Contagion

Complex Networks | @networksvox
CSYS/MATH 303, Spring, 2016

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Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

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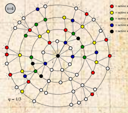

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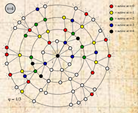


Outline

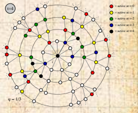
Basic Contagion Models

Global spreading condition

Social Contagion Models
- Network version
- All-to-all networks

Theory
- Spreading possibility
- Spreading probability
- Physical explanation
- Final size

References

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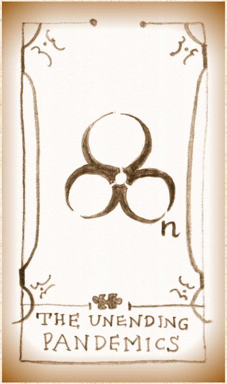
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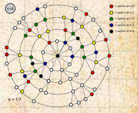


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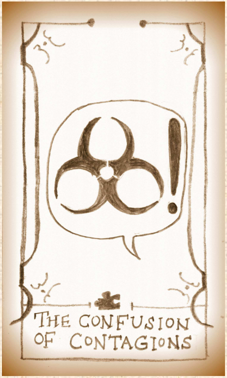
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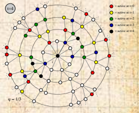

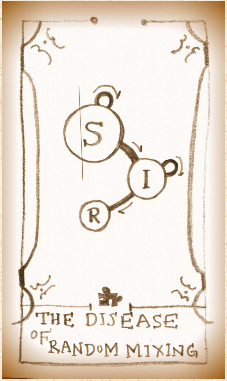
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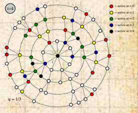


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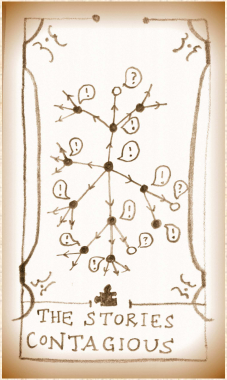
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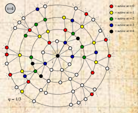


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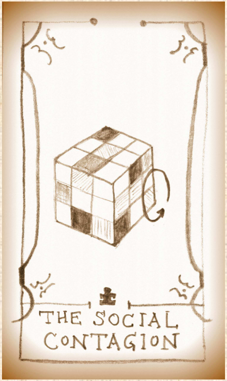
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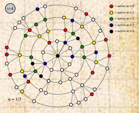


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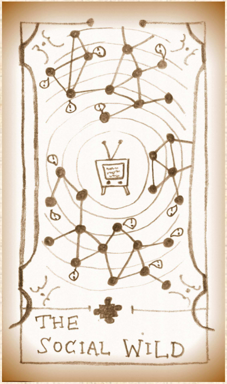
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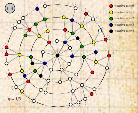


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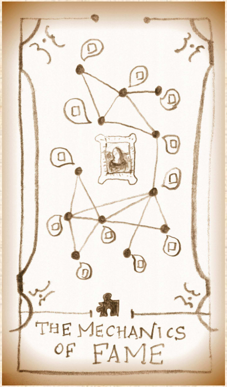
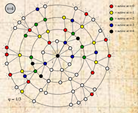

THE MECHANICS OF FAME

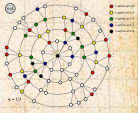


Contagion models

Some large questions concerning network contagion:

1. For a given spreading mechanism on a given network, what's the probability that there will be global spreading?
2. If spreading does take off, how far will it go?
3. How do the details of the network affect the outcome?
4. How do the details of the spreading mechanism affect the outcome?
5. What if the seed is one or many nodes?

Next up: We'll look at some fundamental kinds of spreading on generalized random networks.



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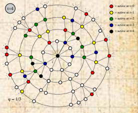

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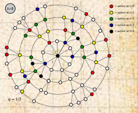

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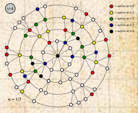

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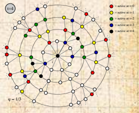

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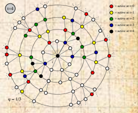

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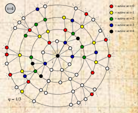
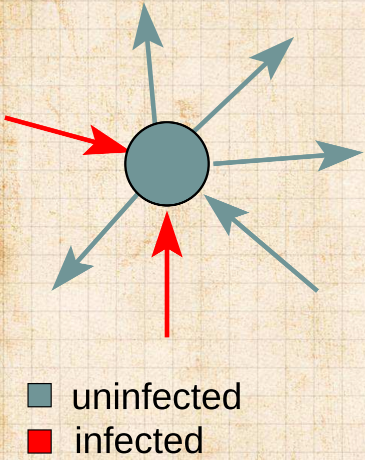
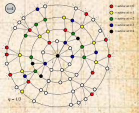


Spreading mechanisms



- General spreading mechanism: State of node i depends on history of i and i's neighbors' states.
- Doses of entity may be stochastic and history-dependent.
- May have multiple, interacting entities spreading at once.



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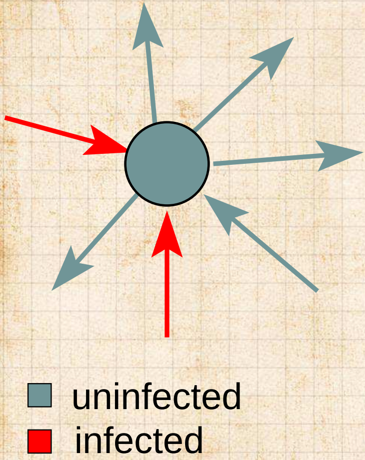

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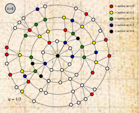

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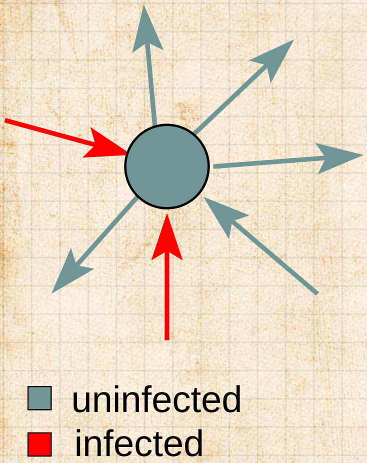

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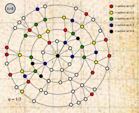
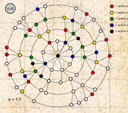


Spreading on Random Networks

- For random networks, we know local structure is pure branching.
- Successful spreading is ∴ contingent on single edges infecting nodes.

- Focus on binary case with edges and nodes either infected or not.
- First big question: for a given network and contagion process, can global spreading from a single seed occur?



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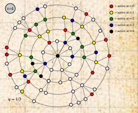

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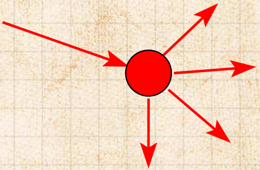
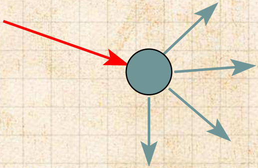


Failure:



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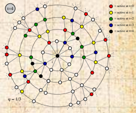

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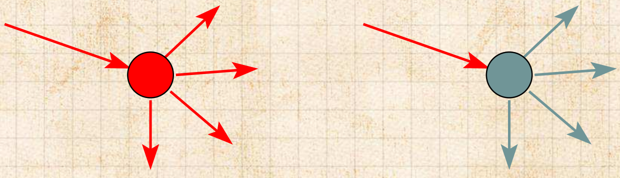

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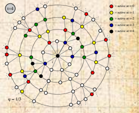

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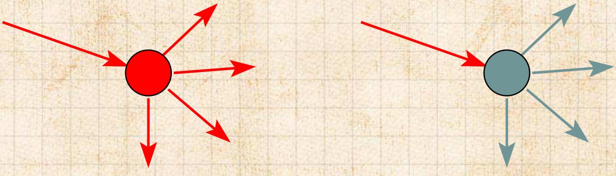

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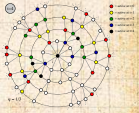
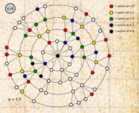


Global spreading condition

- We need to find: [5]
  $\mathbf{R}$ = the average # of infected edges that one random infected edge brings about.
- Call $\mathbf{R}$ the gain ratio.
- Define B_{k1} as the probability that a node of degree k is infected by a single infected edge.
-

$$\mathbf{R} = \sum_{k=0}^{\infty} \underbrace{\frac{kP_k}{\langle k \rangle}}_{\substack{\text{prob. of} \\ \text{connecting to} \\ \text{a degree } k \text{ node}}} \bullet \underbrace{(k-1)}_{\substack{\text{\# outgoing} \\ \text{infected} \\ \text{edges}}} \bullet \underbrace{B_{k1}}_{\substack{\text{Prob. of} \\ \text{infection}}}$$

$$+ \sum_{k=0}^{\infty} \overbrace{\frac{kP_k}{\langle k \rangle}} \bullet \underbrace{0}_{\substack{\text{\# outgoing} \\ \text{infected} \\ \text{edges}}} \bullet \underbrace{(1 - B_{k1})}_{\substack{\text{Prob. of} \\ \text{no infection}}}$$



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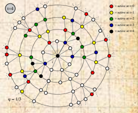

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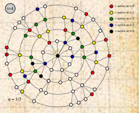

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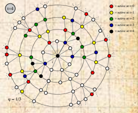

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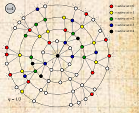

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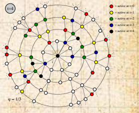

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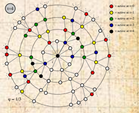

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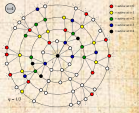

The UNIVERSITY of VERMONT

Global spreading condition

- Our global spreading condition is then:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

- Case 1: If $B_{k1} = 1$ then

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) = \frac{\langle k(k-1) \rangle}{\langle k \rangle} > 1.$$

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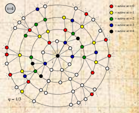

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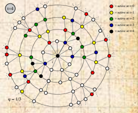

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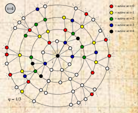

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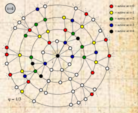

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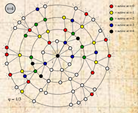

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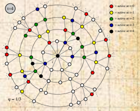
- Aka bond percolation.
- Resulting degree distribution $\tilde{P}_k$:

$$\tilde{P}_k = \beta^k \sum_{i=k}^{\infty} \binom{i}{k} (1-\beta)^{i-k} P_i.$$

Insert question from assignment 9

- We can show $F_{\tilde{P}}(x) = F_P(\beta x + 1 - \beta)$.



