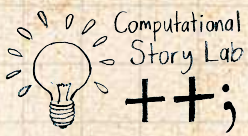
Social Contagion

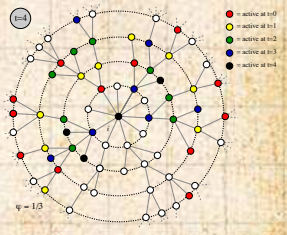
Principles of Complex Systems | @pocsvox
CSYS/MATH 300, Fall, 2015 | #FallPoCS2015

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont



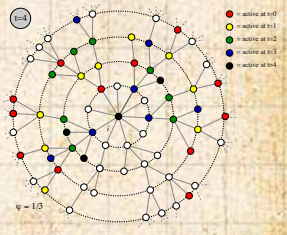
PoCS
Principles of
Complex Systems
@pocsvox
What's the Story?



Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

These slides are brought to you by:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



The UNIVERSITY of VERMONT

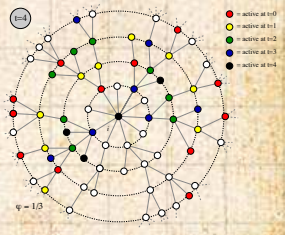
Outline

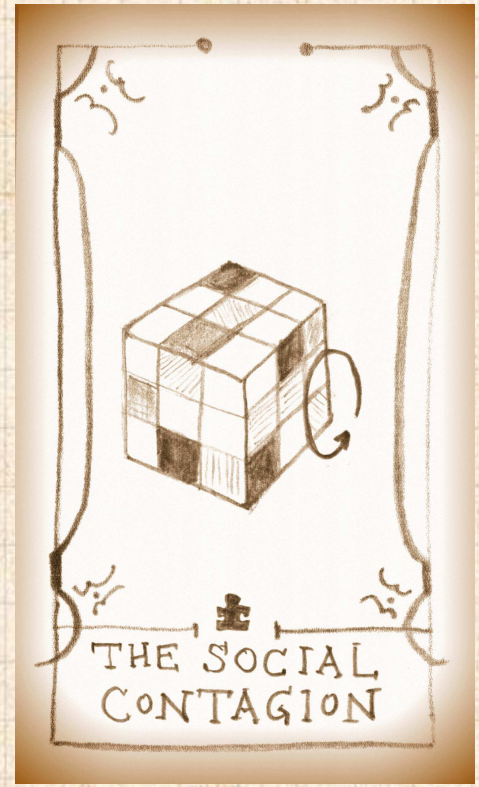
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Social Contagion Models

- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

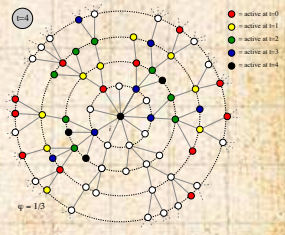
References





Social Contagion Models
- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

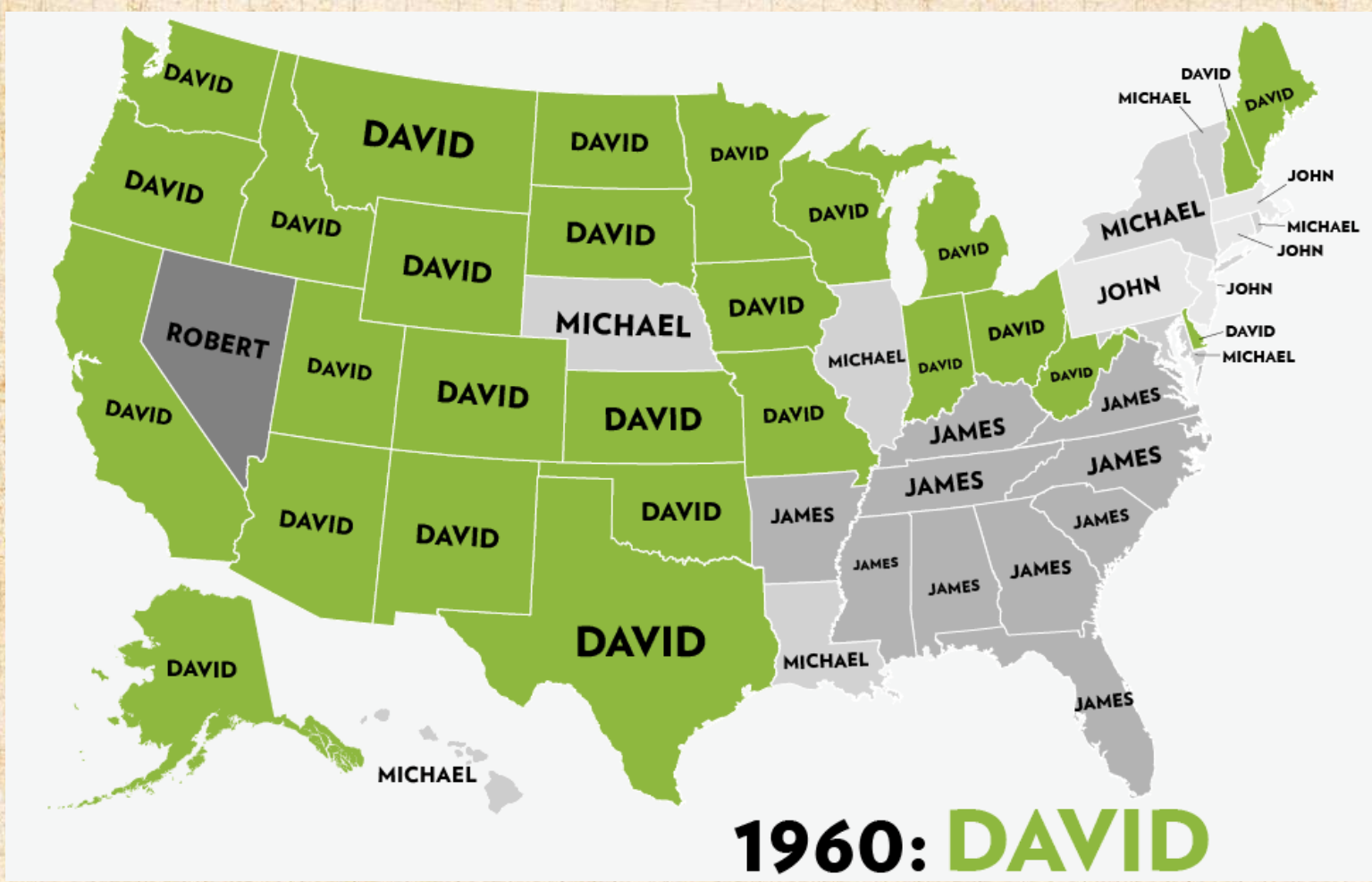
References



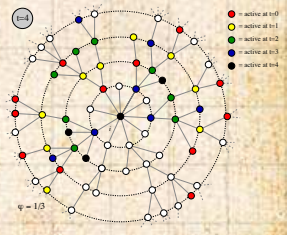
The UNIVERSITY of VERMONT

Social Contagion Models
- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

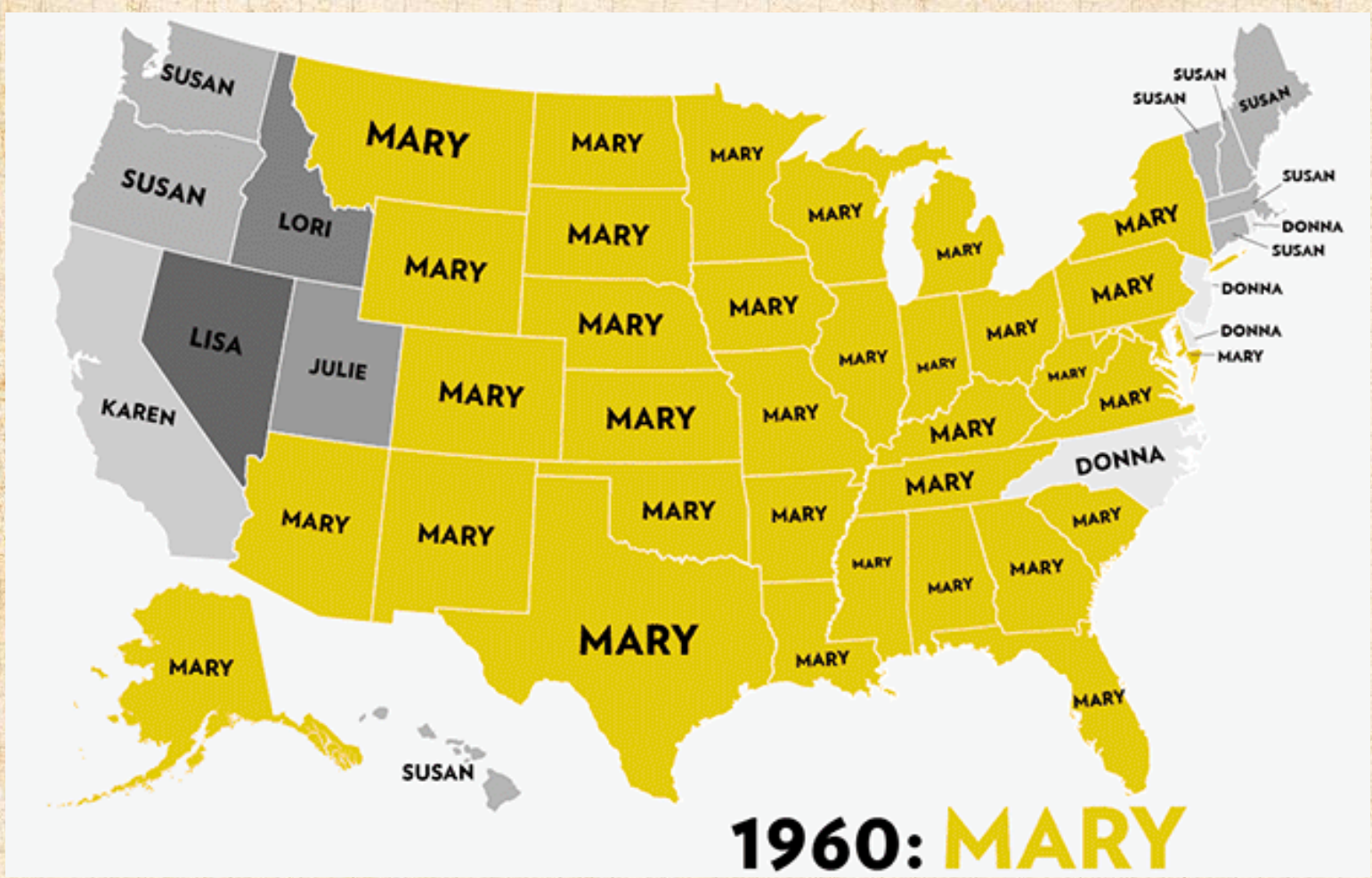
References



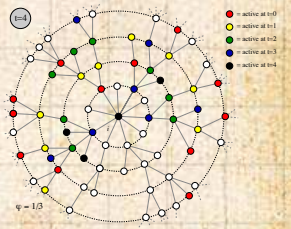
From the Atlantic



Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



From the Atlantic



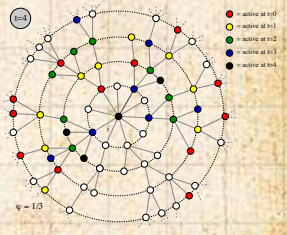
Things that spread well:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

buzzfeed.com:

- Dangerously self aware: 11 Elements that make a perfect viral video.

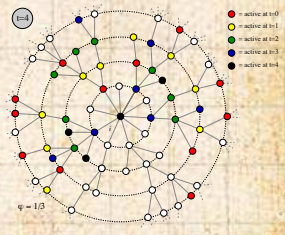
+ News ...



LOL + cute + fail + wtf:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

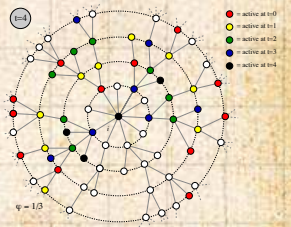
Please try reloading this page. If the problem persists let us know.



The whole lolcats thing:

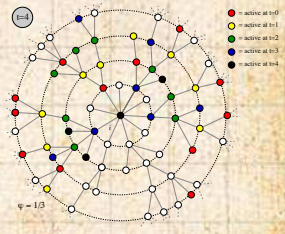
Social Contagion Models
- Background
- Granovetter's model
- Network version
- Final size
- Spreading success
- Groups

References



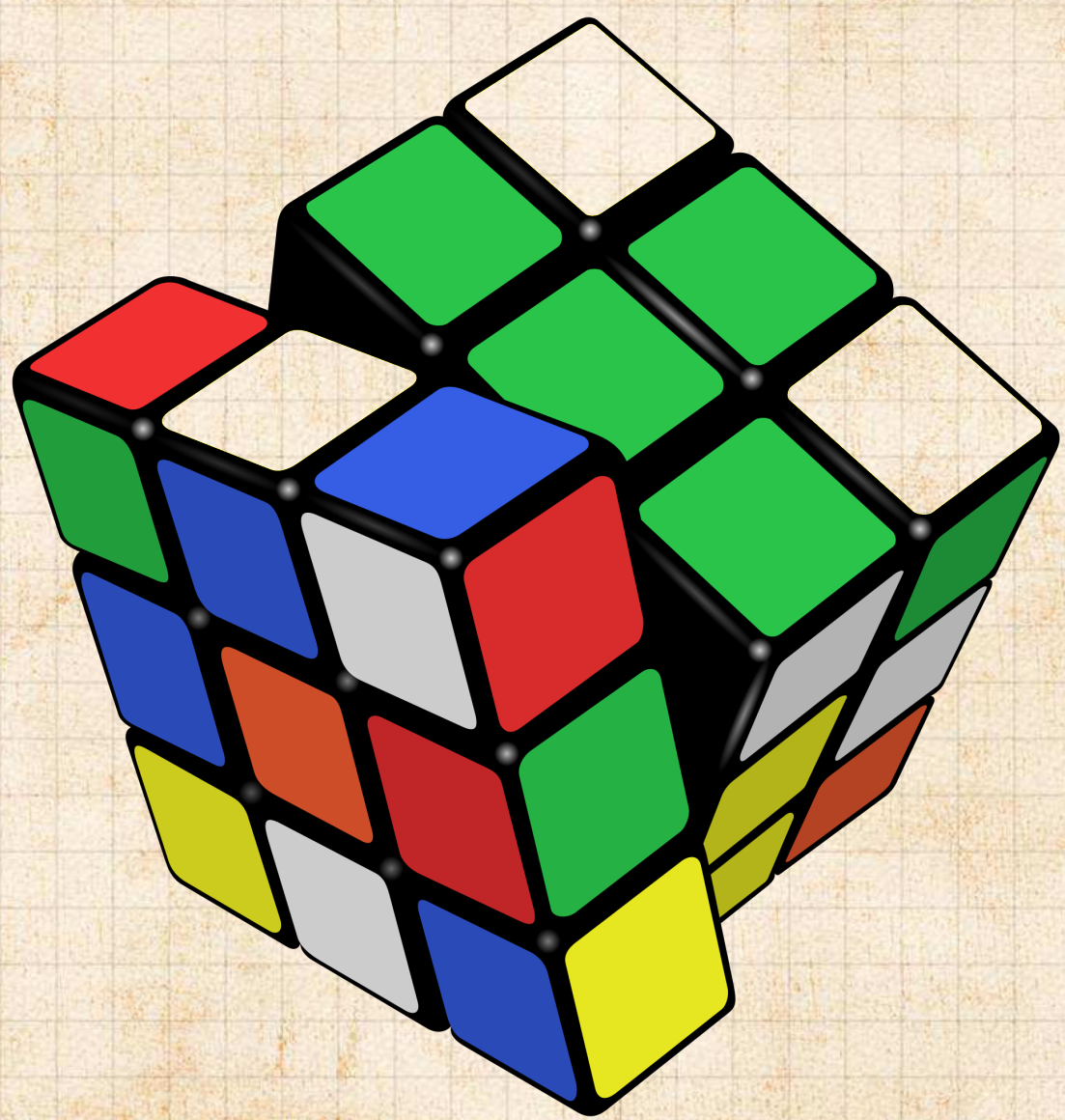
Some things really stick:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

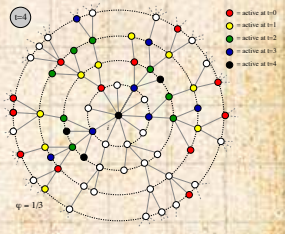


The UNIVERSITY of VERMONT

wtf + geeky + omg:

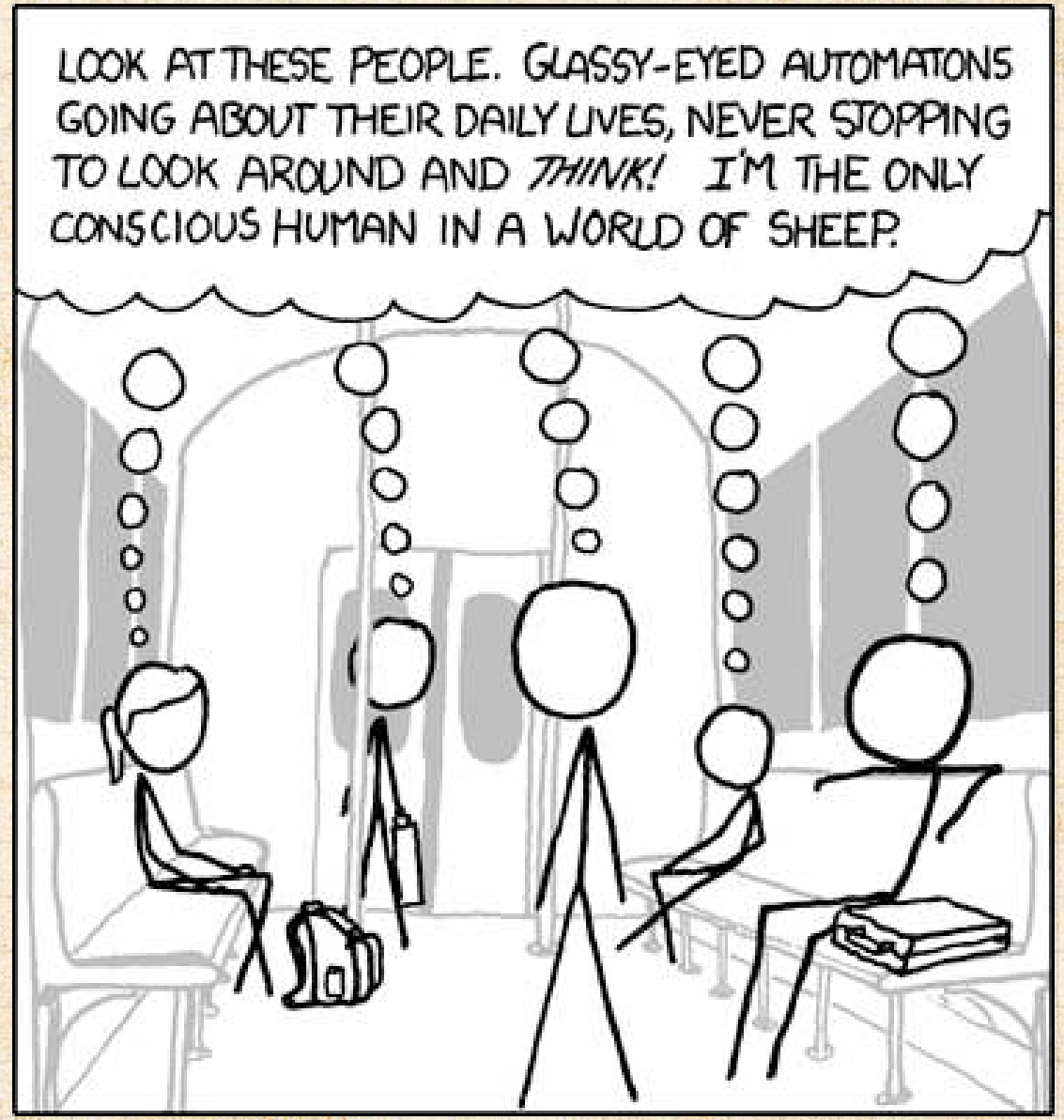


Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

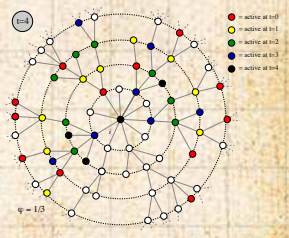


Why social contagion works so well:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

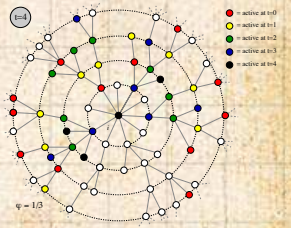


http://xkcd.com/610/



Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



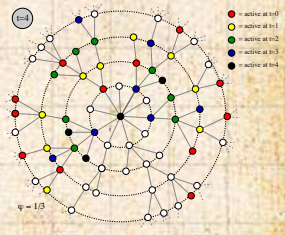
Social Contagion

Examples abound

- fashion
- striking
- smoking [7]
- residential segregation [22]
- iPhones and iThings
- obesity [6]
- Harry Potter
- voting
- gossip
- Rubik's cube
- religious beliefs
- school shootings
- leaving lectures

SIR and SIRS type contagion possible

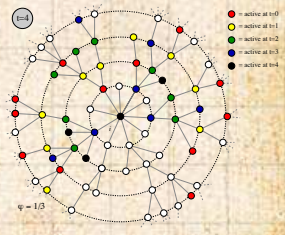
- Classes of behavior versus specific behavior : dieting, horror movies, getting married, invading countries, ...



Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Mixed messages: Please copy, but also, don't copy ...

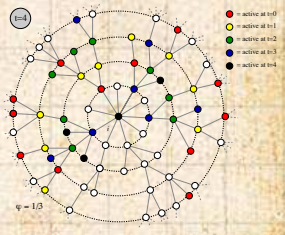
- Cindy Harrell appeared in the (terrifying) music video for Ray Parker Jr.'s Ghostbusters.
- Misframing: Appeals only to seed on exponential growth.



Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Market much?

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Framingham heart study:

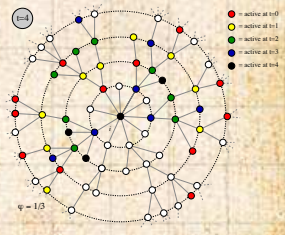
Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking [7]
- The spread of spreading [6]
- Also: happiness [11], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives

Controversy:

- Are your friends making you fat? (Clive Thomspon, NY Times, September 10, 2009).
- Everything is contagious—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Social Contagion

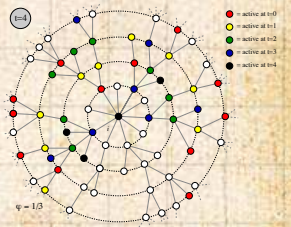
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

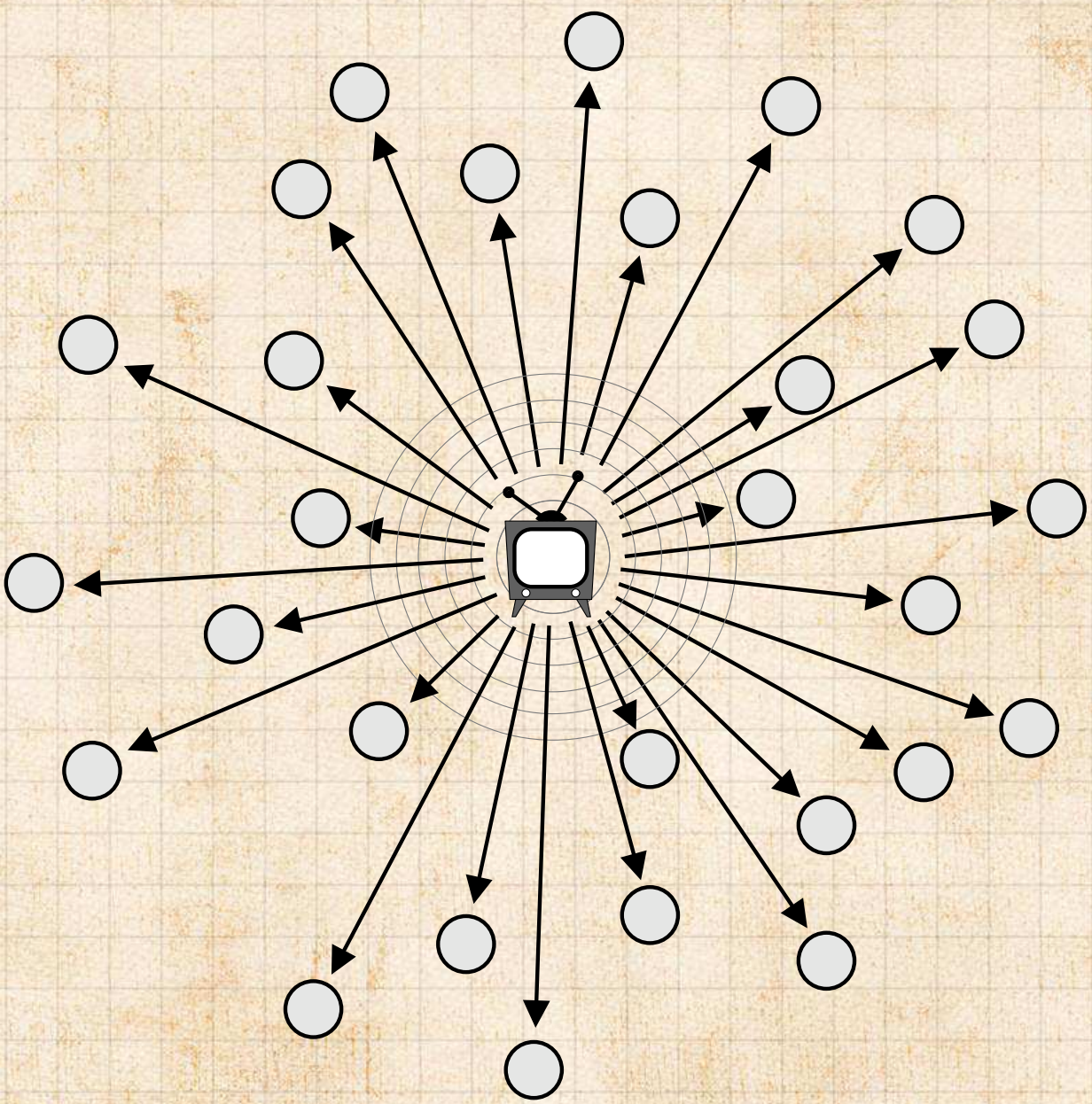
We need to understand influence

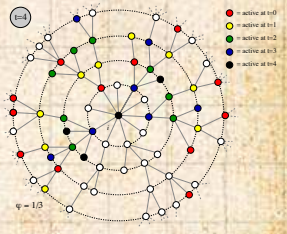
- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers? Highly popularized by Gladwell [12] as 'connectors'
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [19]



The hypodermic model of influence

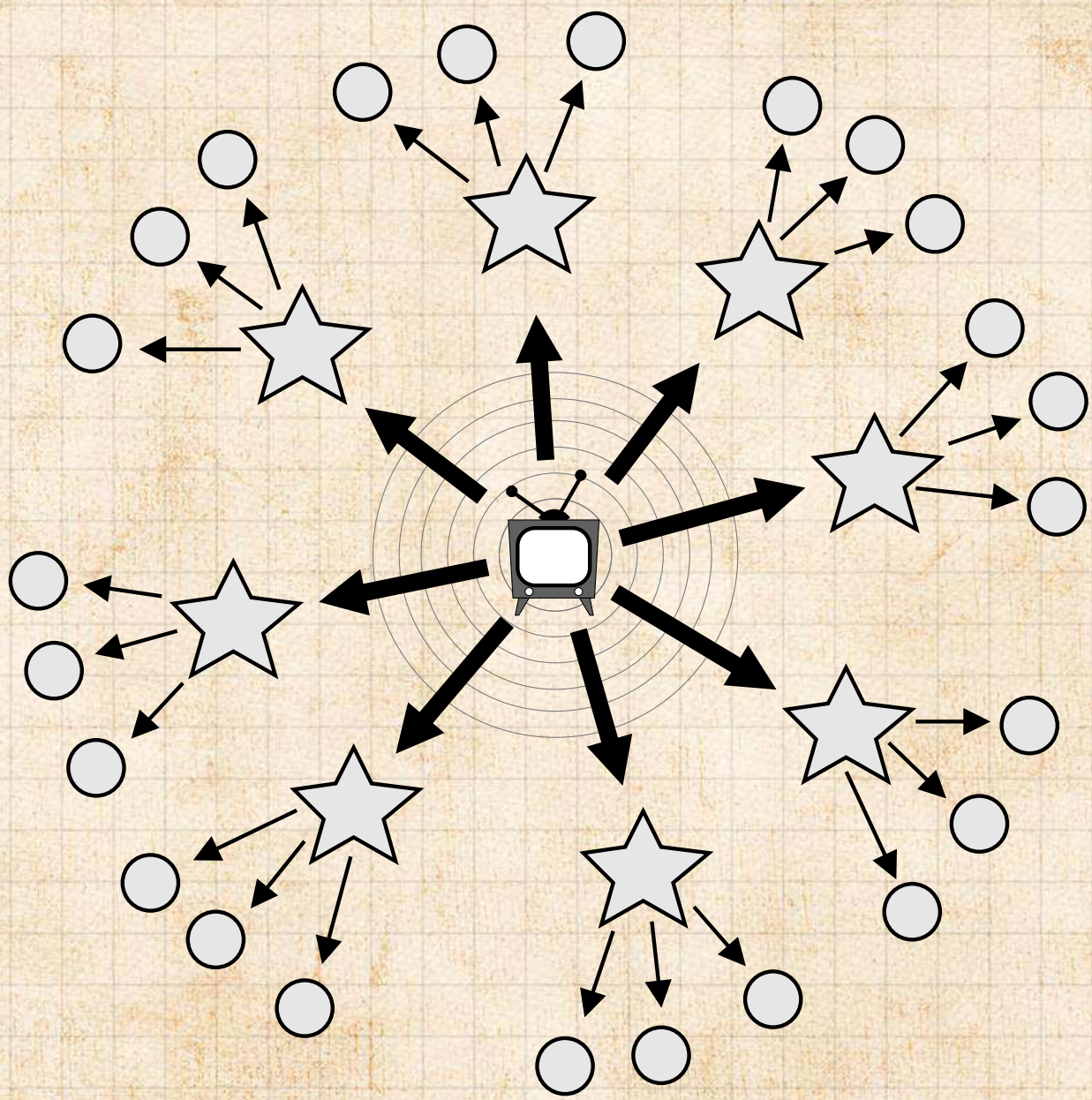
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

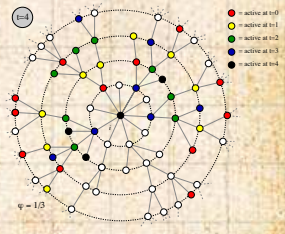




The two step model of influence [19]

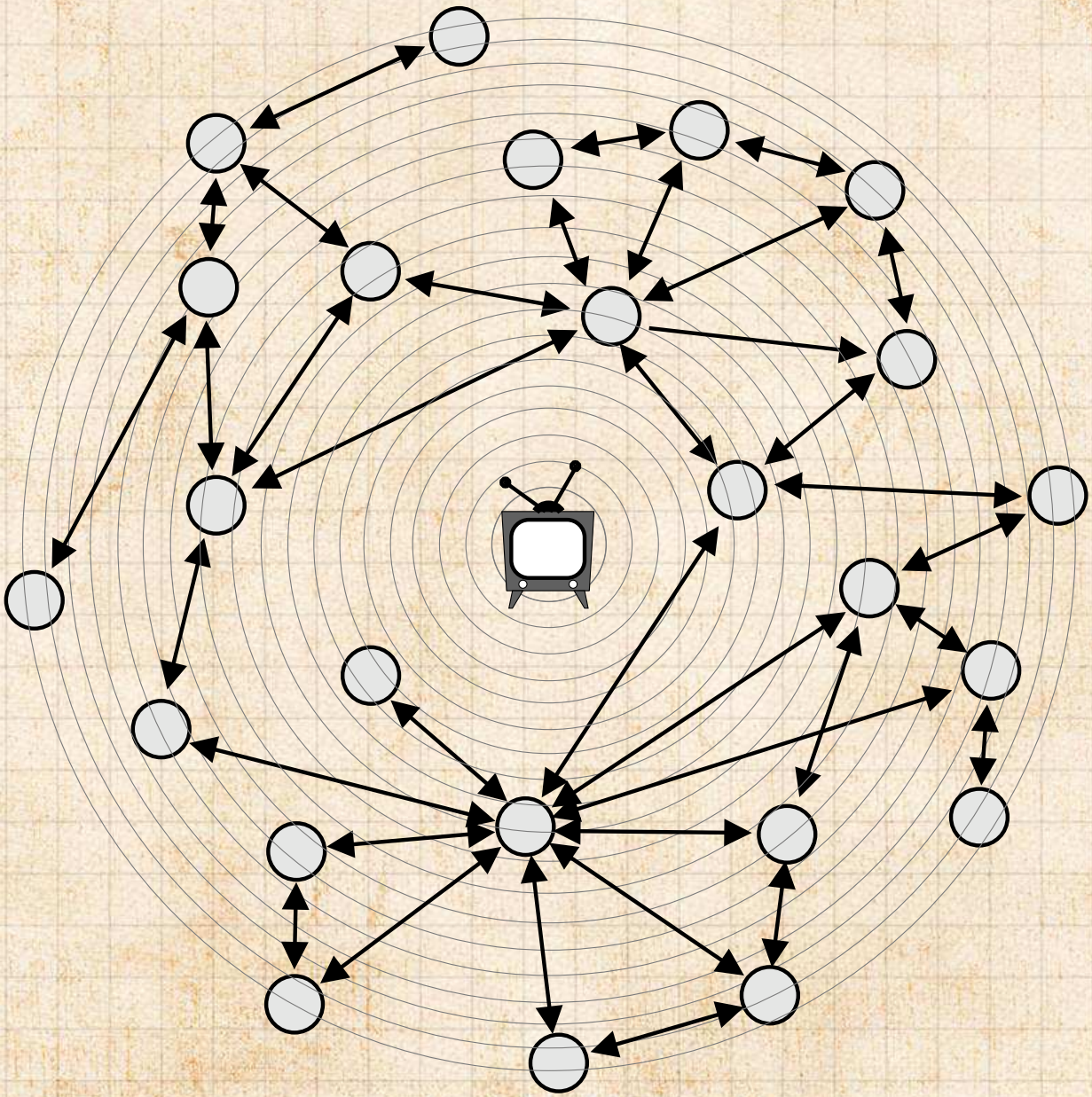
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

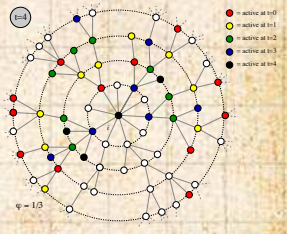




The general model of influence: the Social Wild

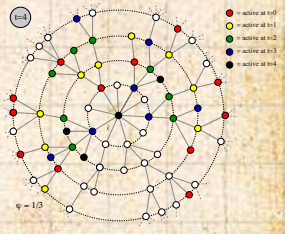
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References





Talking about the social wild:

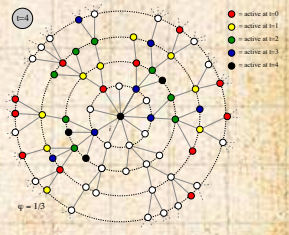
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Why do things spread socially?

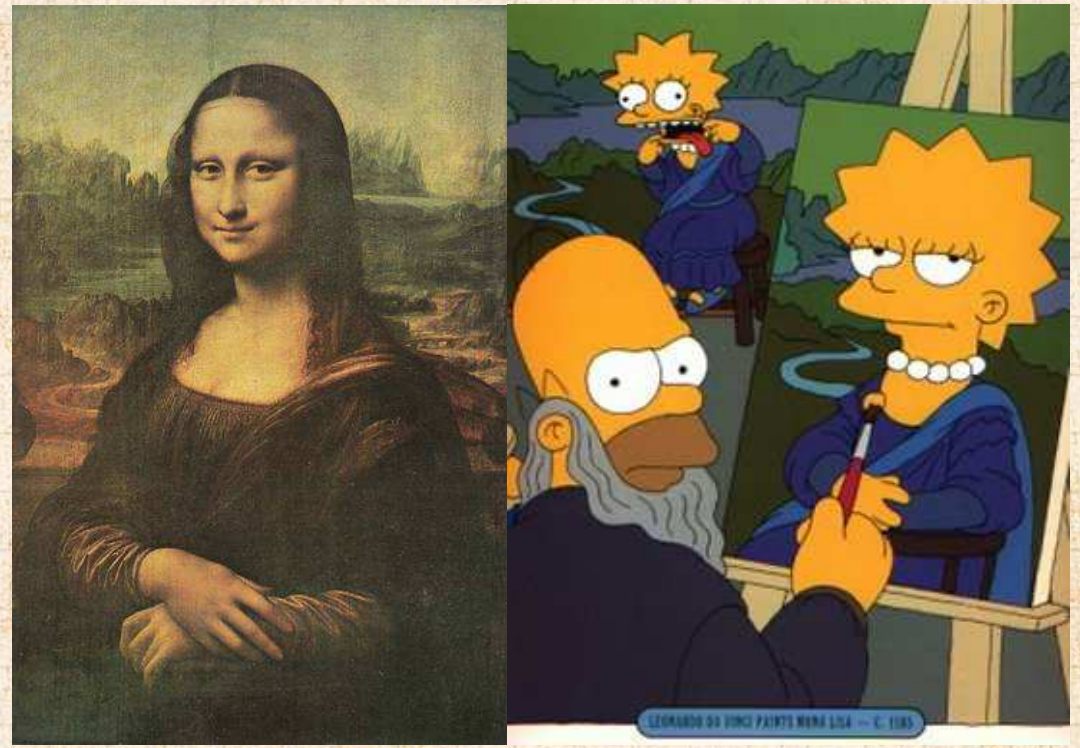
- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are storytellers: *homo narrativus*.
- We like to think things happened for reasons ...
- Reasons for success are usually ascribed to intrinsic properties (examples next).
- Teleological stories of fame are often easy to generate and believe.
- System/group dynamics harder to understand because most of our stories are built around individuals.
- Always good to examine what is said before and after the fact ...

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

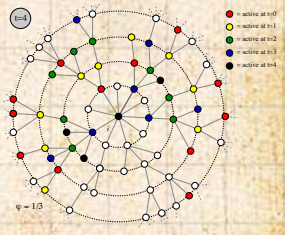


The Mona Lisa

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



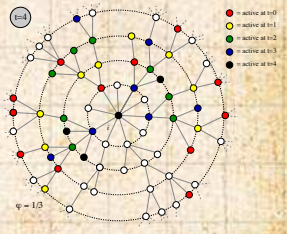
- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism, parody, ...



‘Tattooed Guy’ Was Pivotal in Armstrong Case [nytimes]

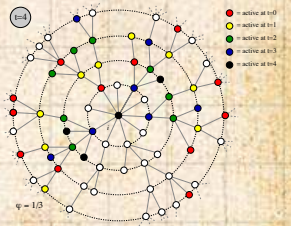
- “... Leogrande’s doping sparked a series of events ...”

