Chaotic Contagion: The Idealized Hipster Effect

Complex Networks | @networksvox
CSYS/MATH 303, Spring, 2016

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

These slides are brought to you by:

CocoNuTs
Complex Networks
@networksvox
Everything is connected

The UNIVERSITY of VERMONT

Outline

CocoNuTs
Complex Networks
@networksvox
Everything is connected

Chaotic Contagion on Networks:

"Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability"
Dodds, Harris, and Danforth,
Phys. Rev. Lett., **110**, 158701, 2013. [1]

"Dynamical influence processes on networks: General theory and applications to social contagion"
Harris, Danforth, and Dodds,
Phys. Rev. E, **88**, 022816, 2013. [2]

Outline

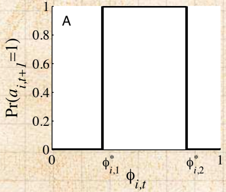
Chaotic contagion:

- What if individual response functions are not monotonic?
- Consider a simple deterministic version:
  - Node i has an 'activation threshold' $\phi_{i,1}$
    ...and a 'de-activation threshold' $\phi_{i,2}$
  - Nodes like to imitate but only up to a limit—they don't want to be like everyone else.

Chaotic contagion:

- What if individual response functions are not monotonic?
- Consider a simple deterministic version:
  - Node i has an 'activation threshold' $\phi_{i,1}$

    ...and a 'de-activation threshold' $\phi_{i,2}$
  - Nodes like to imitate but only up to a limit—they don't want to be like everyone else.

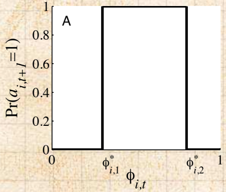


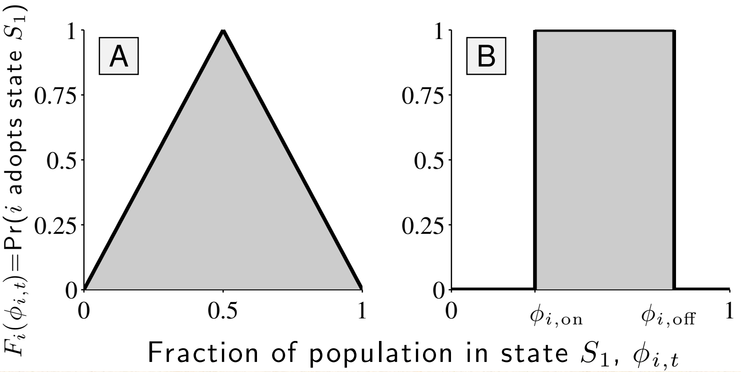
Chaotic contagion:

- What if individual response functions are not monotonic?
- Consider a simple deterministic version:

- Node i has an 'activation threshold' $\phi_{i,1}$

  ...and a 'de-activation threshold' $\phi_{i,2}$
- Nodes like to imitate but only up to a limit—they don't want to be like everyone else.




A
B
1
0.75
0.5
0.25
0
0
0.5
1
$\phi_{i,\mathrm{on}}$
$\phi_{i,\mathrm{off}}$
$F_i(\phi_{i,t})$=Pr(i adopts state S_1)
Fraction of population in state S_1, $\phi_{i,t}$

Chaotic contagion

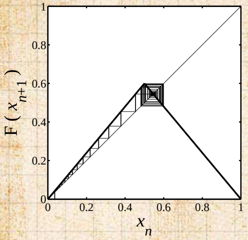
Definition of the tent map:

$$F(x) = \begin{cases} rx \text{ for } 0 \le x \le \frac{1}{2}, \\ r(1-x) \text{ for } \frac{1}{2} \le x \le 1. \end{cases}$$

- The usual business: look at how F iteratively maps the unit interval $[0, 1]$.

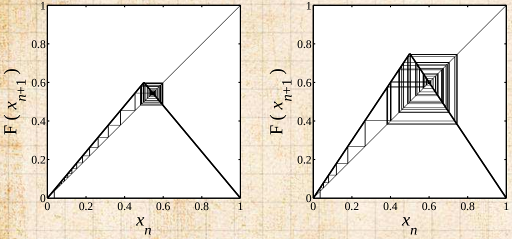
The tent map

Effect of increasing r from 1 to 2.