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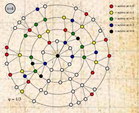

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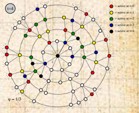

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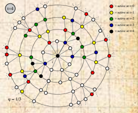

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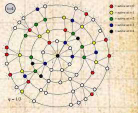

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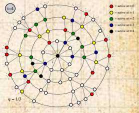

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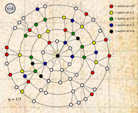

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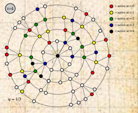

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- **Cases 3, 4, 5, ...:** Now allow B_{k1} to depend on k
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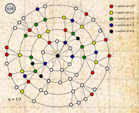

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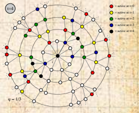

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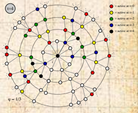

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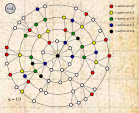

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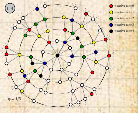

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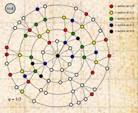

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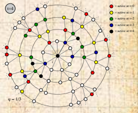

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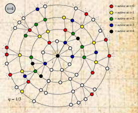

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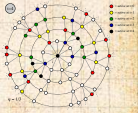
- Cases 3, 4, 5, ...: Now allow B_{k1} to depend on k
- Asymmetry: Transmission along an edge depends on node's degree at other end.
- Possibility: B_{k1} increases with k... unlikely.
- Possibility: B_{k1} is not monotonic in k... unlikely.
- Possibility: B_{k1} decreases with k... hmmm.
- $B_{k1} \searrow$ is a plausible representation of a simple kind of social contagion.
- The story: More well connected people are harder to influence.



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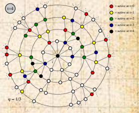

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- Example: $B_{k1} = 1/k$.
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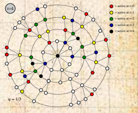
- Since $\mathbf{R}$ is always less than 1, no spreading can occur for this mechanism.
- Decay of B_{k1} is too fast.
- Result is independent of degree distribution.



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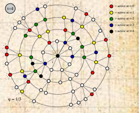

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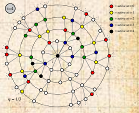

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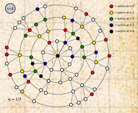

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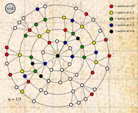

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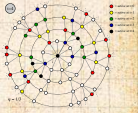

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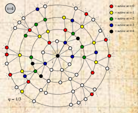

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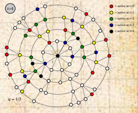

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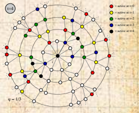

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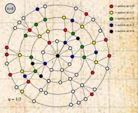

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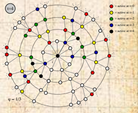

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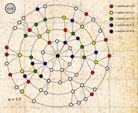

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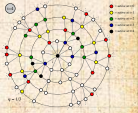

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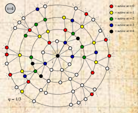

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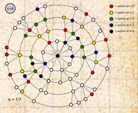

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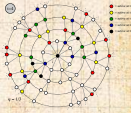
- The uniform threshold model global spreading condition:

$$\mathbf{R} = \sum_{k=1}^{\lfloor \frac{1}{\phi} \rfloor} (k-1) \bullet \frac{kP_k}{\langle k \rangle} > 1.$$

- As $\phi \to 1$, all nodes become resilient and $r \to 0$.
- As $\phi \to 0$, all nodes become vulnerable and the contagion condition matches up with the giant component condition.
- Key: If we fix ϕ and then vary $\langle k \rangle$, we may see two phase transitions.
- Added to our standard giant component transition, we will see a cut off in spreading as nodes become more connected.



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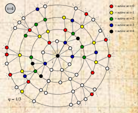

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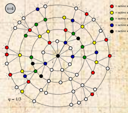

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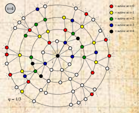

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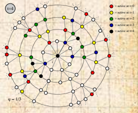
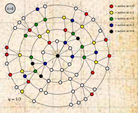


Virtual contagion: Corrupted Blood, a 2005 virtual plague in World of Warcraft:



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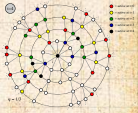

Social Contagion

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- Tipping models—Schelling (1971) [11, 12, 13]
  - Simulation on checker boards.
  - Idea of thresholds.
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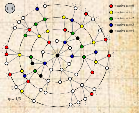

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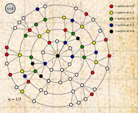

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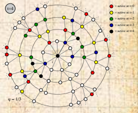

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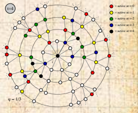

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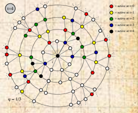

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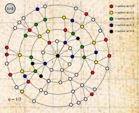

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A simple model of global cascades on random networks

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- Mean field Granovetter model → network model
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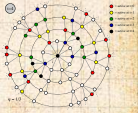

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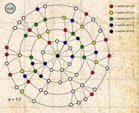

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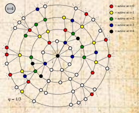

Threshold model on a network

- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when number of active contacts $a_i \geq \phi_i k_i$
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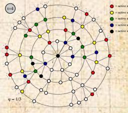

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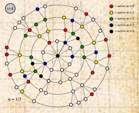

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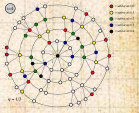

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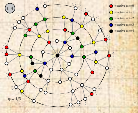

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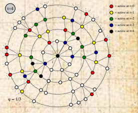

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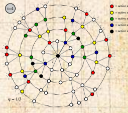

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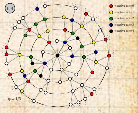

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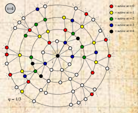

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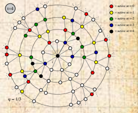

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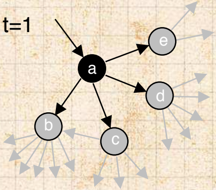

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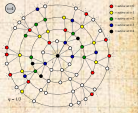

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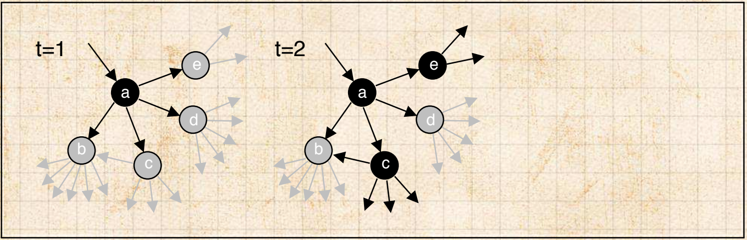

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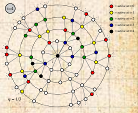

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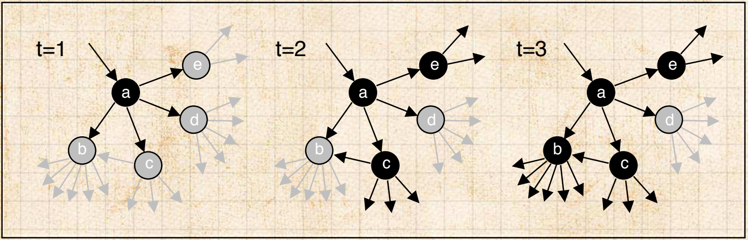

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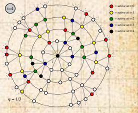

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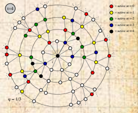
Vulnerables:

- Recall definition: individuals who can be activated by just one contact being active are vulnerables.
- The vulnerability condition for node i: $1/k_i \geq \phi_i$.
- Means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$.
- Key: For global spreading events (cascades) on random networks, must have a *global component of vulnerables* [15]
- For a uniform threshold ϕ, our global spreading condition tells us when such a component exists:

$$\mathbf{R} = \sum_{k=1}^{\lfloor \frac{1}{\phi} \rfloor} \frac{k P_k}{\langle k \rangle} \bullet (k-1) > 1.$$



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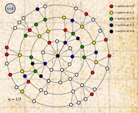

The UNIVERSITY of VERMONT

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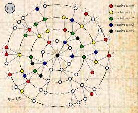

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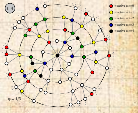

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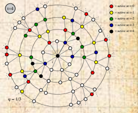

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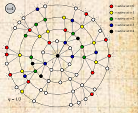
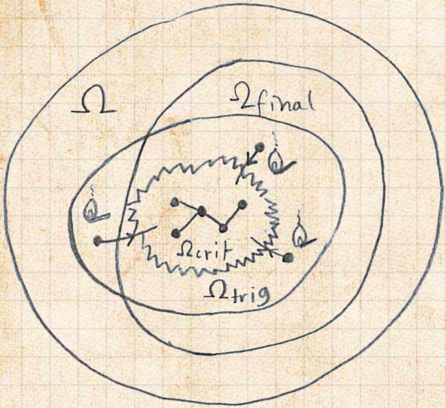


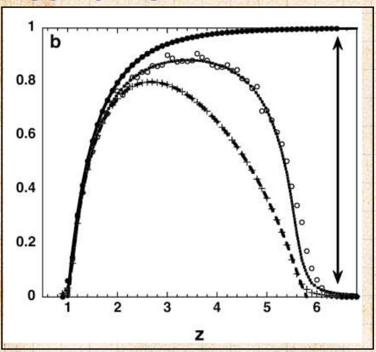
Example random network structure:



- Ω_{crit} = critical mass = global vulnerable component
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network

$\Omega_{crit} \subset \Omega_{trig}$; $\Omega_{crit} \subset \Omega_{final}$; and $\Omega_{trig}, \Omega_{final} \subset \Omega$.