Social Contagion Models
Background
Granovetter’s model
Network version
Final size
Spreading success
Groups
References



The completely unpredicted fall of Eastern Europe

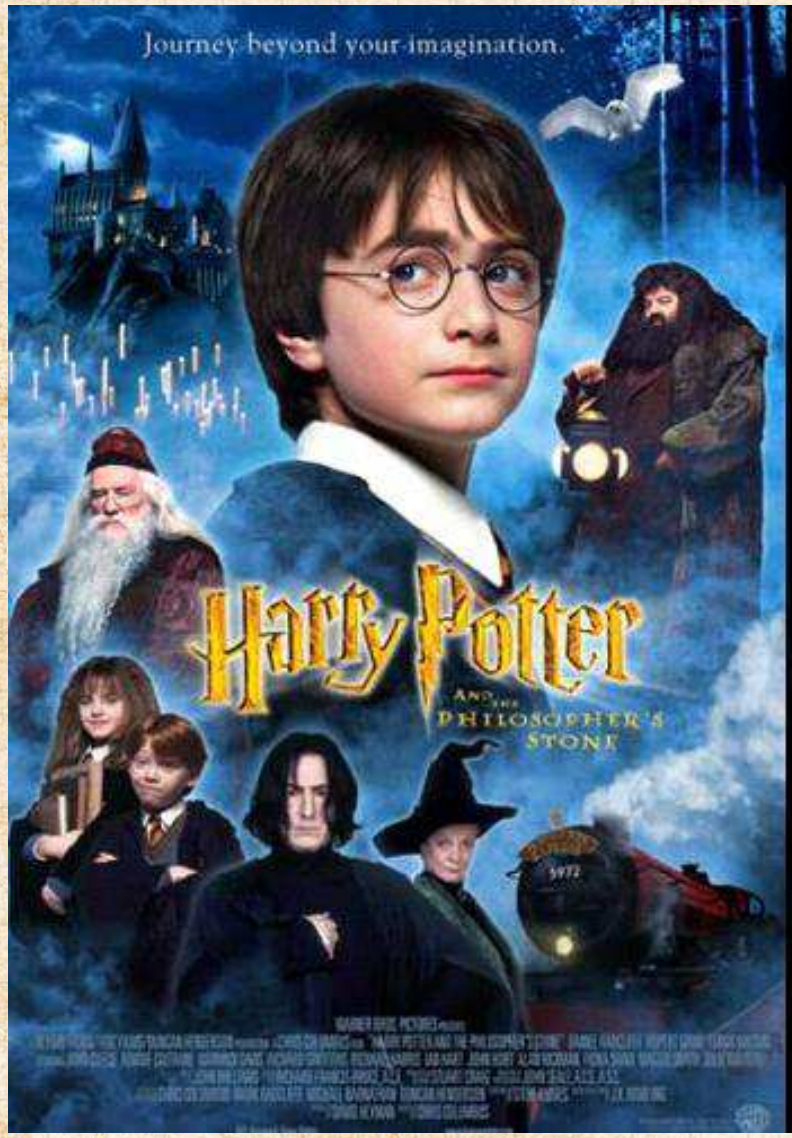
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

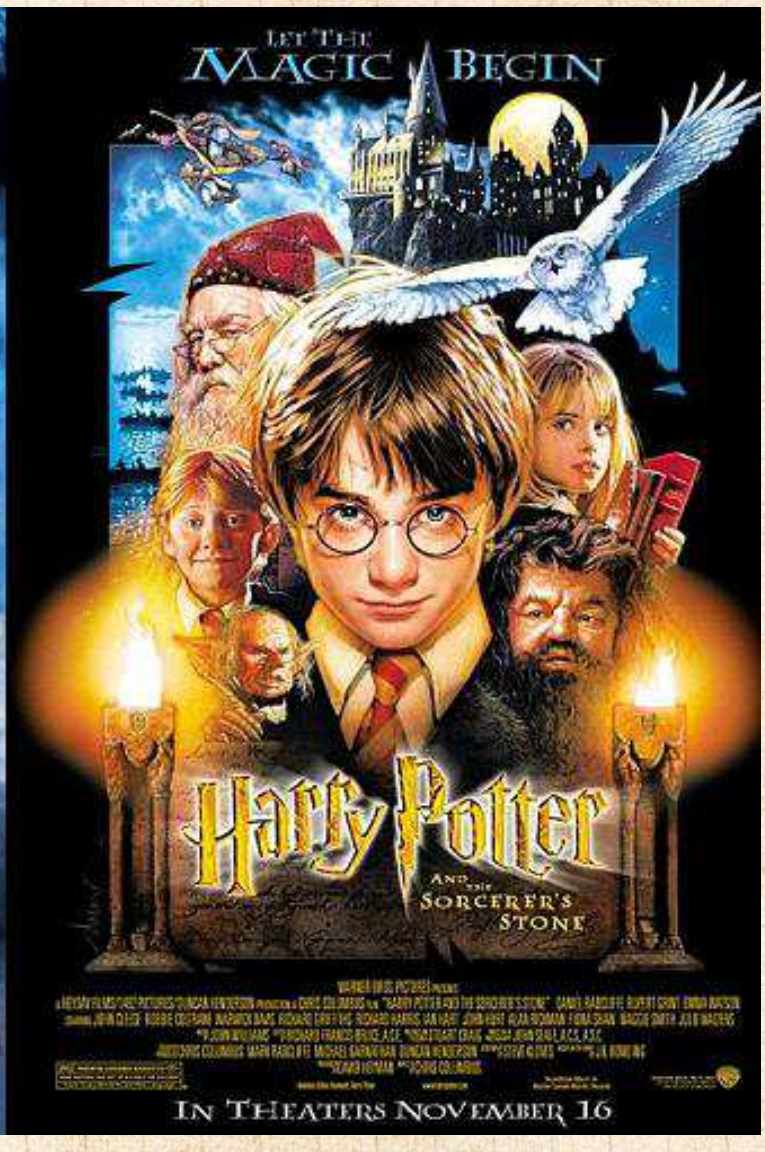


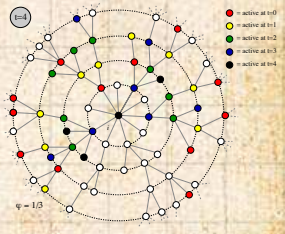
Timur Kuran: [20, 21] "Now Out of Never: The Element of Surprise in the East European Revolution of 1989"

The dismal predictive powers of editors...

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References







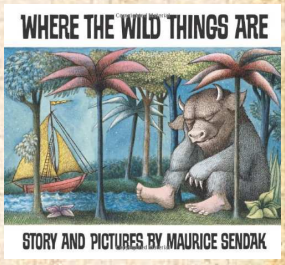
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

From a 2013 Believer Magazine interview with Maurice Sendak:

BLVR: Did the success of Where the Wild Things Are ever feel like an albatross?

MS: It's a nice book. It's perfectly nice. I can't complain about it. I remember Herman Melville said, "When I die no one is going to mention Moby-Dick. They're all going to talk about my first book, about f∗∗∗ing maidens in Tahiti." He was right. No mention of Moby-Dick then. Everyone wanted another Tahitian book, a beach book. But then he kept writing deeper and deeper and then came Moby-Dick and people hated it. The only ones who liked it were Mr. and Mrs. Nathaniel Hawthorne. Moby-Dick didn't get famous until 1930.

- Sendak named his dog Herman.
- The essential Colbert interview: Pt. 1 and Pt. 2.



Drafting success in the NFL:

Top Players by Round, 1995-2012

1ST ROUND	2ND ROUND	3RD ROUND	4TH ROUND	5TH ROUND	6TH ROUND	7TH ROUND
Peyton Manning	**Drew Brees**	**Terrell Owens**	**Jared Allen**	**Zach Thomas**	**Tom Brady**	**Donald Driver**
1ST OVER ALL, 1998	32ND PICK, 2001	89TH PICK, 1996	126TH PICK, 2004	154TH PICK, 1996	199TH PICK, 2000	213TH PICK, 1999

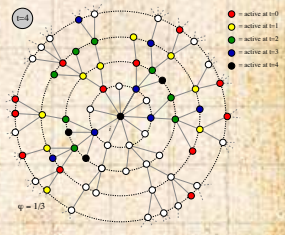
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Messing with social connections

- Ads based on message content (e.g., Google and email)
- BzzAgent
- One of Facebook's early advertising attempts: Beacon
- All of Facebook's advertising attempts.



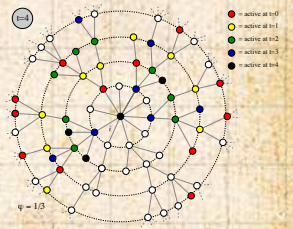
Getting others to do things for you

A very good book: 'Influence' [8] by Robert Cialdini

Six modes of influence:

1. Reciprocation: *The Old Give and Take... and Take*; e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*; e.g., Hazing.
3. Social Proof: *Truths Are Us*; e.g., Jonestown, Kitty Genovese (contested).
4. Liking: *The Friendly Thief*; e.g., Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*; e.g., Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*; e.g., Prohibition.

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



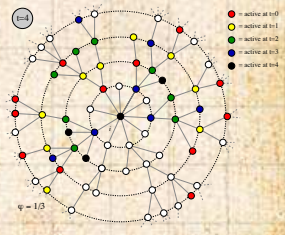
Social contagion

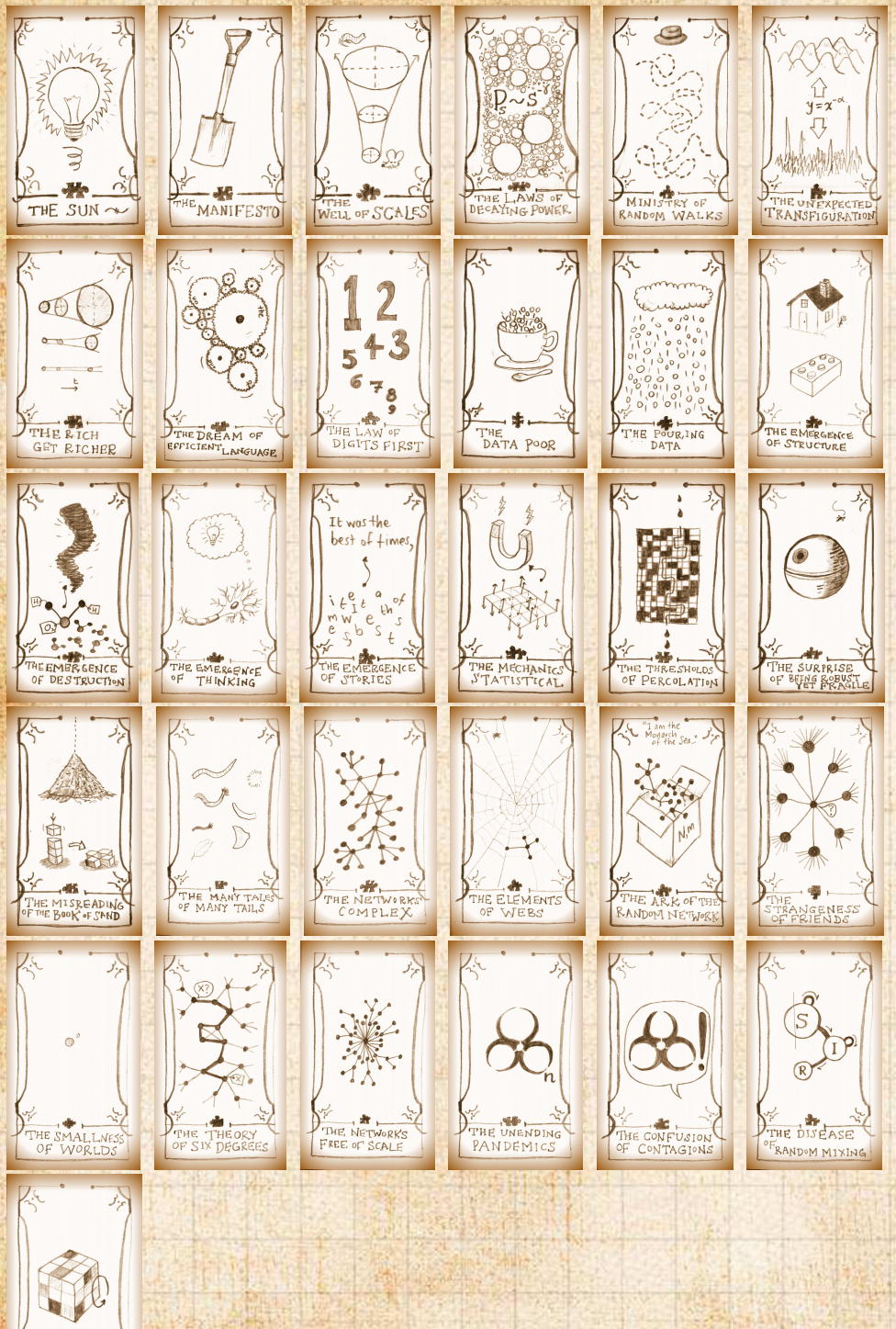
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

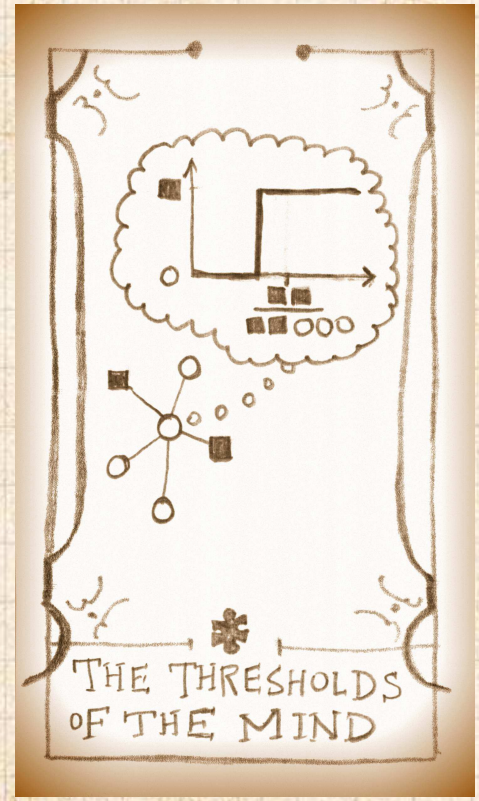
- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)



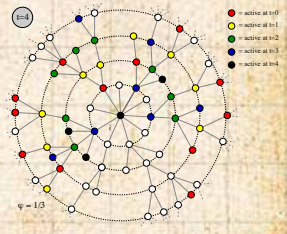




Social Contagion Models

Background
Granovetter's model
Network version
Final size
Spreading success
Groups

References

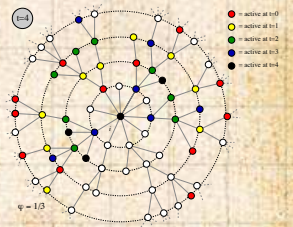


Social Contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Some important models:

- Tipping models—Schelling (1971) [22, 23, 24]
  - Simulation on checker boards
  - Idea of thresholds
  - Polygon-themed online visualization. (Includes optional diversity-seeking proclivity.)
  - Explore the Netlogo online implementation [29]
- Threshold models—Granovetter (1978) [15]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [2, 3]
  - Social learning theory, Informational cascades,...

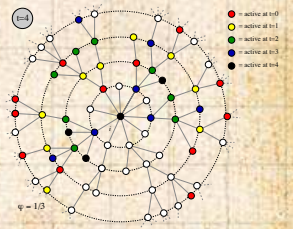


Social contagion models

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter... (unrealistic).
- Assumption: level of influence per person is uniform (unrealistic).

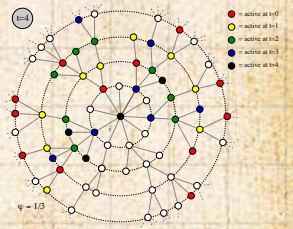


Social Contagion

Some possible origins of thresholds:

- Inherent, evolution-devised inclination to coordinate, to conform, to imitate. [1]
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
  - Externalities = Effects on others not directly involved in a transaction
  - Examples: telephones, fax machine, Facebook, operating systems
  - An individual's utility increases with the adoption level among peers and the population in general

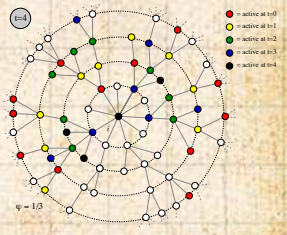
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Neural reboot (NR):

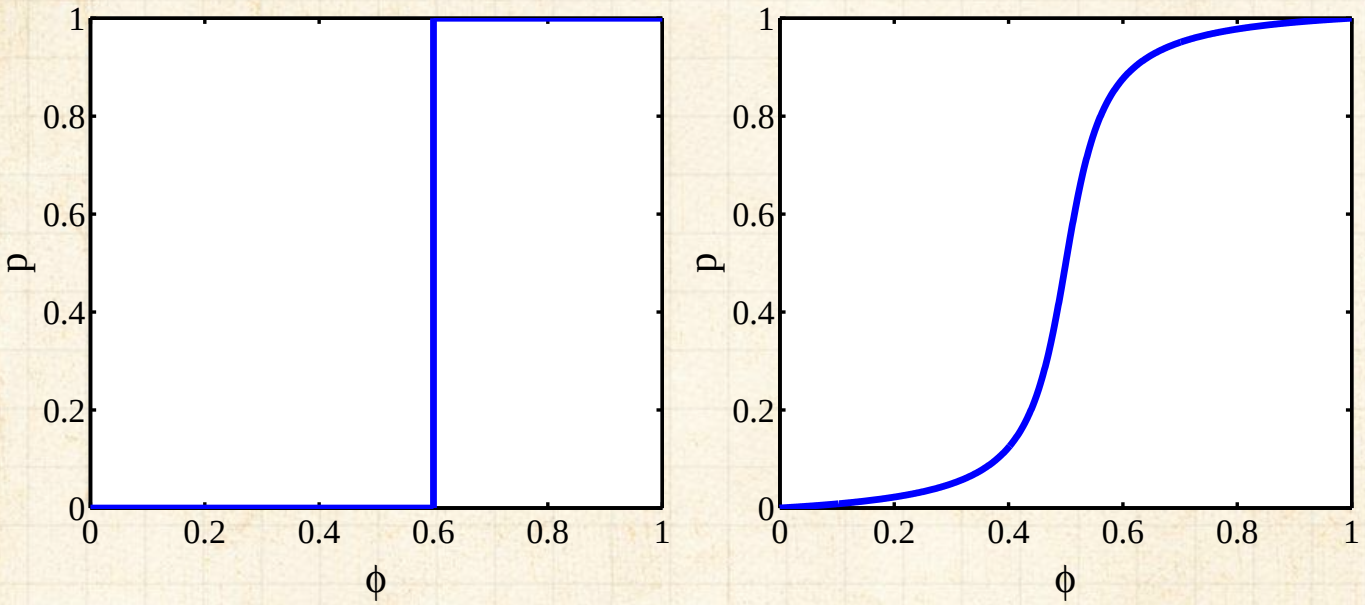
Shareworthy interlude

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

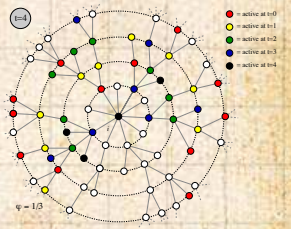


Threshold models—response functions

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



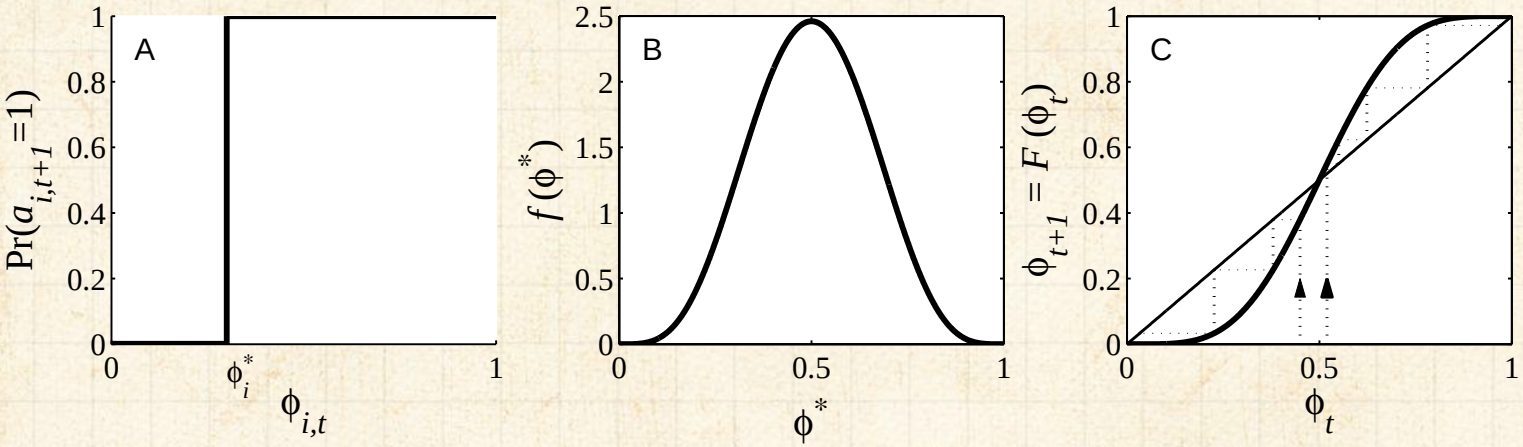
- Example threshold influence response functions: deterministic and stochastic
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Two states: S and I.



