

Social Contagion

Principles of Complex Systems | @pocsvox
 CSYS/MATH 300, Fall, 2015 | #FallPoCS2015

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 Vermont Advanced Computing Core | University of Vermont



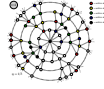
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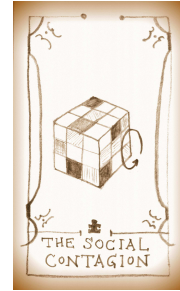
Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
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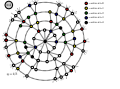


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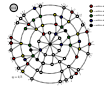


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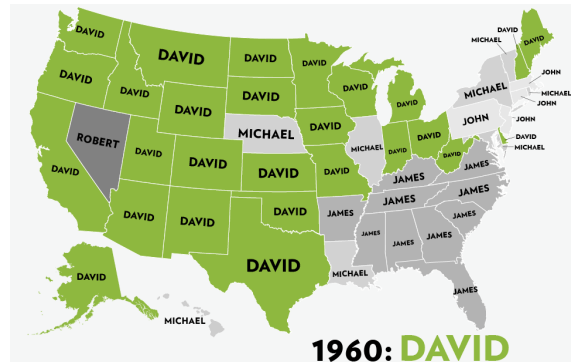
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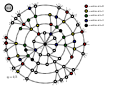
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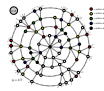
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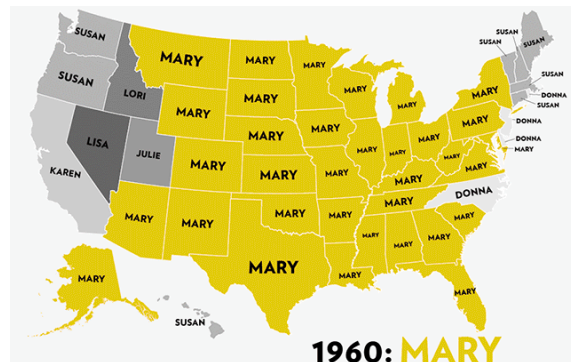
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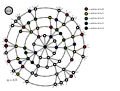
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Things that spread well:

buzzfeed.com



► Dangerously self aware: 11 Elements that make a perfect viral video.

+ News ...

LOL + cute + fail + wtf:

Oopsie!



BUZZFEED FELL DOWN AND WENT BOOM.

Please try reloading this page. If the problem persists [let us know](#).

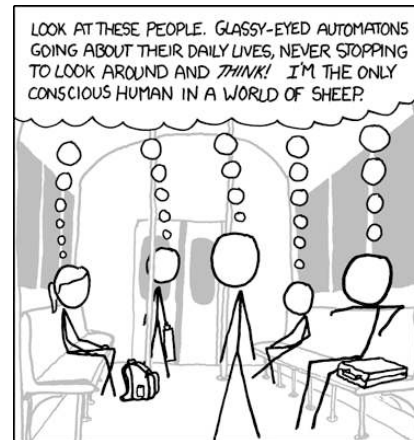
The whole lolcats thing:



Some things really stick:



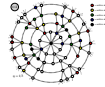
Why social contagion works so well:



<http://xkcd.com/610/>

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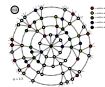
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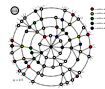
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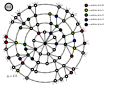
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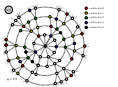
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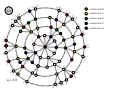
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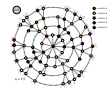
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Framingham heart study:

Evolving network stories (Christakis and Fowler):

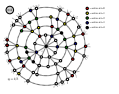
- ▶ The spread of quitting smoking [7]
- ▶ The spread of spreading [6]
- ▶ Also: happiness [11], loneliness, ...
- ▶ The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives

Controversy:

- ▶ Are your friends making you fat? (Clive Thomson, NY Times, September 10, 2009).
- ▶ Everything is contagious—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

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Examples abound

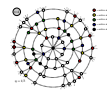
- ▶ fashion
- ▶ striking
- ▶ smoking [7]
- ▶ residential segregation [22]
- ▶ iPhones and iThings
- ▶ obesity [6]
- ▶ Harry Potter
- ▶ voting
- ▶ gossip
- ▶ Rubik's cube
- ▶ religious beliefs
- ▶ school shootings
- ▶ **leaving lectures**

SIR and SIRS type contagion possible

- ▶ Classes of behavior versus specific behavior : **dieting, horror movies, getting married, invading countries, ...**

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Two focuses for us

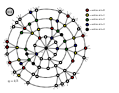
- ▶ Widespread media influence
- ▶ Word-of-mouth influence

We need to understand influence

- ▶ Who influences whom? Very hard to measure...
- ▶ What kinds of influence response functions are there?
- ▶ Are some individuals super influencers? Highly popularized by Gladwell [12] as 'connectors'
- ▶ The infectious idea of opinion leaders (Katz and Lazarsfeld) [19]

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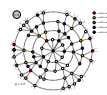


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Mixed messages: Please copy, but also, don't copy ...

