Generalized Contagion Complex Networks | @networksvox CSYS/MATH 303, Spring, 2016

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont













COCONUTS

Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Appendix

References







Licensed under the Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License.

290 1 of 63

These slides are brought to you by:

Sealie & Lambie Productions

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

20f63

Outline

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 3 of 63

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix

References



VERMONT

Basic questions about contagion

How many types of contagion are there? How can we categorize real-world contagions? Can we connect models of disease-like and soc contagion?

Focus: mean field models.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix

References



VERMONT 8

Basic questions about contagion
 How many types of contagion are there?
 How can we categorize real-world contagion
 Can we connect models of disease-like and s contagion?
 Encus, mean field models

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 4 of 63

Basic questions about contagion

- How many types of contagion are there?
 How can we categorize real-world contagions?
 - Can we connect models of disease-like and socia contagion? Focus: mean field models.

Basic questions about contagion

How many types of contagion are there?
How can we categorize real-world contagions?
Can we connect models of disease-like and social contagion?

cus: mean field models.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix References

CocoNuTs Complex Networks Onetworksvox



20 4 of 63

Basic questions about contagion

- How many types of contagion are there?
- How can we categorize real-world contagions?
 - Can we connect models of disease-like and social contagion?
- Focus: mean field models.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix References

CocoNuTs Complex Networks Onetworksvox



The standard SIR model ^[10] = basic model of disease contag

> S(t) + I(t) + R(t) = 1Presumes random interactions (mass-actic principle) Interactions are independent (no memory)

Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





The standard SIR model ^[10]

🚳 = basic model of disease contagion

S(t) + I(t) + R(t) = 1Presumes random interactions (mass-actic principle)

Interactions are independent (no memory Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





The standard SIR model [10]

= basic model of disease contagion
 Three states:

S(t) + I(t) + R(t) = 1Presumes random interactions (mass-actio principle)

Interactions are independent (no memory Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





The standard SIR model [10]

- 🗞 = basic model of disease contagion
- \lambda Three states:
 - 1. S = Susceptible

Presumes random interactions (mass-actio principle)

Interactions are independent (no memory Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





The standard SIR model [10]

- 🚳 = basic model of disease contagion
- 🚓 Three states:
 - S = Susceptible
 I = Infective/Infectious

S(t) + I(t) + R(t) = 1Presumes random interactions (mass-actio principle)

Interactions are independent (no memory Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





The standard SIR model [10]

- 🚳 = basic model of disease contagion
- 🚳 Three states:
 - 1. S = Susceptible
 - 2. I = Infective/Infectious
 - 3. R = Recovered or Removed or Refractory

Presumes random interactions (mass-actio principle)

Interactions are independent (no memory Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

- Interdependent interaction models
- Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





The standard SIR model [10]

- 🚳 = basic model of disease contagion
- 🚳 Three states:
 - 1. S = Susceptible
 - 2. I = Infective/Infectious
 - 3. R = Recovered or Removed or Refractory

Presumes random interactions (mass-actio principle)

Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





The standard SIR model [10]

- 🚳 = basic model of disease contagion
- 🚳 Three states:
 - 1. S = Susceptible
 - 2. I = Infective/Infectious
 - 3. R = Recovered or Removed or Refractory

$$S(t) + I(t) + R(t) = 1$$

Presumes random interactions (mass-actio principle)

Interactions are independent (no memory Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

- Interdependent interaction models
- Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





The standard SIR model [10]

- 🚳 = basic model of disease contagion
- \lambda Three states:
 - 1. S = Susceptible
 - 2. I = Infective/Infectious
 - 3. R = Recovered or Removed or Refractory

 Presumes random interactions (mass-action principle)

Interactions are independent (no memory Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

- Interdependent interaction models
- Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





The standard SIR model [10]

- 🚳 = basic model of disease contagion
- \lambda Three states:
 - 1. S = Susceptible
 - 2. I = Infective/Infectious
 - 3. R = Recovered or Removed or Refractory

$${ { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } {$$

Presumes random interactions (mass-action principle)

lnteractions are independent (no memory)

Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





The standard SIR model [10]

- 🚳 = basic model of disease contagion
- \lambda Three states:
 - 1. S = Susceptible
 - 2. I = Infective/Infectious
 - 3. R = Recovered or Removed or Refractory

$${ { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } { l } {$$

- Presumes random interactions (mass-action principle)
- lnteractions are independent (no memory)
 - Discrete and continuous time versions

COcoNuTS

Introduction

Independent Interaction models

- Interdependent interaction models
- Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Discrete time automata example:



COCONUTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

200 6 of 63

Discrete time automata example:



Transition Probabilities:

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 6 of 63

Discrete time automata example:



Transition Probabilities:

 β for being infected given contact with infected

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Discrete time automata example:



Transition Probabilities:

 β for being infected given contact with infected r for recovery

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix

References





200 6 of 63

Discrete time automata example:



Transition Probabilities:

 β for being infected given contact with infected r for recovery ρ for loss of immunity COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix





Original models attributed to

1920's: Reed and Frost 1920's/1930's: Kermack and McKendrick Coupled differential equations with a massprinciple

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 7 of 63

Original models attributed to

A 1920's: Reed and Frost

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

200 7 of 63

Original models attributed to

1920's: Reed and Frost
 1920's/1930's: Kermack and McKendrick^[7, 9, 8]
 Coupled differential equations with a mass-action



COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Original models attributed to

- 🚳 1920's: Reed and Frost
- 🗞 1920's/1930's: Kermack and McKendrick ^[7, 9, 8]
- Coupled differential equations with a mass-action principle



COCONUTS

Independent

Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Differential equations for continuous model

$$\frac{d}{dt}S = -\beta IS + \rho R$$
$$\frac{d}{dt}I = \beta IS - rI$$
$$\frac{d}{dt}R = rI - \rho R$$

 β , r, and ρ are now rates.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Differential equations for continuous model

$$\frac{d}{dt}S = -\beta IS + \rho R$$
$$\frac{d}{dt}I = \beta IS - rI$$
$$\frac{d}{dt}R = rI - \rho R$$

 β , r, and ρ are now rates.

Reproduction Number R_0 :

 R_0 = expected number of infected individuals resulting from a single initial infective Epidemic threshold: If $R_0 > 1$, 'epidemic' occu

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Differential equations for continuous model

$$\frac{d}{dt}S = -\beta IS + \rho R$$
$$\frac{d}{dt}I = \beta IS - rI$$
$$\frac{d}{dt}R = rI - \rho R$$

$$\beta$$
, r , and ρ are now rates.

Reproduction Number R_0 :

 R_0 = expected number of infected individuals resulting from a single initial infective

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Differential equations for continuous model

- $\frac{d}{dt}S = -\beta IS + \rho R$ $\frac{d}{dt}I = \beta IS rI$ $\frac{d}{dt}R = rI \rho R$
- β , r, and ρ are now rates.

Reproduction Number R₀:

R₀ = expected number of infected individuals resulting from a single initial infective
 Epidemic threshold: If R₀ > 1, 'epidemic' occurs.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Reproduction Number R_0

Discrete version:

Set up: One Infective in a randomly mixing population of Susceptibles

At time t = 0, single infective random bumps into Susceptible

Probability of transmission = β At time t = 1, single infective remains infected w probability 1 - r

At time t = k, single Infective remains infected with probability $(1 - r)^k$ COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





990f63

Reproduction Number R_0

Discrete version:

- Set up: One Infective in a randomly mixing population of Susceptibles
- At time t = 0, single infective random bumps into a Susceptible
 - Probability of transmission = β At time t = 1, single Infective remains infected with probability 1 - r
 - At time t = k, single infective remains infected with probability $(1 - r)^k$

COcoNuTS

Introduction

Independent Interaction models

- Interdependent interaction models
- Generalized Model Homogeneous version Heterogeneous version
- Nutshel
- Appendix
- References





Reproduction Number R_0

Discrete version:

- Set up: One Infective in a randomly mixing population of Susceptibles
- At time t = 0, single infective random bumps into a Susceptible
- \mathfrak{F} Probability of transmission = β
 - At time t = 1, single infective remains infected with probability 1 + r
 - At time t = k, single infective remains infected with probability $(1 - r)^k$

COcoNuTS

Introduction

Independent Interaction models

- Interdependent interaction models
- Generalized Model Homogeneous version Heterogeneous version
- Nutshell
- Appendix
- References





Reproduction Number R₀

Discrete version:

- Set up: One Infective in a randomly mixing population of Susceptibles
- At time t = 0, single infective random bumps into a Susceptible
- \mathfrak{S} Probability of transmission = β
- At time t = 1, single Infective remains infected with probability 1 r

At time t = k, single infective remains infected with probability $(1 - r)^k$ COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix




Reproduction Number R₀

Discrete version:

- Set up: One Infective in a randomly mixing population of Susceptibles
- At time t = 0, single infective random bumps into a Susceptible
- \mathfrak{S} Probability of transmission = β
- At time t = 1, single Infective remains infected with probability 1 r
- At time t = k, single Infective remains infected with probability $(1 - r)^k$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix





Discrete version:

Expected number infected by original Infective:

$$R_0 = \beta + (1-r)\beta + (1-r)^2\beta + (1-r)^3\beta + \dots$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 10 of 63

Discrete version:

Expected number infected by original Infective:

$$R_0 = \beta + (1-r)\beta + (1-r)^2\beta + (1-r)^3\beta + \dots$$

$$=\beta\left(1+(1-r)+(1-r)^2+(1-r)^3+\ldots\right)$$

Similar story for continuous mode

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 10 of 63

Discrete version:

Expected number infected by original Infective:

$$R_0 = \beta + (1-r)\beta + (1-r)^2\beta + (1-r)^3\beta + \dots$$

$$= \beta \left(1 + (1-r) + (1-r)^2 + (1-r)^3 + \dots \right)$$
$$= \beta \frac{1}{1 - (1-r)}$$

Similar story for continuous mode

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Discrete version:

Expected number infected by original Infective:

$$R_0 = \beta + (1-r)\beta + (1-r)^2\beta + (1-r)^3\beta + \dots$$

$$=\beta\left(1+(1-r)+(1-r)^2+(1-r)^3+\ldots\right)$$

$$=\beta\frac{1}{1-(1-r)}=\beta/r$$

Similar story for continuous mode

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 10 of 63

Discrete version:

Expected number infected by original Infective:

$$R_0 = \beta + (1-r)\beta + (1-r)^2\beta + (1-r)^3\beta + \dots$$

$$= \beta \left(1 + (1-r) + (1-r)^2 + (1-r)^3 + \dots \right)$$
$$= \beta \frac{1}{1-(1-r)} = \beta/r$$

🚳 Similar story for continuous model.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 10 of 63

Independent Interaction models



Introduction

COcoNuTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References

CocoNuTs

200 11 of 63

NIVERSITY

Independent Interaction models



Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Continuous phase transition.