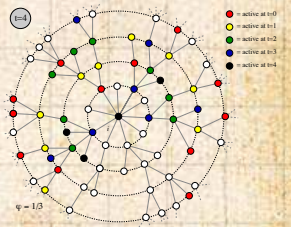
Threshold models

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Action based on perceived behavior of others:



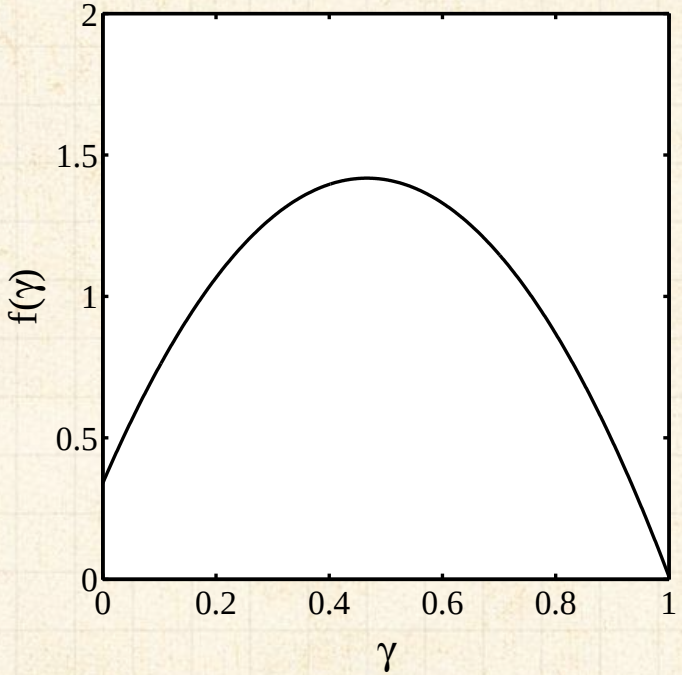
- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a Critical mass model

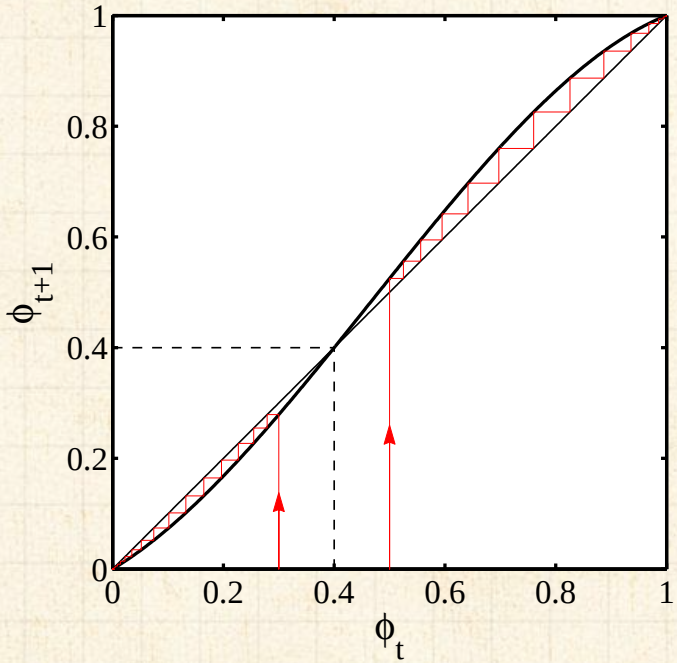


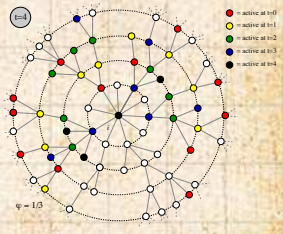
Threshold models

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Another example of critical mass model:



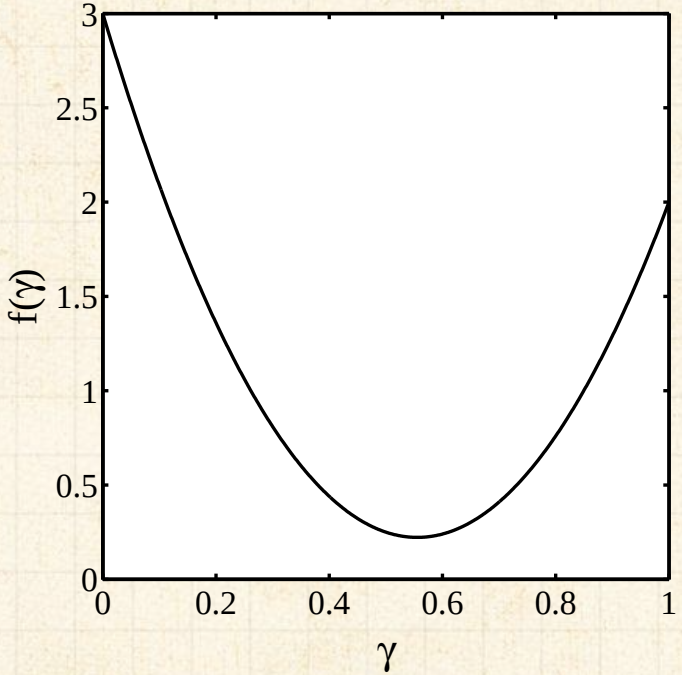


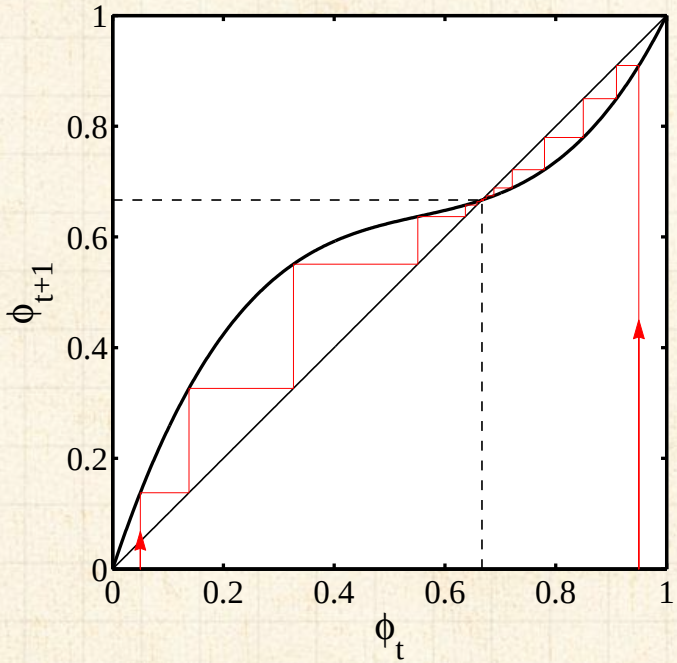


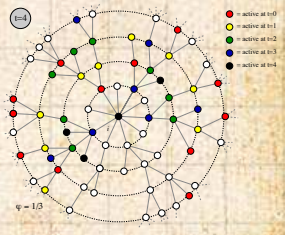
Threshold models

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Example of single stable state model:

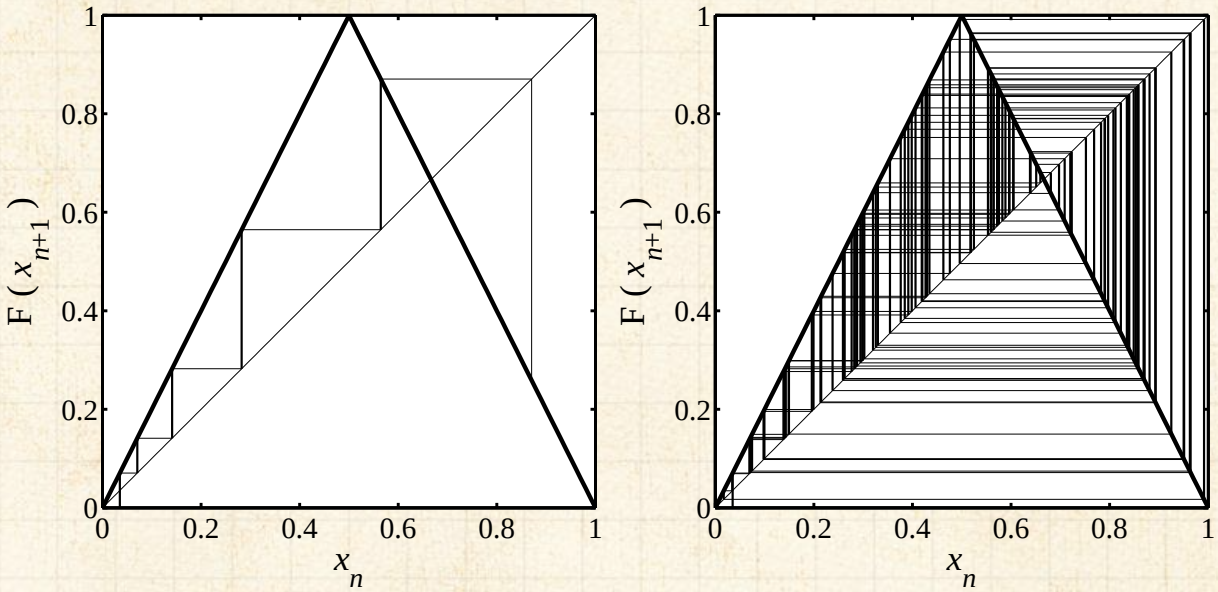




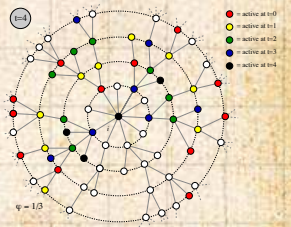


Threshold models

Chaotic behavior possible [17, 16, 9, 18]



- Period doubling arises as map amplitude r is increased.
- Synchronous update assumption is crucial



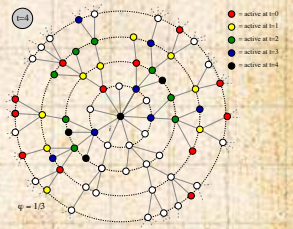
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Threshold models—Nutshell

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Implications for collective action theory:

1. Collective uniformity $\not\Rightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes
3. The stories/dynamics of complex systems are conceptually inaccessible for individual-centric narratives.
4. System stories live in left null space of our stories—we can't even see them.
5. But we happily impose simplistic, individual-centric stories—we can't help ourselves.



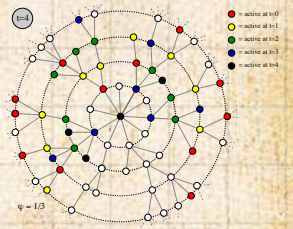
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Many years after Granovetter and Soong's work:

- "A simple model of global cascades on random networks"
  D. J. Watts. Proc. Natl. Acad. Sci., 2002 [26]
  - Mean field model $\rightarrow$ network model
  - Individuals now have a limited view of the world

We'll also explore:

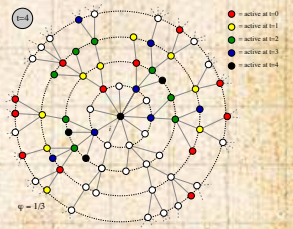
- "Seed size strongly affects cascades on random networks" [14]
  Gleeson and Cahalane, Phys. Rev. E, 2007.
- "Direct, phyiscally motivated derivation of the contagion condition for spreading processes on generalized random networks" [10] Dodds, Harris, and Payne, Phys. Rev. E, 2011
- "Influentials, Networks, and Public Opinion Formation" [27]
  Watts and Dodds, J. Cons. Res., 2007.



Threshold model on a network

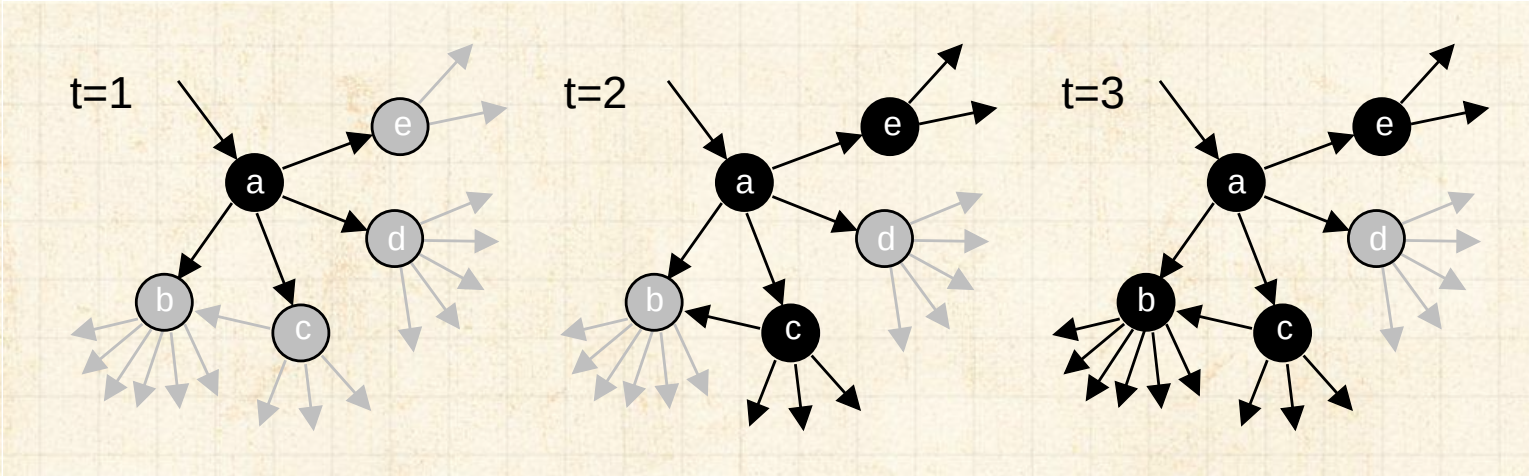
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

- Interactions between individuals now represented by a network.
- Network is sparse.
- Individual i has k_i contacts.
- Influence on each link is reciprocal and of unit weight.
- Each individual i has a fixed threshold ϕ_i.
- Individuals repeatedly poll contacts on network.
- Synchronous, discrete time updating.
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$.
- Individuals remain active when switched (no recovery = SI model).

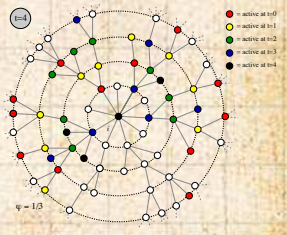


Threshold model on a network

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



- All nodes have threshold $\phi = 0.2$.



Snowballing

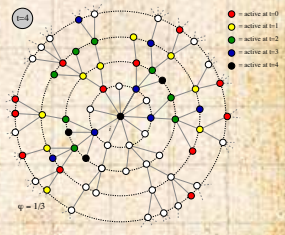
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

First study random networks:

- Start with N nodes with a degree distribution P_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition

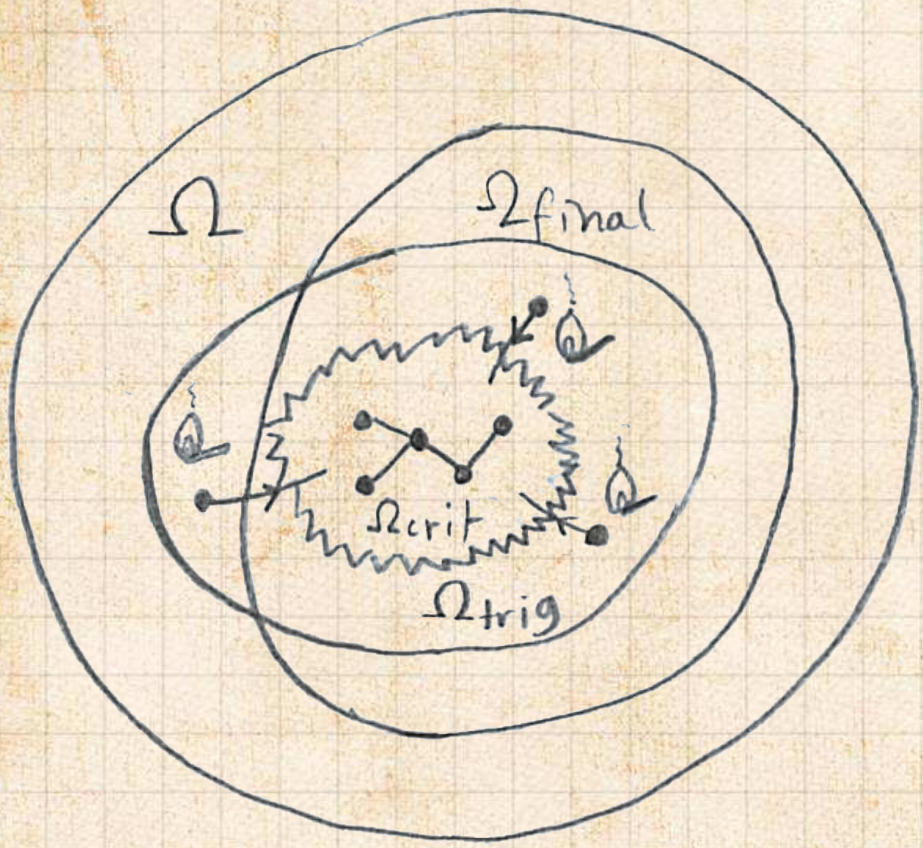
The Cascade Condition:

1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?

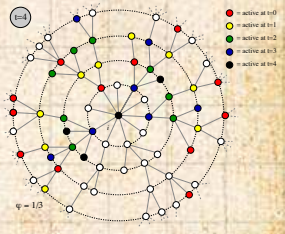


Example random network structure:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



- $\Omega_{crit} = \Omega_{vuln}$ = critical mass = global vulnerable component
- Ω_{trig} = triggering component
- Ω_{final} = potential extent of spread
- Ω = entire network



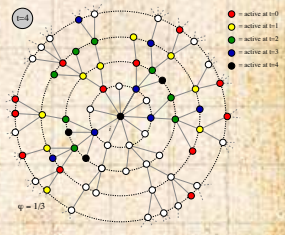
$\Omega_{crit} \subset \Omega_{trig}$; $\Omega_{crit} \subset \Omega_{final}$; and $\Omega_{trig}, \Omega_{final} \subset \Omega$.

Snowballing

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to at least 1 more infected link, then activation spreads.
- We need to understand which nodes can be activated when only one of their neigbors becomes active.



The most gullible

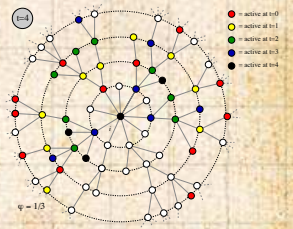
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables* [26]
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.



Cascade condition

Back to following a link:

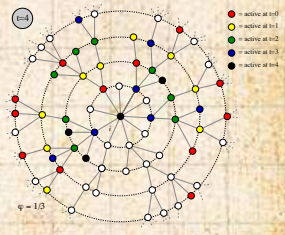
- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k.
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Cascade condition

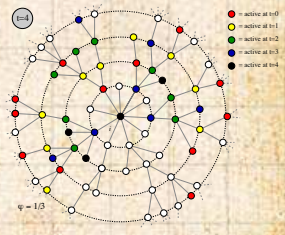
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Next: Vulnerability of linked node

- Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*)\mathrm{d}\phi'_*$$

- If linked node is vulnerable, it produces $k-1$ new outgoing active links
- If linked node is not vulnerable, it produces no active links.



Cascade condition

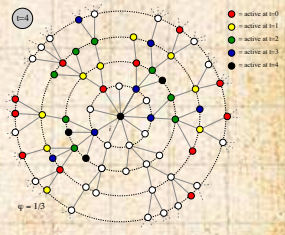
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Putting things together:

- Expected number of active edges produced by an active edge:

$$R = \left[\sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1-\beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}} \right]$$

$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$



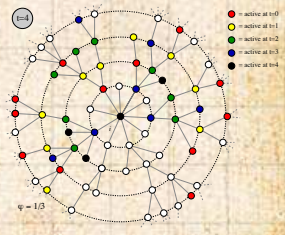
Cascade condition

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

So... for random networks with fixed degree distributions, cacades take off when:

$$\sum_{k=1}^{\infty}(k-1)\cdot\beta_k\cdot\frac{kP_k}{\langle k\rangle}>1.$$

- $\beta_k =$ probability a degree k node is vulnerable.
- $P_k =$ probability a node has degree k.