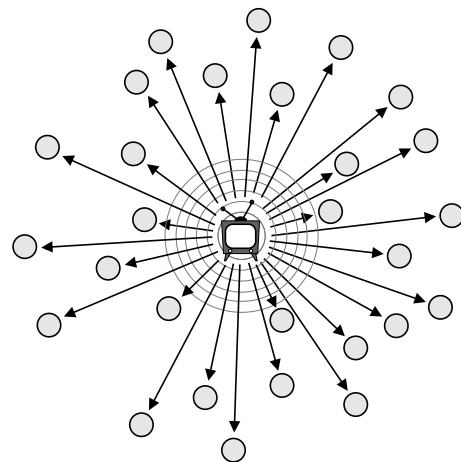
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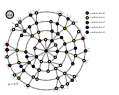
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The hypodermic model of influence



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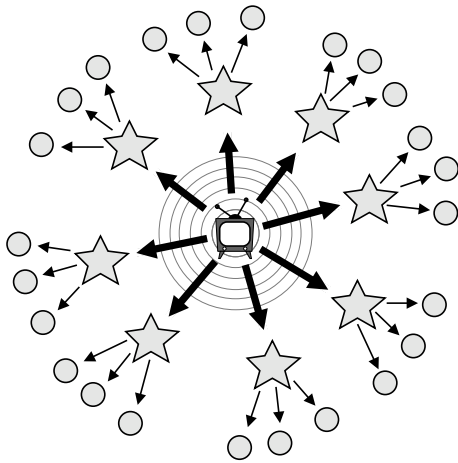
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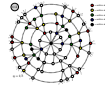
- ▶ Cindy Harrell appeared in the (terrifying) music video for Ray Parker Jr.'s Ghostbusters.
- ▶ Misframing: Appeals only to seed on exponential growth.

The two step model of influence ^[19]



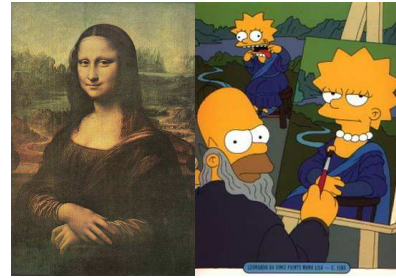
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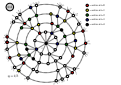
The Mona Lisa



- ▶ "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- ▶ Not the world's greatest painting from the start...
- ▶ Escalation through theft, vandalism, **parody**, ...

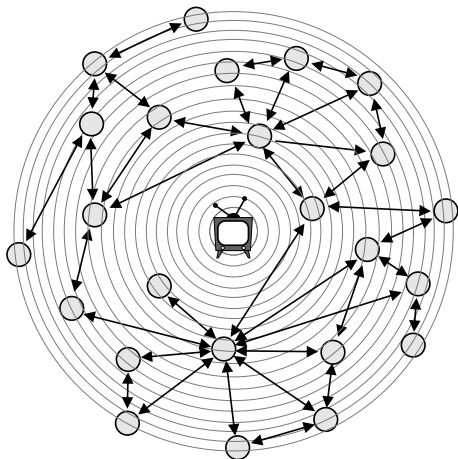
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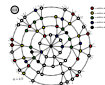
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The general model of influence: the Social Wild



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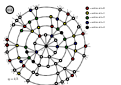
'Tattooed Guy' Was Pivotal in Armstrong Case [\[nytimes\]](#)



- ▶ "... Leogrande's doping sparked a series of events ..."

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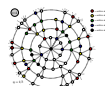
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Why do things spread socially?

- ▶ Because of properties of special individuals?
- ▶ Or system level properties?
- ▶ Is the match that lights the fire important?
- ▶ Yes. But only because we are storytellers: *homo narrativus* [\[1\]](#).
- ▶ We like to think things happened for reasons ...
- ▶ Reasons for success are usually ascribed to intrinsic properties (examples next).
- ▶ Teleological stories of fame are often easy to generate and believe.
- ▶ System/group dynamics harder to understand because most of our stories are built around individuals.
- ▶ Always good to examine what is said before and after the fact ...

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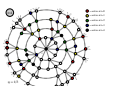
The completely unpredicted fall of Eastern Europe



Timur Kuran: ^[20, 21] "Now Out of Never: The Element of Surprise in the East European Revolution of 1989"

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The dismal predictive powers of editors...



From a 2013 Believer Magazine interview with Maurice Sendak:

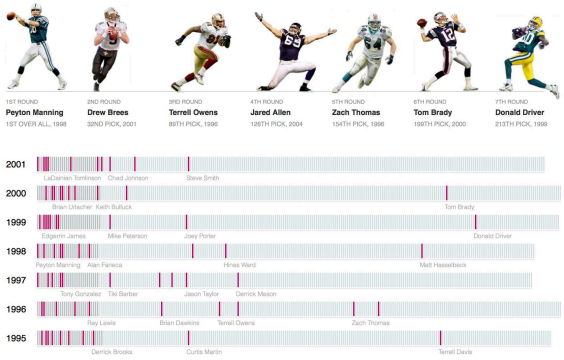
BLVR: Did the success of *Where the Wild Things Are* ever feel like an albatross?

MS: It's a nice book. It's perfectly nice. I can't complain about it. I remember Herman Melville said, "When I die no one is going to mention Moby-Dick. They're all going to talk about my first book, about f***ing maidens in Tahiti." He was right. No mention of Moby-Dick then. Everyone wanted another Tahitian book, a beach book. But then he kept writing deeper and deeper and then came Moby-Dick and people hated it. The only ones who liked it were Mr. and Mrs. Nathaniel Hawthorne. Moby-Dick didn't get famous until 1930.

- ▶ Sendak named his dog Herman.
- ▶ The essential Colbert interview: [Pt. 1](#) and [Pt. 2](#).

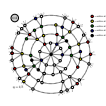
Drafting success in the NFL:

Top Players by Round, 1995-2012



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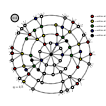
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Messing with social connections

- ▶ Ads based on message content (e.g., Google and email)
- ▶ BzzAgent
- ▶ One of Facebook's early advertising attempts: Beacon
- ▶ All of Facebook's advertising attempts.