200 11 of 63

Independent Interaction models



Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References

CocoNuTs Complex Networks ©networksvox werything is connected



Continuous phase transition.
 Fine idea from a simple model.

Valiant attempts to use SIR and co. elsewhere: Adoption of ideas/beliefs (Goffman & Newell, 1964) Spread of rumors (Daley & Kendall, 1964, 1965) Diffusion of innovations (Bass, 1969) Spread of fanatical behavior (Castillo-Chávez &

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 12 of 63

Valiant attempts to use SIR and co. elsewhere:

- Adoption of ideas/beliefs (Goffman & Newell, 1964)^[6]
 - Spread of rumors (Daley & Kendall, 1964, 1965). Diffusion of innovations (Bass, 1969) Spread of fanatical behavior (Castillo-Chávez Song 2003)

Introduction

COCONUTS

Independent Interaction models

- Interdependent interaction models
- Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





nac 12 of 63

Valiant attempts to use SIR and co. elsewhere:

- Adoption of ideas/beliefs (Goffman & Newell, 1964)^[6]
- Spread of rumors (Daley & Kendall, 1964, 1965)^[2, 3]

Diffusion of innovations (Bass, 1969) Spread of fanatical behavior (Castillo-Chávez & Song, 2003)

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 12 of 63

Valiant attempts to use SIR and co. elsewhere:

- Adoption of ideas/beliefs (Goffman & Newell, 1964)^[6]
- Spread of rumors (Daley & Kendall, 1964, 1965)^[2, 3]
- liffusion of innovations (Bass, 1969)^[1]

Spread of fanatical behavior (Castillo-Chávez 8 Song, 2003)

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





nac 12 of 63

Valiant attempts to use SIR and co. elsewhere:

- Adoption of ideas/beliefs (Goffman & Newell, 1964)^[6]
- Spread of rumors (Daley & Kendall, 1964, 1965)^[2, 3]
- 💫 Diffusion of innovations (Bass, 1969) [1]
- Spread of fanatical behavior (Castillo-Chávez & Song, 2003)

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Granovetter's model (recap of recap)

🚳 Action based on perceived behavior of others.



Two states: S and I.
 Recovery now possible (SIS).
 φ = fraction of contacts 'on' (e.g., rioting).
 Discrete time, synchronous update.
 This is a Critical mass model.
 Interdependent interaction model.

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version

Nutshel

Appendix

References





na @ 13 of 63

Disease models assume independence of infectious events.

Threshold models only involve proportions. $3/10 \equiv 30/100$. Threshold models ignore exact sequence of influences Threshold models assume immediate pollin Mean-field models neglect network structur Network effects only part of story.

media, advertising, direct marketing.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Disease models assume independence of infectious events.

Threshold models only involve proportions: $3/10 \equiv 30/100.$

Threshold models ignore exact sequence o influences

Threshold models assume immediate pollin Mean-field models neglect network structur Network effects only part of story: media, advertising, direct marketing.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 14 of 63

Disease models assume independence of infectious events.

- Threshold models only involve proportions: $3/10 \equiv 30/100$.
- Threshold models ignore exact sequence of influences

Threshold models assume immediate polling Mean-field models neglect network structure Network effects only part of story: media, advertising, direct marketing.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Disease models assume independence of infectious events.

- Threshold models only involve proportions: $3/10 \equiv 30/100$.
- Threshold models ignore exact sequence of influences
- 🚳 Threshold models assume immediate polling.

Mean-field models neglect network structur Network effects only part of story: media, advertising, direct marketing.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Disease models assume independence of infectious events.

- Threshold models only involve proportions: $3/10 \equiv 30/100$.
- Threshold models ignore exact sequence of influences
- 🚳 Threshold models assume immediate polling.
- 🚳 Mean-field models neglect network structure

Network effects only part of story: media, advertising, direct marketing.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Disease models assume independence of infectious events.

- Threshold models only involve proportions: $3/10 \equiv 30/100$.
- Threshold models ignore exact sequence of influences
- 🚳 Threshold models assume immediate polling.
- 🚳 Mean-field models neglect network structure
- Network effects only part of story: media, advertising, direct marketing.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Basic ingredients:

Incorporate memory of a contagious element ^[4, 5]
Population of *N* individuals, each in state S, I, or R
Each individual randomly contacts another at each time step.
\$\overline{\phi}\$, = fraction infected at time t

= probability of <u>contact</u> with infected individu With probability p, contact with infective leads to an **exposure**.

If exposed, individual receives a dose of size d drawn from distribution f. Otherwise d = 0.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix





Basic ingredients:

Incorporate memory of a contagious element ^[4, 5]
 Population of N individuals, each in state S, I, or R.
 Each individual randomly contacts another at each time step.

= probability of <u>contact</u> with infected individu With probability *p*, contact with infective leads to an **exposure**.

If exposed, individual receives a dose of size a drawn from distribution f. Otherwise d = 0.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix





Basic ingredients:

- Incorporate memory of a contagious element^[4, 5]
- Solution of N individuals, each in state S, I, or R.
- Each individual randomly contacts another at each time step.
 - ϕ_t = fraction infected at time t= probability of <u>contact</u> with infected individu With probability p_i contact with infective leads to an **exposure**.
 - If exposed, individual receives a dose of size a drawn from distribution f. Otherwise d = 0.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





nac 15 of 63

Basic ingredients:

- Incorporate memory of a contagious element^[4, 5]
- \Im Population of N individuals, each in state S, I, or R.
- Each individual randomly contacts another at each time step.
- $\phi_t =$ fraction infected at time t= probability of <u>contact</u> with infected individual

With probability *p*, contact with infective leads to an exposure.

If exposed, individual receives a dose of size d drawn from distribution f. Otherwise d = 0.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Basic ingredients:

- Incorporate memory of a contagious element^[4, 5]
- \Im Population of N individuals, each in state S, I, or R.
- Each individual randomly contacts another at each time step.
- $\phi_t =$ fraction infected at time t= probability of <u>contact</u> with infected individual
- With probability *p*, contact with infective leads to an exposure.

If exposed, individual receives a dose of size d drawn from distribution f. Otherwise d = 0.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Basic ingredients:

- Incorporate memory of a contagious element^[4, 5]
- \Im Population of N individuals, each in state S, I, or R.
- Each individual randomly contacts another at each time step.
- $\phi_t =$ fraction infected at time t= probability of <u>contact</u> with infected individual
- With probability *p*, contact with infective leads to an exposure.
- If exposed, individual receives a dose of size d drawn from distribution f. Otherwise d = 0.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





 $S \Rightarrow I$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

990 16 of 63

 $S \Rightarrow I$

Individuals 'remember' last T contacts: $D_{t,i} = \sum_{t'=t-T+1}^{t} d_i(t')$ Infection occurs if individual i's 'threshold' is

Threshold d_i^* drawn from arbitrary distribution at t = 0.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





 $S \Rightarrow I$

lndividuals 'remember' last T contacts:

$$D_{t,i} = \sum_{t'=t-T+1}^{t} d_i(t')$$

Infection occurs if individual i's 'threshold' is exceeded:

$$D_{t,i} \geq d_i^*$$

Threshold d_i^* drawn from arbitrary distribution at t = 0.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Individuals 'remember' last T contacts:

$$D_{t,i} = \sum_{t'=t-T+1}^{t} d_i(t')$$

A Infection occurs if individual i's 'threshold' is exceeded:

$$D_{t,i} \ge d_i^*$$

 $S \Rightarrow I$

 $\underset{i}{\bigotimes}$ Threshold d_i^* drawn from arbitrary distribution g at t = 0.

COCONUTS

Introduction

Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Appendix





$I \Rightarrow R$

When $D_{t,i} < d_i^*$, individual *i* recovers to state R with probability *r*.

Once in state R, individuals become susceptible aga with probability ho.

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 17 of 63

$I \Rightarrow R$

When $D_{t,i} < d_i^*$, individual *i* recovers to state R with probability *r*.

$R \Rightarrow S$

Once in state R, individuals become susceptible again with probability ρ .

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 17 of 63

A visual explanation





Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version

Heterogeneous version

Nutshell

Appendix

References

Complex Networks @networksvox bweything is connected



ク へ 18 of 63



1 if $D_{t,i} < d_i^*$

 $r\rho$ if $D_{t,i} < d_i^*$

 $r(1-\rho)$ if $D_{t,i} < d_i^*$

 $-\rho$

1 if $D_{t,i} \ge d_i^*$

 $1 - r \text{ if } D_{t\,i} < d_i^*$

1 if $D_{t,i} \ge d_i^*$

S

Generalized mean-field model

Study SIS-type contagion first:

Recovered individuals are immediately susceptible again:

Look for steady-state behavior as a function exposure probability p. Denote fixed points by ϕ^* .

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 P 19 of 63

Generalized mean-field model

Study SIS-type contagion first:

Recovered individuals are immediately susceptible again:

 $\rho = 1.$

Look for steady-state behavior as a function of exposure probability p. Denote fixed points by ϕ^* . COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 P 19 of 63
Study SIS-type contagion first:

Recovered individuals are immediately susceptible again:

 $\rho = 1.$

Look for steady-state behavior as a function of exposure probability p.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 19 of 63

Study SIS-type contagion first:

Recovered individuals are immediately susceptible again:

 $\rho = 1.$

Look for steady-state behavior as a function of exposure probability *p*.
 Denote fixed points by *φ**.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Study SIS-type contagion first:

Recovered individuals are immediately susceptible again:

 $\rho = 1.$

Look for steady-state behavior as a function of exposure probability *p*.
 Denote fixed points by *o**.

Homogeneous version: All individuals have threshold a All dose sizes are equal: d = 1

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Study SIS-type contagion first:

Recovered individuals are immediately susceptible again:

 $\rho = 1.$

Look for steady-state behavior as a function of exposure probability *p*.
 Denote fixed points by *o**.