Cascade condition

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

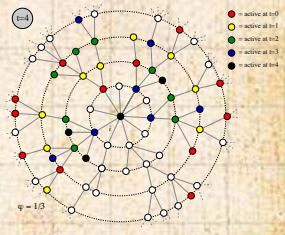
Two special cases:

- (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

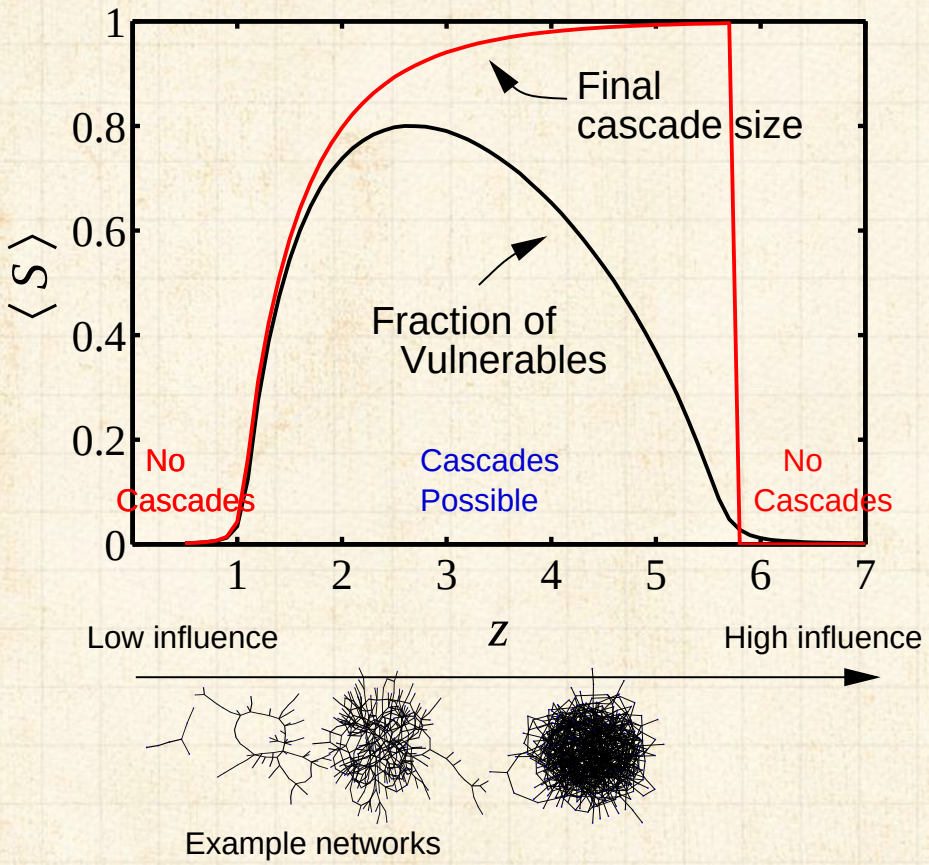
- (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} > 1.$$

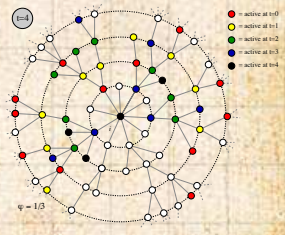


Cascades on random networks

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

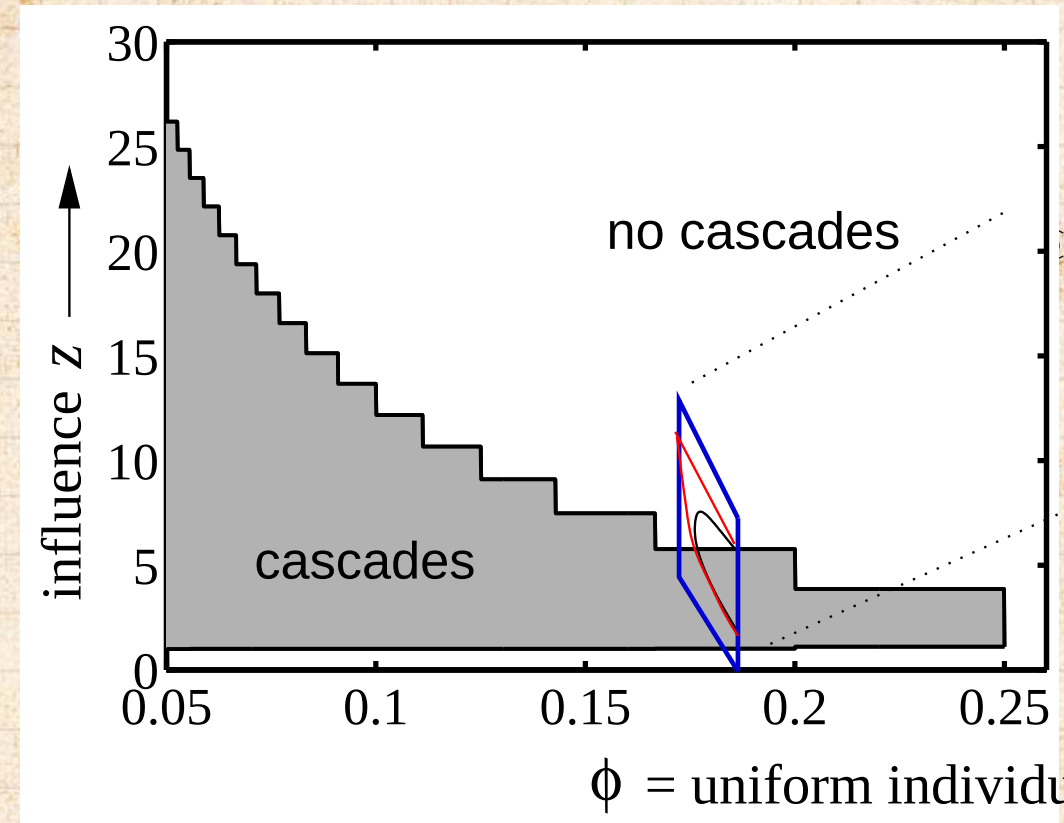


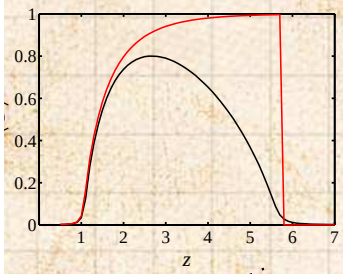
- Cascades occur only if size of max vulnerable cluster > 0.
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

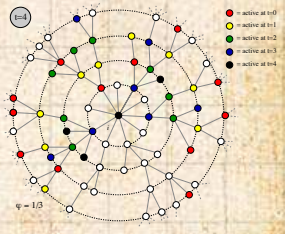


Cascade window for random networks

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

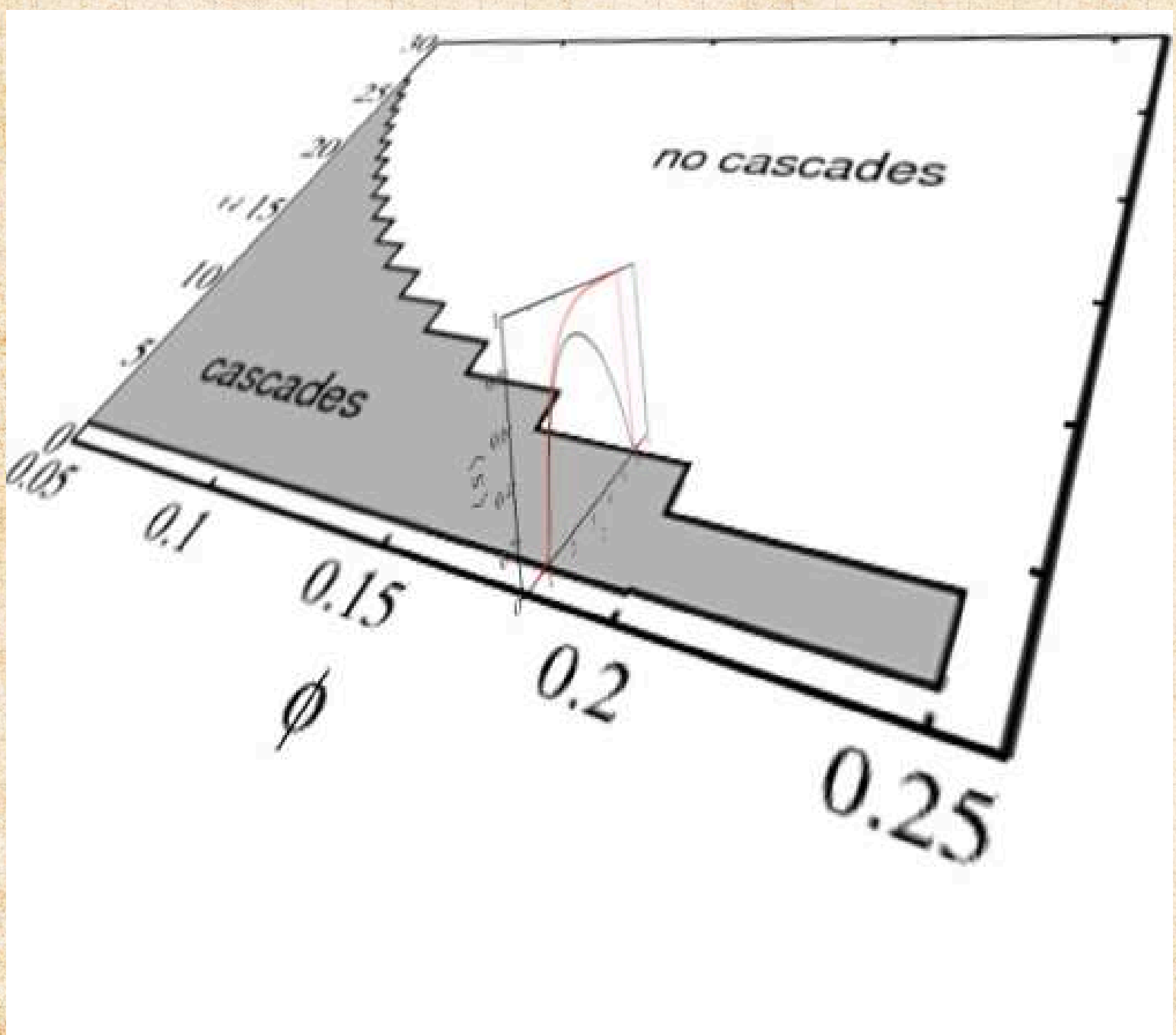




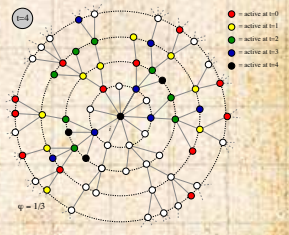


- 'Cascade window' widens as threshold ϕ decreases.
- Lower thresholds enable spreading.

Cascade window for random networks

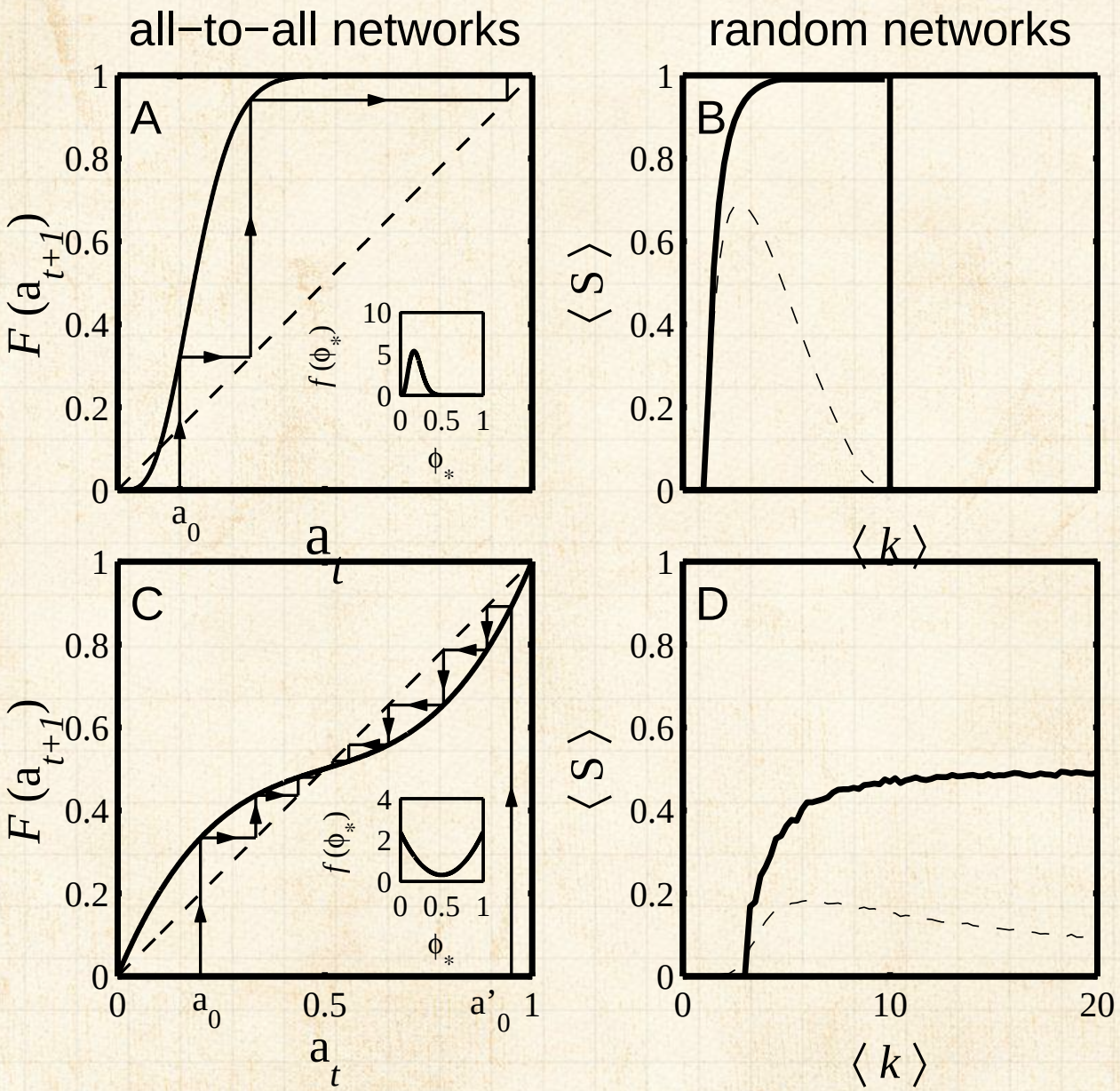


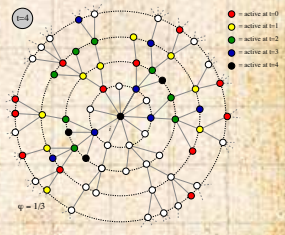
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



All-to-all versus random networks

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



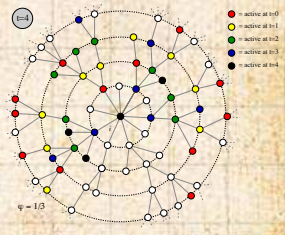


Cascade window—summary

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

For our simple model of a uniform threshold:

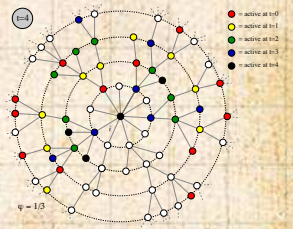
1. Low $\langle k \rangle$: No cascades in poorly connected networks.
   No global clusters of any kind.
2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists.
   Cascades are possible in "Cascade window."



Threshold contagion on random networks

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

- Next: Find expected fractional size of spread.
- Not obvious even for uniform threshold problem.
- Difficulty is in figuring out if and when nodes that need ≥ 2 hits switch on.
- Problem beautifully solved for infinite seed case by Gleeson and Cahalane:
  "Seed size strongly affects cascades on random networks," Phys. Rev. E, 2007. [14]
- Developed further by Gleeson in "Cascades on correlated and modular random networks," Phys. Rev. E, 2008. [13]

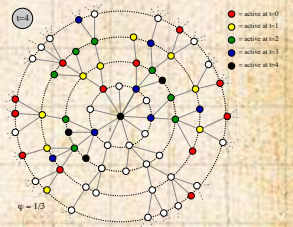


Determining expected size of spread:

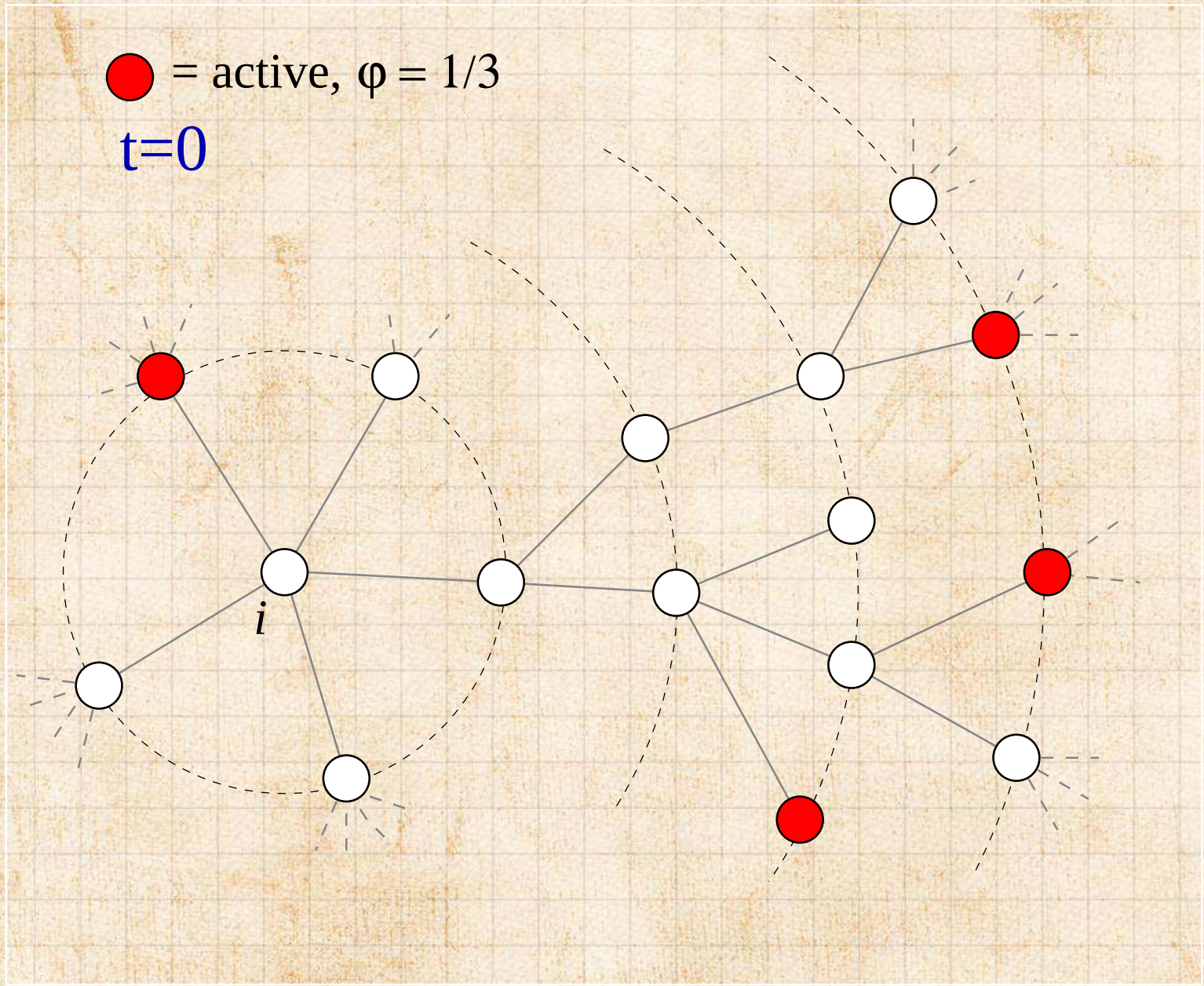
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

- Randomly turn on a fraction ϕ_0 of nodes at time $t = 0$
- Capitalize on local branching network structure of random networks (again)
- Now think about what must happen for a specific node i to become active at time t:

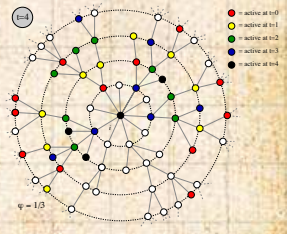
- $t = 0$: i is one of the seeds (prob = ϕ_0)
- $t = 1$: i was not a seed but enough of i's friends switched on at time $t = 0$ so that i's threshold is now exceeded.
- $t = 2$: enough of i's friends and friends-of-friends switched on at time $t = 0$ so that i's threshold is now exceeded.
- $t = n$: enough nodes within n hops of i switched on at $t = 0$ and their effects have propagated to reach i.