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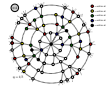
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Getting others to do things for you

A very good book: 'Influence' by Robert Cialdini

Six modes of influence:

1. **Reciprocation:** *The Old Give and Take... and Take;* e.g., Free samples, Hare Krishnas.
2. **Commitment and Consistency:** *Hobgoblins of the Mind;* e.g., Hazing.
3. **Social Proof:** *Truths Are Us;* e.g., Jonestown, Kitty Genovese (contested).
4. **Liking:** *The Friendly Thief;* e.g., Separation into groups is enough to cause problems.
5. **Authority:** *Directed Deference;* e.g., Milgram's obedience to authority experiment.
6. **Scarcity:** *The Rule of the Few;* e.g., Prohibition.



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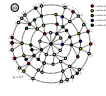
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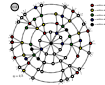
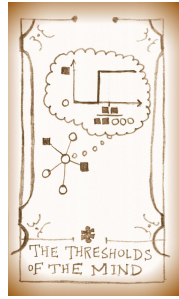
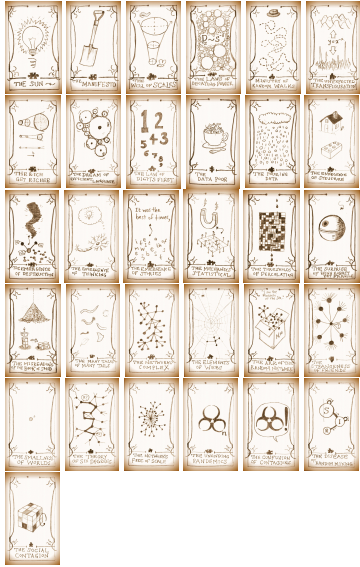
- ▶ Cialdini's modes are heuristics that help up us get through life.
- ▶ Useful but can be leveraged...

Other acts of influence:

- ▶ Conspicuous Consumption (Veblen, 1912)
- ▶ Conspicuous Destruction (Potlatch)



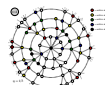
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Some important models:

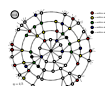
- ▶ Tipping models—Schelling (1971) [22, 23, 24]
 - ▶ Simulation on checker boards
 - ▶ Idea of thresholds
 - ▶ Polygon-themed online visualization. (Includes optional diversity-seeking proclivity.)
 - ▶ Explore the Netlogo online implementation
- ▶ Threshold models—Granovetter (1978) [15]
- ▶ Herding models—Bikhchandani, Hirschleifer, Welch (1992) [2, 3]
 - ▶ Social learning theory, Informational cascades,...



Social contagion models

Thresholds

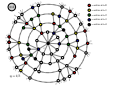
- ▶ Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- ▶ 'Others' may be everyone in a population, an individual's close friends, any reference group.
- ▶ Response can be probabilistic or deterministic.
- ▶ Individual thresholds can vary
- ▶ Assumption: order of others' adoption does not matter... (unrealistic).
- ▶ Assumption: level of influence per person is uniform (unrealistic).



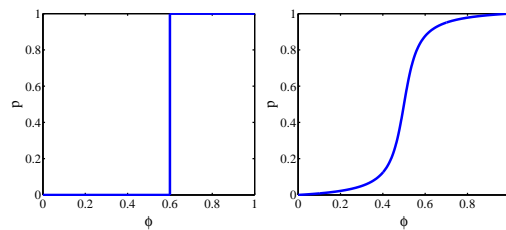
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Some possible origins of thresholds:

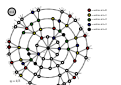
- ▶ Inherent, evolution-devised inclination to coordinate, to conform, to imitate. [1]
- ▶ Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- ▶ Economics: Network effects or network externalities
 - ▶ Externalities = Effects on others not directly involved in a transaction
 - ▶ Examples: telephones, fax machine, Facebook, operating systems
 - ▶ An individual's utility increases with the adoption level among peers and the population in general



Threshold models—response functions

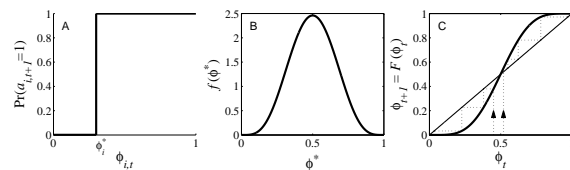


- ▶ Example threshold influence response functions: deterministic and stochastic
- ▶ ϕ = fraction of contacts 'on' (e.g., rioting)
- ▶ Two states: S and I.

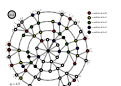


Threshold models

Action based on perceived behavior of others:

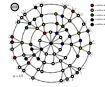
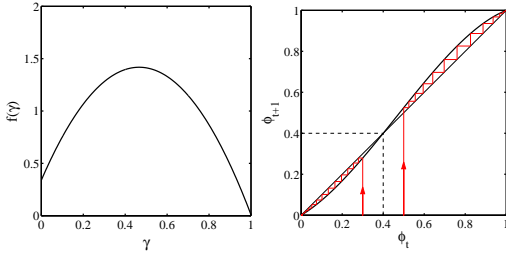


- ▶ Two states: S and I.
- ▶ ϕ = fraction of contacts 'on' (e.g., rioting)
- ▶ Discrete time update (strong assumption!)
- ▶ This is a Critical mass model



Threshold models

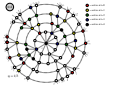
Another example of critical mass model:



Threshold models—Nutshell

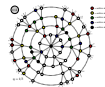
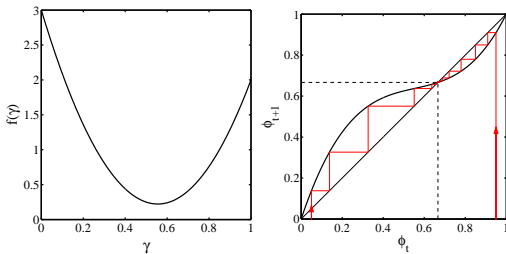
Implications for collective action theory:

1. Collective uniformity \Rightarrow individual uniformity
2. Small individual changes \Rightarrow large global changes
3. The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
4. System stories live in left null space of our stories—we can't even see them.
5. But we happily impose simplistic, individual-centric stories—we can't help ourselves.