Global spreading events on random networks [15]



$z = \langle k \rangle$

- Top curve: final fraction infected if successful.
- Middle curve: chance of starting a global spreading event (cascade).
- Bottom curve: fractional size of vulnerable subcomponent.[15]

- Global spreading events occur only if size of vulnerable subcomponent > 0.
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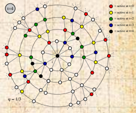
- 'Ignorance' facilitates spreading.



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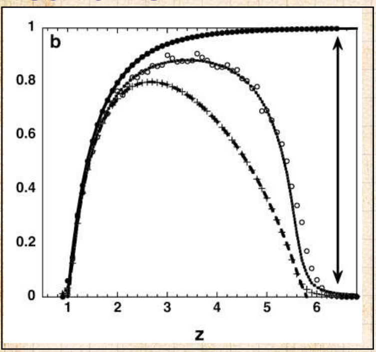

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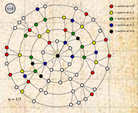

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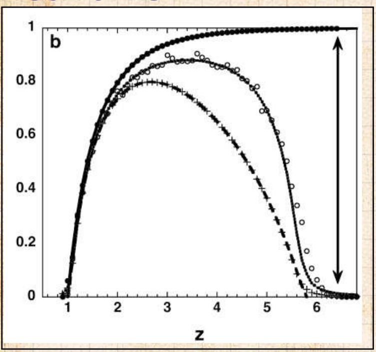

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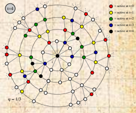

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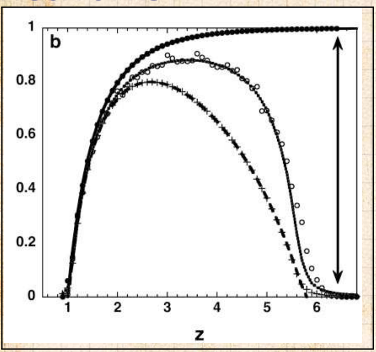

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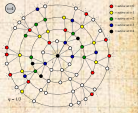

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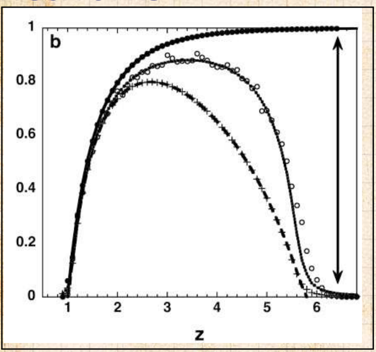

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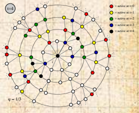

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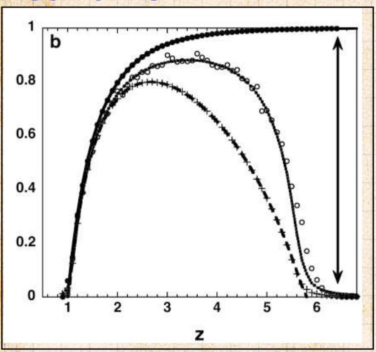

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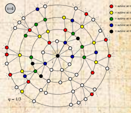
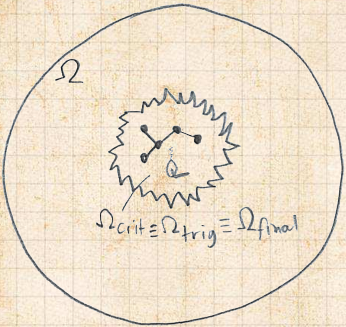
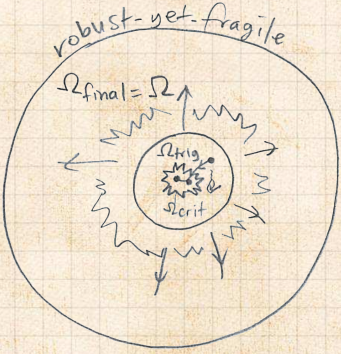
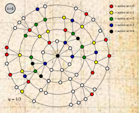


Cascades on random networks





- Above lower phase transition
- Just below upper phase transition



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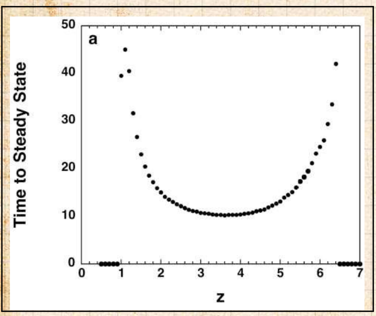
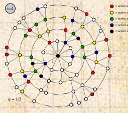


(n.b., $z = \langle k \rangle$)

- Time taken for cascade to spread through network. [15]
- Two phase transitions.
- Largest vulnerable component = critical mass.
- Now have endogenous mechanism for spreading from an individual to the critical mass and then beyond.



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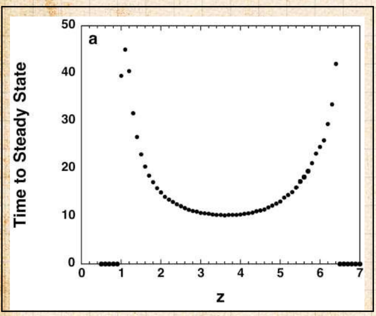

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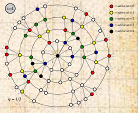
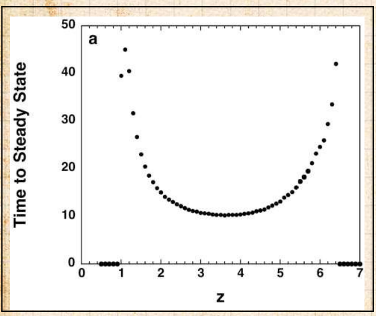


Cascades on random networks



(n.b., $z = \langle k \rangle$)

- Time taken for cascade to spread through network. [15]
- Two phase transitions.

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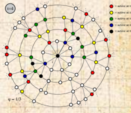

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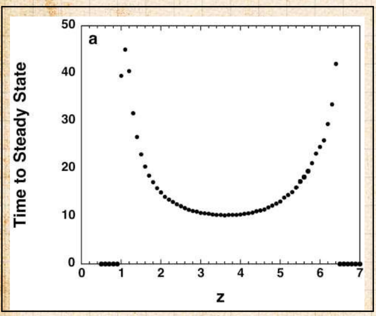

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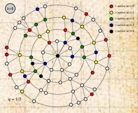
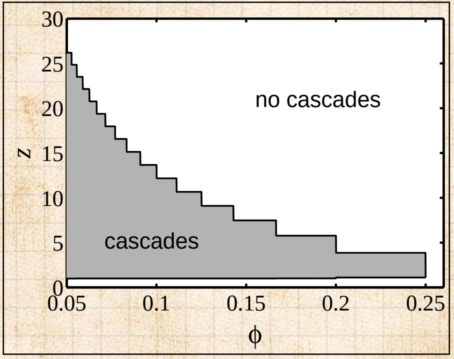


Cascade window for random networks



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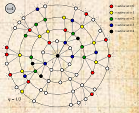
- Outline of cascade window for random networks.



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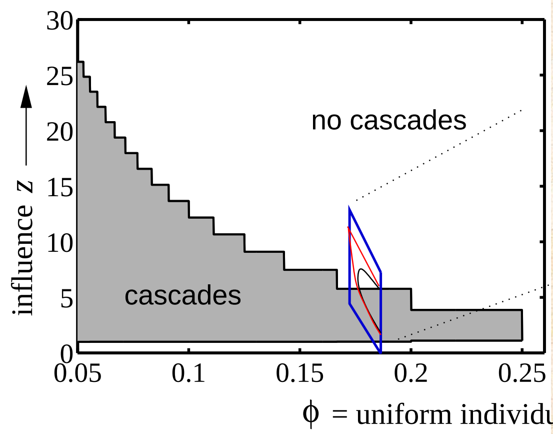

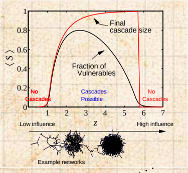

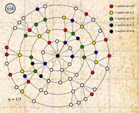

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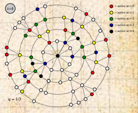
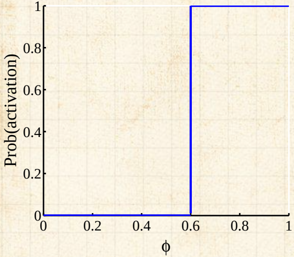


Social Contagion

Granovetter's Threshold model—recap



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- ϕ_* = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.
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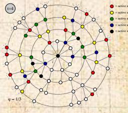

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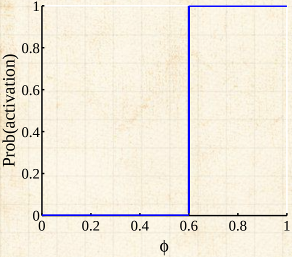

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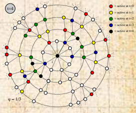

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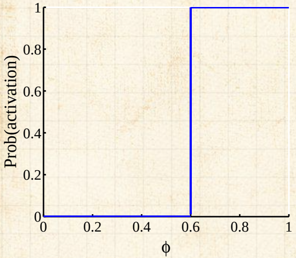

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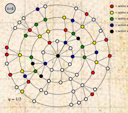

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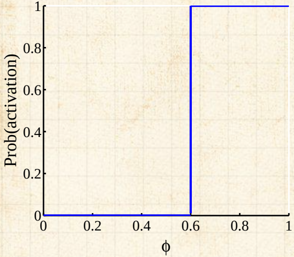

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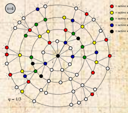

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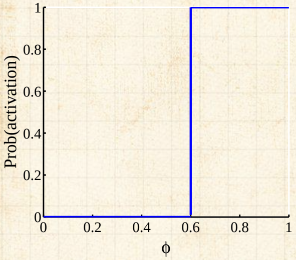

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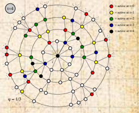
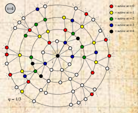


Social Sciences—Threshold models

- At time $t + 1$, fraction rioting = fraction with $\phi_* \leq \phi_t$.
- $$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*)\mathrm{d}\phi_* = F(\phi_*)|_0^{\phi_t} = F(\phi_t)$$
- $\Rightarrow$ Iterative maps of the unit interval $[0, 1]$.