Homogeneous version:



COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Study SIS-type contagion first:

Recovered individuals are immediately susceptible again:

 $\rho = 1.$

Look for steady-state behavior as a function of exposure probability *p*.
 Denote fixed points by *o**.

Homogeneous version:

All individuals have threshold d^* All dose sizes are equal: d = 1

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Outline

COCONUTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



UNIVERSITY SVERMONT

20 of 63

Generalized Model Homogeneous version

Fixed points for r < 1, $d^* = 1$, and T = 1:

T = 1 means recovery is probabilistic.
 T = 1 means individuals forget past interactions.
 d = 1 means one positive interaction will infect a individual.

Evolution of infection level:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 0 21 of 63

Fixed points for r < 1, $d^* = 1$, and T = 1: r < 1 means recovery is probabilistic.

T = 1 means individuals forget past interactions. $d^{2} = 1$ means one positive interaction will infect a individual.

Evolution of infection level:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 0 21 of 63

Fixed points for r < 1, $d^* = 1$, and T = 1:

r < 1 means recovery is probabilistic.
 T = 1 means individuals forget past interactions.

 I means one positive interaction will infect a dividual.

Evolution of infection level:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

nac 21 of 63

Fixed points for r < 1, $d^* = 1$, and T = 1:

- $rac{1}{2}$ means recovery is probabilistic.
- T = 1 means individuals forget past interactions.
- $d^* = 1$ means one positive interaction will infect an individual.

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r < 1, $d^* = 1$, and T = 1:

- $rac{1}{2}$ means recovery is probabilistic.
- $rac{1}{2}$ T = 1 means individuals forget past interactions.
- $d^* = 1$ means one positive interaction will infect an individual.

Evolution of infection level:

$$\phi_{t+1} = p\phi_t + \phi_t (1 - p\phi_t) (1 - r).$$

 Fraction infected between t and t + 1, independent of past state or recovery.
 Probability of being infected and not being reinfected.
 Probability of not recovering

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r < 1, $d^* = 1$, and T = 1:

- $rac{1}{2}$ means recovery is probabilistic.
- $rac{1}{2}$ T = 1 means individuals forget past interactions.
- $d^* = 1$ means one positive interaction will infect an individual.
- Evolution of infection level:

$$\phi_{t+1} = \underbrace{p\phi_t}_{\mathsf{a}}$$

a: Fraction infected between t and t + 1, independent of past state or recovery.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r < 1, $d^* = 1$, and T = 1:

- $rac{1}{2}$ means recovery is probabilistic.
- T = 1 means individuals forget past interactions.
- $d^* = 1$ means one positive interaction will infect an individual.
- Evolution of infection level:

$$\phi_{t+1} = \underbrace{p\phi_t}_{\mathsf{a}} + \underbrace{\phi_t(1 - p\phi_t)}_{\mathsf{b}}$$

- a: Fraction infected between t and t + 1, independent of past state or recovery.
- b: Probability of being infected and not being reinfected.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r < 1, $d^* = 1$, and T = 1:

- $rac{1}{2}$ means recovery is probabilistic.
- T = 1 means individuals forget past interactions.
- $d^* = 1$ means one positive interaction will infect an individual.
- Evolution of infection level:

$$\phi_{t+1} = \underbrace{p\phi_t}_{\mathsf{a}} + \underbrace{\phi_t(1-p\phi_t)}_{\mathsf{b}} \underbrace{(1-r)}_{\mathsf{C}}.$$

- a: Fraction infected between t and t + 1, independent of past state or recovery.
- b: Probability of being infected and not being reinfected.
- c: Probability of not recovering.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r < 1, $d^* = 1$, and T = 1:

Set
$$\phi_t = \phi^*$$
:

Critical point at $p = p_c = r$. Spreading takes off if p/r > 1Find continuous phase transition as for SIR mo Goodness. Matches $R_o = \beta/\gamma > 1$ condition.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r < 1, $d^* = 1$, and T = 1: Set $\phi_* = \phi^*$:

$$\phi^* = p \phi^* + (1 - p \phi^*) \phi^* (1 - r)$$

Critical point at $p = p_c = r$. Spreading takes off if p/r > 1Find continuous phase transition as for SIR mo Goodness: Matches $R_o = \beta/\gamma > 1$ condition:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





22 of 63

Fixed points for r < 1, $d^* = 1$, and T = 1: Set $\phi_* = \phi^*$:

$$\phi^* = p\phi^* + (1 - p\phi^*)\phi^*(1 - r)$$

$$\Rightarrow 1=p+(1-p\phi^*)(1-r), \quad \phi^*\neq 0,$$

Critical point at $p = p_c = r$. Spreading takes off if p/r > 1Find continuous phase transition as for SIR mod Goodness. Matches $R_o = B/\gamma > 1$ condition:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r < 1, $d^* = 1$, and T = 1: Set $\phi_t = \phi^*$:

$$\phi^* = p\phi^* + (1 - p\phi^*)\phi^*(1 - r)$$

$$\Rightarrow 1=p+(1-p\phi^*)(1-r), \quad \phi^*\neq 0,$$

$$\Rightarrow \phi^* = rac{1-r/p}{1-r}$$
 and $\phi^* = 0.$

Critical point at $p = p_c = r$. Spreading takes off if p/r > 1Find continuous phase transition as for SIR mod Goodness. Matches $R_o = \beta/\gamma > 1$ condition.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





na @ 22 of 63

Fixed points for r < 1, $d^* = 1$, and T = 1: Set $\phi_t = \phi^*$:

$$\phi^* = p\phi^* + (1 - p\phi^*)\phi^*(1 - r)$$

$$\Rightarrow 1 = p + (1 - p\phi^*)(1 - r), \quad \phi^* \neq 0,$$

$$\Rightarrow \phi^* = rac{1-r/p}{1-r}$$
 and $\phi^* = 0.$

Critical point at $p = p_c = r$. Spreading takes off if p/r > 1Find continuous phase transition as for SIF Goodness. Matches $R_o = B/\alpha > 1$ conditio

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





22 of 63

Fixed points for r < 1, $d^* = 1$, and T = 1: Set $\phi_t = \phi^*$:

$$\phi^* = p\phi^* + (1 - p\phi^*)\phi^*(1 - r)$$

$$\Rightarrow 1 = p + (1 - p\phi^*)(1 - r), \quad \phi^* \neq 0,$$

$$\Rightarrow \phi^* = rac{1-r/p}{1-r}$$
 and $\phi^* = 0.$



Find continuous phase transition as for SIR mode Goodness: Matches $R_o = \beta/\gamma > 1$ condition:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r < 1, $d^* = 1$, and T = 1: Set $\phi_t = \phi^*$:

$$\phi^* = p\phi^* + (1 - p\phi^*)\phi^*(1 - r)$$

$$\Rightarrow 1 = p + (1 - p\phi^*)(1 - r), \quad \phi^* \neq 0,$$

$$\Rightarrow \phi^* = rac{1-r/p}{1-r}$$
 and $\phi^* = 0$.

Critical point at p = p_c = r.
Spreading takes off if p/r > 1
Find continuous phase transition as for SIR model.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





nac 22 of 63

Fixed points for r < 1, $d^* = 1$, and T = 1: Set $\phi_t = \phi^*$:

$$\phi^* = p\phi^* + (1 - p\phi^*)\phi^*(1 - r)$$

$$\Rightarrow 1=p+(1-p\phi^*)(1-r), \quad \phi^*\neq 0,$$

$$\Rightarrow \phi^* = rac{1-r/p}{1-r}$$
 and $\phi^* = 0$.

Critical point at p = p_c = r.
Spreading takes off if p/r > 1
Find continuous phase transition as for SIR model.
Goodness: Matches R_o = β/γ > 1 condition.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r = 1, $d^* = 1$, and T > 1

T = 1 means recovery is immediate.
 T > 1 means individuals remember at least 2 interactions.

d = 1 means only one positive interaction in pas
 T interactions will infect individual.
 Effect of individual interactions is independent
 from effect of others.
 Call o the steady state level of infection.

Pr(infected) = 1 - Pr(uninfected):

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshel

Appendix

References





Fixed points for r = 1, $d^* = 1$, and T > 1

 $rac{1}{2}$ = 1 means recovery is immediate.

T > 1 means individuals remember at least 2 interactions.

d = 1 means only one positive interaction in pas *T* interactions will infect individual.
 Effect of individual interactions is independent from effect of others.
 Call of the steady state level of infection.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for r = 1, $d^* = 1$, and T > 1

r = 1 means recovery is immediate. T > 1 means individuals remember at least 2 interactions.

d = 1 means only one positive interaction in pa
 T interactions will infect individual.
 Effect of individual interactions is independent from effect of others.
 Call o the steady state level of infection.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshel

Appendix

References





Fixed points for r = 1, $d^* = 1$, and T > 1

- $rac{1}{2}$ = 1 means recovery is immediate.
- T > 1 means individuals remember at least 2 interactions.
- $d^* = 1$ means only one positive interaction in past *T* interactions will infect individual.
 - Effect of individual interactions is independen from effect of others. Call the steady state level of infection. Pr(infected) = 1 - Pr(uninfected):

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for r = 1, $d^* = 1$, and T > 1

- $rac{1}{3}$ r = 1 means recovery is immediate.
- T > 1 means individuals remember at least 2 interactions.
- $d^* = 1$ means only one positive interaction in past *T* interactions will infect individual.
- Effect of individual interactions is independent from effect of others.