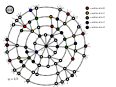
Threshold models

Example of single stable state model:



Many years after Granovetter and Soong's work:

- ▶ "A simple model of global cascades on random networks"
D. J. Watts. Proc. Natl. Acad. Sci., 2002 [26]
 - ▶ Mean field model \rightarrow network model
 - ▶ Individuals now have a limited view of the world

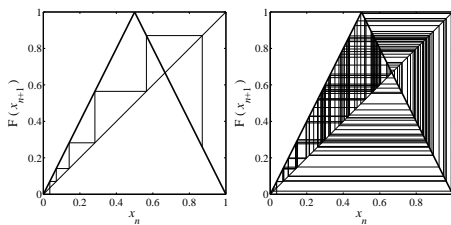


We'll also explore:

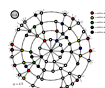
- ▶ "Seed size strongly affects cascades on random networks" [14]
Gleeson and Cahalane, Phys. Rev. E, 2007.
- ▶ "Direct, physically motivated derivation of the contagion condition for spreading processes on generalized random networks" [10]
Dodds, Harris, and Payne, Phys. Rev. E, 2011
- ▶ "Influentials, Networks, and Public Opinion Formation" [27]
Watts and Dodds, J. Cons. Res., 2007.
- ▶ "Threshold models of Social Influence" [28]
Watts and Dodds, The Oxford Handbook of Analytical Sociology, 2009.

Threshold models

Chaotic behavior possible [17, 16, 9, 18]

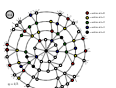


- ▶ Period doubling arises as map amplitude r is increased.
- ▶ Synchronous update assumption is crucial

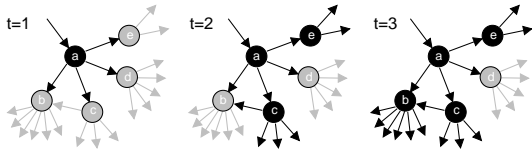


Threshold model on a network

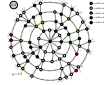
- ▶ Interactions between individuals now represented by a network.
- ▶ Network is **sparse**.
- ▶ Individual i has k_i contacts.
- ▶ Influence on each link is **reciprocal** and of **unit weight**.
- ▶ Each individual i has a fixed threshold ϕ_i .
- ▶ Individuals repeatedly poll contacts on network.
- ▶ Synchronous, discrete time updating.
- ▶ Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$.
- ▶ Individuals remain active when switched (no recovery = SI model).



Threshold model on a network



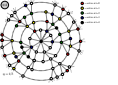
▶ All nodes have threshold $\phi = 0.2$.



Snowballing

Follow active links

- ▶ An active link is a link connected to an activated node.
- ▶ If an infected link leads to **at least 1 more infected link**, then **activation spreads**.
- ▶ We need to understand which nodes can be activated when only one of their neighbors becomes active.



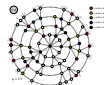
Snowballing

First study random networks:

- ▶ Start with N nodes with a degree distribution P_k
- ▶ Nodes are randomly connected (carefully so)
- ▶ Aim: Figure out when activation will propagate
- ▶ Determine a **cascade condition**

The Cascade Condition:

1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?



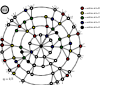
The most gullible

Vulnerables:

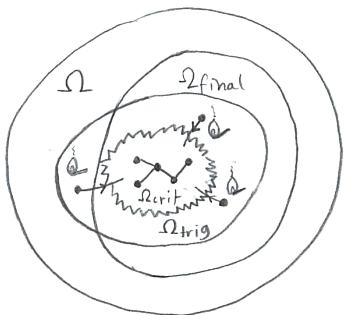
- ▶ We call individuals who can be activated by just one contact being active **vulnerables**
- ▶ The vulnerability condition for node i :

$$1/k_i \geq \phi_i$$

- ▶ Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- ▶ For global cascades on random networks, must have a **global cluster of vulnerables** [26]
- ▶ **Cluster of vulnerables = critical mass**
- ▶ Network story: 1 node \rightarrow critical mass \rightarrow everyone.

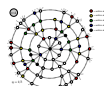


Example random network structure:



- ▶ $\Omega_{crit} = \Omega_{vuln} =$ critical mass = global vulnerable component
- ▶ $\Omega_{trig} =$ triggering component
- ▶ $\Omega_{final} =$ potential extent of spread
- ▶ $\Omega =$ entire network

$$\Omega_{crit} \subset \Omega_{trig}; \Omega_{crit} \subset \Omega_{final}; \text{ and } \Omega_{trig}, \Omega_{final} \subset \Omega.$$



Cascade condition

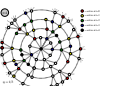
Back to following a link:

- ▶ A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- ▶ Follows from there being k ways to connect to a node with degree k .
- ▶ Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- ▶ So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$



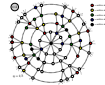
Cascade condition

Next: Vulnerability of linked node

- ▶ Linked node is **vulnerable** with probability

$$\beta_k = \int_{\phi'_k=0}^{1/k} f(\phi'_k) d\phi'_k$$

- ▶ If linked node is **vulnerable**, it produces $k - 1$ new outgoing active links
- ▶ If linked node is **not vulnerable**, it produces **no** active links.