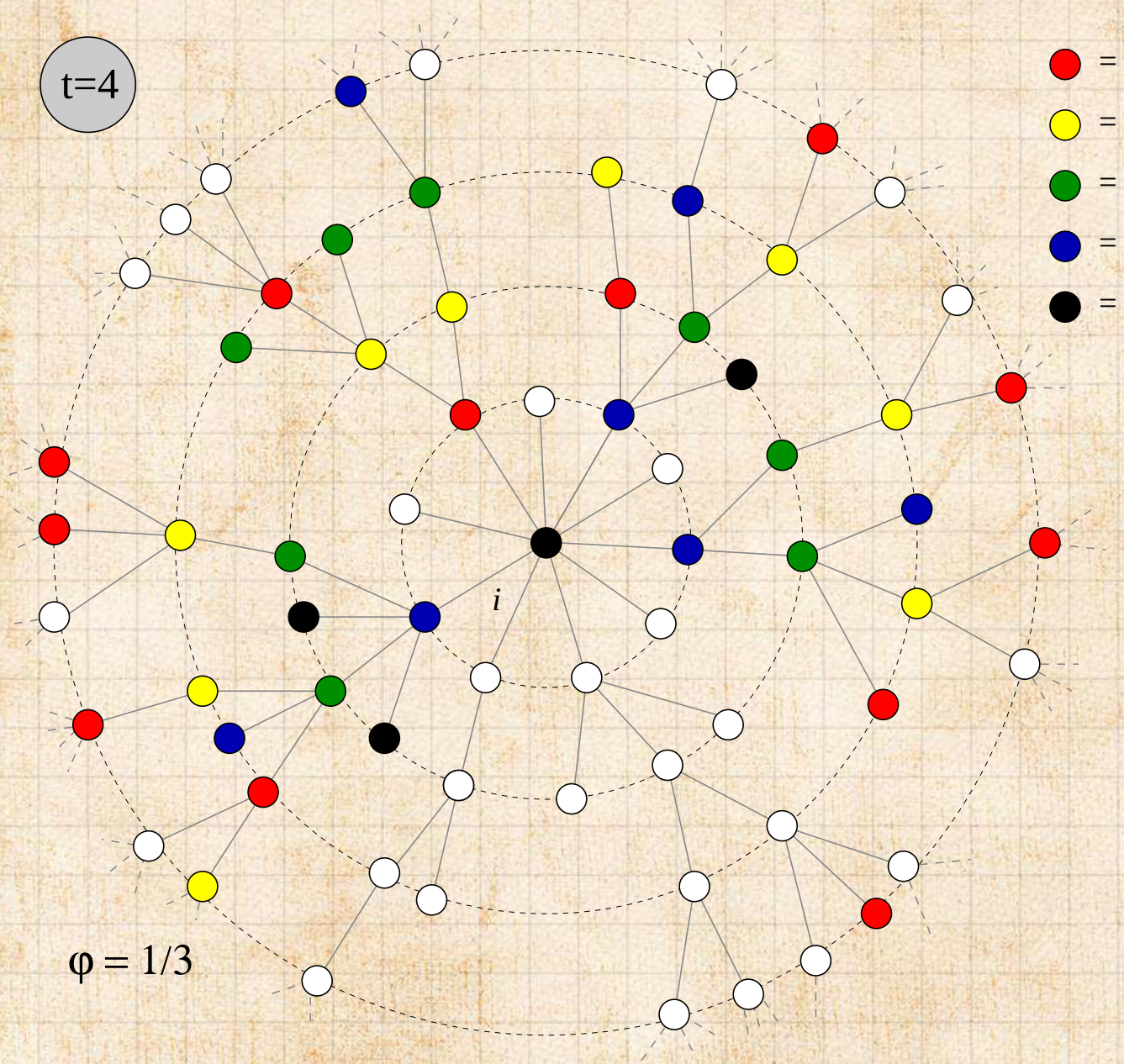
Expected size of spread



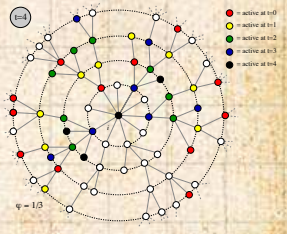
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Expected size of spread



Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

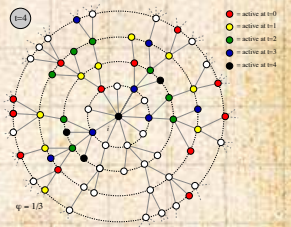


Expected size of spread

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Notes:

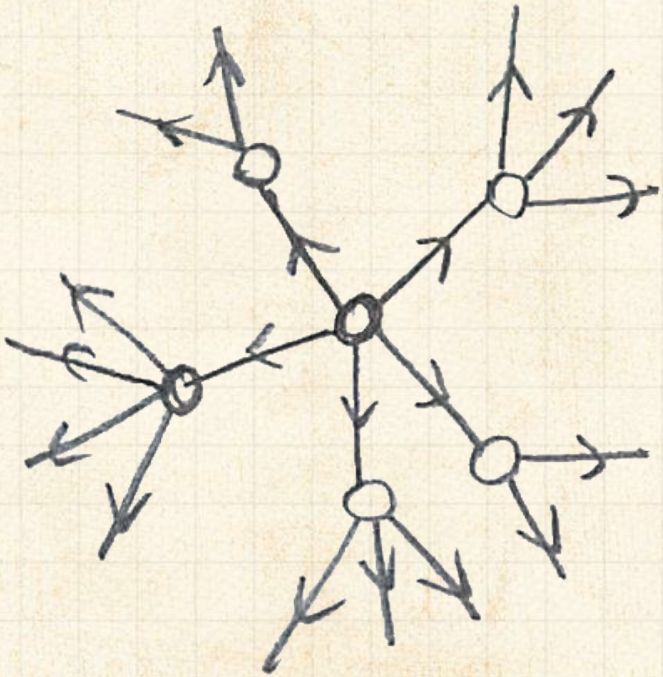
- Calculations are possible if nodes do not become inactive (strong restriction).
- Not just for threshold model—works for a wide range of contagion processes.
- We can analytically determine the entire time evolution, not just the final size.
- We can in fact determine $\mathbf{Pr}$(node of degree k switching on at time t).
- Asynchronous updating can be handled too.

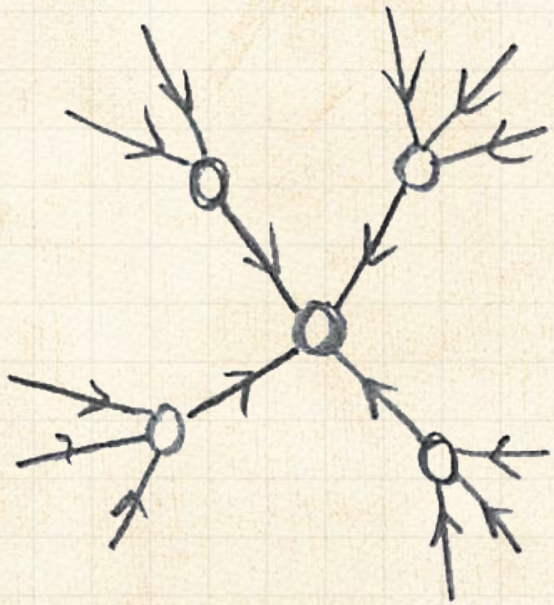


Expected size of spread

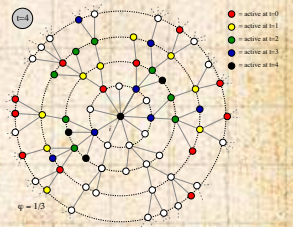
Pleasantness:

- Taking off from a single seed story is about expansion away from a node.
- Extent of spreading story is about contraction at a node.





Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

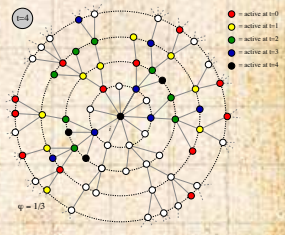


Expected size of spread

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

- Notation:
  $\phi_{k,t} = \mathbf{Pr}(\text{a degree } k \text{ node is active at time } t)$.
- Notation: $B_{kj} = \mathbf{Pr}$ (a degree k node becomes active if j neighbors are active).
- Our starting point: $\phi_{k,0} = \phi_0$.
- $\binom{k}{j}\phi_0^j(1-\phi_0)^{k-j}$ = $\mathbf{Pr}$ (j of a degree k node's neighbors were seeded at time $t = 0$).
- Probability a degree k node was a seed at $t = 0$ is ϕ_0 (as above).
- Probability a degree k node was not a seed at $t = 0$ is $(1-\phi_0)$.
- Combining everything, we have:

$$\phi_{k,1} = \phi_0 + (1-\phi_0)\sum_{j=0}^{k}\binom{k}{j}\phi_0^j(1-\phi_0)^{k-j}B_{kj}.$$



Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

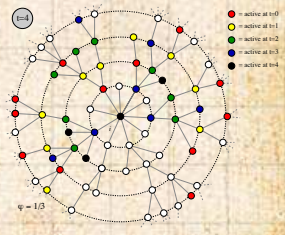
- For general t, we need to know the probability an edge coming into a degree k node at time t is active.
- Notation: call this probability θ_t.
- We already know $\theta_0 = \phi_0$.
- Story analogous to $t = 1$ case. For node i:

$$\phi_{i,t+1} = \phi_0 + (1 - \phi_0) \sum_{j=0}^{k_i} \binom{k_i}{j} \theta_t^j (1 - \theta_t)^{k_i - j} B_{k_i j}.$$

- Average over all nodes to obtain expression for ϕ_{t+1}:

$$\phi_{t+1} = \phi_0 + (1 - \phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} \binom{k}{j} \theta_t^j (1 - \theta_t)^{k - j} B_{kj}.$$

- So we need to compute θ_t... massive excitement...



Expected size of spread

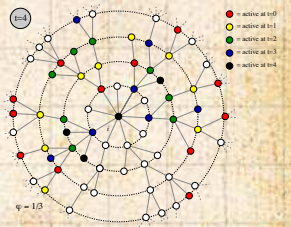
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

First connect θ_0 to θ_1:

- $\theta_1 = \phi_0 +$

$$(1-\phi_0)\sum_{k=1}^{\infty}\frac{kP_k}{\langle k\rangle}\sum_{j=0}^{k-1}\binom{k-1}{j}\theta_0^{\,j}(1-\theta_0)^{k-1-j}B_{kj}$$

- $\frac{kP_k}{\langle k\rangle} = R_k$ = $\mathbf{Pr}$(edge connects to a degree k node).
- $\sum_{j=0}^{k-1}$ piece gives $\mathbf{Pr}$(degree node k activates) of its neighbors $k-1$ incoming neighbors are active.
- ϕ_0 and $(1-\phi_0)$ terms account for state of node at time $t=0$.
- See this all generalizes to give θ_{t+1} in terms of θ_t...



Expected size of spread

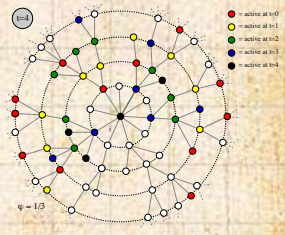
Two pieces: edges first, and then nodes

1. $\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1-\phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^j (1-\theta_t)^{k-1-j} B_{kj}}_{\text{social effects}}$

   with $\theta_0 = \phi_0$.

2. $\phi_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1-\phi_0) \underbrace{\sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} \binom{k}{j} \theta_t^j (1-\theta_t)^{k-j} B_{kj}}_{\text{social effects}}.$

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



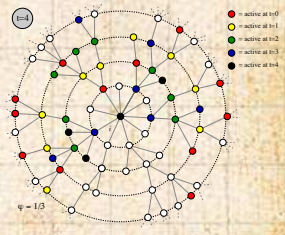
Expected size of spread

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Iterative map for θ_t is key:

$$\theta_{t+1} = \underbrace{\phi_0}_{\text{exogenous}} + (1-\phi_0) \underbrace{\sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} \theta_t^{\,j} (1-\theta_t)^{k-1-j} B_{kj}}_{\text{social effects}}$$

$$= G(\theta_t; \phi_0)$$



Expected size of spread:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

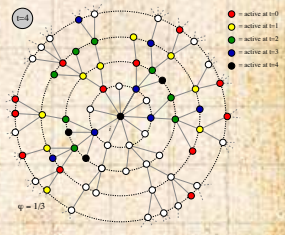
- Retrieve cascade condition for spreading from a single seed in limit $\phi_0 \to 0$.
- Depends on map $\theta_{t+1} = G(\theta_t; \phi_0)$.
- First: if self-starters are present, some activation is assured:

$$G(0; \phi_0) = \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet B_{k0} > 0.$$

meaning $B_{k0} > 0$ for at least one value of $k \geq 1$.

- If $\theta = 0$ is a fixed point of G (i.e., $G(0; \phi_0) = 0$) then spreading occurs if

$$G'(0; \phi_0) = \sum_{k=0}^{\infty} \frac{kP_k}{\langle k \rangle} \bullet (k-1) \bullet B_{k1} > 1.$$



Expected size of spread:

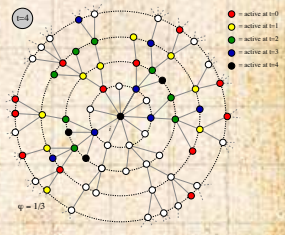
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

In words:

- If $G(0;\phi_0) > 0$, spreading must occur because some nodes turn on for free.
- If G has an unstable fixed point at $\theta = 0$, then cascades are also always possible.

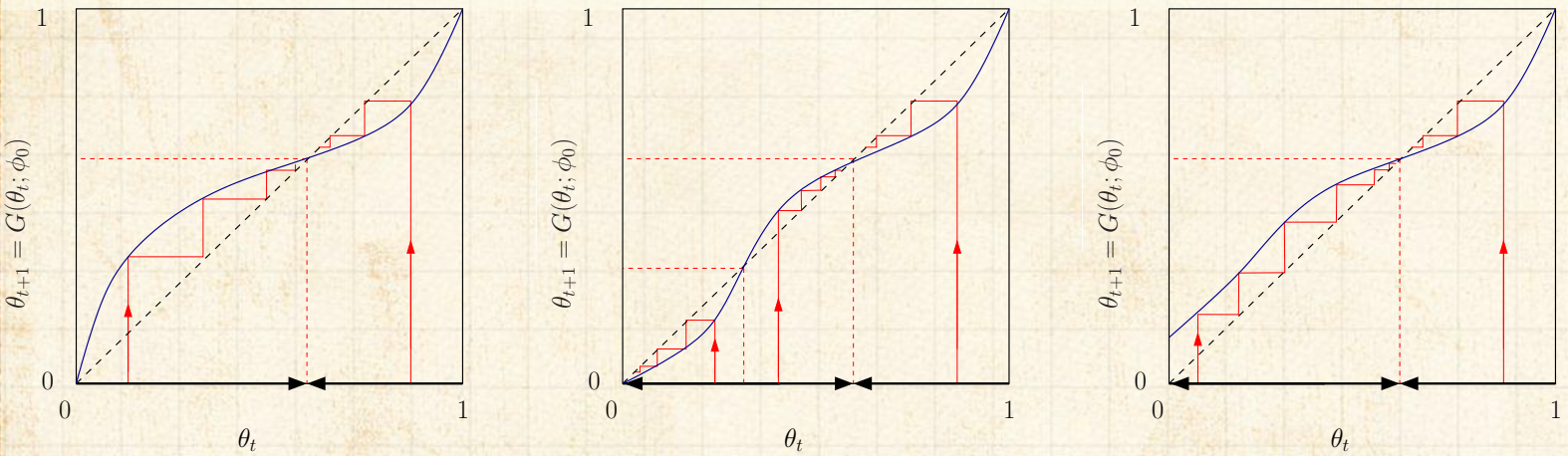
Non-vanishing seed case:

- Cascade condition is more complicated for $\phi_0 > 0$.
- If G has a stable fixed point at $\theta = 0$, and an unstable fixed point for some $0 < \theta_* < 1$, then for $\theta_0 > \theta_*$, spreading takes off.
- Tricky point: G depends on ϕ_0, so as we change ϕ_0, we also change G.
- A version of a critical mass model again.

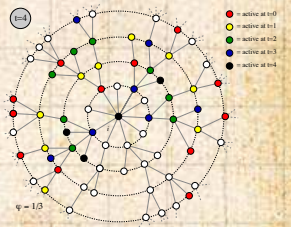


General fixed point story:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



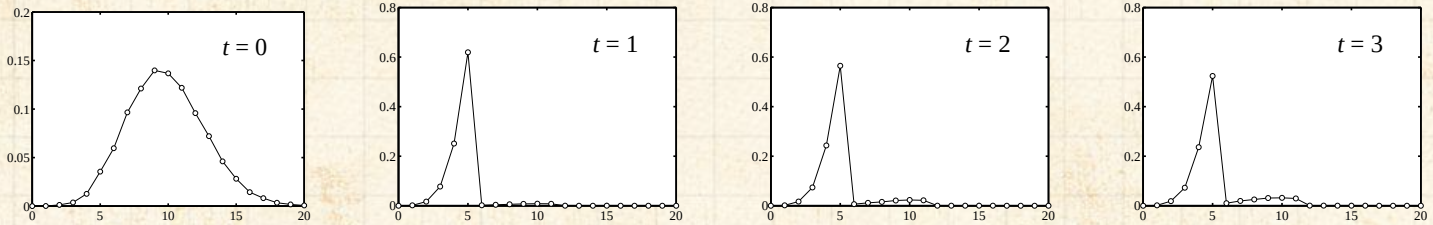
- Given $\theta_0 (= \phi_0)$, θ_∞ will be the nearest stable fixed point, either above or below.
- n.b., adjacent fixed points must have opposite stability types.
- Important: Actual form of G depends on ϕ_0.
- So choice of ϕ_0 dictates both G and starting point—can't start anywhere for a given G.