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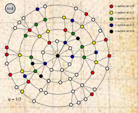

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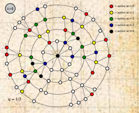
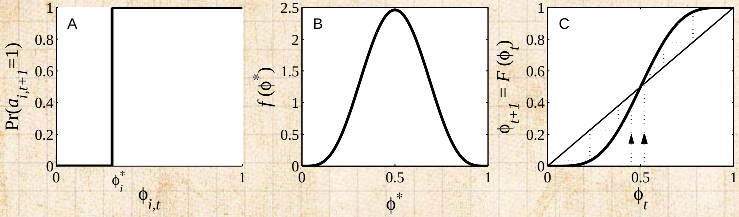
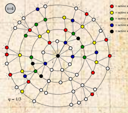


Social Sciences—Threshold models

Action based on perceived behavior of others.



- Two states: S and I
- Recover now possible (SIS)
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time, synchronous update (strong assumption!)
- This is a Critical mass model



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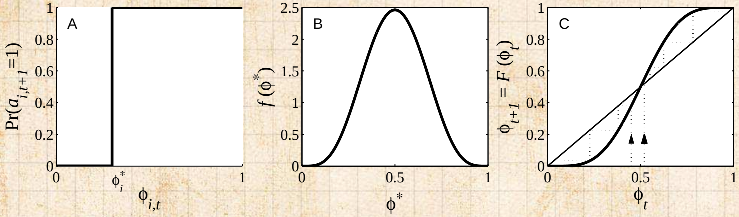

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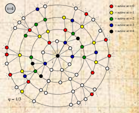

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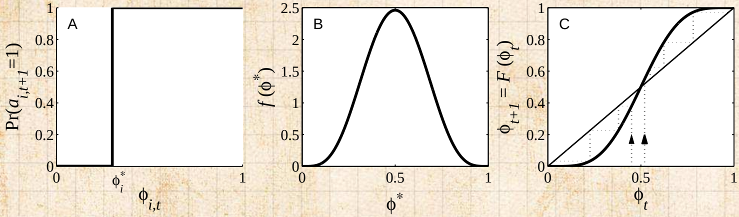

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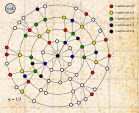

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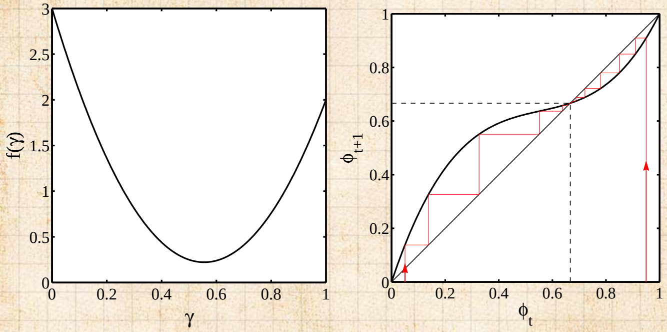
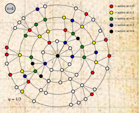


- Example of single stable state model



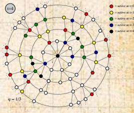
Social Sciences—Threshold models

Implications for collective action theory:

1. Collective uniformity $\not\Rightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes

Next:

- Connect mean-field model to network model.
- Single seed for network model: $1/N \to 0$.
- Comparison between network and mean-field model sensible for vanishing seed size for the latter.



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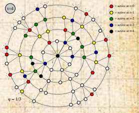

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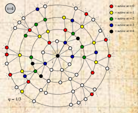

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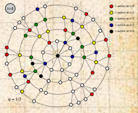

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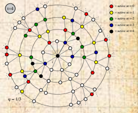

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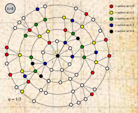

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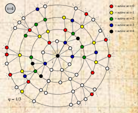
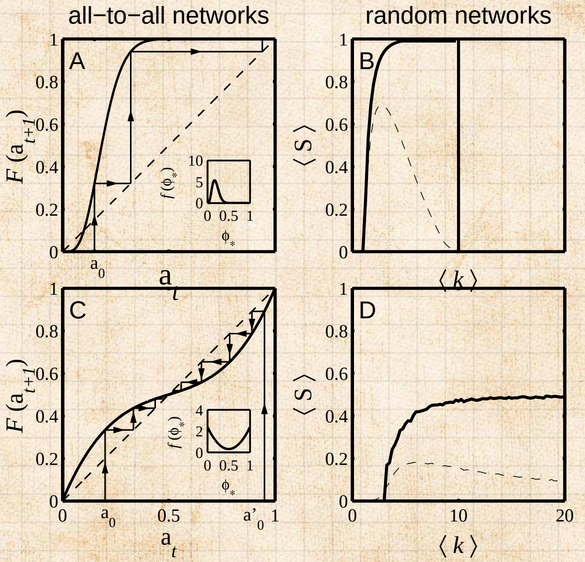
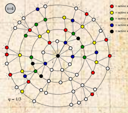


All-to-all versus random networks



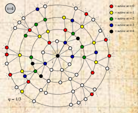


Spreadworthiness: Cat videos

Bowling with Ragdolls:

https://www.youtube.com/v/XX-g2nmqL9Q?rel=0

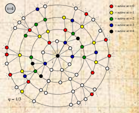
- Organic extreme outlier?
- Success did not spread to other videos.



Threshold contagion on random networks

Three key pieces to describe analytically:

1. The fractional size of the largest subcomponent of vulnerable nodes, S_{vuln}.
2. The chance of starting a global spreading event, $P_{\text{trig}} = S_{\text{trig}}$.
3. The expected final size of any successful spread, S.
   - n.b., the distribution of S is almost always bimodal.



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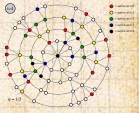

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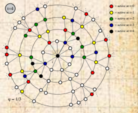

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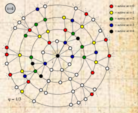

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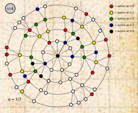
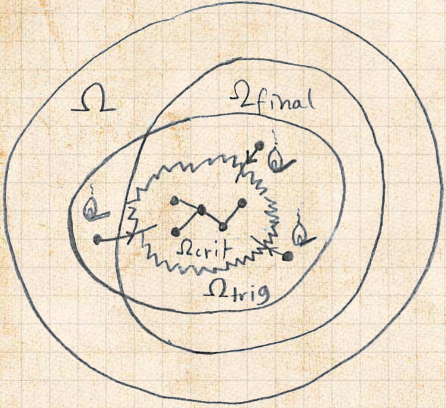
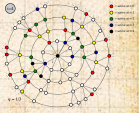


Example random network structure:



- $\Omega_{\text{crit}} = \Omega_{\text{vuln}} =$ critical mass = global vulnerable component
- $\Omega_{\text{trig}} =$ triggering component
- $\Omega_{\text{final}} =$ potential extent of spread
- $\Omega =$ entire network

$\Omega_{\text{crit}} \subset \Omega_{\text{trig}}$; $\Omega_{\text{crit}} \subset \Omega_{\text{final}}$; and $\Omega_{\text{trig}}, \Omega_{\text{final}} \subset \Omega$.



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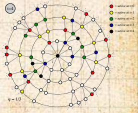


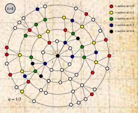
Threshold contagion on random networks

- **First goal:** Find the largest component of vulnerable nodes.
- Recall that for finding the giant component's size, we had to solve:

$$F_\pi(x) = xF_P(F_\rho(x)) \text{ and } F_\rho(x) = xF_R(F_\rho(x))$$

- We'll find a similar result for the subset of nodes that are vulnerable.
- This is a node-based percolation problem.
- For a general monotonic threshold distribution $f(\phi)$, a degree k node is vulnerable with probability

$$B_{k1} = \int_0^{1/k} f(\phi)\mathrm{d}\phi.$$



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- First goal: Find the largest component of vulnerable nodes.
- Recall that for finding the giant component's size, we had to solve:

$$F_\pi(x) = xF_P\left(F_\rho(x)\right) \text{ and } F_\rho(x) = xF_R\left(F_\rho(x)\right)$$

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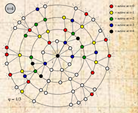

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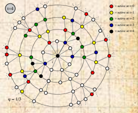

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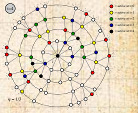

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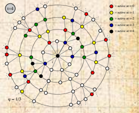

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- We now have a generating function for the probability that a randomly chosen node is vulnerable and has degree k:

$$F_P^{(\text{vuln})}(x) = \sum_{k=0}^{\infty} P_k B_{k1} x^k.$$

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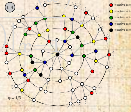

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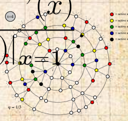

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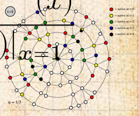

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The UNIVERSITY of VERMONT

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- Functional relations for component size g.f.'s are almost the same ...

$$F_\pi^{(\text{vuln})}(x) = \underbrace{1 - F_P^{(\text{vuln})}(1)}_{\substack{\text{central node} \\ \text{is not} \\ \text{vulnerable}}} + x F_P^{(\text{vuln})}\left(F_\rho^{(\text{vuln})}(x)\right)$$

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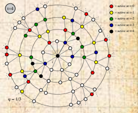

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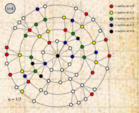

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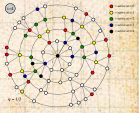

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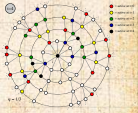

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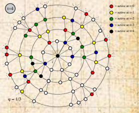

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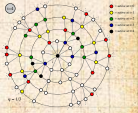

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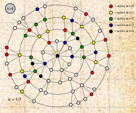

Threshold contagion on random networks

- Second goal: Find probability of triggering largest vulnerable component.
- Assumption is first node is randomly chosen.
- Same set up as for vulnerable component except now we don't care if the initial node is vulnerable or not:

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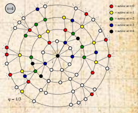

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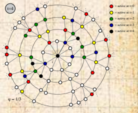

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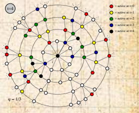

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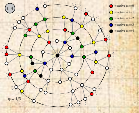

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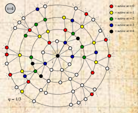
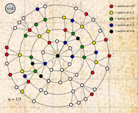


Physical derivation of possibility and probability of global spreading:

- Possibility: binary indicator of phase. Global spreading events are either possible or can never happen.
- For random networks, global spreading possibility is understood as meaning a giant component of vulnerable nodes exists.
- Next: what's the probability that a randomly infected node will cause a global spreading event?
- Call this P_{trig}.
- As usual, it's all about edges and we need to first determine the probability that an infected edge leads to a global spreading event.
- Call this Q_{trig}.
- Later: Generalize to more complex networks involving assortativity of all kinds.



