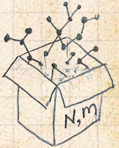

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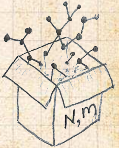

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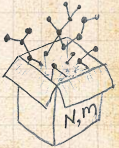


G.f.'s for component size distributions:

- $$F_\pi(x) = \sum_{n=0}^{\infty} \pi_n x^n \text{ and } F_\rho(x) = \sum_{n=0}^{\infty} \rho_n x^n$$

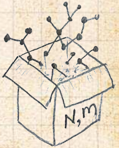
The largest component:

- Subtle key: $F_\pi(1)$ is the probability that a node belongs to a finite component.
- Therefore: $S_1 = 1 - F_\pi(1)$.

Our mission, which we accept:

- Determine and connect the four generating functions

$$F_P, F_R, F_\pi, \text{ and } F_\rho.$$



Outline

Generating Functions
- Definitions
- Basic Properties
- Giant Component Condition
- Component sizes
- Useful results
- Size of the Giant Component
- Average Component Size

References

Useful results we'll need for g.f.'s

Sneaky Result 1:

- Consider two random variables U and V whose values may be $0, 1, 2, \ldots$
- Write probability distributions as U_k and V_k and g.f.'s as F_U and F_V.
- SR1: If a third random variable is defined as

$$W = \sum_{i=1}^{U} V^{(i)} \text{ with each } V^{(i)} \stackrel{d}{=} V$$

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$$\boxed{F_W(x) = F_U\left(F_V(x)\right)}$$

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THE JOHN DORY
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THE NETWORKS COMPLEX
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THE NETWORKS FREE OF SCALE
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE UNKNOWN MECHANISM
THE BRAID OR THE BRANCH
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THE CHURCH OF QUARTEROLOGY
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THE TRUTHICIDE
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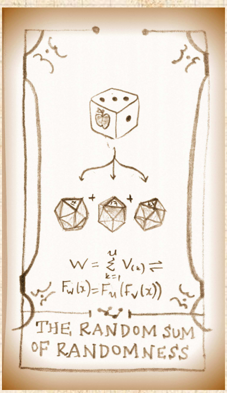
THE ARK OF THE RANDOM NETWORK
THE DARK ART OF GENERATING FUNCTIONS


$W = \sum_{k=1}^{U} V_{(k)} \rightleftarrows$
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THE RANDOM SUM OF RANDOMNESS


THE DARK ART OF GENERATING FUNCTIONS

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Write probability that variable W has value k as W_k.

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$$= \sum_{j=0}^{\infty} U_j \left(F_V(x)\right)^j$$

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- Alternate, groovier proof in the accompanying assignment.

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Sneaky Result 2:

- Start with a random variable U with distribution U_k $(k = 0, 1, 2, \ldots)$
- SR2: If a second random variable is defined as

$$V = U + 1 \text{ then } F_V(x) = xF_U(x)$$

- Reason: $V_k = U_{k-1}$ for $k \geq 1$ and $V_0 = 0$.
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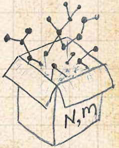
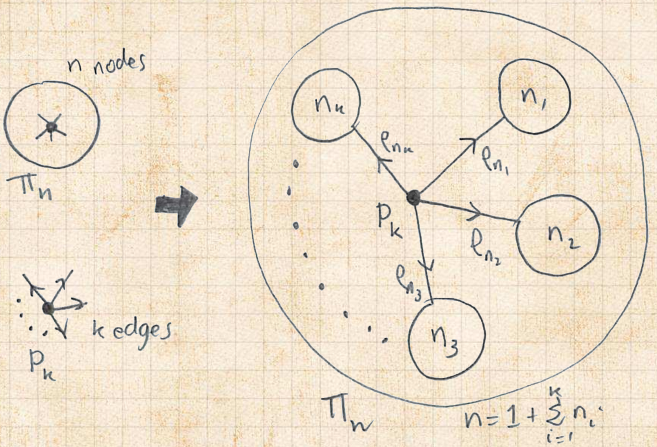


Connecting generating functions:

- Goal: figure out forms of the component generating functions, F_π and F_ρ.



- Relate π_n to P_k and ρ_n through one step of recursion.