Cascade condition

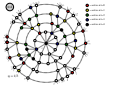
Two special cases:

- ▶ (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

- ▶ (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$



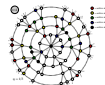
Cascade condition

Putting things together:

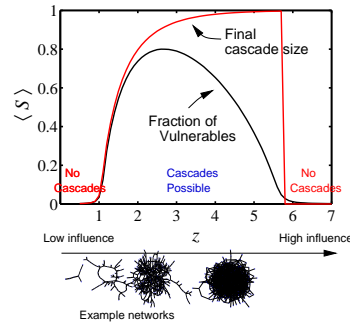
- ▶ Expected number of active edges produced by an active edge:

$$R = \left[\sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1-\beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}} \right]$$

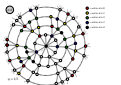
$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$



Cascades on random networks



- ▶ Cascades occur only if size of max vulnerable cluster > 0.
- ▶ System may be 'robust-yet-fragile'.
- ▶ 'Ignorance' facilitates spreading.

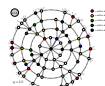


Cascade condition

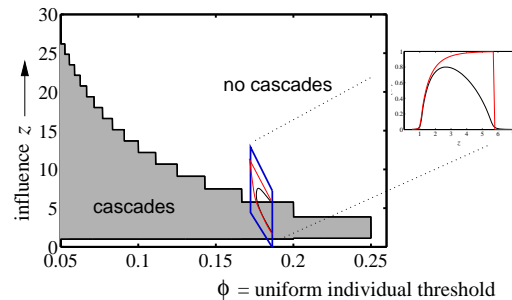
So... for random networks with fixed degree distributions, cascades take off when:

$$\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

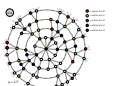
- ▶ β_k = probability a degree k node is vulnerable.
- ▶ P_k = probability a node has degree k .



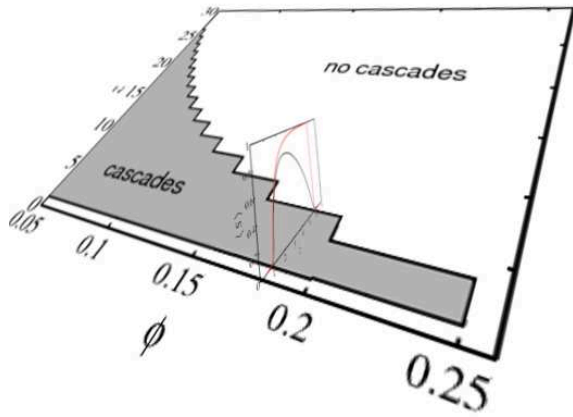
Cascade window for random networks



- ▶ 'Cascade window' widens as threshold ϕ decreases.
- ▶ Lower thresholds enable spreading.

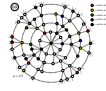


Cascade window for random networks



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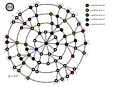
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Threshold contagion on random networks

- ▶ **Next:** Find expected fractional size of spread.
- ▶ Not obvious even for uniform threshold problem.
- ▶ Difficulty is in figuring out if and when nodes that need ≥ 2 hits switch on.
- ▶ Problem **beautifully solved** for infinite seed case by Gleeson and Cahalane: "Seed size strongly affects cascades on random networks," Phys. Rev. E, 2007.^[14]
- ▶ Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008.^[13]

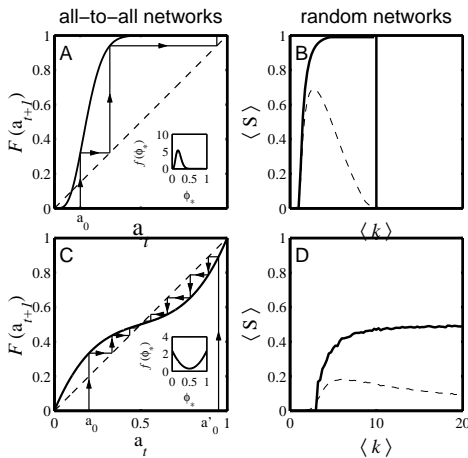
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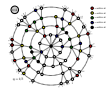
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All-to-all versus random networks



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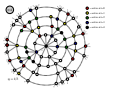
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Determining expected size of spread:

- ▶ Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$
- ▶ Capitalize on local branching network structure of random networks (again)
- ▶ Now think about what must happen for a specific node i to become active at time t :
 - $t = 0$: i is one of the seeds (prob = ϕ_0)
 - $t = 1$: i was not a seed but enough of i 's friends switched on at time $t = 0$ so that i 's threshold is now exceeded.
 - $t = 2$: enough of i 's friends and friends-of-friends switched on at time $t = 0$ so that i 's threshold is now exceeded.
 - $t = n$: enough nodes within n hops of i switched on at $t = 0$ and their effects have propagated to reach i .

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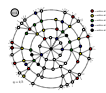
Cascade window—summary

For our simple model of a uniform threshold:

1. Low $\langle k \rangle$: No cascades in poorly connected networks.
No global clusters of any kind.
2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists.
Cascades are possible in "Cascade window."

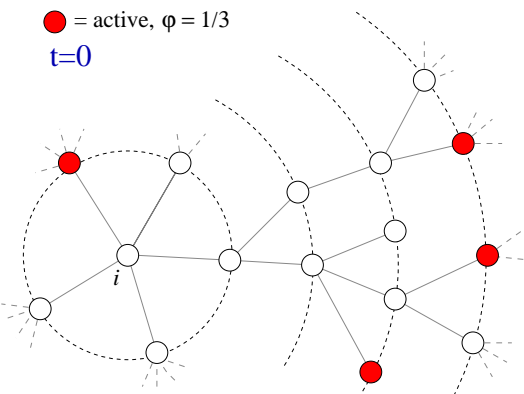
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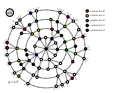
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Expected size of spread