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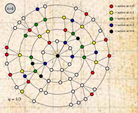

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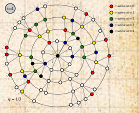

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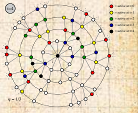

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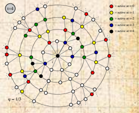

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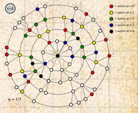

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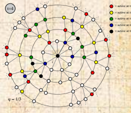

Probability an infected edge leads to a global spreading event:

- Q_{trig} must satisfying a one-step recursion relation.
- Follow an infected edge and use three pieces:
  1. Probability of reaching a degree k node is $Q_k = \frac{kP_k}{\langle k \rangle}$.
  2. The node reached is vulnerable with probability B_{k1}.
  3. At least one of the node's outgoing edges leads to a global spreading event = 1 − probability no edges do so = $1 - (1 - Q_{\text{trig}})^{k-1}$.
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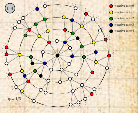

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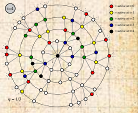

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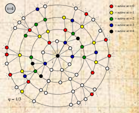

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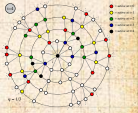

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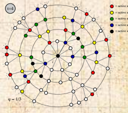

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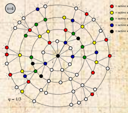

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$$S_{\mathrm{vuln}} = \sum_k P_k \bullet B_{k1} \bullet \left[1 - (1 - Q_{\mathrm{trig}})^k\right] > 0.$$

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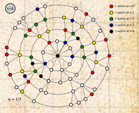

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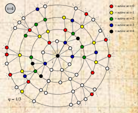

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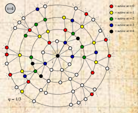

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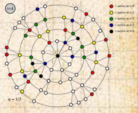

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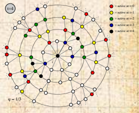

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- We found that $F_\rho^{(\text{vuln})}(1)$—the probability that a random edge leads to a finite vulnerable component—satisfies

$$F_\rho^{(\text{vuln})}(1) = 1 - F_R^{(\text{vuln})}(1) + 1 \cdot F_R^{(\text{vuln})}\left(F_\rho^{(\text{vuln})}(1)\right).$$

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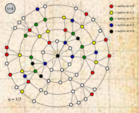

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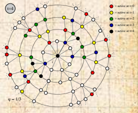

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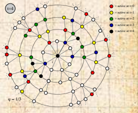

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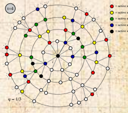

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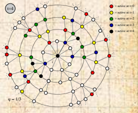

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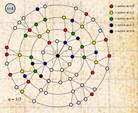

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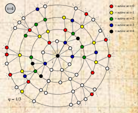

Triggering probability for single-seed global spreading events:

- Slight adjustment to the vulnerable component calculation.
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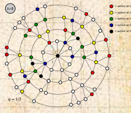

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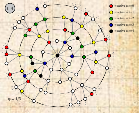

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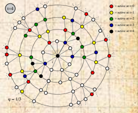

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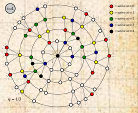

Connection to simple gain ratio argument:

- Earlier, we showed the global spreading condition follows from the gain ratio $\mathbf{R} > 1$:

$$\mathbf{R} = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

- We would very much like to see that $\mathbf{R} > 1$ matches up with $Q_{\text{trig}} > 0$.
- It really would be just so totally awesome.
- Must come from our basic edge triggering probability equation:

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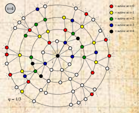
- We need to find out what happens as $Q_{\text{trig}} \to 0$.[9]



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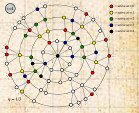

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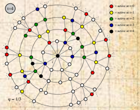

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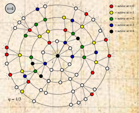

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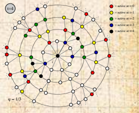

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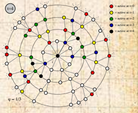

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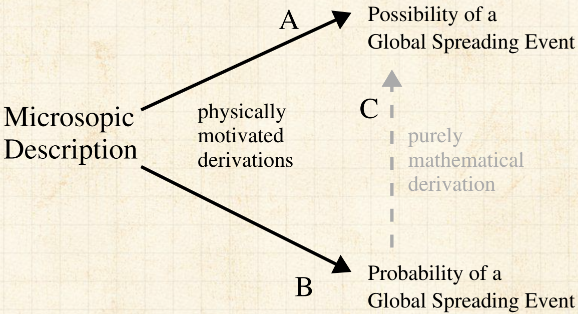

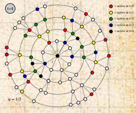

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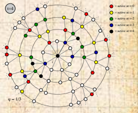

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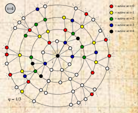

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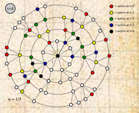

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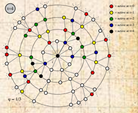

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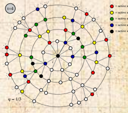

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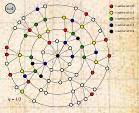

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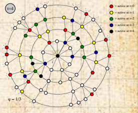
Outline



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- Third goal: Find expected fractional size of spread.
- Not obvious even for uniform threshold problem.
- Difficulty is in figuring out if and when nodes that need ≥ 2 hits switch on.
- Problem solved for infinite seed case by Gleeson and Cahalane:
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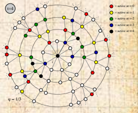

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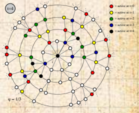

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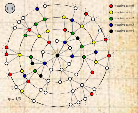

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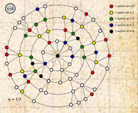

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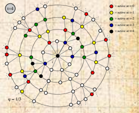
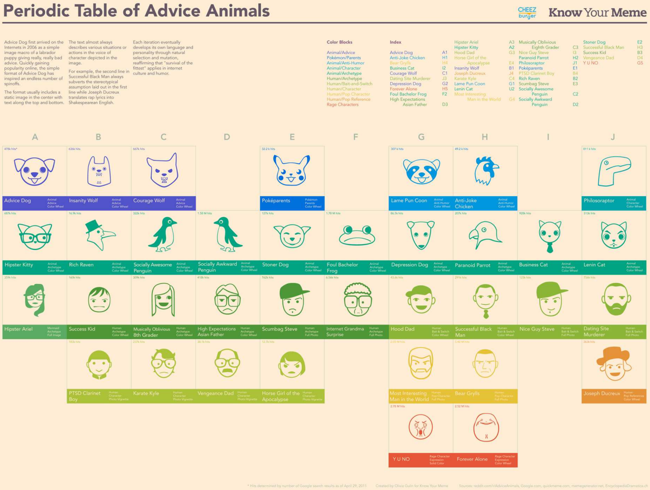
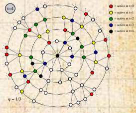


Meme species:



More here at http://knowyourmeme.com



Expected size of spread

Idea:

- Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$
- Capitalize on local branching network structure of random networks (again)
- Now think about what must happen for a specific node i to become active at time t:
  - $t = 0$: i is one of the seeds (prob = ϕ_0)
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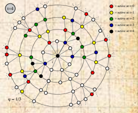

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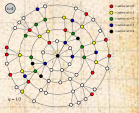

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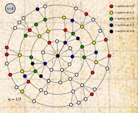

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- Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$
- Capitalize on local branching network structure of random networks (again)
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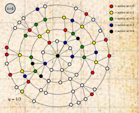

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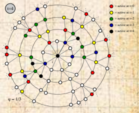

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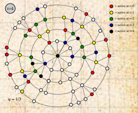

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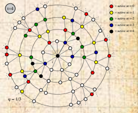

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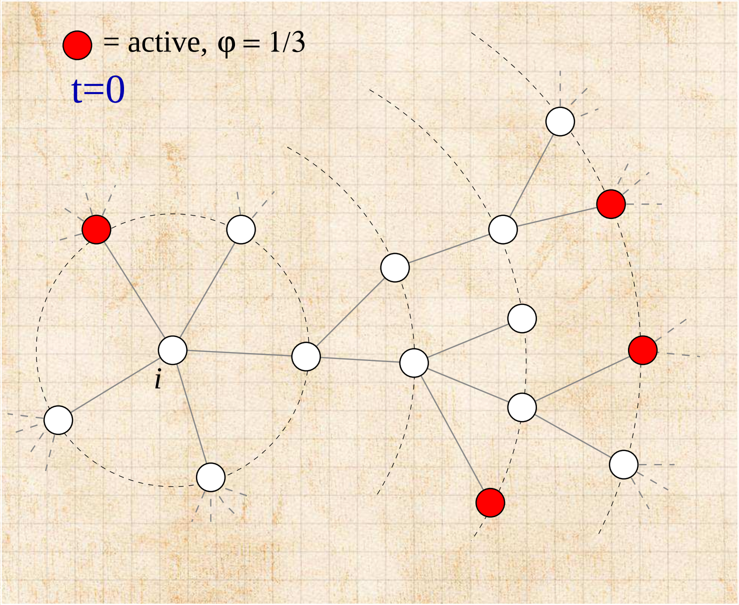