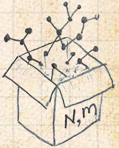
Connecting generating functions:

- π_n = probability that a random node belongs to a finite component of size n

$$= \sum_{k=0}^{\infty} P_k \times \Pr\begin{pmatrix} \text{sum of sizes of subcomponents} \\ \text{at end of } k \text{ random links} = n-1 \end{pmatrix}$$

- Therefore: $F_\pi(x) = x \, F_P(F_\rho(x))$

- Extra factor of x accounts for random node itself.



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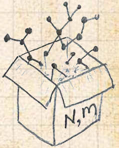

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- Therefore: $$F_\pi(x) = \underbrace{x}_{\text{SR2}} \underbrace{F_P(F_\rho(x))}_{\text{SR1}}$$

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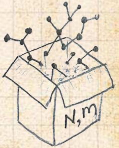

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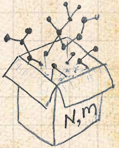

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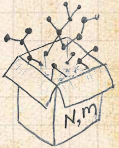

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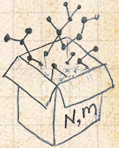

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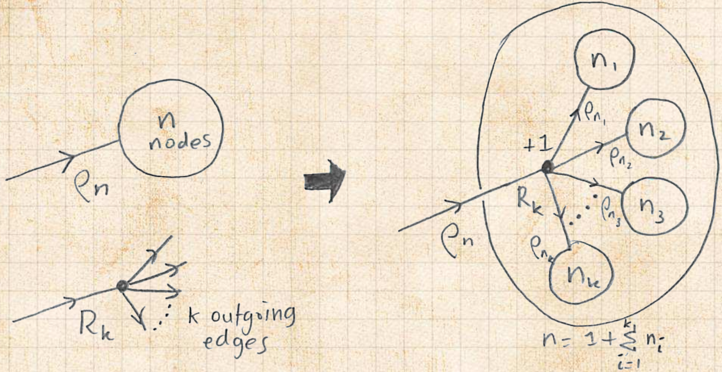

- Relate ρ_n to R_k and ρ_n through one step of recursion.

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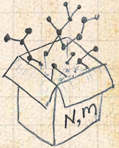

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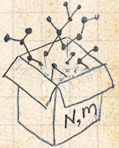

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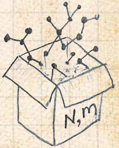

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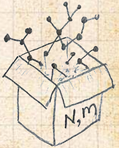

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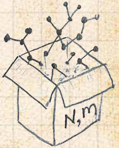

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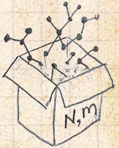

The University of Vermont

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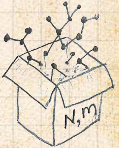

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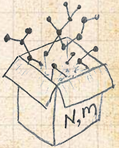

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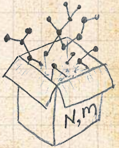

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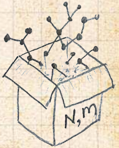

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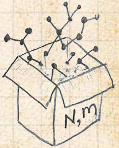

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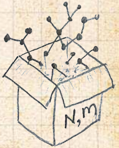

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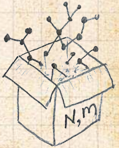

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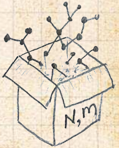

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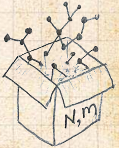

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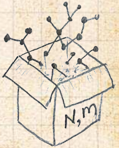

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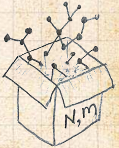

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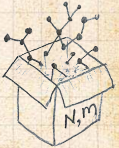

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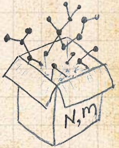

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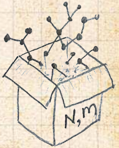

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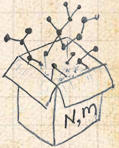

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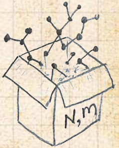

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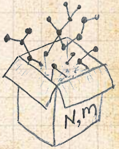

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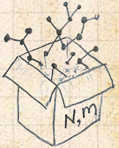

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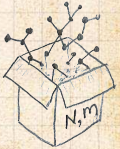