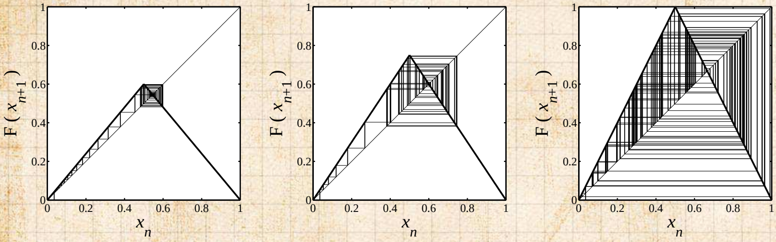
The tent map

Effect of increasing r from 1 to 2.



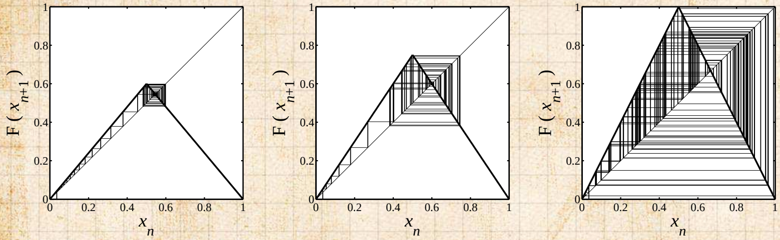
The tent map

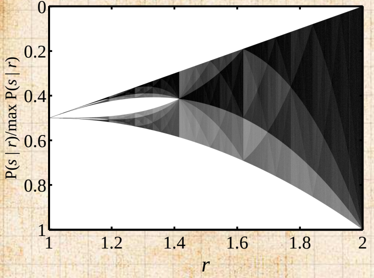
Effect of increasing r from 1 to 2.



The tent map

Effect of increasing r from 1 to 2.



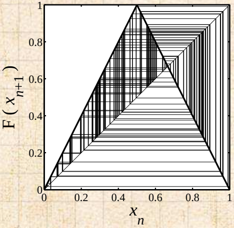


Orbit diagram:

Chaotic behavior increases as map slope r is increased.

Chaotic behavior

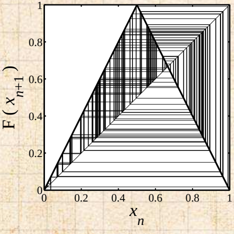
Take $r = 2$ case:



- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information.

Chaotic behavior

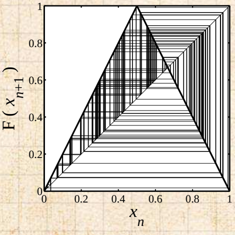
Take $r = 2$ case:



- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

Chaotic behavior

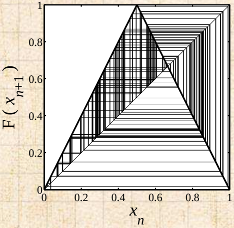
Take $r = 2$ case:



- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

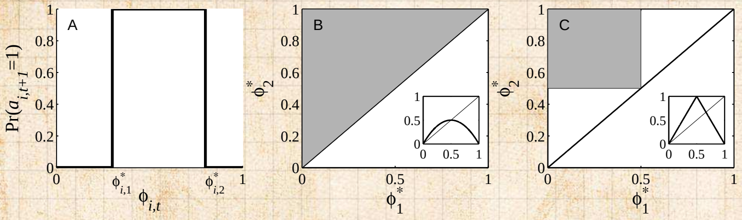
Chaotic behavior

Take $r = 2$ case:



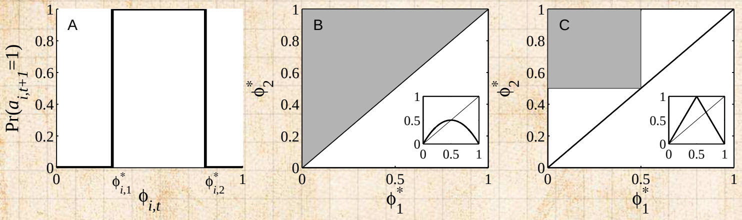
- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

Two population examples:



- Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- Insets show composite response function averaged over population.
- We'll consider plot C's example: the tent map.

Two population examples:



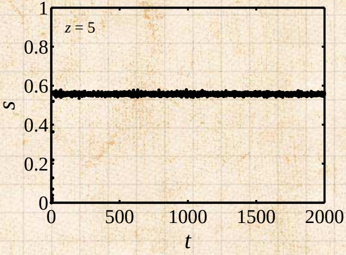
- Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- Insets show composite response function averaged over population.
- We'll consider plot C's example: the tent map.