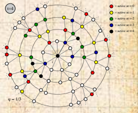

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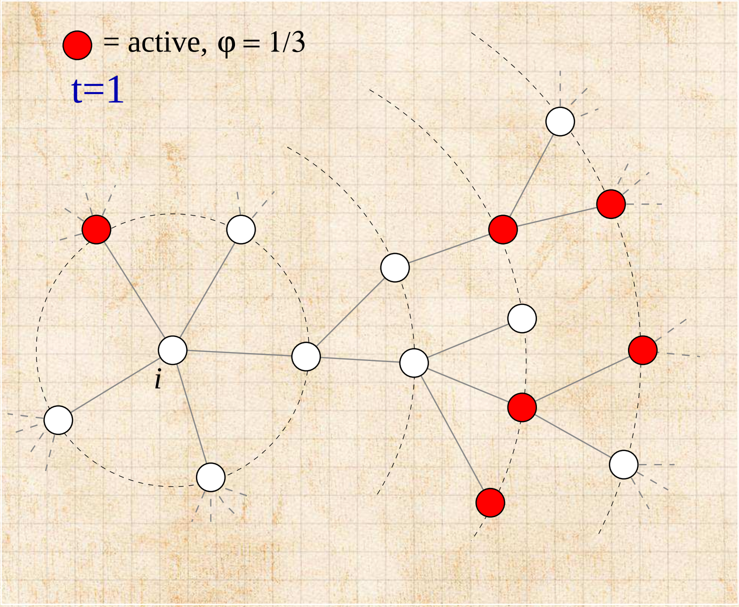

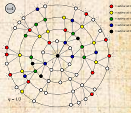

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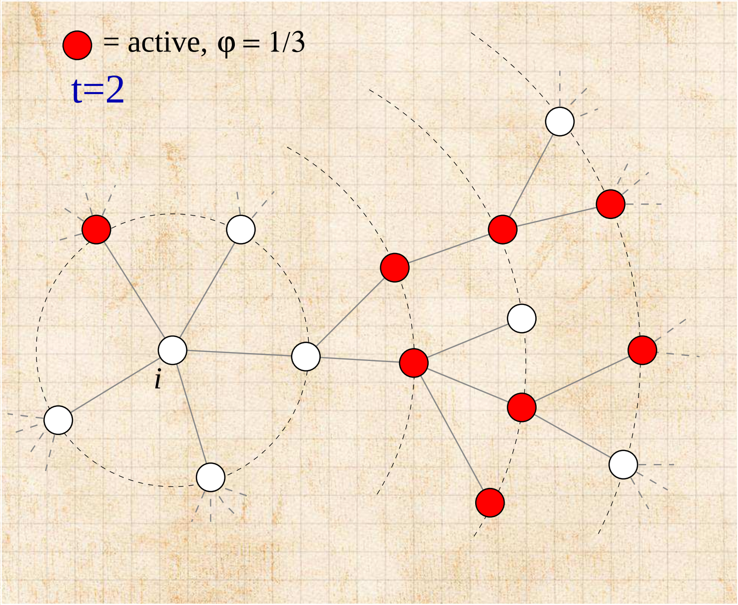

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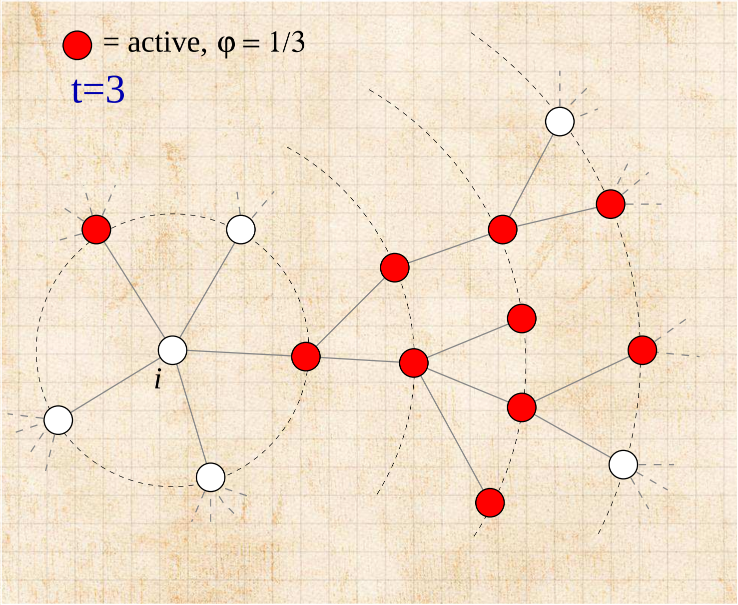

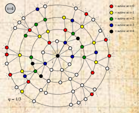

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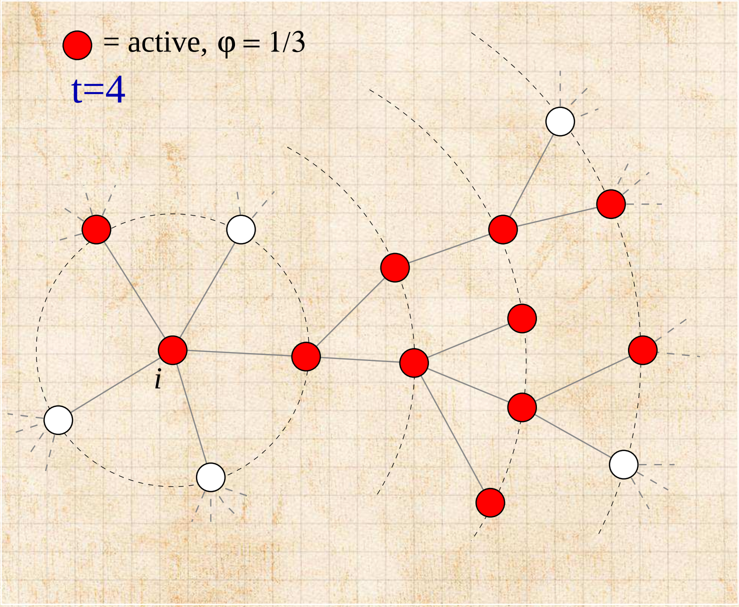

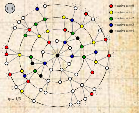

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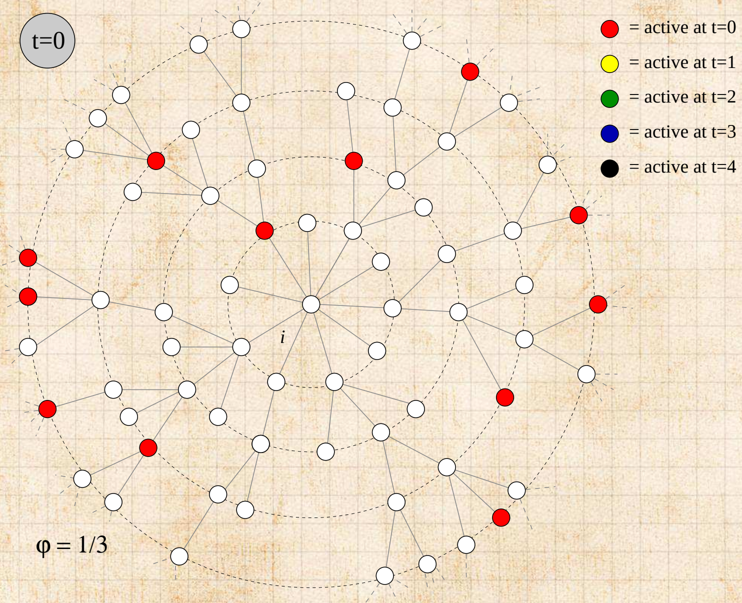

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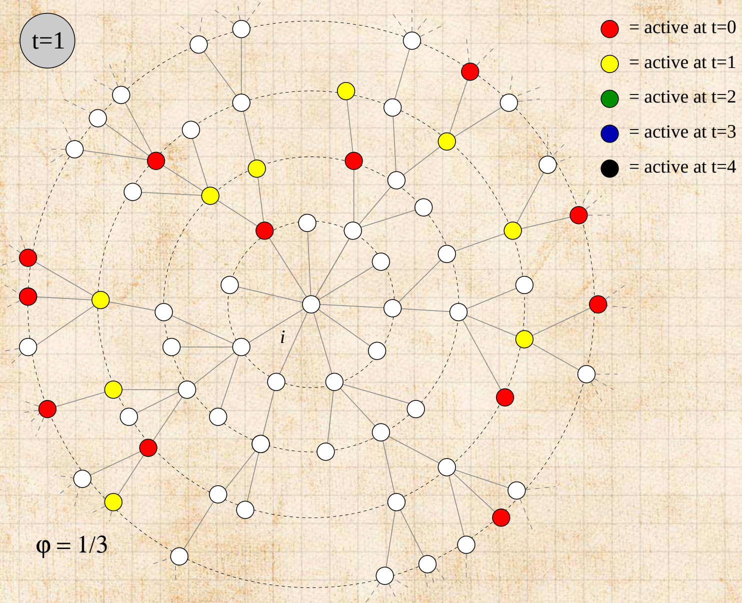

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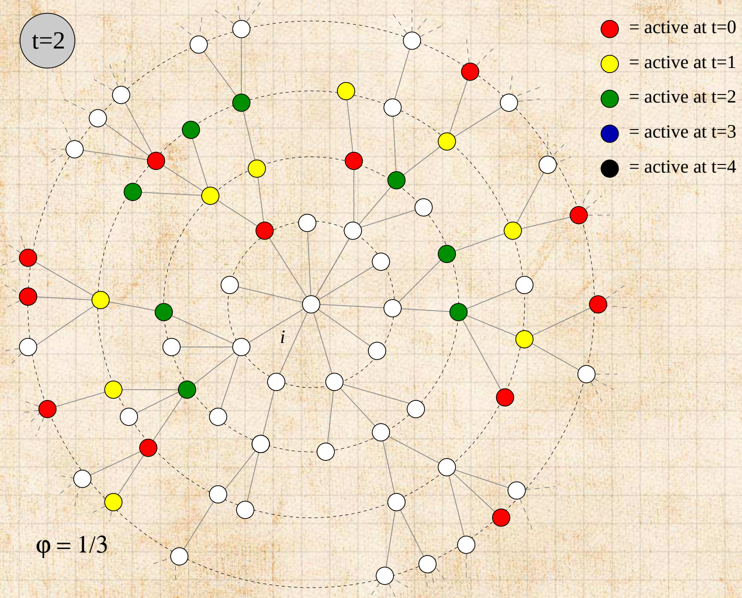