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$$= e^{-\langle k \rangle (1-x)} = F_P(x) \qquad \text{...aha!}$$

- RHS's of our two equations are the same.
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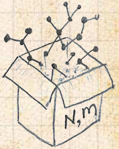

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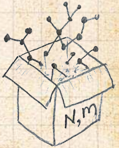

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- We are down to
  $F_\pi(x) = xF_R(F_\pi(x))$ and $F_R(x) = e^{-\langle k \rangle (1-x)}$.
- $$\therefore F_\pi(x) = x e^{-\langle k \rangle (1 - F_\pi(x))}$$
- We're first after $S_1 = 1 - F_\pi(1)$ so set $x = 1$ and replace $F_\pi(1)$ by $1 - S_1$:

$$1 - S_1 = e^{-\langle k \rangle S_1}$$

Or: $\langle k \rangle = \frac{1}{S_1} \ln \frac{1}{1 - S_1}$

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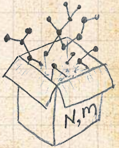

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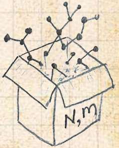

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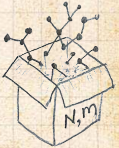

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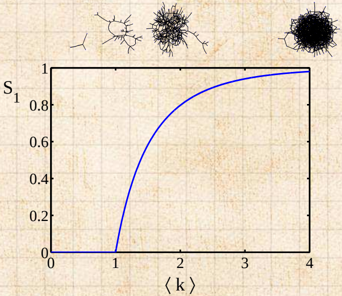

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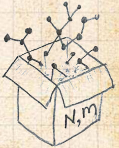

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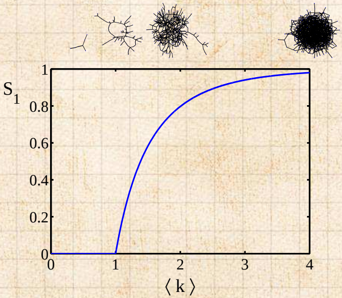

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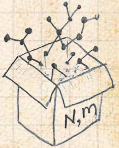

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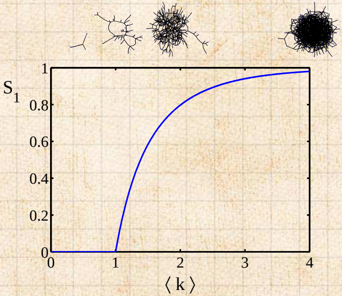

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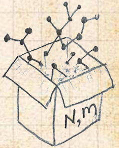

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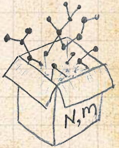

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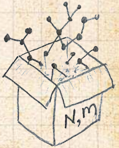

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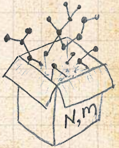

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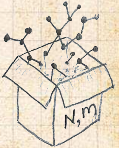

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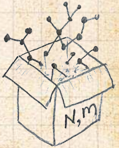

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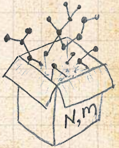

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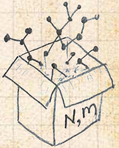

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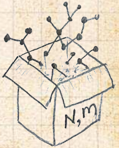

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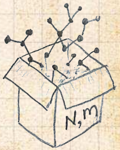

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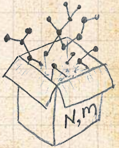

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$$P_k = \frac{1}{2}\delta_{k1} + \frac{1}{2}\delta_{k3}.$$

- We find (two ways): $R_k = \frac{1}{4}\delta_{k0} + \frac{3}{4}\delta_{k2}.$
- A giant component exists because: $\langle k \rangle_R = 0 \times 1/4 + 2 \times 3/4 = 3/2 > 1.$
- Generating functions for P_k and R_k:

$$F_P(x) = \frac{1}{2}x + \frac{1}{2}x^3 \text{ and } F_R(x) = \frac{1}{4}x^0 + \frac{3}{4}x^2$$

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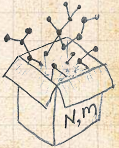

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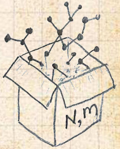

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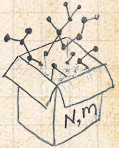

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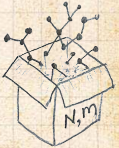