Call of the steady state level of infection Pr(infected) = 1 - Pr(uninfected):

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





23 of 63

Fixed points for r = 1, $d^* = 1$, and T > 1

- $rac{1}{3}$ r=1 means recovery is immediate.
- T > 1 means individuals remember at least 2 interactions.
- $d^* = 1$ means only one positive interaction in past *T* interactions will infect individual.
- Effect of individual interactions is independent from effect of others.
- \mathfrak{S} Call ϕ^* the steady state level of infection.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

Fixed points for r = 1, $d^* = 1$, and T > 1

- $rac{1}{3}$ r=1 means recovery is immediate.
- T > 1 means individuals remember at least 2 interactions.
- $d^* = 1$ means only one positive interaction in past *T* interactions will infect individual.
- Effect of individual interactions is independent from effect of others.
- \mathfrak{B} Call ϕ^* the steady state level of infection.
- Pr(infected) = 1 Pr(uninfected):



Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





23 of 63

Fixed points for r = 1, $d^* = 1$, and T > 1

- $rac{1}{3}$ r=1 means recovery is immediate.
- T > 1 means individuals remember at least 2 interactions.
- $d^* = 1$ means only one positive interaction in past *T* interactions will infect individual.
- Effect of individual interactions is independent from effect of others.
- \mathfrak{B} Call ϕ^* the steady state level of infection.
- Pr(infected) = 1 Pr(uninfected):

$$\phi^* = 1 - (1 - p\phi^*)^T$$



Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshel

Appendix





$$\phi^* = 1 - (1-p\phi^*)^T$$

Again find continuous phase transition. Note: we can solve for p but not ϕ^* :

 $p \models (\phi^*)^{-1} [1 - (1 - \phi^*)^{1/2}]$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





24 of 63

$$\phi^* = 1-(1-p\phi^*)^T$$

 \bigotimes Look for critical infection probability p_c .

Again find continuous phase transition Note: we can solve for p but not ϕ^* :



COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





24 of 63

$$\phi^* = 1 - (1-p\phi^*)^T$$

♣ Look for critical infection probability p_c . ♣ As $\phi^* \rightarrow 0$, we see

$$\phi^* \simeq pT\phi^*$$

Again find continuous phase transition. Note: we can solve for p but not ϕ^* : $p \models (\phi^*)^{-1}[1 - (1 - \phi^*)^{1/T}].$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





990 24 of 63

$$\phi^* = 1-(1-p\phi^*)^T$$

♣ Look for critical infection probability p_c . ♣ As $\phi^* \rightarrow 0$, we see

$$\phi^* \simeq pT \phi^* \ \Rightarrow p_c = 1/T$$

Again find continuous phase transition. Note: we can solve for *p* but not ϕ^* : $p = (\phi^*)^{-1}[1 - (1 - \phi^*)^{1/T}].$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





$$\phi^* = 1 - (1-p\phi^*)^T$$

♣ Look for critical infection probability p_c . ♣ As $\phi^* \rightarrow 0$, we see

$$\phi^* \simeq pT \phi^* \ \Rightarrow p_c = 1/T$$

🚳 Again find continuous phase transition...

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





24 of 63

$$\phi^* = 1 - (1-p\phi^*)^T$$

♣ Look for critical infection probability p_c . ♣ As $\phi^* \rightarrow 0$, we see

$$\phi^* \simeq pT \phi^* \ \Rightarrow p_c = 1/T$$

Again find continuous phase transition... Note: we can solve for p but not ϕ^* :

$$p = (\phi^*)^{-1} [1 - (1 - \phi^*)^{1/T}].$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix




Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$



Start with r = 1, $d^* = 1$, and $T \ge 1$ case we have just examined:

$$\phi^* = 1 - (1 - p\phi^*)^T.$$

COCONUTS

Introduction

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix

References





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$



Start with r = 1, $d^* = 1$, and $T \ge 1$ case we have just examined:

$$\phi^* = 1 - (1 - p\phi^*)^T.$$

So For r < 1, add to right hand side fraction who:

COCONUTS

Introduction

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$



Start with r = 1, $d^* = 1$, and $T \ge 1$ case we have just examined:

$$\phi^* = 1 - (1 - p\phi^*)^T.$$

So For r < 1, add to right hand side fraction who: 1. Did not receive any infections in last T time steps,

COCONUTS

Introduction

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$



Start with r = 1, $d^* = 1$, and $T \ge 1$ case we have just examined:

$$\phi^* = 1 - (1 - p\phi^*)^T.$$

Sor r < 1, add to right hand side fraction who: 1. Did not receive any infections in last T time steps, 2. And did not recover from a previous infection.

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix

References





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$



Start with r = 1, $d^* = 1$, and $T \ge 1$ case we have just examined:

 $\phi^* = 1 - (1 - p\phi^*)^T.$

So For r < 1, add to right hand side fraction who: 1. Did not receive any infections in last T time steps, 2. And did not recover from a previous infection.

Define corresponding dose histories. Example: AA.

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$



Start with r = 1, $d^* = 1$, and $T \ge 1$ case we have just examined:

 $\phi^* = 1 - (1 - p\phi^*)^T.$

Sor r < 1, add to right hand side fraction who: 1. Did not receive any infections in last T time steps, 2. And did not recover from a previous infection.

Define corresponding dose histories. Example:

$$H_1 = \{\dots, d_{t-T-2}, d_{t-T-1}, 1, \underbrace{0, 0, \dots, 0, 0}_{T \text{ 0's}}\},\$$

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$



Start with r = 1, $d^* = 1$, and $T \ge 1$ case we have just examined:

 $\phi^* = 1 - (1 - p\phi^*)^T.$

Sor r < 1, add to right hand side fraction who: 1. Did not receive any infections in last T time steps, 2. And did not recover from a previous infection.

Define corresponding dose histories. Example:

$$H_1 = \{\dots, d_{t-T-2}, d_{t-T-1}, 1, \underbrace{0, 0, \dots, 0, 0}_{T \text{ 0's}}\},\$$



COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix





$$H_{m+1} = \{\dots, d_{t-T-m-1}, 1, \underbrace{0, 0, \dots, 0, 0}_{m \text{ 0's }}, \underbrace{0, 0, \dots, 0, 0}_{T \text{ 0's }}\}$$

Overall probabilities for dose histories occurring

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

$$H_{m+1} = \{\dots, d_{t-T-m-1}, 1, \underbrace{0, 0, \dots, 0, 0}_{m \text{ 0's }}, \underbrace{0, 0, \dots, 0, 0}_{T \text{ 0's }}\}$$

Overall probabilities for dose histories occurring:

$$P(H_1) = p\phi^*(1 - p\phi^*)^T(1 - r),$$

Pr(infection T + m + 1 time steps ago) Pr(no doses received in T + m time steps since Pr(no recovery in m chances) COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





$$H_{m+1} = \{\dots, d_{t-T-m-1}, 1, \underbrace{0, 0, \dots, 0, 0}_{m \text{ 0's }}, \underbrace{0, 0, \dots, 0, 0}_{T \text{ 0's }}\}$$

Overall probabilities for dose histories occurring:

$$P(H_1) = p \phi^* (1 - p \phi^*)^T (1 - r),$$

 $P(H_{m+1}) = p\phi^* (1 - p\phi^*)^{T+m} (1 - r)^{m+1}$

Pr(infection T + m + 1 time steps ago) Pr(no doses received in T + m time steps since Pr(no recovery in m chances) COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





$$H_{m+1} = \{\dots, d_{t-T-m-1}, 1, \underbrace{0, 0, \dots, 0, 0}_{m \text{ 0's }}, \underbrace{0, 0, \dots, 0, 0}_{T \text{ 0's }}\}$$

Overall probabilities for dose histories occurring:

$$P(H_1) = p \phi^* (1 - p \phi^*)^T (1 - r)$$

$$P(H_{m+1}) = \underbrace{p\phi^*}_{\circ}$$

a: Pr(infection T + m + 1 time steps ago)

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Nutshell

Appendix





$$H_{m+1} = \{\dots, d_{t-T-m-1}, 1, \underbrace{0, 0, \dots, 0, 0}_{m \text{ 0's }}, \underbrace{0, 0, \dots, 0, 0}_{T \text{ 0's }}\}$$

Overall probabilities for dose histories occurring:

$$P(H_1) = p \phi^* (1 - p \phi^*)^T (1 - r)$$

$$P(H_{m+1}) = \underbrace{p\phi^*}_{a} \underbrace{(1 - p\phi^*)^{T+m}}_{b}$$

a: Pr(infection T + m + 1 time steps ago)
b: Pr(no doses received in T + m time steps since)

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Nutshell

Appendix

References





$$H_{m+1} = \{\dots, d_{t-T-m-1}, 1, \underbrace{0, 0, \dots, 0, 0}_{m \text{ 0's }}, \underbrace{0, 0, \dots, 0, 0}_{T \text{ 0's }}\}$$

Overall probabilities for dose histories occurring:

$$P(H_1) = p \phi^* (1-p \phi^*)^T (1-r),$$

$$P(H_{m+1}) = \underbrace{p\phi^*}_a \underbrace{(1-p\phi^*)^{T+m}}_b \underbrace{(1-r)^{m+1}}_c$$

a: Pr(infection T + m + 1 time steps ago)
b: Pr(no doses received in T + m time steps since)
c: Pr(no recovery in m chances)

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Pr(recovery) = Pr(seeing no doses for at least T time steps and recovering) COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Pr(recovery) = Pr(seeing no doses for at least T time steps and recovering)

$$= r \sum_{m=0}^{\infty} P(H_{T+m})$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Pr(recovery) = Pr(seeing no doses for at least T time steps and recovering)

$$= r \sum_{m=0}^{\infty} P(H_{T+m}) = r \sum_{m=0}^{\infty} p \phi^* (1 - p \phi^*)^{T+m} (1 - r)^m$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





うへで 27 of 63

Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Pr(recovery) = Pr(seeing no doses for at least T time steps and recovering)

$$= r \sum_{m=0}^{\infty} P(H_{T+m}) = r \sum_{m=0}^{\infty} p \phi^* (1 - p \phi^*)^{T+m} (1 - r)^m$$

$$=r\frac{p\phi^*(1-p\phi^*)^T}{1-(1-p\phi^*)(1-r)}$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Pr(recovery) = Pr(seeing no doses for at least T time steps and recovering)

$$= r \sum_{m=0}^{\infty} P(H_{T+m}) = r \sum_{m=0}^{\infty} p \phi^* (1 - p \phi^*)^{T+m} (1 - r)^m$$

$$=r\frac{p\phi^*(1-p\phi^*)^T}{1-(1-p\phi^*)(1-r)}$$

A Fixed point equation:

$$\phi^* = 1 - \frac{r(1-p\phi^*)^T}{1-(1-p\phi^*)(1-r)}.$$

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Fixed point equation (again):

$$\phi^* = 1 - \frac{r(1 - p\phi^*)^T}{1 - (1 - p\phi^*)(1 - r)}$$

Find critical exposure probability by examinin, above as $\phi^* \rightarrow 0$.

where $\tau =$ mean recovery time for simple relaxation process. Decreasing r keeps individuals infected for k and decreases p_c .