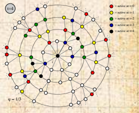

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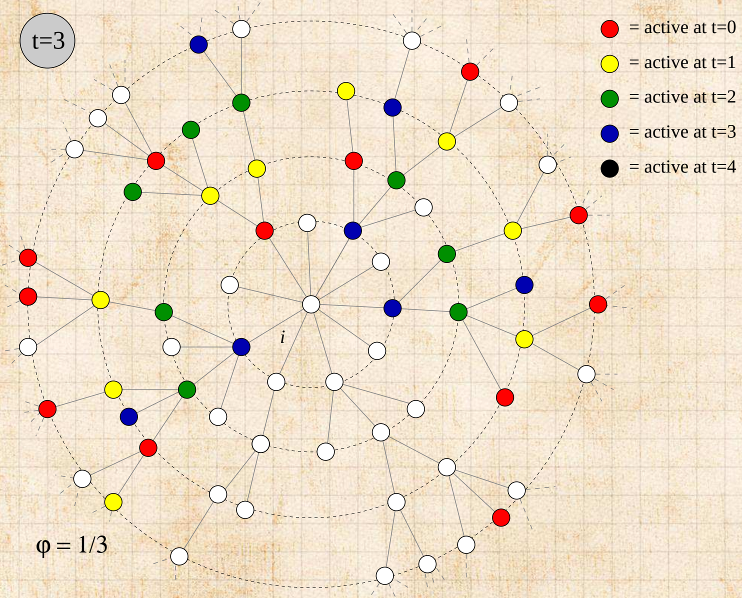

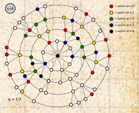

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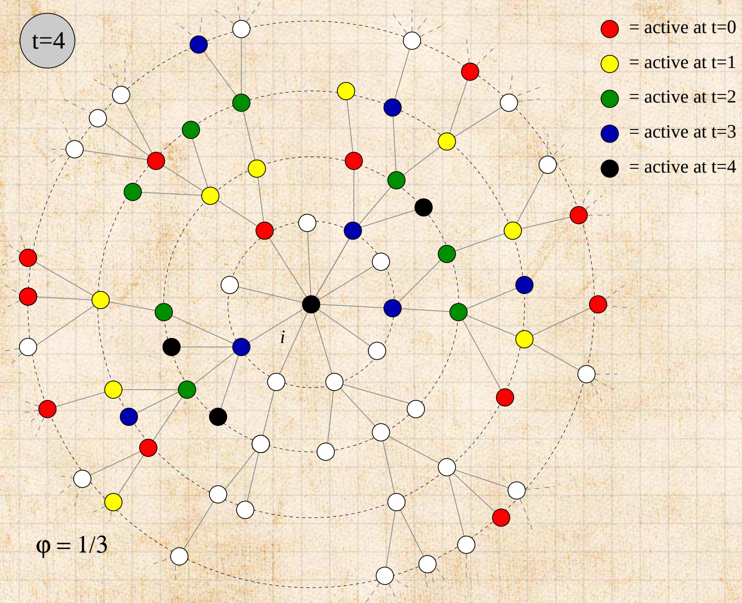

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Notes:

- Calculations presume nodes do not become inactive (strong restriction, liftable)
- Not just for threshold model—works for a wide range of contagion processes.
- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determine $\mathbf{Pr}$(node of degree k switches on at time t).
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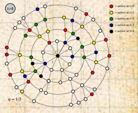

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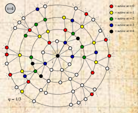

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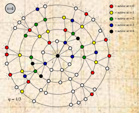

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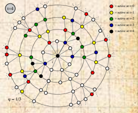

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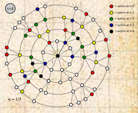

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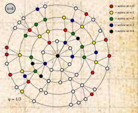

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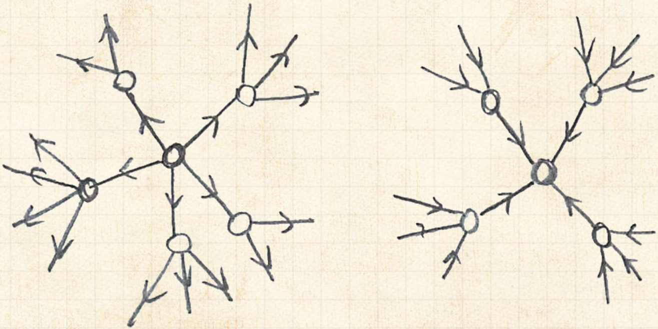
Pleasantness:

- Taking off from a single seed story is about expansion away from a node.
- Extent of spreading story is about contraction at a node.



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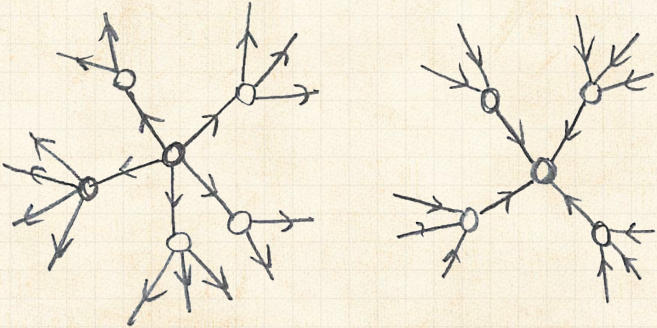

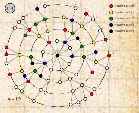

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- **Notation:** $\phi_{k,t} = \mathbf{Pr}$(a degree k node is active at time t).
- **Notation:** $B_{kj} = \mathbf{Pr}$ (a degree k node becomes active if j neighbors are active).
- Our starting point: $\phi_{k,0} = \phi_0$.
- $\binom{k}{j}\phi_0^j(1-\phi_0)^{k-j} = \mathbf{Pr}$ (j of a degree k node's neighbors were seeded at time $t=0$).
- Probability a degree k node was a seed at $t=0$ is ϕ_0 (as above).
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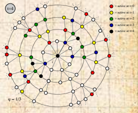

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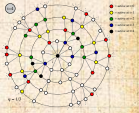

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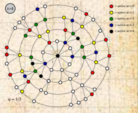

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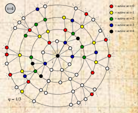

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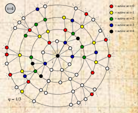

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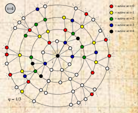

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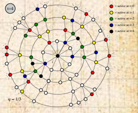

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- For general t, we need to know the probability an edge coming into a degree k node at time t is active.
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- We already know $\theta_0 = \phi_0$.
- Story analogous to $t = 1$ case. For specific node i:

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- Average over all nodes with degree k to obtain expression for ϕ_{t+1}:

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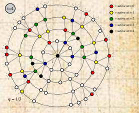

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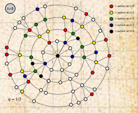

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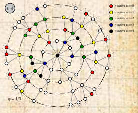

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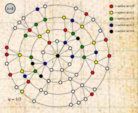

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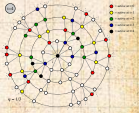

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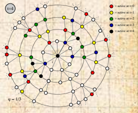

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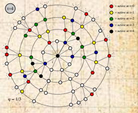

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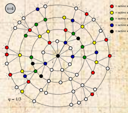

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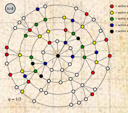

Expected size of spread

Two pieces: edges first, and then nodes

1. $$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + \underbrace{(1-\phi_0)\sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\sum_{j=0}^{k-1}\binom{k-1}{j}\theta_t^{\,j}(1-\theta_t)^{k-1-j}B_{kj}}_{\text{social effects}}$$

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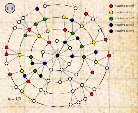
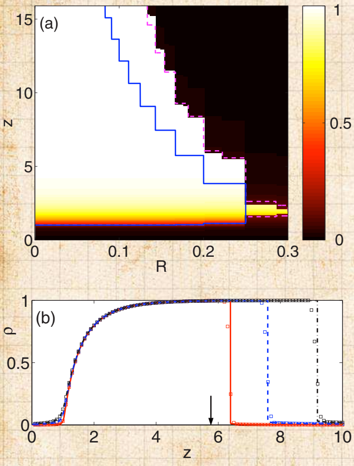


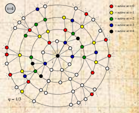
Comparison between theory and simulations



From Gleeson and Cahalane [7]

- Pure random networks with simple threshold responses
- R = uniform threshold (our ϕ_*); z = average degree; $\rho = \phi$; $q = \theta$; $N = 10^5$.
- $\phi_0 = 10^{-3}$, 0.5×10^{-2}, and 10^{-2}.
- Cascade window is for $\phi_0 = 10^{-2}$ case.
- Sensible expansion of cascade window as ϕ_0 increases.

Basic Contagion Models
Global spreading condition
Social Contagion Models
Network version
All-to-all networks
Theory
Spreading possibility
Spreading probability
Physical explanation
Final size
References



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meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.

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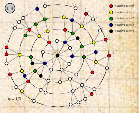
Insert question from assignment 10



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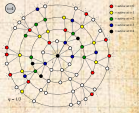

The University of Vermont

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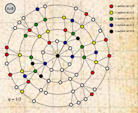

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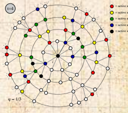

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- If G has an unstable fixed point at $\theta = 0$, then cascades are also always possible.