A joyful example:

$$P_k = \frac{1}{2}\delta_{k1} + \frac{1}{2}\delta_{k3}.$$

- We find (two ways): $R_k = \frac{1}{4}\delta_{k0} + \frac{3}{4}\delta_{k2}$.
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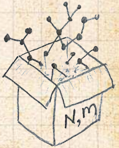

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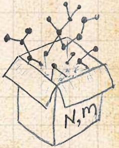

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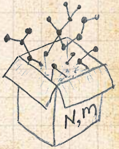
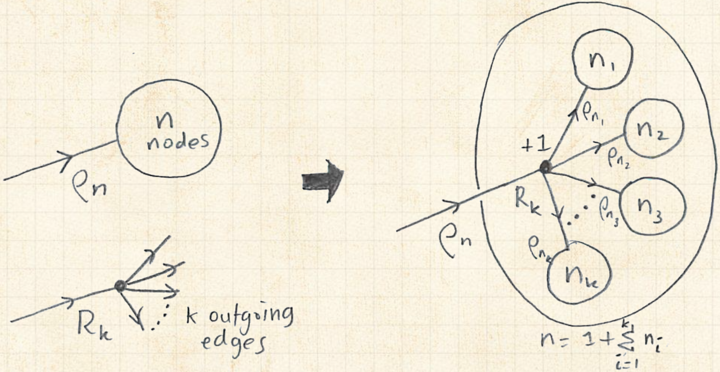


Find $F_\rho(x)$ first:

- We know:

$$F_\rho(x) = xF_R\left(F_\rho(x)\right).$$



Sticking things in things, we have:

$$F_\rho(x) = x\left(\frac{1}{4} + \frac{3}{4}\left[F_\rho(x)\right]^2\right).$$

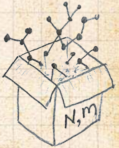
Rearranging:

$$3x\left[F_\rho(x)\right]^2 - 4F_\rho(x) + x = 0.$$

Please and thank you:

$$F_\rho(x) = \frac{2}{3x}\left(1 \pm \sqrt{1 - \frac{3}{4}x^2}\right)$$

- Time for a Taylor series expansion.
- The promise: non-negative powers of x with non-negative coefficients.
- First: which sign do we take?



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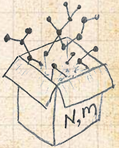

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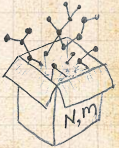

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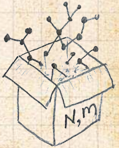

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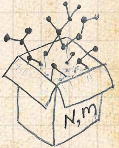

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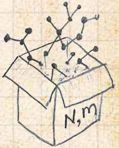

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Thinking about the limit $x \to 0$ in

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we see that the positive sign solution blows to smithereens, and the negative one is okay.

So we must have:

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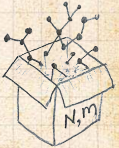
We can now deploy the Taylor expansion:

$$(1+z)^\theta = \binom{\theta}{0}z^0 + \binom{\theta}{1}z^1 + \binom{\theta}{2}z^2 + \binom{\theta}{2}z^3 + \ldots$$



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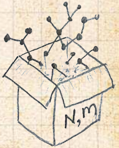

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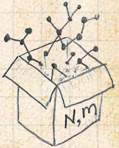

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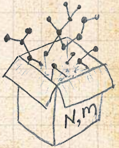


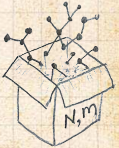
Let's define a binomial for arbitrary θ and $k = 0, 1, 2, \ldots$:

$$\binom{\theta}{k} = \frac{\Gamma(\theta + 1)}{\Gamma(k+1)\Gamma(\theta - k + 1)}$$

For $\theta = \frac{1}{2}$, we have:

$$(1+z)^{\frac{1}{2}} = \binom{\frac{1}{2}}{0} z^0 + \binom{\frac{1}{2}}{1} z^1 + \binom{\frac{1}{2}}{2} z^2 + \ldots$$

Note: $(1+z)^\theta \sim 1 + \theta z$ always.