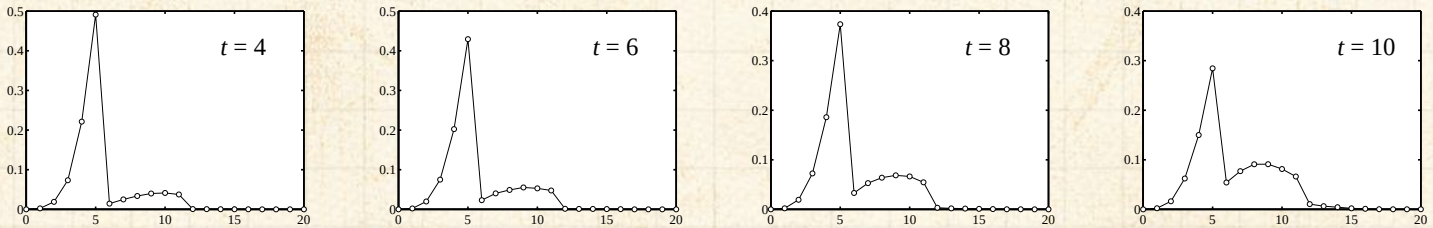
Early adopters—degree distributions

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

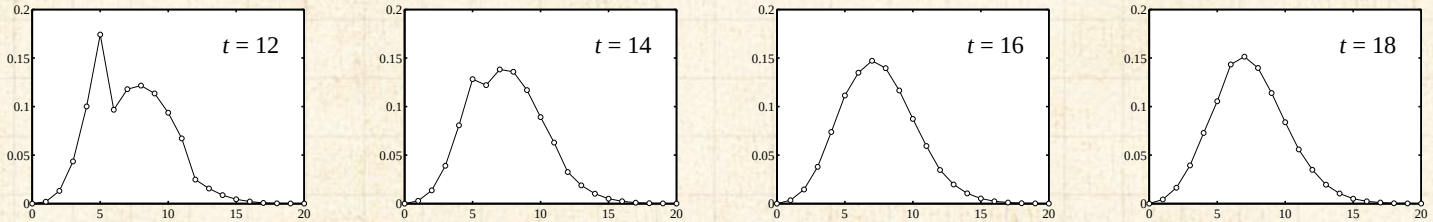
$t = 0$ $t = 1$ $t = 2$ $t = 3$



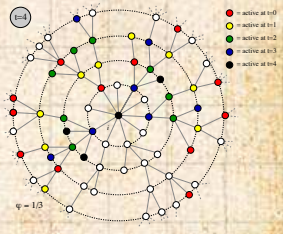
$t = 4$ $t = 6$ $t = 8$ $t = 10$



$t = 12$ $t = 14$ $t = 16$ $t = 18$

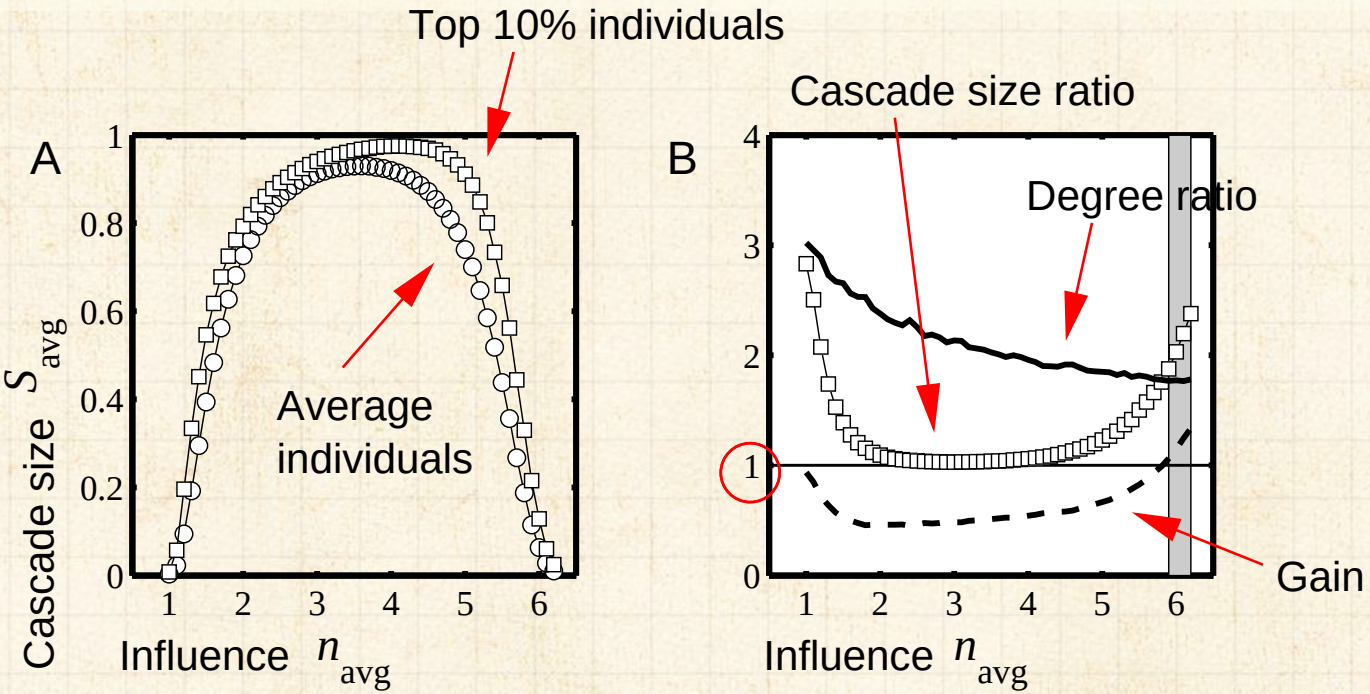


$P_{k,t}$ versus k

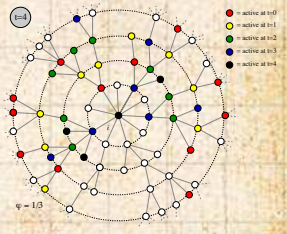


The multiplier effect:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

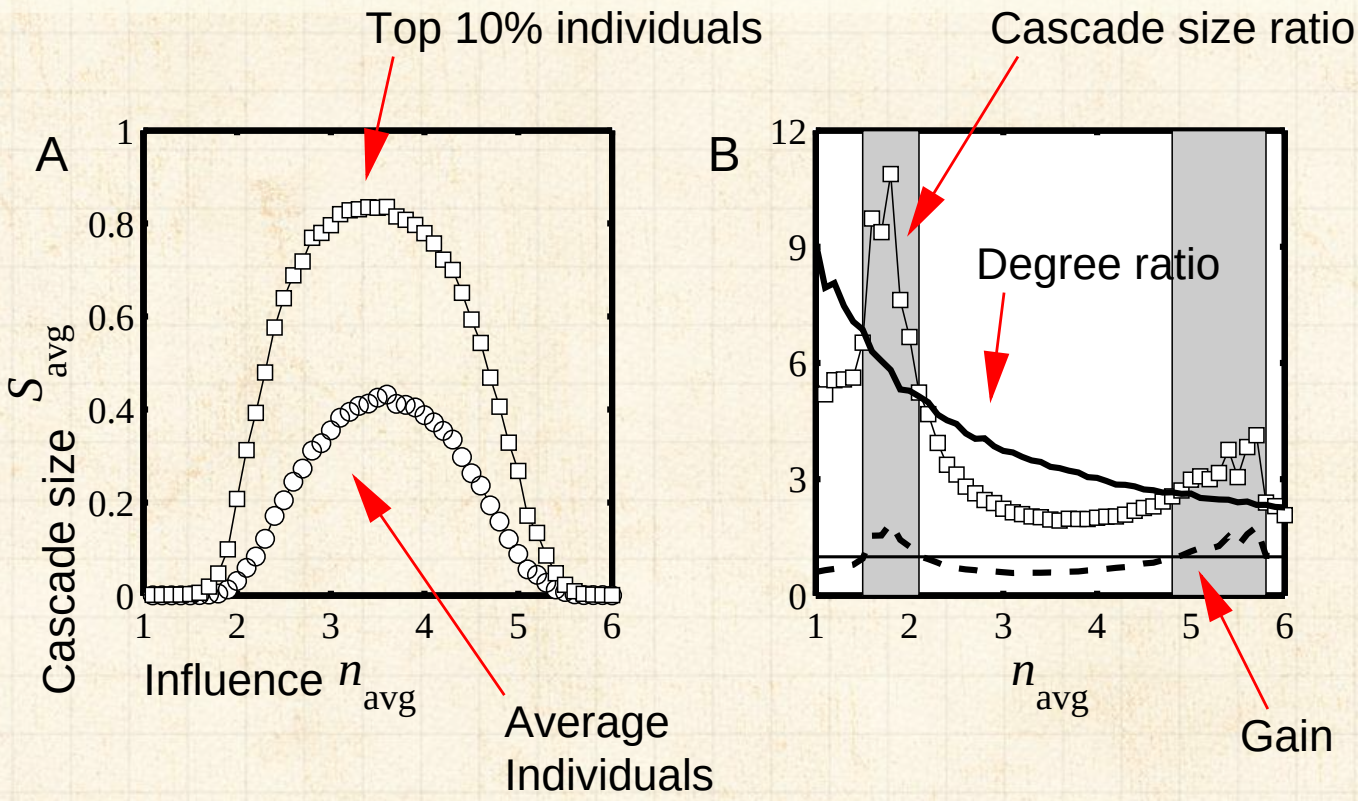


- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.

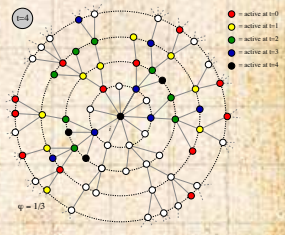


The multiplier effect:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

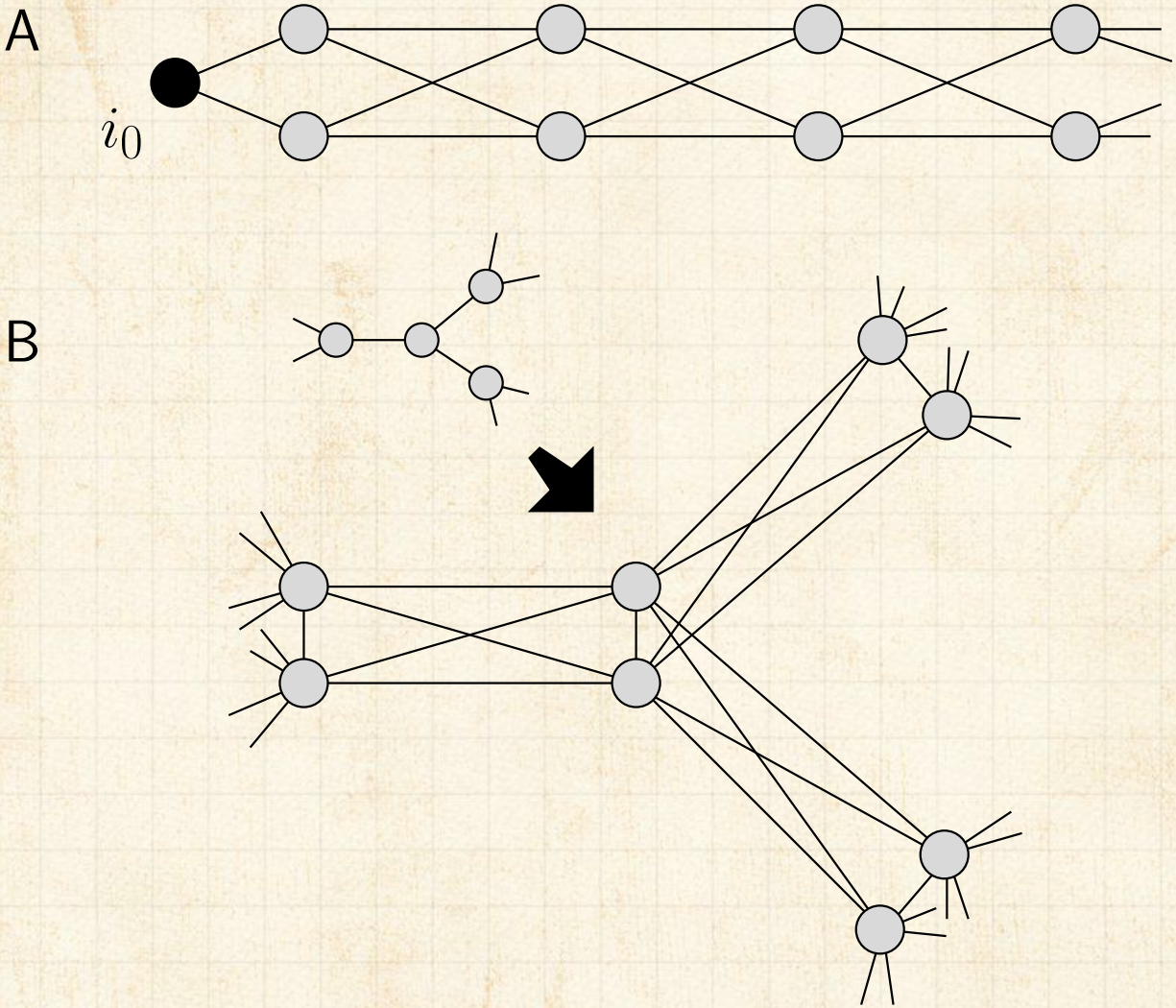


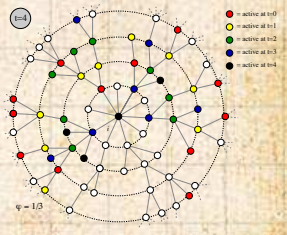
- Skewed influence distribution example.



Special subnetworks can act as triggers

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

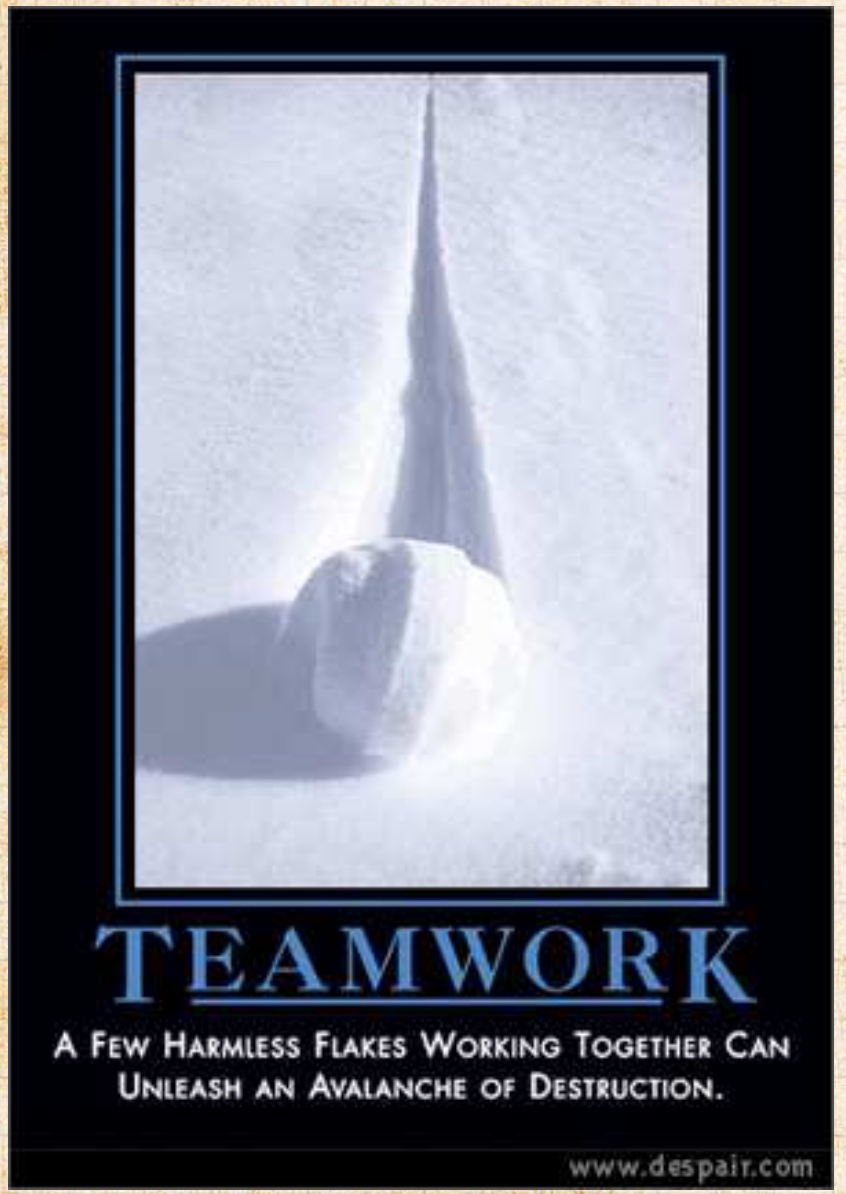




- $\phi = 1/3$ for all nodes

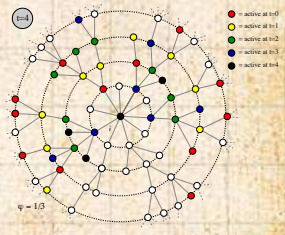
The power of groups...

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



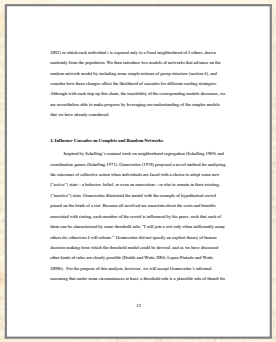
despair.com

"A few harmless flakes working together can unleash an avalanche of destruction."



Extensions

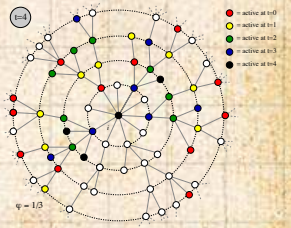
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



"Threshold Models of Social Influence"

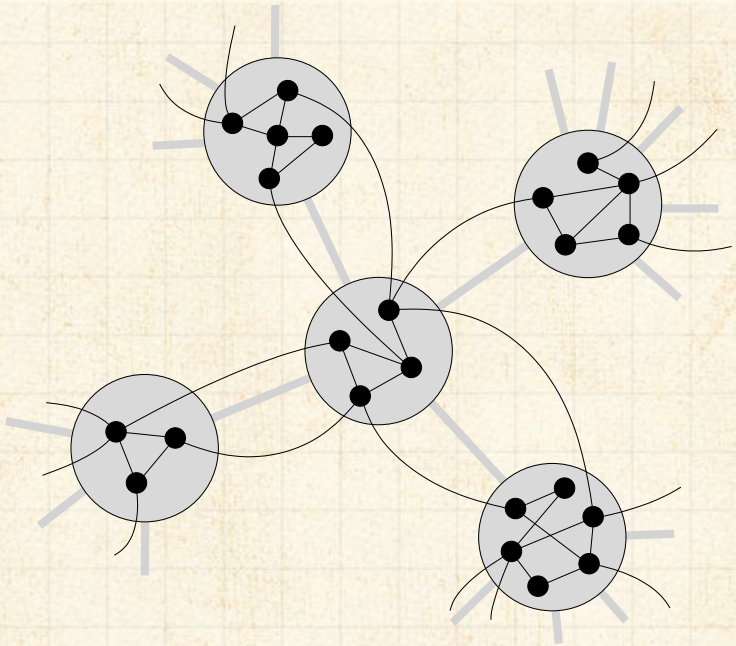
Watts and Dodds,
The Oxford Handbook of Analytical Sociology, , 475–497, 2009. [28]

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: group structure

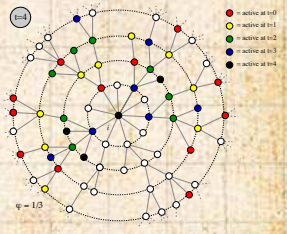


Group structure—Ramified random networks

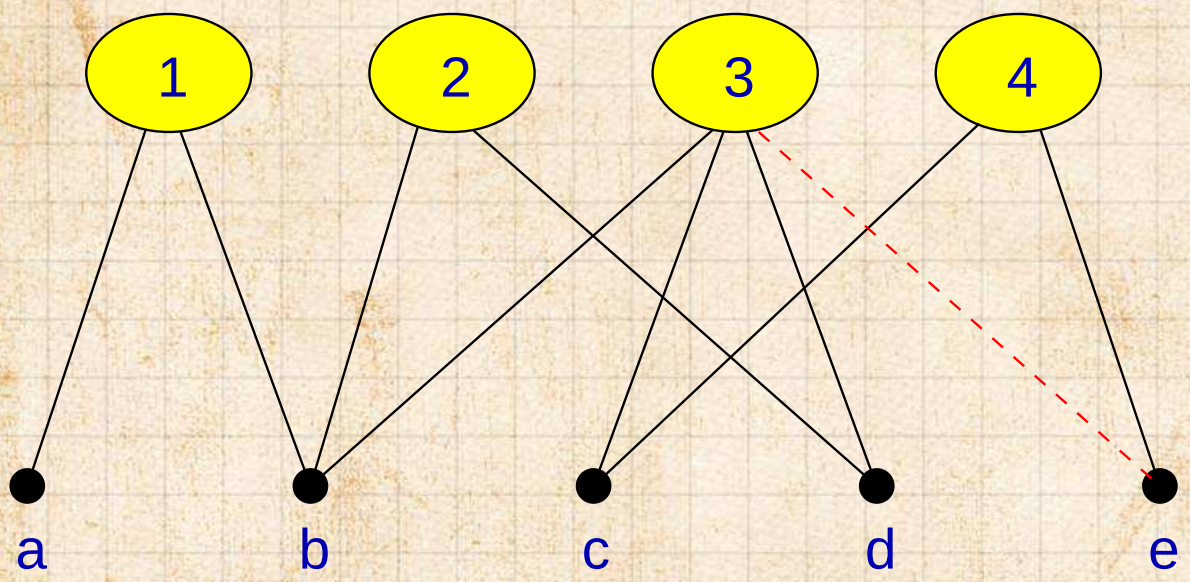
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



p = intergroup connection probability
q = intragroup connection probability.