Outline

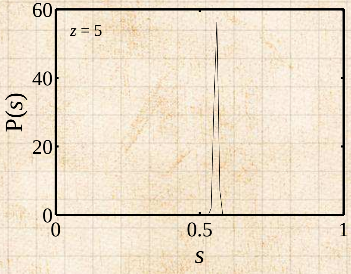
CocoNuTs
Complex Networks
@networksvox
Everything is connected

The UNIVERSITY of VERMONT

Invariant densities—stochastic response functions

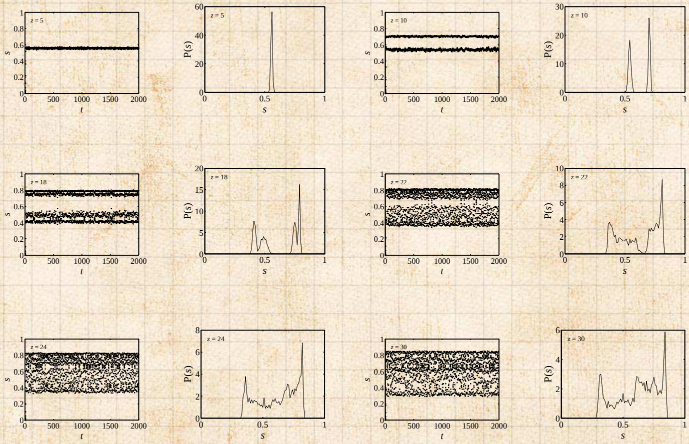


activation time series

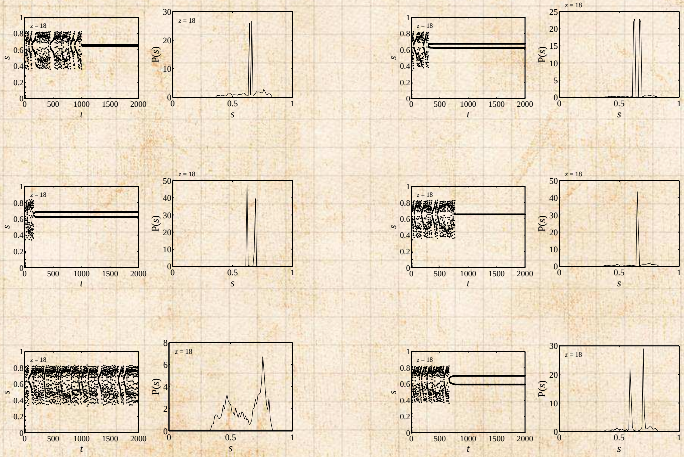


activation density

Invariant densities—stochastic response functions

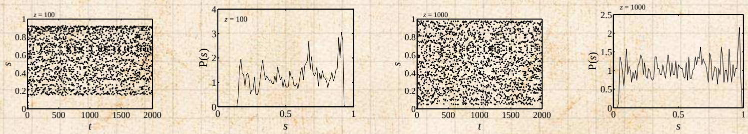


Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$



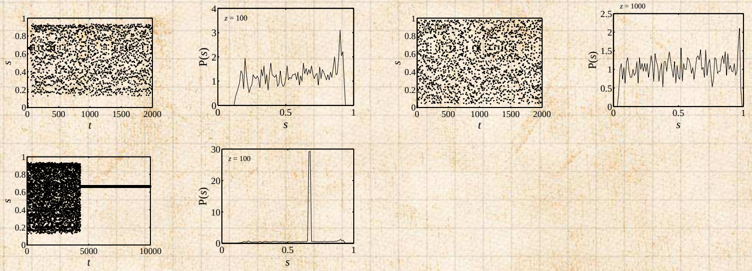
CocoNuTs
Complex Networks
@networksvox
Everything is connected

Invariant densities—stochastic response functions



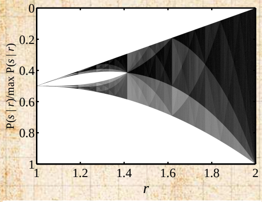
Trying out higher values of $\langle k \rangle$...