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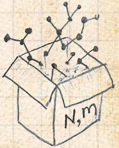

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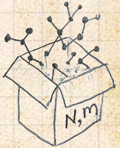

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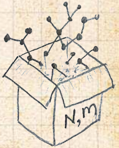

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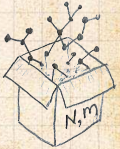

- Totally psyched, we go back to here:

$$F_\rho(x) = \frac{2}{3x}\left(1 - \sqrt{1 - \frac{3}{4}x^2}\right).$$

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$$F_\rho(x) = \sum_{n=0}^{\infty} \rho_n x^n =$$

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- We can now find $F_\pi(x)$ with:

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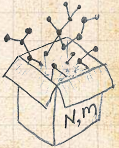

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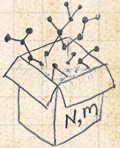

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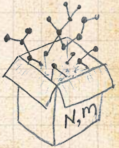

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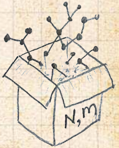

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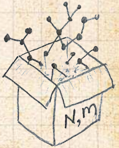

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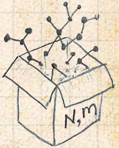

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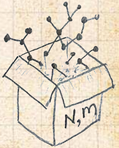

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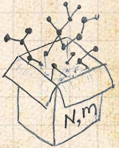

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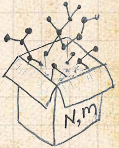

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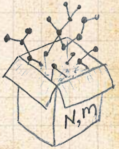

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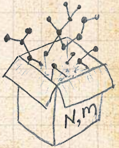

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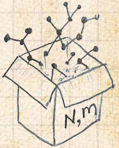

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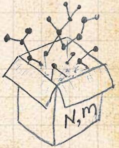

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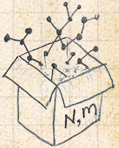
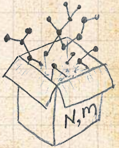


Outline



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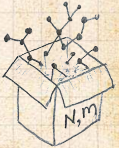

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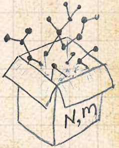

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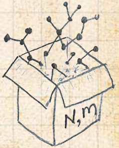

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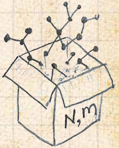

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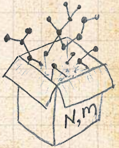

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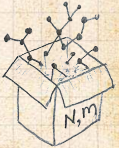

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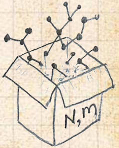

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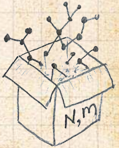

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- Use fact that $F_P = F_R$ and $F_\pi = F_\rho$.
- Two differentiated equations reduce to only one:

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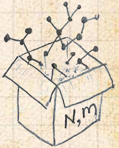
Rearrange: $$F'_\pi(x) = \frac{F_P(F_\pi(x))}{1 - xF'_P(F_\pi(x))}$$

- Simplify denominator using $F'_P(x) = \langle k \rangle F_P(x)$
- Replace $F_P(F_\pi(x))$ using $F_\pi(x) = xF_P(F_\pi(x))$.
- Set $x = 1$ and replace $F_\pi(1)$ with $1 - S_1$.

End result: $$\langle n \rangle = F'_\pi(1) = \frac{(1 - S_1)}{1 - \langle k \rangle (1 - S_1)}$$



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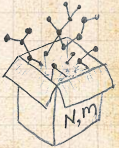

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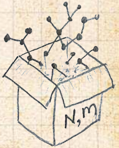

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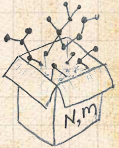

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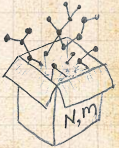

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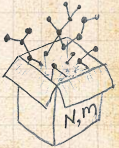

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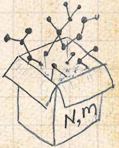

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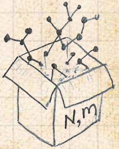

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- Our result for standard random networks:

$$\langle n \rangle = F'_\pi(1) = \frac{(1 - S_1)}{1 - \langle k \rangle (1 - S_1)}$$