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Fixed point equation (again):

$$\phi^* = 1 - \frac{r(1 - p\phi^*)^T}{1 - (1 - p\phi^*)(1 - r)}.$$

Find critical exposure probability by examining above as $\phi^* \rightarrow 0$.

where $\tau =$ mean recovery time for simple relaxation process. Decreasing r keeps individuals infected for lo and decreases p_c .

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Fixed point equation (again):

2

$$\phi^* = 1 - \frac{r(1 - p\phi^*)^T}{1 - (1 - p\phi^*)(1 - r)}$$

Find critical exposure probability by examining above as $\phi^* \rightarrow 0$.

$$\Rightarrow \quad p_c = \frac{1}{T + 1/r - 1} = \frac{1}{T + \tau}$$

where τ = mean recovery time for simple relaxation process.

Decreasing *r* keeps individuals infected for long and decreases *p*_c.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for $r \leq 1$, $d^* = 1$, and $T \geq 1$

Fixed point equation (again):

3

$$\phi^* = 1 - \frac{r(1 - p\phi^*)^T}{1 - (1 - p\phi^*)(1 - r)}.$$

Sind critical exposure probability by examining above as $\phi^* \rightarrow 0$.

$$\Rightarrow \quad p_c = \frac{1}{T+1/r-1} = \frac{1}{T+\tau}$$

where τ = mean recovery time for simple relaxation process.

Solution Decreasing r keeps individuals infected for longer and decreases p_c .

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Epidemic threshold:

Fixed points for $d^* = 1$, $r \leq 1$, and $T \geq 1$

$$\begin{split} & & & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\$$



Introduction

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References

Solution Example details: $T = 2 \& r = 1/2 \Rightarrow p_c = 1/3$. Solution Blue = stable, red = unstable, fixed points. Solution $\tau = 1/r - 1$ = characteristic recovery time = 1. Solution $T + \tau \simeq$ average memory in system = 3.

CocoNuTs Complex Networks Onetworksvox Everything is connected



Epidemic threshold:

Fixed points for $d^* = 1$, $r \le 1$, and $T \ge 1$

$$\begin{split} & & & & & & \\ & & & & \\ & & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\ & & & \\$$



Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References

CocoNuTs Complex Networks @networksvox Everything is connected



Example details: T = 2 & r = 1/2 ⇒ p_c = 1/3.
Blue = stable, red = unstable, fixed points.
τ = 1/r - 1 = characteristic recovery time = 1.
T + τ ≃ average memory in system = 3.
Phase transition can be seen as a transcritical bifurcation. ^[11]

All right: $d^* = 1$ models correspond to simple disease spreading models.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 CP 30 of 63

3

All right: $d^* = 1$ models correspond to simple disease spreading models. 3 What if we allow $d^* \geq 2?$

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix

References





29 CP 30 of 63

All right: $d^* = 1$ models correspond to simple disease spreading models.

 $\textcircled{What if we allow } d^* \geq 2?$

3

Again first consider SIS with immediate recovery (r = 1)

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





- All right: $d^* = 1$ models correspond to simple disease spreading models.
- $\textcircled{What if we allow } d^* \geq 2?$
- Again first consider SIS with immediate recovery (r = 1)
- Also continue to assume unit dose sizes $(f(d) = \delta(d-1))$.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 0 30 of 63

- All right: $d^* = 1$ models correspond to simple disease spreading models.
- $\textcircled{What if we allow } d^* \geq 2?$
- Again first consider SIS with immediate recovery (r = 1)
- Also continue to assume unit dose sizes $(f(d) = \delta(d-1))$.
- To be infected, must have at least d^* exposures in last T time steps.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





- All right: $d^* = 1$ models correspond to simple disease spreading models.
- $\textcircled{What if we allow } d^* \geq 2?$
- Again first consider SIS with immediate recovery (r = 1)
- Also continue to assume unit dose sizes $(f(d) = \delta(d-1)).$
- To be infected, must have at least d* exposures in last T time steps.
 - Fixed point equation:

$$\phi^* = \sum_{i=d^*}^T {T \choose i} (p\phi^*)^i (1-p\phi^*)^{T-i}.$$

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





- All right: $d^* = 1$ models correspond to simple disease spreading models.
- $\textcircled{What if we allow } d^* \geq 2?$
- Again first consider SIS with immediate recovery (r = 1)
- Also continue to assume unit dose sizes $(f(d) = \delta(d-1))$.
- To be infected, must have at least d^* exposures in last T time steps.

Fixed point equation:

$$\phi^* = \sum_{i=d^*}^T {T \choose i} (p\phi^*)^i (1-p\phi^*)^{T-i}.$$

As always, $\phi^* = 0$ works too.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Fixed points for r = 1, $d^* > 1$, and $T \ge 1$

e.g., for $d^* = 2, T = 3$:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for r = 1, $d^* > 1$, and $T \ge 1$

 \bigotimes Exactly solvable for small T.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for r = 1, $d^* > 1$, and $T \ge 1$ \clubsuit Exactly solvable for small T.

3 e.g., for $d^* = 2, T = 3$:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for r = 1, $d^* > 1$, and $T \ge 1$

Exactly solvable for small T. e.g., for $d^* = 2$, T = 3:

Fixed point equation: $\phi^* =$ $3p^2 \phi^{*2} (1 - p \phi^*) + p^3 \phi^{*3}$

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Fixed points for r = 1, $d^* > 1$, and $T \ge 1$ Solve the second seco

Fixed point equation:
 \$\phi^* = \$\$\$ 3p^2 \phi^{*2} (1 - p \phi^*) + p^3 \phi^{*3}\$
 See new structure: a saddle node bifurcation [11] appears as p increases.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References




Homogeneous, multi-hit models:

Fixed points for r = 1, $d^* > 1$, and $T \ge 1$ Solvable for small T. Solvable for small T.



Fixed point equation:
 \$\phi^* = \$\$\$ 3p^2 \phi^{*2} (1 - p \phi^*) + p^3 \phi^{*3}\$
 See new structure: a saddle node bifurcation [11] appears as p increases.

 $\textcircled{b} (p_b,\phi^*) = (8/9,27/32).$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Homogeneous, multi-hit models:

Fixed points for r = 1, $d^* > 1$, and $T \ge 1$ Solvable for small T. Solvable $d^* = 2$, T = 3:



Fixed point equation:
 \$\phi^* = \$\$\$ 3p^2 \phi^{*2}(1 - p \phi^*) + p^3 \phi^{*3}\$
 See new structure: a saddle node bifurcation [11] appears as p increases.

$$(p_b, \phi^*) = (8/9, 27/32).$$

Behavior akin to output of Granovetter's threshold model.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Homogeneous, multi-hit models:

Another example:



COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



 $r = 1, d^* = 3, T = 12$ Saddle-node bifurcation.



20 0 32 of 63



 $\begin{array}{l} \textcircled{3} \quad d^* = 1 \rightarrow d^* > 1; \\ \begin{array}{c} \text{jump between} \\ \text{continuous} \\ \text{phase transition} \\ \text{and pure critical} \\ \text{mass model.} \\ \end{array}$ $\begin{array}{c} \textcircled{3} \quad \text{Unstable curve} \\ \text{for } d^* = 2 \text{ does} \\ \text{not hit } \phi^* = 0. \end{array}$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





NIVERSITY



 $\begin{array}{l} \textcircled{3} d^* = 1 \rightarrow d^* > 1; \\ \begin{array}{c} \text{jump between} \\ \text{continuous} \\ \text{phase transition} \\ \text{and pure critical} \\ \text{mass model.} \\ \end{array}$ $\begin{array}{c} \textcircled{3} \\ \begin{array}{c} \textbf{Unstable curve} \\ \text{for } d^* = 2 \text{ does} \\ \text{not hit } \phi^* = 0. \end{array}$

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Vutshell

Appendix

References





See either simple phase transition or saddle-node bifurcation, nothing in between.

na @ 33 of 63

Sifurcation points for example fixed T, varying d^* :



$$\begin{array}{c} \textcircled{6}{\ } T = 96 \ (\). \\ \textcircled{6}{\ } T = 24 \ (\triangleright), \\ \textcircled{6}{\ } T = 12 \ (\triangleleft), \\ \textcircled{6}{\ } T = 6 \ (\Box), \\ \textcircled{6}{\ } T = 3 \ (\bigcirc), \end{array}$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



Vermont

200 34 of 63

For r < 1, need to determine probability of recovering as a function of time since dose load last dropped below threshold.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 35 of 63

For r < 1, need to determine probability of recovering as a function of time since dose load last dropped below threshold.
 Partially summed random walks:

$$D_i(t) = \sum_{t'=t-T+1}^t d_i(t')$$

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 35 of 63

For r < 1, need to determine probability of recovering as a function of time since dose load last dropped below threshold.
 Partially summed random walks:

$$D_i(t) = \sum_{t'=t-T+1}^t d_i(t')$$

Solution Example for T = 24, $d^* = 14$:

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 0 35 of 63

Sor r < 1, need to determine probability of recovering as a function of time since dose load last dropped below threshold. Partially summed random walks:

$$D_i(t) = \sum_{t'=t-T+1}^{t} d_i(t')$$

Example for
$$T = 24$$
, $d^* = 14$:

2

R



COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Appendix

References



NIVERSITY 6 29 CP 35 of 63

Define γ_m as fraction of individuals for whom D(t) last equaled, and has since been below, their threshold m time steps ago,

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 36 of 63

Define γ_m as fraction of individuals for whom D(t) last equaled, and has since been below, their threshold m time steps ago,
 Fraction of individuals below threshold but not recovered:

$$\Gamma(p,\phi^*;r) = \sum_{m=1}^\infty (1-r)^m \gamma_m(p,\phi^*).$$

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 36 of 63

Define γ_m as fraction of individuals for whom D(t) last equaled, and has since been below, their threshold m time steps ago,
 Fraction of individuals below threshold but not

recovered:

$$\Gamma(p,\phi^*;r) = \sum_{m=1}^\infty (1-r)^m \gamma_m(p,\phi^*).$$

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References

CocoNuTs Complex Networks @networksvox Everything is connected



990 36 of 63

Fixed point equation:

$$\phi^* = \Gamma(p, \phi^*; r) + \sum_{i=d^*}^T {T \choose i} (p\phi^*)^i (1 - p\phi^*)^{T-i}.$$

Want to examine how dose load can drop below threshold of $d^* = 2$:

Iwo subsequences do this:

Note: second sequence includes an extra 0 since this is necessary to stay below $d^* = 2$.