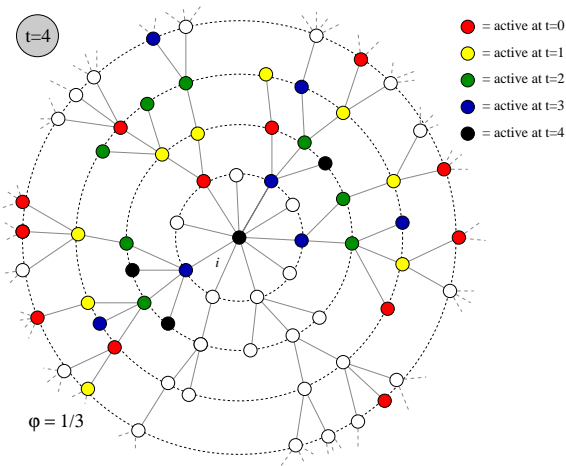
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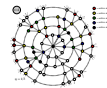
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Expected size of spread



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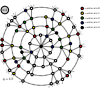
Expected size of spread

- ▶ **Notation:**
 $\phi_{k,t} = \Pr(\text{a degree } k \text{ node is active at time } t).$
- ▶ **Notation:** $B_{kj} = \Pr(\text{a degree } k \text{ node becomes active if } j \text{ neighbors are active}).$
- ▶ Our starting point: $\phi_{k,0} = \phi_0.$
- ▶ $\binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} = \Pr(j \text{ of a degree } k \text{ node's neighbors were seeded at time } t = 0).$
- ▶ Probability a degree k node was a seed at $t = 0$ is ϕ_0 (as above).
- ▶ Probability a degree k node was not a seed at $t = 0$ is $(1 - \phi_0).$
- ▶ Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^k \binom{k}{j} \phi_0^j (1 - \phi_0)^{k-j} B_{kj}.$$

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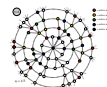
Expected size of spread

Notes:

- ▶ Calculations are possible if nodes do not become inactive (strong restriction).
- ▶ Not just for threshold model—works for a wide range of contagion processes.
- ▶ We can analytically determine the entire time evolution, not just the final size.
- ▶ We can in fact determine $\Pr(\text{node of degree } k \text{ switching on at time } t).$
- ▶ Asynchronous updating can be handled too.

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- ▶ For general t , we need to know the probability an edge coming into a degree k node at time t is active.
- ▶ **Notation:** call this probability $\theta_t.$
- ▶ We already know $\theta_0 = \phi_0.$
- ▶ Story analogous to $t = 1$ case. For node i :

$$\phi_{i,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{k_i} \binom{k_i}{j} \theta_t^j (1 - \theta_t)^{k_i-j} B_{k_i,j}.$$

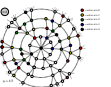
- ▶ Average over all nodes to obtain expression for $\phi_{t+1}:$

$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{k,j}.$$

- ▶ So we need to compute $\theta_t \dots$ massive excitement...

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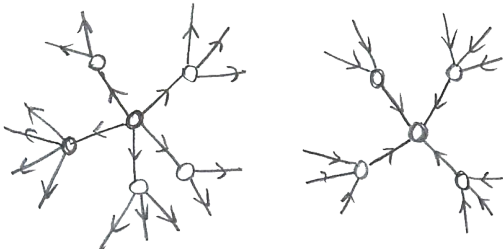


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Expected size of spread

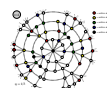
Pleasantness:

- ▶ Taking off from a single seed story is about **expansion** away from a node.
- ▶ Extent of spreading story is about **contraction** at a node.



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Expected size of spread

First connect θ_0 to θ_1 :

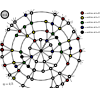
- ▶ $\theta_1 = \phi_0 +$

$$(1 - \phi_0) \sum_{k=1}^{\infty} \frac{k P_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_0^j (1 - \theta_0)^{k-1-j} B_{k,j}$$

- ▶ $\frac{k P_k}{\langle k \rangle} = R_k = \Pr(\text{edge connects to a degree } k \text{ node}).$
- ▶ $\sum_{j=0}^{k-1}$ piece gives $\Pr(\text{degree node } k \text{ activates})$ of its neighbors $k - 1$ incoming neighbors are active.
- ▶ ϕ_0 and $(1 - \phi_0)$ terms account for state of node at time $t = 0.$
- ▶ See this all generalizes to give θ_{t+1} in terms of $\theta_t \dots$

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Expected size of spread

Two pieces: edges first, and then nodes

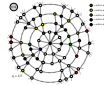
$$1. \theta_{t+1} = \underbrace{\phi_0 + (1 - \phi_0) \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{kj}}_{\text{social effects}}$$

with $\theta_0 = \phi_0$.

$$2. \phi_{t+1} = \underbrace{\phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^k \binom{k}{j} \theta_t^j (1 - \theta_t)^{k-j} B_{kj}}_{\text{social effects}}$$

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Expected size of spread:

In words:

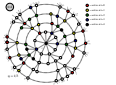
- ▶ If $G(0; \phi_0) > 0$, spreading must occur because some nodes turn on for free.
- ▶ If G has an **unstable fixed point** at $\theta = 0$, then cascades are also always possible.

Non-vanishing seed case:

- ▶ Cascade condition is more complicated for $\phi_0 > 0$.
- ▶ If G has a **stable fixed point** at $\theta = 0$, and an **unstable fixed point** for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
- ▶ Tricky point: G depends on ϕ_0 , so as we change ϕ_0 , we also change G .
- ▶ A version of a critical mass model again.