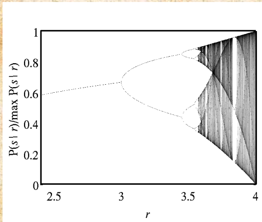
Invariant densities—deterministic response functions



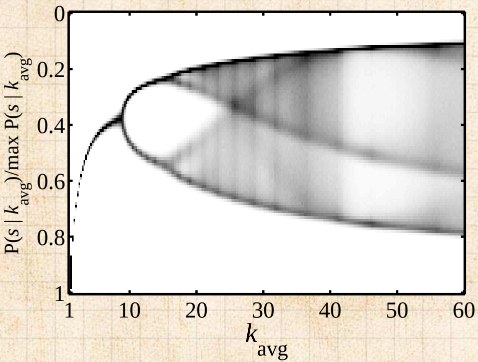
Trying out higher values of $\langle k \rangle$...

Connectivity leads to chaos:

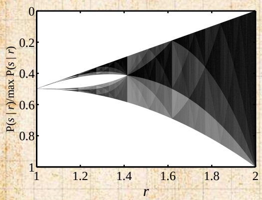


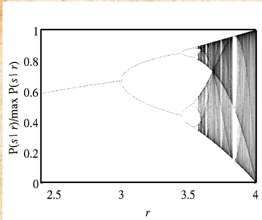


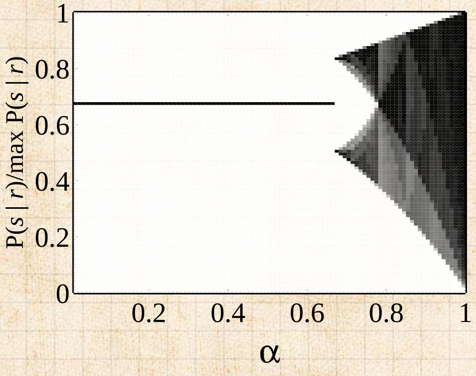
Stochastic response functions:



Bifurcation diagram: Asynchronous updating







Bifurcation diagram: Asynchronous updating

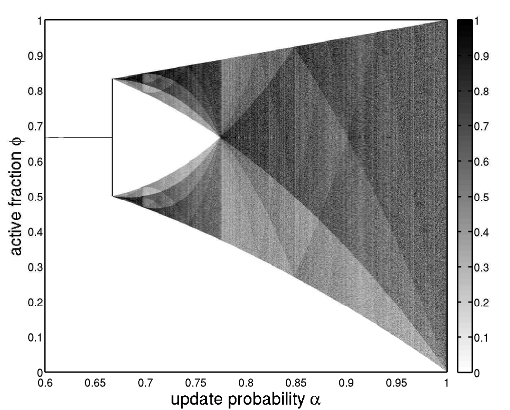


FIG. 3. Bifurcation diagram for the dense map $\Phi(\phi;\alpha)$, Eqn. (18). This was generated by iterating the map at 1000 α values between 0 and 1. The iteration was carried out with 3 random initial conditions for 10000 time steps each, discarding the first 1000. The ϕ-axis contains 1000 bins and the invariant density, shown by the grayscale value, is normalized by the maximum for each α. With $\alpha < 2/3$, all trajectories go to the fixed point at $\phi = 2/3$.

https://www.youtube.com/v/7JHrZyyq870?rel=0

How the bifurcation diagram changes with increasing average degree $\langle k \rangle$ as a function of the synchronicity parameter α for the stochastic response (tent map) case.

https://www.youtube.com/v/_zwK6poIBvc?rel=0

How the bifurcation diagram changes with increasing α, the synchronicity parameter as a function of average degree $\langle k \rangle$ for the stochastic response (tent map) case.

https://www.youtube.com/v/3bo4fzp4Snw?rel=0

LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. Average degree = 6, update synchronicity parameter α = 1. The macroscopic behavior is period-1, plus noisy fluctuations.

https://www.youtube.com/v/7UCuIa_ktmw?rel=0

LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. Average degree = 11, update synchronicity parameter $\alpha = 1$. The macroscopic behavior is period-2, plus noisy fluctuations.

https://www.youtube.com/v/oWKt8Zj1Ccw?rel=0

LIC dynamics on a fixed graph with a shared stochastic (tent map) response function. $\langle k \rangle = 30$, update synchronicity parameter $\alpha = 1$. The macroscopic behavior is chaotic.