- Recall that $\langle k \rangle = 1$ is the critical value of average degree for standard random networks.
- Look at what happens when we increase $\langle k \rangle$ to 1 from below.
- We have $S_1 = 0$ for all $\langle k \rangle < 1$ so

$$\langle n \rangle = \frac{1}{1 - \langle k \rangle}$$

- This blows up as $\langle k \rangle \to 1$.
- Reason: we have a power law distribution of component sizes at $\langle k \rangle = 1$.
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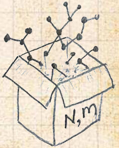

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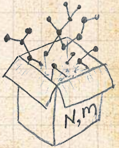

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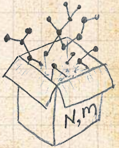

The University of Vermont

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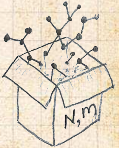

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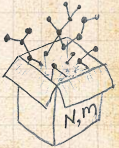

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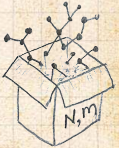

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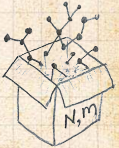

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Average component size

- Limits of $\langle k \rangle = 0$ and ∞ make sense for

$$\langle n \rangle = F'_{\pi}(1) = \frac{(1 - S_1)}{1 - \langle k \rangle (1 - S_1)}$$

- As $\langle k \rangle \to 0$, $S_1 = 0$, and $\langle n \rangle \to 1$.
- All nodes are isolated.
- As $\langle k \rangle \to \infty$, $S_1 \to 1$ and $\langle n \rangle \to 0$.
- No nodes are outside of the giant component.

Extra on largest component size:

- For $\langle k \rangle = 1$, $S_1 \sim N^{2/3}/N$.
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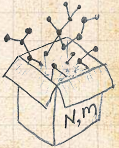

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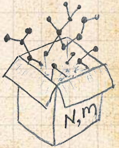

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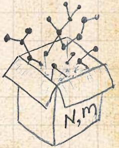

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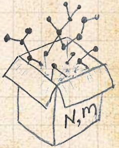

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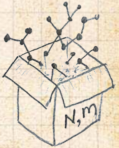

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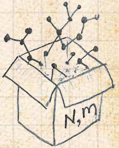

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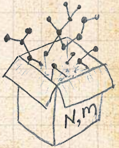

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$$\langle n \rangle = F'_\pi(1) = F_P\left(F_\rho(1)\right) + F'_\rho(1) F'_P\left(F_\rho(1)\right)$$

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$$F_P(x) = \frac{1}{2}x + \frac{1}{2}x^3 \text{ and } F_R(x) = \frac{1}{4}x^0 + \frac{3}{4}x^2.$$

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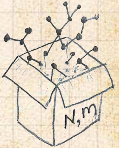

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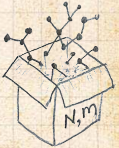

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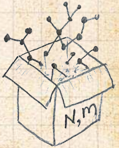

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- Place stick between teeth, and recall that we have:

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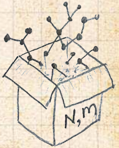

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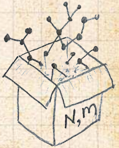

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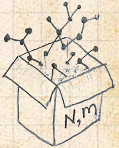

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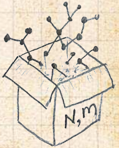

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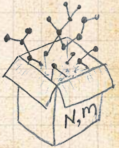

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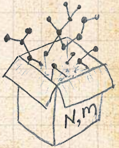

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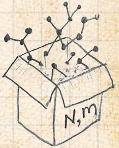

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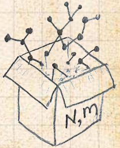

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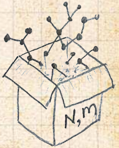

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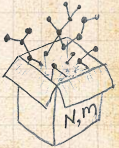

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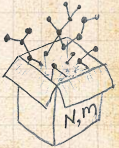

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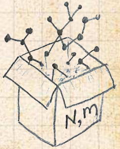
- We'll find generating functions useful for contagion.
- But we'll also see that more direct, physics-bearing calculations are possible.



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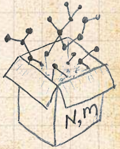

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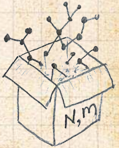

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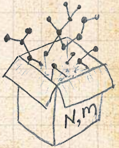
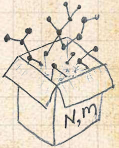


Neural reboot (NR):

Elevation:



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