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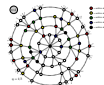
Expected size of spread

Iterative map for θ_t is key:

$$\theta_{t+1} = \underbrace{\phi_0 + (1 - \phi_0) \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1 - \theta_t)^{k-1-j} B_{kj}}_{\text{social effects}} = G(\theta_t; \phi_0)$$

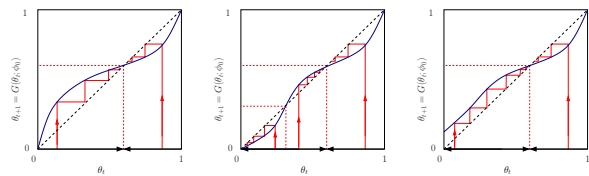
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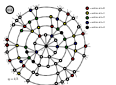
General fixed point story:



- ▶ Given $\theta_0 (= \phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.
- ▶ n.b., adjacent fixed points must have opposite stability types.
- ▶ **Important:** Actual form of G depends on ϕ_0 .
- ▶ So choice of ϕ_0 dictates both G and starting point—can't start anywhere for a given G .

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Expected size of spread:

- ▶ Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \rightarrow 0$.
- ▶ Depends on map $\theta_{t+1} = G(\theta_t; \phi_0)$.
- ▶ First: if self-starters are present, some activation is assured:

$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet B_{k0} > 0.$$

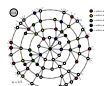
meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.

- ▶ If $\theta = 0$ is a fixed point of G (i.e., $G(0; \phi_0) = 0$) then spreading occurs if

$$G'(0; \phi_0) = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$

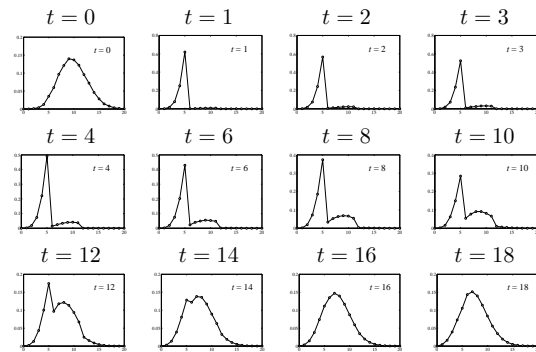
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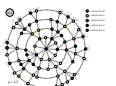
Early adopters—degree distributions



$P_{k,t}$ versus k

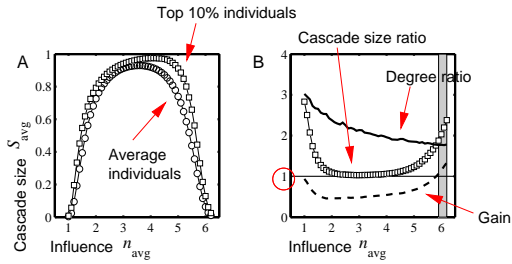
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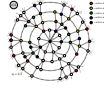
The multiplier effect:



- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.

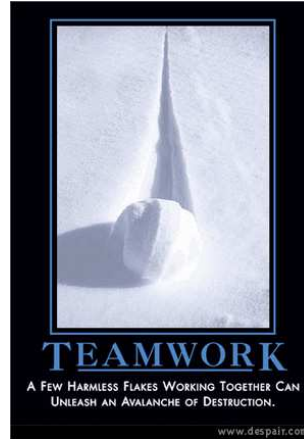
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The power of groups...

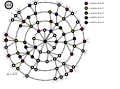


"A few harmless flakes working together can unleash an avalanche of destruction."

despair.com

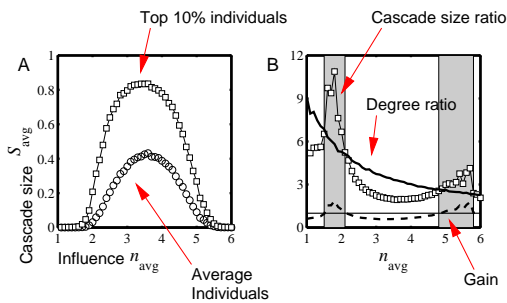
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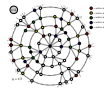
The multiplier effect:



- Skewed influence distribution example.

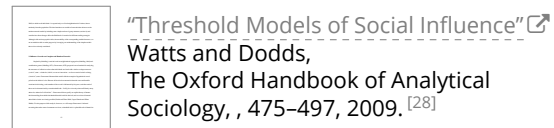
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Extensions

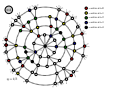


"Threshold Models of Social Influence"
Watts and Dodds,
The Oxford Handbook of Analytical
Sociology, , 475-497, 2009. [28]

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: **group structure**

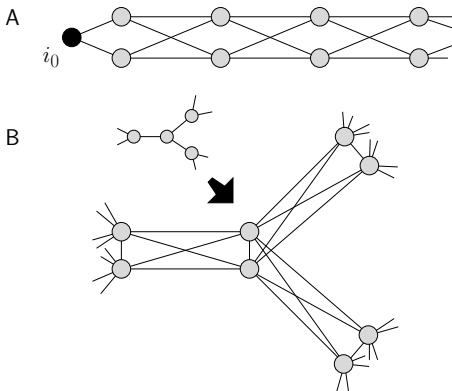
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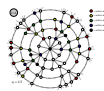
Special subnetworks can act as triggers



- $\phi = 1/3$ for all nodes

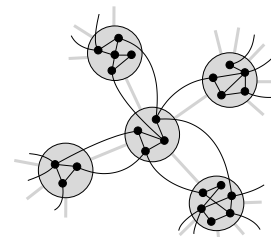
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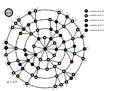
Group structure—Ramified random networks



p = intergroup connection probability
 q = intragroup connection probability.