https://www.youtube.com/v/AfhUlkIOiOU?rel=0

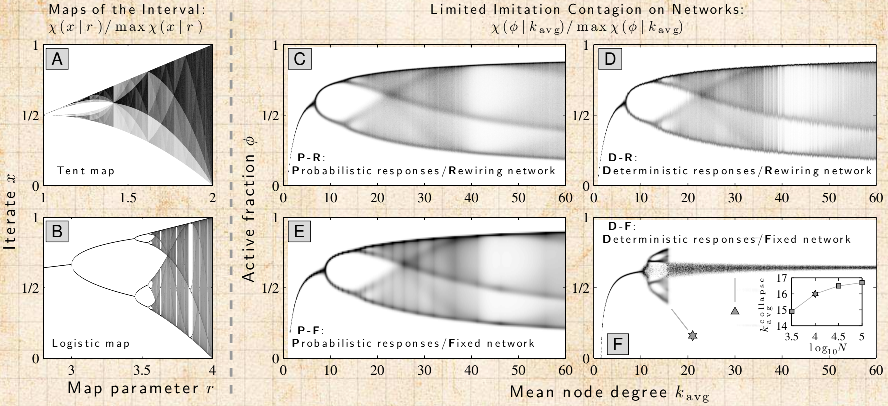
LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 30, update synchronicity parameter α = 1. Shown are nodes which continue changing (703/1000) after the transient chaotic behavior has "collapsed."

https://www.youtube.com/v/ZwY0hTstJ2M?rel=0

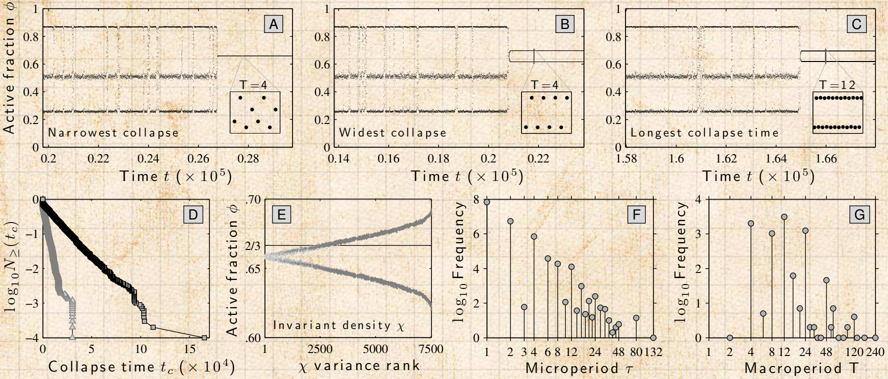
LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 30, update synchronicity parameter α = 1. The dynamics exhibit transient chaotic behavior before collapsing to a fixed point.

https://www.youtube.com/v/YDhjmFyBSn4?rel=0

LIC dynamics on a fixed graph with fixed, deterministic response functions. Average degree = 17, update synchronicity parameter α = 1. The dynamics exhibit transient chaotic behavior before collapsing to a period-4 orbit.



Maps of the Interval:
$\chi(x \mid r)/\max\chi(x \mid r)$
Limited Imitation Contagion on Networks:
$\chi(\phi \mid k_{avg})/\max\chi(\phi \mid k_{avg})$
A
Tent map
B
Logistic map
Iterate x
Map parameter r
C
P-R:
Probabilistic responses/Rewiring network
D
D-R:
Deterministic responses/Rewiring network
E
P-F:
Probabilistic responses/Fixed network
F
D-F:
Deterministic responses/Fixed network
$k_{avg}^{collapse}$
$\log_{10}N$
Active fraction ϕ
Mean node degree k_{avg}



A
B
C
D
E
F
G
Narrowest collapse
Widest collapse
Longest collapse time
T = 4
T = 4
T = 12
Active fraction ϕ
Time t ($\times 10^5$)
$\log_{10} N_{\geq}(t_c)$
Collapse time t_c ($\times 10^4$)
Invariant density χ
χ variance rank
$\log_{10}$ Frequency
Microperiod τ
Macroperiod T

References I

[1] P. S. Dodds, K. D. Harris, and C. M. Danforth.
Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability.
Phys. Rev. Lett., 110:158701, 2013. pdf

[2] K. D. Harris, C. M. Danforth, and P. S. Dodds.
Dynamical influence processes on networks: General theory and applications to social contagion.
Phys. Rev. E, 88:022816, 2013. pdf