To stay below threshold, observe acceptable following sequences may be composed of any combination of two subsequences:

)} and $b = \{1, 0\}$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





990 37 of 63



🚳 Want to examine how dose load can drop below threshold of $d^* = 2$:

$$D_n = 2 \Rightarrow D_{n+1} = 1$$

COCONUTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix

References





2 0 0 37 of 63



🚳 Want to examine how dose load can drop below threshold of $d^* = 2$:

$$D_n = 2 \Rightarrow D_{n+1} = 1$$



🚳 Two subsequences do this:

COCONUTS

Introduction

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix







🚳 Want to examine how dose load can drop below threshold of $d^* = 2$:

$$D_n = 2 \Rightarrow D_{n+1} = 1$$



🚳 Two subsequences do this: $\{d_{n-2}, d_{n-1}, d_n, d_{n+1}\} = \{1, 1, 0, 0\}$

COCONUTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix

References





DQ @ 37 of 63



🚳 Want to examine how dose load can drop below threshold of $d^* = 2$:

$$D_n = 2 \Rightarrow D_{n+1} = 1$$

🚳 Two subsequences do this: $\{d_{n-2}, d_{n-1}, d_n, d_{n+1}\} = \{1, 1, 0, 0\}$ and $\{d_{n-2}, d_{n-1}, d_n, d_{n+1}, d_{n+2}\} = \{1, 0, 1, 0, 0\}.$

COCONUTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Appendix

References





29 CP 37 of 63



🚳 Want to examine how dose load can drop below threshold of $d^* = 2$:

$$D_n = 2 \Rightarrow D_{n+1} = 1$$

🚳 Two subsequences do this: $\{d_{n-2}, d_{n-1}, d_n, d_{n+1}\} = \{1, 1, 0, 0\}$ and $\{d_{n-2}, d_{n-1}, d_n, d_{n+1}, d_{n+2}\} = \{1, 0, 1, 0, 0\}.$ Note: second sequence includes an extra 0 since this is necessary to stay below $d^* = 2$.

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Appendix

References



2 a a 37 of 63

UNIVERSITY 6



🚳 Want to examine how dose load can drop below threshold of $d^* = 2$:

$$D_n = 2 \Rightarrow D_{n+1} = 1$$

🚳 Two subsequences do this: $\{d_{n-2}, d_{n-1}, d_n, d_{n+1}\} = \{1, 1, 0, 0\}$ and $\{d_{n-2}, d_{n-1}, d_n, d_{n+1}, d_{n+2}\} = \{1, 0, 1, 0, 0\}.$ 🚳 Note: second sequence includes an extra 0 since this is necessary to stay below $d^* = 2$. line stay below threshold, observe acceptable following sequences may be composed of any combination of two subsequences:

$$a = \{0\}$$
 and $b = \{1, 0, 0\}.$

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Appendix





Determine number of sequences of length m that keep dose load below $d^* = 2$.

where [1] means floor. Corresponding possible values fo

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Determine number of sequences of length m that keep dose load below $d^* = 2$.

 N_a = number of $a = \{0\}$ subsequences.

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Determine number of sequences of length m that keep dose load below $d^* = 2$.

- N_a = number of $a = \{0\}$ subsequences.
- N_b = number of $b = \{1, 0, 0\}$ subsequences.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Determine number of sequences of length m that keep dose load below $d^* = 2$.

 N_a = number of $a = \{0\}$ subsequences.

 N_b = number of $b = \{1, 0, 0\}$ subsequences.

$$m = N_a \cdot 1 + N_b \cdot 3$$

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Determine number of sequences of length m that keep dose load below $d^* = 2$.

 N_a = number of $a = \{0\}$ subsequences.

 $\Im N_b$ = number of $b = \{1, 0, 0\}$ subsequences.

$$m = N_a \cdot 1 + N_b \cdot 3$$

Possible values for N_b :

$$0, 1, 2, \ldots, \left\lfloor \frac{m}{3}
ight
floor$$
 .

where [.] means floor.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Determine number of sequences of length m that keep dose load below $d^* = 2$.

 $N_a =$ number of $a = \{0\}$ subsequences.

 $\Im N_b$ = number of $b = \{1, 0, 0\}$ subsequences.

$$m = N_a \cdot 1 + N_b \cdot 3$$

Possible values for N_b :

$$0, 1, 2, \ldots, \left\lfloor \frac{m}{3}
ight
floor$$
.

where $\lfloor \cdot \rfloor$ means floor. Sourcesponding possible values for N_a :

$$m, m-3, m-6, \ldots, m-3\left\lfloor \frac{m}{3} \right\rfloor.$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix



\aleph How many ways to arrange $N_a a$'s and $N_b b$'s?

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 CP 39 of 63

How many ways to arrange N_a a's and N_b b's?
 Think of overall sequence in terms of subsequences:

$$\{Z_1,Z_2,\ldots,Z_{N_a+N_b}\}$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 C 39 of 63

How many ways to arrange N_a a's and N_b b's?
 Think of overall sequence in terms of subsequences:

$$\{Z_1,Z_2,\ldots,Z_{N_a+N_b}\}$$

 $\gtrsim N_a + N_b$ slots for subsequences.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 39 of 63

How many ways to arrange N_a a's and N_b b's?
 Think of overall sequence in terms of subsequences:

$$\{Z_1,Z_2,\ldots,Z_{N_a+N_b}\}$$

 $N_a + N_b$ slots for subsequences.
 Choose positions of either *a*'s or *b*'s:

$${N_a+N_b\choose N_a}={N_a+N_b\choose N_b}$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Total number of allowable sequences of length m:

$$\sum_{N_b=0}^{\lfloor m/3\rfloor} {N_b+N_a \choose N_b} = \sum_{k=0}^{\lfloor m/3\rfloor} {m-2k \choose k}$$

where $k = N_b$ and we have used $m = N_a + 3N_b$.

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 40 of 63

Total number of allowable sequences of length m:

$$\sum_{N_b=0}^{\lfloor m/3\rfloor} {N_b+N_a \choose N_b} = \sum_{k=0}^{\lfloor m/3\rfloor} {m-2k \choose k}$$

where $k = N_b$ and we have used $m = N_a + 3N_b$. $P(a) = (1 - p\phi^*)$ and $P(b) = p\phi^*(1 - p\phi^*)^2$

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Total number of allowable sequences of length m:

$$\sum_{N_b=0}^{\lfloor m/3\rfloor} {N_b+N_a \choose N_b} = \sum_{k=0}^{\lfloor m/3\rfloor} {m-2k \choose k}$$

where $k = N_b$ and we have used $m = N_a + 3N_b$. $P(a) = (1 - p\phi^*)$ and $P(b) = p\phi^*(1 - p\phi^*)^2$ Total probability of allowable sequences of length m:

$$\chi_m(p,\phi^*) = \sum_{k=0}^{\lfloor m/3 \rfloor} {m-2k \choose k} (1-p\phi^*)^{m-k} (p\phi^*)^k$$

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Vutshell

Appendix





Total number of allowable sequences of length m:

$$\sum_{N_b=0}^{\lfloor m/3\rfloor} {N_b+N_a \choose N_b} = \sum_{k=0}^{\lfloor m/3\rfloor} {m-2k \choose k}$$

where $k = N_b$ and we have used $m = N_a + 3N_b$. $P(a) = (1 - p\phi^*)$ and $P(b) = p\phi^*(1 - p\phi^*)^2$ Total probability of allowable sequences of length m:

$$\chi_m(p,\phi^*) = \sum_{k=0}^{\lfloor m/3 \rfloor} {m-2k \choose k} (1-p\phi^*)^{m-k} (p\phi^*)^k$$

Solution: Write a randomly chosen sequence of a's and b's of length m as $D_m^{a,b}$.

ntroduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





Nearly there... must account for details of sequence endings.
 Three endings ⇒ Six possible sequences:

 $D_1 = \{1, 1, 0, 0, D_{m-1}^{a, b}\}$ $D_2 = \{1, 1, 0, 0, D_{m-2}^{a, b}, 1\}$ $D_3 = \{1, 1, 0, 0, D_m^{a, b}, 1, 0\}$ $D_4 = \{1, 0, 1, 0, 0, D_{m-2}^{a, b}\}$ $D_5 = \{1, 0, 1, 0, 0, D_{m-3}^{a, b}, 1\}$ $D_6 = \{1, 0, 1, 0, 0, D_m^{a, b}, 1, 0\}$ COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 Al of 63

Nearly there... must account for details of sequence endings.
 Three endings ⇒ Six possible sequences:

 $D_1 = \{1, 1, 0, 0, D_{m-1}^{a, b}\}$ $D_2 = \{1, 1, 0, 0, D_{m-2}^{a, b}, 1\}$ $D_3 = \{1, 1, 0, 0, D_m^{a, b}, 1, 0\}$ $D_4 = \{1, 0, 1, 0, 0, D_{m-2}^{a, b}\}$ $D_5 = \{1, 0, 1, 0, 0, D_{m-3}^{a, b}, 1\}$ $D_6 = \{1, 0, 1, 0, 0, D_m^{a, b}, 1, 0\}$ COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 Al of 63
Sequence endings.
 Six possible sequences:

$$\begin{array}{ll} D_1 = \{1,1,0,0,D_{m-1}^{a,b}\} & P_1 = (p\phi)^2(1-p\phi)^2\chi_{m-1}(p,\phi) & \begin{array}{l} & \end{array}{l} \\ & \begin{array}{l} & \end{array}{l} \\ & D_2 = \{1,1,0,0,D_{m-2}^{a,b},1\} \\ & D_3 = \{1,1,0,0,D_{m-3}^{a,b},1,0\} \\ & D_4 = \{1,0,1,0,0,D_{m-2}^{a,b}\} \\ & D_5 = \{1,0,1,0,0,D_{m-3}^{a,b},1\} \\ & D_6 = \{1,0,1,0,0,D_{m-4}^{a,b},1,0\} \\ & \end{array} \right) \\ \end{array}$$

990 41 of 63

COCONUTS

Introduction

Interaction models

Interdependent interaction models

F.P. Eq:
$$\phi^* = \Gamma(p, \phi^*; r) + \sum_{i=d^*}^T {T \choose i} (p\phi^*)^i (1 - p\phi^*)^{T-i}.$$

where $\Gamma(p, \phi^*; r) =$

$$(1-r)(p\phi)^{2}(1-p\phi)^{2} + \sum_{m=1}^{\infty} (1-r)^{m}(p\phi)^{2}(1-p\phi)^{2} \times \frac{1}{2} + \sum_{m=1}^{\infty} (1-r)^{m}(p\phi)^{2} \times \frac{1}{2$$

$$\begin{split} & [\chi_{m-1} + \chi_{m-2} + 2p\phi(1-p\phi)\chi_{m-3} + p\phi(1-p\phi)^2\chi_{m-4}] \\ & \text{and} \end{split}$$

$$\chi_m(p,\phi^*) = \sum_{k=0}^{\lfloor m/3 \rfloor} {m-2k \choose k} (1-p\phi^*)^{m-k} (p\phi^*)^k.$$

Note: $(1-r)(p\phi)^2(1-p\phi)^2$ accounts for $\{1, 0, 1, 0\}$ sequence.