Non-vanishing seed case:

- Cascade condition is more complicated for $\phi_0 > 0$.
- If G has a stable fixed point at $\theta = 0$, and an unstable fixed point for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
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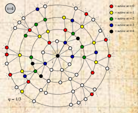

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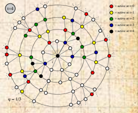

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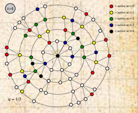

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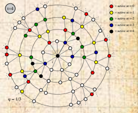

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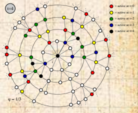
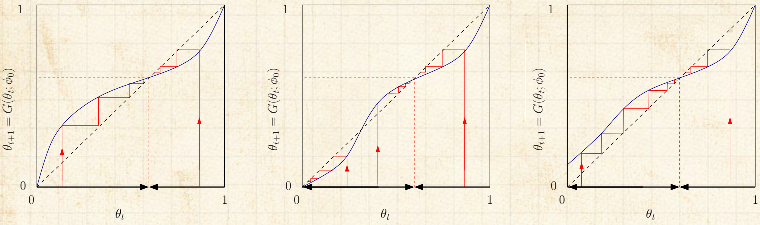


General fixed point story:



- Given $\theta_0 (= \phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.
- n.b., adjacent fixed points must have opposite stability types.
- Important: Actual form of G depends on ϕ_0.
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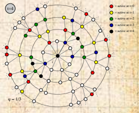

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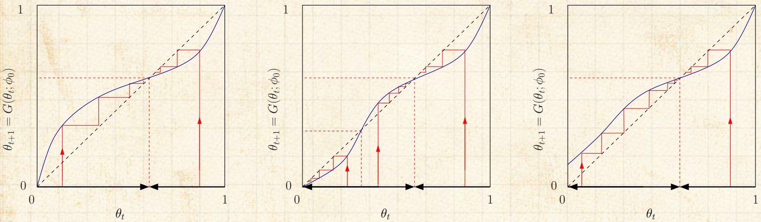

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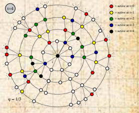

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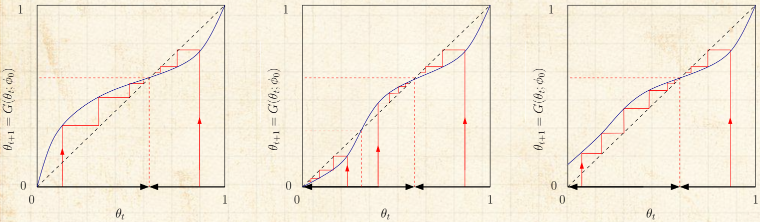

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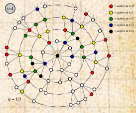

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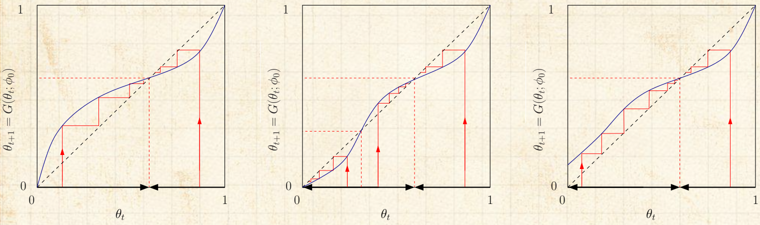

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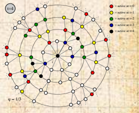

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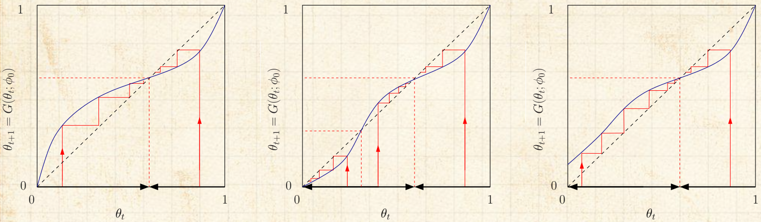

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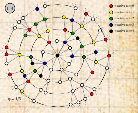

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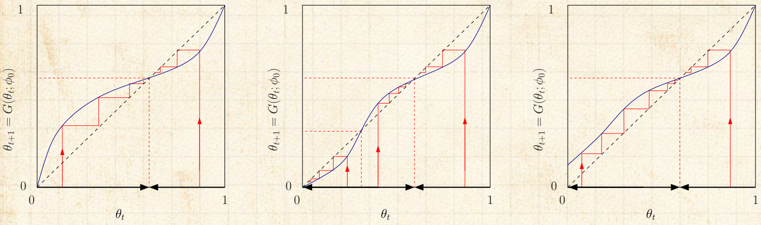

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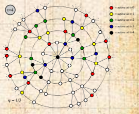
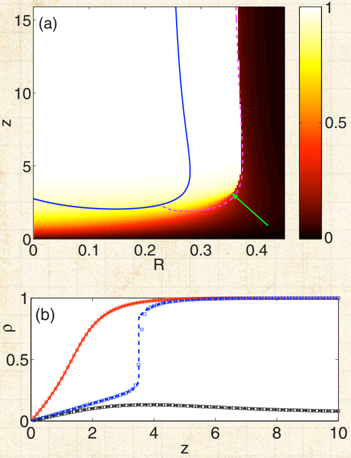
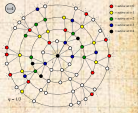


Interesting behavior:



From Gleeson and Cahalane [7]

- Now allow thresholds to be distributed according to a Gaussian with mean R.
- R = 0.2, 0.362, and 0.38; $\sigma = 0.2$.
- $\phi_0 = 0$ but some nodes have thresholds ≤ 0 so effectively $\phi_0 > 0$.
- Now see a (nasty) discontinuous phase transition for low $\langle k \rangle$.



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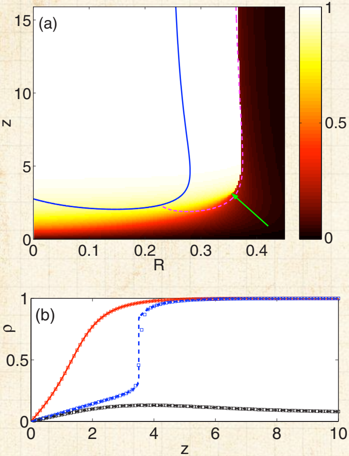

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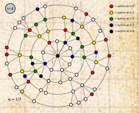

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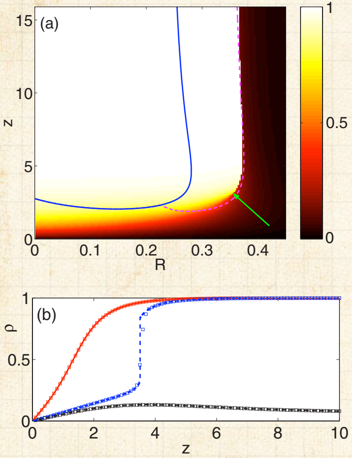

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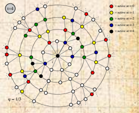

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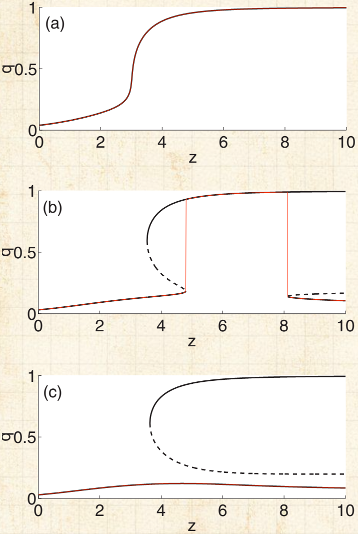
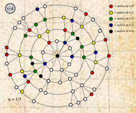
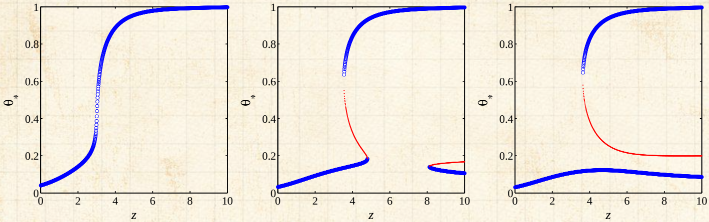


From Gleeson and Cahalane [7]

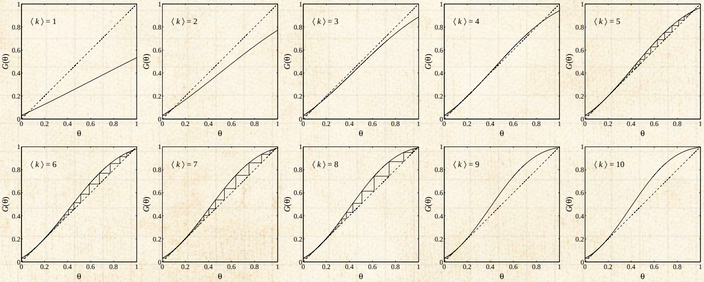
- Plots of stability points for $\theta_{t+1} = G(\theta_t; \phi_0)$.
- n.b.: 0 is not a fixed point here: $\theta_0 = 0$ always takes off.
- Top to bottom: R = 0.35, 0.371, and 0.375.
- Saddle node bifurcations appear and merge (b and c).

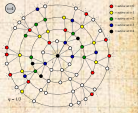


What's happening:



- Fixed points slip above and below the $\theta_{t+1} = \theta_t$ line:





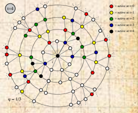
Time-dependent solutions

Synchronous update

- Done: Evolution of ϕ_t and θ_t given exactly by the maps we have derived.

Asynchronous updates

- Update nodes with probability α.
- As $\alpha \to 0$, updates become effectively independent.
- Now can talk about $\phi(t)$ and $\theta(t)$.



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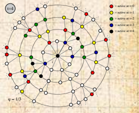

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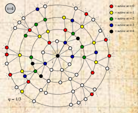

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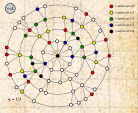

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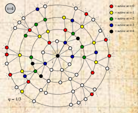

Nutshell:

- Solid dive into understanding contagion on generalized random networks.
- Threshold model leads to idea of vulnerables and a critical mass. [16, 8]
- Generating function approaches provided first breakthroughs and gave possibility and probability of spreading. [10, 16]
- Later: A probabilistic, physical method solved the whole story for a fractional seed—final size, dynamics, ... [7, 6]
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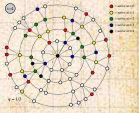

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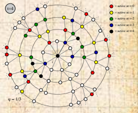

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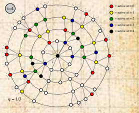

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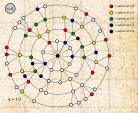

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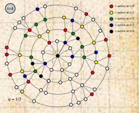

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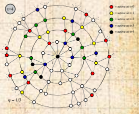

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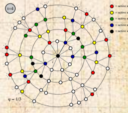
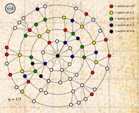


Neural reboot (NR):

Pangolin happiness:

https://www.youtube.com/v/LMiYjkG4onM?rel=0



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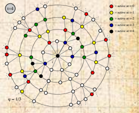

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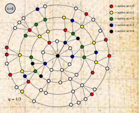

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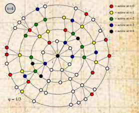

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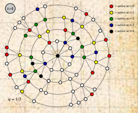

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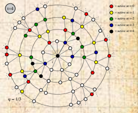

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