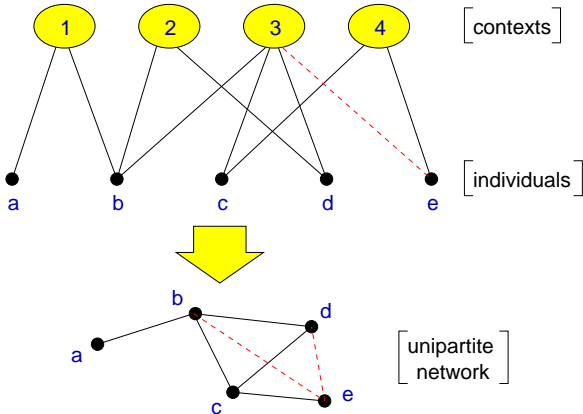
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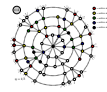
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Bipartite networks



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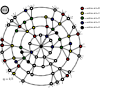
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Generalized affiliation model networks with triadic closure

- ▶ Connect nodes with probability $\propto \exp^{-\alpha d}$ where α = homophily parameter and d = distance between nodes (height of lowest common ancestor)
- ▶ τ_1 = intergroup probability of friend-of-friend connection
- ▶ τ_2 = intragroup probability of friend-of-friend connection

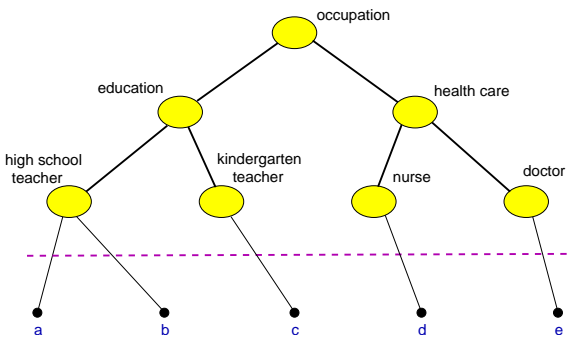
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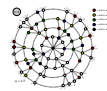
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Context distance



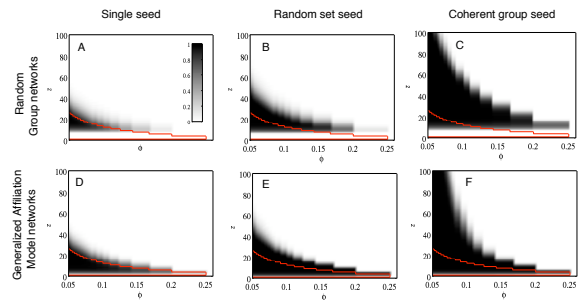
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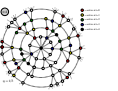
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Cascade windows for group-based networks



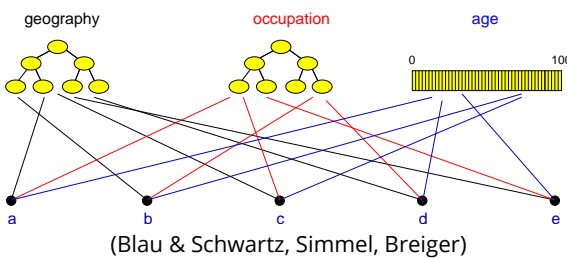
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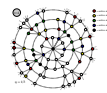
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Generalized affiliation model



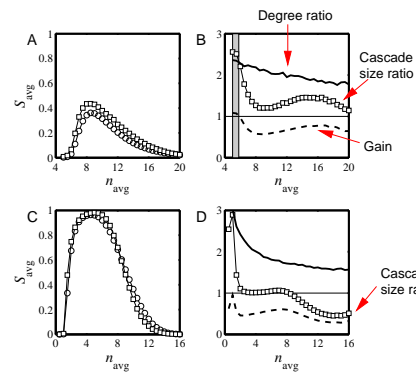
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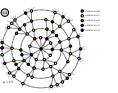
Multiplier effect for group-based networks:



- ▶ Multiplier almost always below 1.

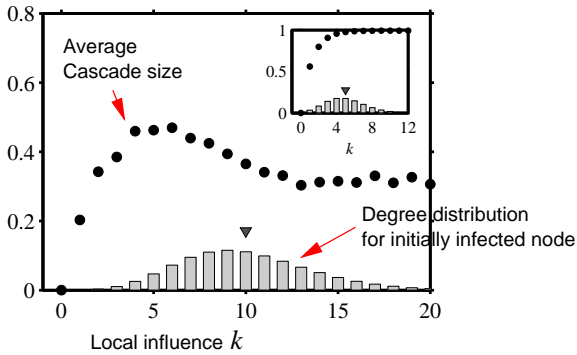
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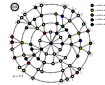
Assortativity in group-based networks



- ▶ The most connected nodes aren't always the most 'influential.'
- ▶ Degree assortativity is the reason.

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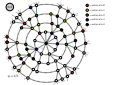
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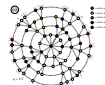
Social contagion

Summary

- ▶ 'Influential vulnerables' are key to spread.
- ▶ Early adopters are mostly vulnerables.
- ▶ Vulnerable nodes important but not necessary.
- ▶ Groups may greatly facilitate spread.
- ▶ Seems that cascade condition is a global one.
- ▶ Most extreme/unexpected cascades occur in highly connected networks
- ▶ 'Influentials' are posterior constructs.
- ▶ Many potential influentials exist.

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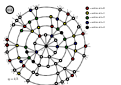
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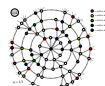
Social contagion

Implications

- ▶ Focus on the influential vulnerables.
- ▶ Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- ▶ Only simple ideas can spread by word-of-mouth. (Idea of opinion leaders spreads well...)
- ▶ Want enough individuals who will adopt and display.
- ▶ Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- ▶ Entities can be novel or designed to combine with others, e.g. block another one.

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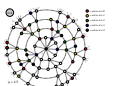
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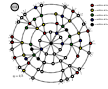
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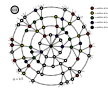
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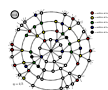
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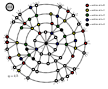
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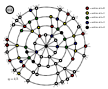
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Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities.
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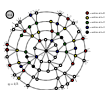
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