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix





 $T = 3, d^* = 2$



 $r = 0.01, 0.05, 0.10, 0.15, 0.20, \dots, 1.00.$

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix







to spreading for $r \gtrsim 0.382$.

200 44 of 63

UNIVERSITY

COCONUTS



 \Im No spreading for $r \gtrsim 0.382$.

200 44 of 63

UNIVERSITY

COCONUTS

🙈 Two kinds of contagion processes:

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 A 45 of 63

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





Two kinds of contagion processes:
 1. Continuous phase transition: SIR-like.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References





990 45 of 63

🚳 Two kinds of contagion processes:

- 1. Continuous phase transition: SIR-like.
- 2. Saddle-node bifurcation: threshold model-like.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

Two kinds of contagion processes:
 1. Continuous phase transition: SIR-like.
 2. Saddle-node bifurcation: threshold model-like.

 $d^* = 1$: spreading from small seeds possible.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT 8

Two kinds of contagion processes:

 Continuous phase transition: SIR-like.
 Saddle-node bifurcation: threshold model-like.

 d* = 1: spreading from small seeds possible.
 d* > 1: critical mass model.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT SITY

Two kinds of contagion processes:

 Continuous phase transition: SIR-like.
 Saddle-node bifurcation: threshold model-like.

 d* = 1: spreading from small seeds possible.
 d* > 1: critical mass model.
 Are other behaviors possible?

Outline

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Heterogeneous version

Nutshel

Appendix

References



VERMONT

20 A 46 of 63

Generalized Model

Heterogeneous version

Now allow for general dose distributions (f) and threshold distributions (g).

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Heterogeneous version

Nutshell

Appendix

References





20 A 47 of 63

Now allow for general dose distributions (*f*) and threshold distributions (*g*).
 Key quantities:

$$P_k = \int_0^\infty \mathsf{d} d^* g(d^*) P\left(\sum_{j=1}^k d_j \ge d^*\right) ext{ where } 1 \le k \le T_k$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version

Heterogeneous version

Nutshell

Appendix

References





20 A 47 of 63

Now allow for general dose distributions (*f*) and threshold distributions (*g*).
 Key quantities:

$$P_k = \int_0^\infty \mathsf{d} d^* g(d^*) P\left(\sum_{j=1}^k d_j \ge d^*\right) ext{ where } 1 \le k \le T_k$$

 P_k = Probability that the threshold of a randomly selected individual will be exceeded by k doses.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshel

Appendix

References





20 47 of 63

Now allow for general dose distributions (*f*) and threshold distributions (*g*).
 Key quantities:

$$P_k = \int_0^\infty \mathsf{d} d^* g(d^*) P\left(\sum_{j=1}^k d_j \ge d^*
ight) ext{ where } 1 \le k \le T_k$$

 P_k = Probability that the threshold of a randomly selected individual will be exceeded by k doses.

🗞 e.g.,

 P₁ = Probability that <u>one dose</u> will exceed the threshold of a random individual
 = Fraction of most vulnerable individuals.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshel

Appendix





Generalized model—heterogeneity, r = 1

🙈 Fixed point equation:

$$\phi^* = \sum_{k=1}^T {T \choose k} (p\phi^*)^k (1 - p\phi^*)^{T-k} \underline{P_k}$$

COCONUTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Heterogeneous version

Nutshell

Appendix

References





うへへ 48 of 63

 P₁T is the expected number of vulnerables the initial infected individual meets before recovering
 p₁P₁T is - the expected number of successful infections (equivalent to B₀)

Generalized model—heterogeneity, r = 1Fixed point equation:

$$\phi^* = \sum_{k=1}^T {T \choose k} (p\phi^*)^k (1 - p\phi^*)^{T-k} \underline{P_k}$$

Expand around $\phi^* = 0$ to find when spread from single seed is possible:

$$\boxed{pP_1T\geq 1}$$

P₂T is the expected number of vulnerables the initial infected individual meets before recoveri 2. *pP₁T* is - the expected number of successful infections (equivalent to *R₀*).

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Heterogeneous version

Nutshell

Appendix





Generalized model—heterogeneity, r = 1 Fixed point equation:

$$\phi^* = \sum_{k=1}^T {T \choose k} (p\phi^*)^k (1 - p\phi^*)^{T-k} \underline{P_k}$$

Expand around $\phi^* = 0$ to find when spread from single seed is possible:

$$\label{eq:pp1} \boxed{pP_1T \geq 1} \qquad \text{or} \qquad \Rightarrow p_c = 1/(TP_1)$$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Heterogeneous version

Nutshell

Appendix

References





うへ ~ 48 of 63

Generalized model—heterogeneity, r = 1

lixed point equation:

$$\phi^* = \sum_{k=1}^T {T \choose k} (p\phi^*)^k (1 - p\phi^*)^{T-k} \underline{P_k}$$

Expand around $\phi^* = 0$ to find when spread from single seed is possible:

$$pP_1T \ge 1$$
 or $\Rightarrow p_c =$

$$\Rightarrow p_c = 1/(TP_1)$$

🚳 Very good:

 P₁T is the expected number of vulnerables the initial infected individual meets before recovering.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Heterogeneous version

Nutshell

Appendix

References





20 48 of 63

Generalized model—heterogeneity, r = 1 Fixed point equation:

 $\phi^* = \sum_{k=1}^T {T \choose k} (p\phi^*)^k (1 - p\phi^*)^{T-k} \underline{P_k}$

Expand around $\phi^* = 0$ to find when spread from single seed is possible:

$$pP_1T \ge 1$$
 or

$$\Rightarrow p_c = 1/(TP_1)$$

Appendix

COCONUTS

Independent

Interaction models Interdependent interaction models

Generalized

Model Homogeneous version Heterogeneous version

References

🚳 Very good:

- P₁T is the expected number of vulnerables the initial infected individual meets before recovering.
 nP T is the expected number of successful
- 2. pP_1T is \therefore the expected number of successful infections (equivalent to R_0).



CocoNuTs

Generalized model—heterogeneity, r = 1Fixed point equation:

 $\phi^* = \sum_{k=1}^{T} {T \choose k} (p\phi^*)^k (1 - p\phi^*)^{T-k} \underline{P_k}$

S Expand around $\phi^* = 0$ to find when spread from single seed is possible:

$$pP_1T \ge 1$$
 or

$$\Rightarrow p_c = 1/(TP_1)$$

Appendix

References

🚳 Very good:

- 1. P_1T is the expected number of vulnerables the initial infected individual meets before recovering.
- 2. pP_1T is : the expected number of successful infections (equivalent to R_0).



Solution Observe: p, may exceed 1 meaning no spreading from a small seed.





2 a a 48 of 63

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Next: Determine slope of fixed point curve at critical point p_c.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version Heterogeneous version

Nutshel

Appendix

References





20 A 49 of 63

Next: Determine slope of fixed point curve at critical point p_c.
 Expand fixed point equation around (p, φ*) = (p_c, 0).

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshell

Appendix

References



VERMONT 8

う a (~ 49 of 63

Next: Determine slope of fixed point curve at critical point p_c.
 Expand fixed point equation around (p, φ*) = (p_c, 0).
 Find slope depends on (P₁ - P₂/2)^[5] (see Appendix).

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshell

Appendix

References





20 A 49 of 63

3

- Next: Determine slope of fixed point curve at critical point p_c.
 Expand fixed point equation around (p, φ*) = (p_c, 0).
 - Find slope depends on $(P_1 P_2/2)^{[5]}$ (see Appendix).
- Behavior near fixed point depends on whether this slope is

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshell

Appendix

References





20 A 49 of 63

- Solution Next: Determine slope of fixed point curve at critical point p_c .
 - Expand fixed point equation around $(p, \phi^*) = (p_c, 0).$
- Find slope depends on $(P_1 P_2/2)^{[5]}$ (see Appendix).
- Behavior near fixed point depends on whether this slope is
 - 1. positive: $P_1 > P_2/2$ (continuous phase transition)

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshell

Appendix





- Solution Next: Determine slope of fixed point curve at critical point p_c .
 - Expand fixed point equation around $(p, \phi^*) = (p_c, 0)$.
- Find slope depends on $(P_1 P_2/2)^{[5]}$ (see Appendix).
- Behavior near fixed point depends on whether this slope is
 - 1. positive: $P_1 > P_2/2$ (continuous phase transition)
 - 2. negative: $P_1 < P_2/2$ (discontinuous phase transition)

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshel

Appendix



- Solution Next: Determine slope of fixed point curve at critical point p_c .
- Expand fixed point equation around $(p, \phi^*) = (p_c, 0).$
- Find slope depends on $(P_1 P_2/2)^{[5]}$ (see Appendix).
- Behavior near fixed point depends on whether this slope is
 - 1. positive: $P_1 > P_2/2$ (continuous phase transition)
 - 2. negative: $P_1 < P_2/2$ (discontinuous phase transition)
- Now find <u>three</u> basic universal classes of contagion models...

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshel

Appendix





Example configuration:



Bose sizes are lognormally distributed with mean 1 and variance 0.433.

COCONUTS

Introduction

Interaction models

Interdependent interaction models

Generalized Model Homogeneous version

Heterogeneous version

Appendix

References





2 9 0 50 of 63

Example configuration:



Dose sizes are lognormally distributed with mean 1 and variance 0.433.

3 Memory span: T = 10.

COCONUTS

Introduction

Interaction models

Interdependent interaction models

Generalized Model Homogeneous version

Heterogeneous version

Appendix

References





2 9 0 50 of 63

Example configuration:

Dose sizes are lognormally distributed with mean 1 and variance 0.433.

Solution Memory span: T = 10.

🗞 Thresholds are uniformly set at

1.
$$d_* = 0.5$$

2. $d_* = 1.6$
3. $d_* = 3$

Spread of dose sizes matters, details are not important.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version

Heterogeneous version

Nutshell

Appendix

References





20 0 50 of 63

Example configuration:

Dose sizes are lognormally distributed with mean 1 and variance 0.433.

3 Memory span: T = 10.

Thresholds are uniformly set at

1.
$$d_* = 0.5$$

2. $d_* = 1.6$
3. $d_* = 3$



Spread of dose sizes matters, details are not important.

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Appendix





Three universal classes







References





Dac 51 of 63

Three universal classes



Introduction



Epidemic threshold: $P_1 > P_2/2$, $p_c = 1/(TP_1) < 1$





Three universal classes





Interdependent

Homogeneous version Heterogeneous version

models

models

interaction

Generalized Model

Appendix

References



CocoNuTs Complex Networks Onetworksvox Devershing is connected


Three universal classes





Interaction models









Heterogeneous case

Now allow r < 1:



II-III transition generalizes: p_c = 1/[P₁(T + τ)] where τ = 1/r - 1 = expected recovery time
 I-II transition less pleasant analytically.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Heterogeneous version

Nutshell

Appendix

References



VERMONT 8

20 0 52 of 63

More complicated models



Due to heterogeneity in individual thresholds.
 Three classes based on behavior for small seeds.
 Same model classification holds: I, II, and III.

COcoNuTS

ntroduction

Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Heterogeneous version

Nutshel

Appendix

References



VERMONT

nac 53 of 63

Hysteresis in vanishing critical mass models

COcoNuTS



Independent Interaction models

Interdependent interaction models

Generalized Model

Homogeneous version

Heterogeneous version

Nutshell

Appendix

References





200 54 of 63





Memory is a natural ingredient.

COCONUTS

Introduction

Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





2 Q C 55 of 63



Memory is a natural ingredient. Three universal classes of contagion processes:

I. Epidemic Threshold
 II. Vanishing Critical Mass
 III. Critical Mass

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix







Memory is a natural ingredient. Three universal classes of contagion processes:

I. Epidemic Threshold
 II. Vanishing Critical Mass
 III. Critical Mass

🚳 Dramatic changes in behavior possible.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





na (~ 55 of 63

Memory is a natural ingredient. Three universal classes of contagion processes: 1. I. Epidemic Threshold 2. II. Vanishing Critical Mass 3. III. Critical Mass Dramatic changes in behavior possible. -To change kind of model: 'adjust' memory, recovery, fraction of vulnerable individuals (T, r, ρ , P_1 , and/or P_2).

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell Appendix







Memory is a natural ingredient. Three universal classes of contagion processes: 1. I. Epidemic Threshold 2. II. Vanishing Critical Mass 3. III. Critical Mass Dramatic changes in behavior possible. To change kind of model: 'adjust' memory, 4 recovery, fraction of vulnerable individuals (T, r, ρ , P_1 , and/or P_2). 🚳 To change behavior given model: 'adjust' probability of exposure (p) and/or initial number infected (ϕ_0).

COCONUTS

Interaction models

Interdependent interaction models

Generalized Model Homogeneous version

Nutshell Appendix

References





2 a a 55 of 63

Single seed infects others if $pP_1(T + \tau) \ge 1$.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 56 of 63

Single seed infects others if $pP_1(T + \tau) \ge 1$. Key quantity: $p_c = 1/[P_1(T + \tau)]$

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 56 of 63

Single seed infects others if $pP_1(T + \tau) \ge 1$. Key quantity: $p_c = 1/[P_1(T + \tau)]$ If $p_c < 1 \Rightarrow$ contagion can spread from single seed.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





200 56 of 63

Single seed infects others if $pP_1(T + \tau) \ge 1$. Key quantity: $p_c = 1/[P_1(T + \tau)]$ If $p_c < 1 \Rightarrow$ contagion can spread from single seed. Depends only on:

- 1. System Memory $(T + \tau)$.
- 2. Fraction of highly vulnerable individuals (P_1) .

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Single seed infects others if pP₁(T + τ) ≥ 1.
Key quantity: p_c = 1/[P₁(T + τ)]
If p_c < 1 ⇒ contagion can spread from single seed.
Depends only on:

System Memory (T + τ).
Fraction of highly vulnerable individuals (P₁).

Details unimportant: Many threshold and dose distributions give same P_k.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





Single seed infects others if $pP_1(T + \tau) \ge 1$. Key quantity: $p_c = 1/[P_1(T+\tau)]$ $rac{1}{2}$ If $p_c < 1 \Rightarrow$ contagion can spread from single seed. Depends only on: 1. System Memory $(T + \tau)$. 2. Fraction of highly vulnerable individuals (P_1) . Details unimportant: Many threshold and dose distributions give same P_k . Another example of a model where vulnerable/gullible population may be more important than a small group of super-spreaders or influentials.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References



ク へ へ 56 of 63

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix References





 $\phi^* = \sum_{k=1}^T {T \choose k} P_k (p\phi^*)^k (1-p\phi^*)^{T-k},$ $= \sum_{k=1}^T {T \choose k} P_k(p\phi^*)^k \sum_{j=0}^{T-k} {T-k \choose j} (-p\phi^*)^j,$ $= \sum_{k=1}^{T} \sum_{i=0}^{T-k} {T \choose k} {T-k \choose j} P_k (-1)^j (p\phi^*)^{k+j},$ $= \sum_{m=1}^{T} \sum_{k=1}^{m} {T \choose k} {T-k \choose m-k} P_k(-1)^{m-k} (p\phi^*)^m,$ $= ~\sum_{}^{T} C_m (p \phi^*)^m$

Introduction

COcoNuTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix References





$$C_m = (-1)^m {T \choose m} \sum_{k=1}^m (-1)^k {m \choose k} P_k,$$

since

$$\binom{T}{k}\binom{T-k}{m-k}$$

$$\frac{T!}{k!(T-k)!} \frac{(T-k)!}{(m-k)!(T-m)!} \\
\frac{T!}{m!(T-m)!} \frac{m!}{k!(m-k)!} \\
\frac{T}{m!} \frac{m!}{m!} \\
\frac{T}{m!} \frac{m!}{k!(m-k)!} \\
\frac{T}{m} \frac{m}{k}.$$

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix References





20 C 59 of 63

🚳 Linearization gives

 $\phi^*\simeq C_1p\phi^*+C_2p_c^2\phi^{*2}.$

where $C_1 = TP_1(=1/p_c)$ and $C_2 = {T \choose 2}(-2P_1 + P_2)$.

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix References



VERMONT SITY

🚳 Linearization gives

 $\phi^* \simeq C_1 p \phi^* + C_2 p_c^2 {\phi^*}^2.$

where $C_1 = TP_1(=1/p_c)$ and $C_2 = {T \choose 2}(-2P_1 + P_2)$. Using $p_c = 1/(TP_1)$:

 $\phi^* \simeq \frac{C_1}{C_2 p_c^2} (p-p_c) = \frac{T^2 P_1^3}{(T-1)(P_1-P_2/2)} (p-p_c).$

Introduction

COCONUTS

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshel

Appendix References





where $C_1 = TP_1 (= 1/p_c)$ and $C_2 = (\frac{T}{2})(-2P_1 + P_2)$.

Solution Using $p_c = 1/(TP_1)$:

Linearization gives

 $\phi^* \simeq \frac{C_1}{C_2 p_c^2} (p-p_c) = \frac{T^2 P_1^3}{(T-1)(P_1-P_2/2)} (p-p_c).$

 $\phi^* \simeq C_1 p \phi^* + C_2 p_c^2 {\phi^*}^2.$

Sign of derivative governed by $P_1 - P_2/2$.

200 59 of 63

References I

[1] F. Bass. A new product growth model for consumer durables. Manage. Sci., 15:215-227, 1969. pdf D. J. Daley and D. G. Kendall. [2] Epidemics and rumours. Nature, 204:1118, 1964. pdf D. J. Daley and D. G. Kendall. [3] Stochastic rumours. J. Inst. Math. Appl., 1:42-55, 1965. P. S. Dodds and D. J. Watts. [4] Universal behavior in a generalized model of contagion. Phys. Rev. Lett., 92:218701, 2004. pdf

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References



VERMONT

References II

 P. S. Dodds and D. J. Watts. A generalized model of social and biological contagion.
 J. Theor. Biol., 232:587–604, 2005. pdf 2

[6] W. Goffman and V. A. Newill. Generalization of epidemic theory: An application to the transmission of ideas. Nature, 204:225–228, 1964. pdf 7

[7] W. O. Kermack and A. G. McKendrick. A contribution to the mathematical theory of epidemics. Proc. R. Soc. Lond. A, 115:700–721, 1927. pdf C

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





990 61 of 63

References III

 [8] W. O. Kermack and A. G. McKendrick. A contribution to the mathematical theory of epidemics. III. Further studies of the problem of endemicity.
 Proc. R. Soc. Lond. A, 141(843):94–122, 1927. pdf

[9] W. O. Kermack and A. G. McKendrick. Contributions to the mathematical theory of epidemics. II. The problem of endemicity. Proc. R. Soc. Lond. A, 138(834):55–83, 1927. pdf C

[10] J. D. Murray. Mathematical Biology. Springer, New York, Third edition, 2002.

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix





References IV

COcoNuTS

Introduction

Independent Interaction models

Interdependent interaction models

Generalized Model Homogeneous version Heterogeneous version

Nutshell

Appendix

References





20 63 of 63

[11] S. H. Strogatz. Nonlinear Dynamics and Chaos. Addison Wesley, Reading, Massachusetts, 1994.