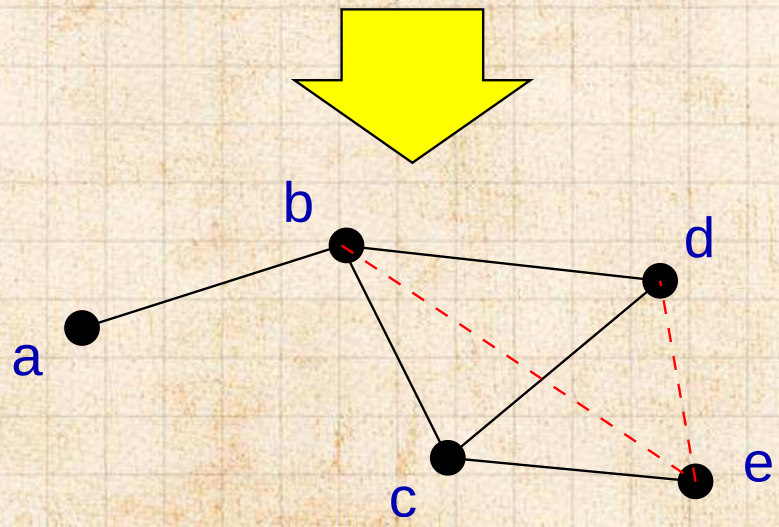


Bipartite networks



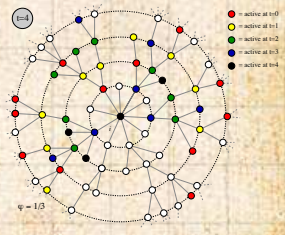
contexts

individuals



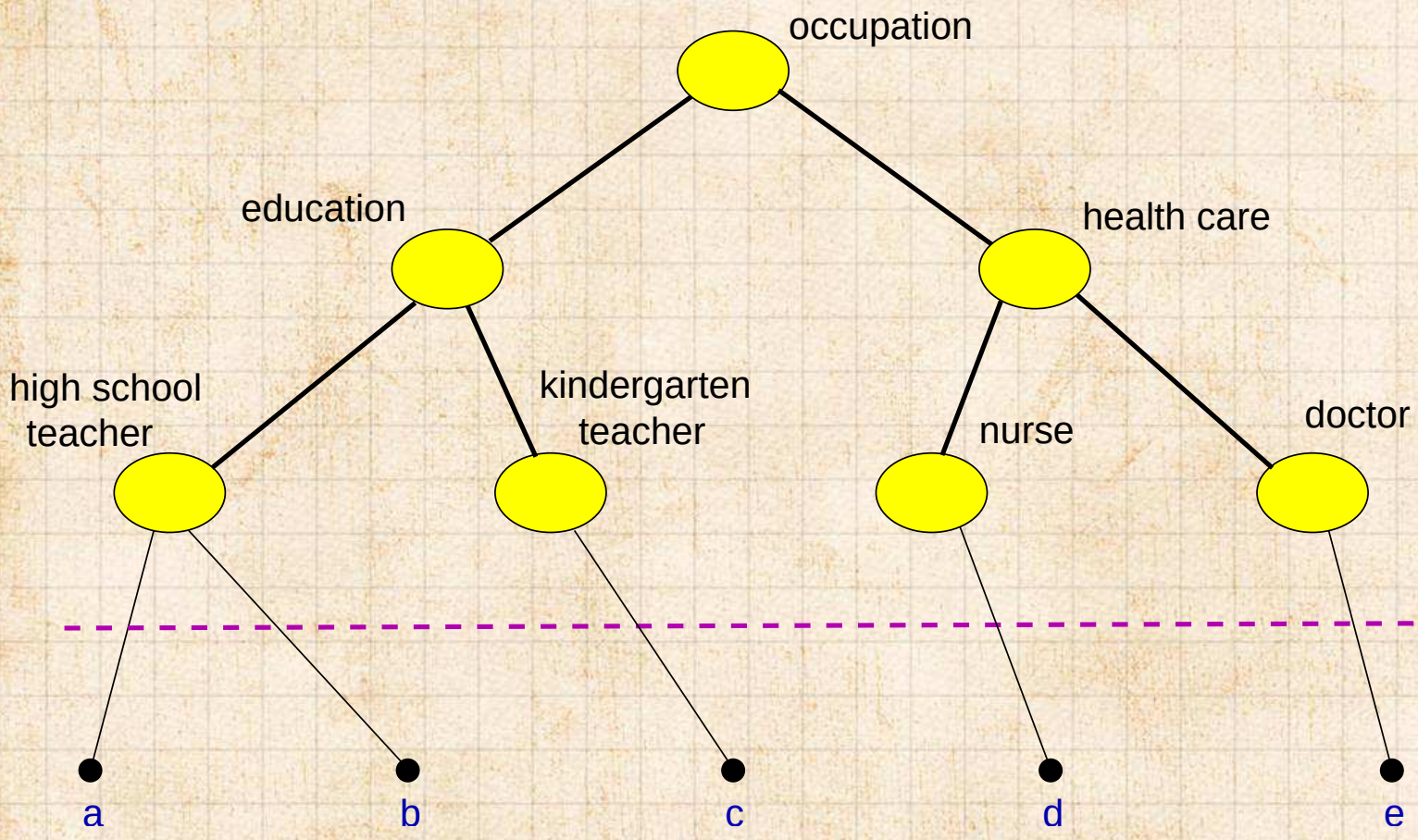
unipartite network

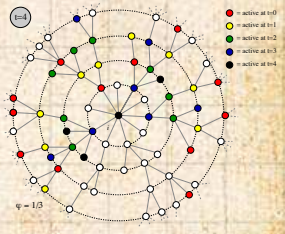
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



Context distance

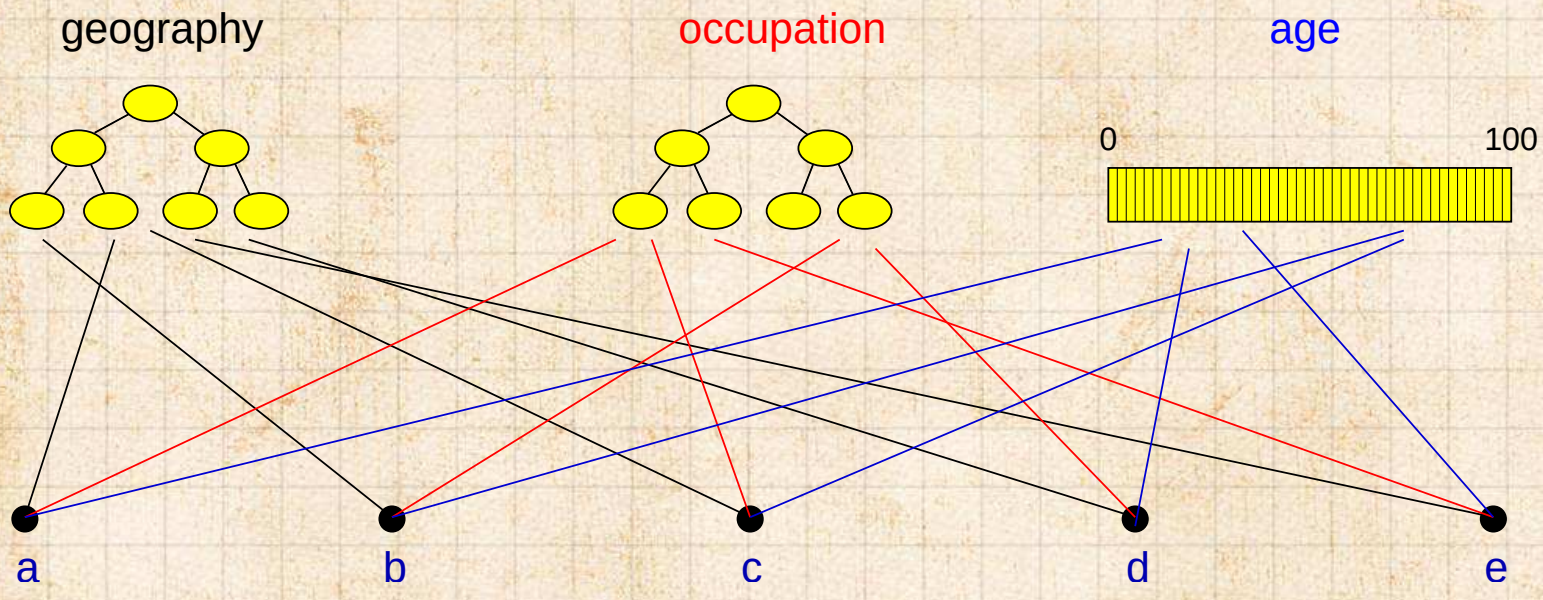
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



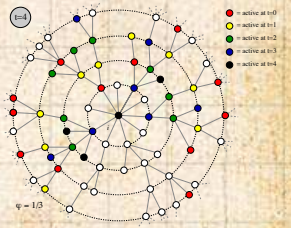


Generalized affiliation model

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



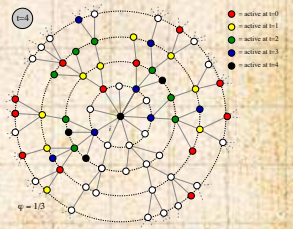
(Blau & Schwartz, Simmel, Breiger)



Generalized affiliation model networks with triadic closure

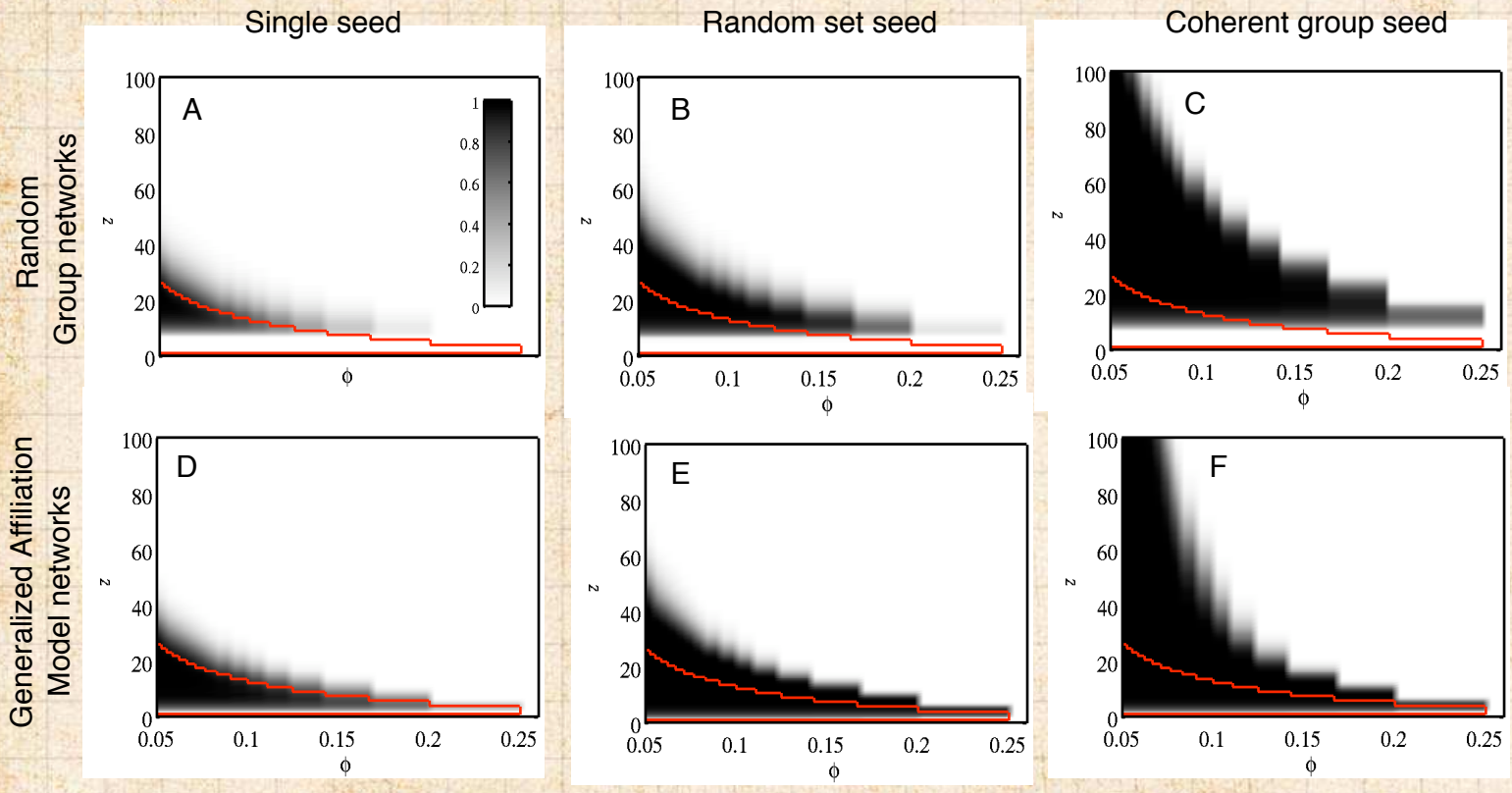
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

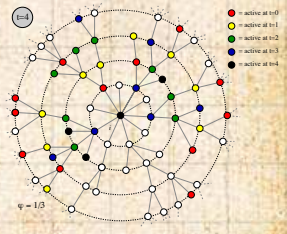
- Connect nodes with probability $\propto \exp^{-\alpha d}$ where
  α = homophily parameter
  and
  d = distance between nodes (height of lowest common ancestor)
- τ_1 = intergroup probability of friend-of-friend connection
- τ_2 = intragroup probability of friend-of-friend connection



Cascade windows for group-based networks

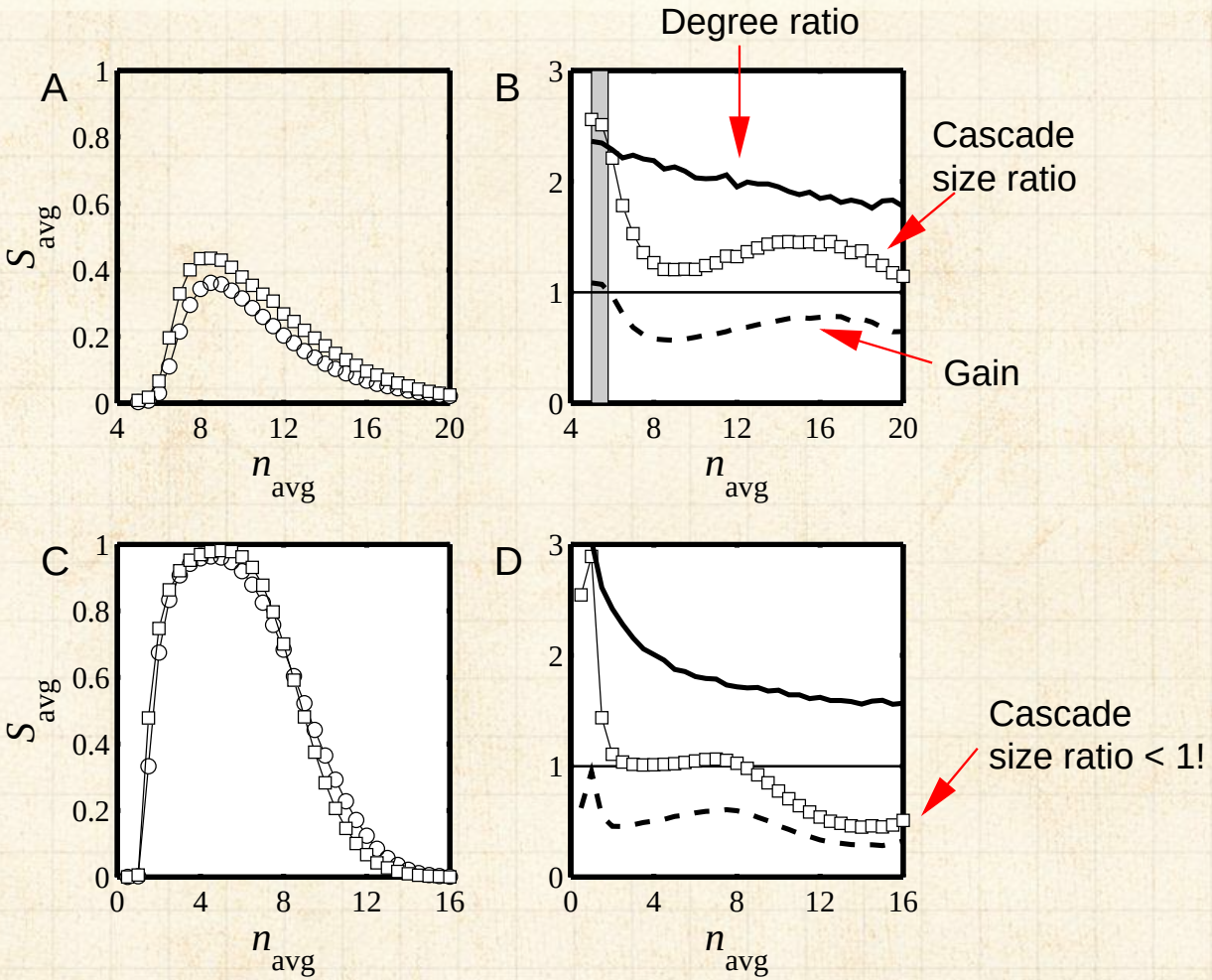
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



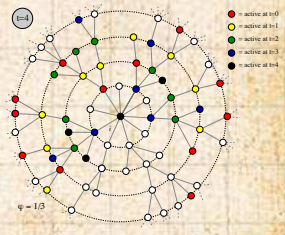


Multiplier effect for group-based networks:

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

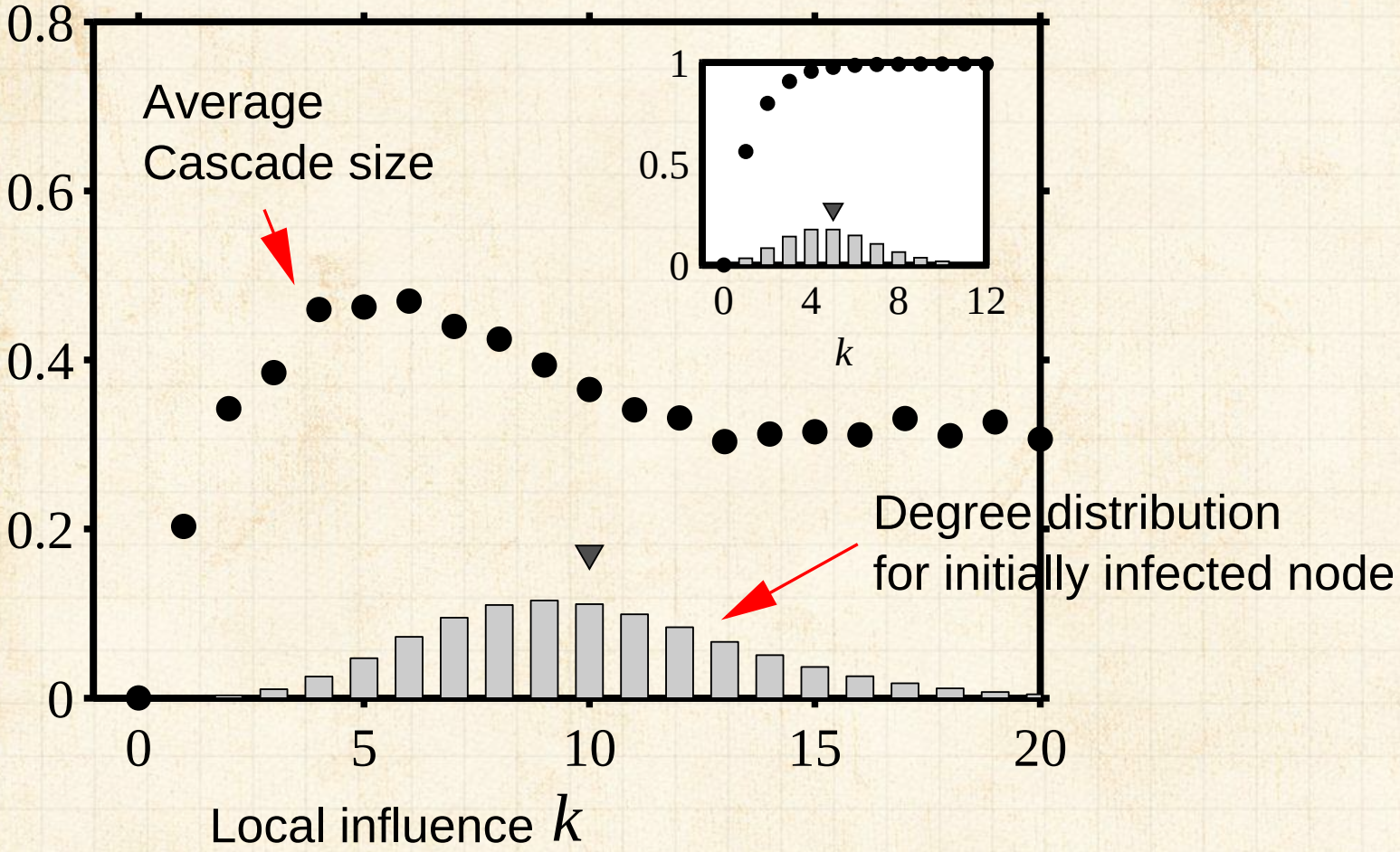


- Multiplier almost always below 1.

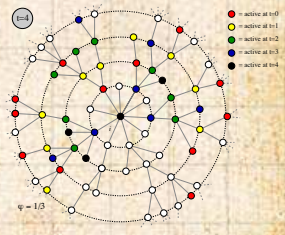


Assortativity in group-based networks

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



- The most connected nodes aren't always the most 'influential.'
- Degree assortativity is the reason.

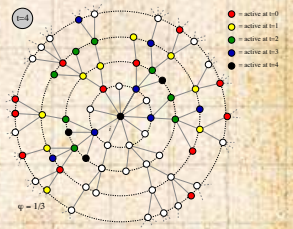


Social contagion

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.

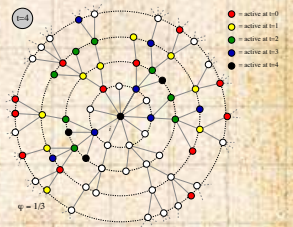


Social contagion

Implications

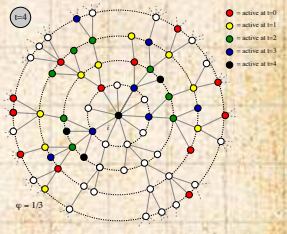
- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one ‘influential.’
- Only simple ideas can spread by word-of-mouth.
  (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo’s, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.

Social Contagion Models
Background
Granovetter’s model
Network version
Final size
Spreading success
Groups
References



Spreading and unspreading: Empires

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References



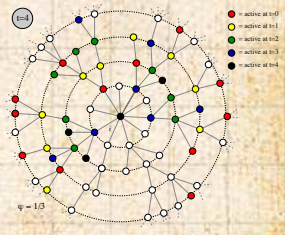
References I

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[1] A. Bentley, M. Earls, and M. J. O'Brien.
I'll Have What She's Having: Mapping Social Behavior.
MIT Press, Cambridge, MA, 2011.

[2] S. Bikhchandani, D. Hirshleifer, and I. Welch.
A theory of fads, fashion, custom, and cultural change as informational cascades.
J. Polit. Econ., 100:992–1026, 1992.

[3] S. Bikhchandani, D. Hirshleifer, and I. Welch.
Learning from the behavior of others: Conformity, fads, and informational cascades.
J. Econ. Perspect., 12(3):151–170, 1998. pdf



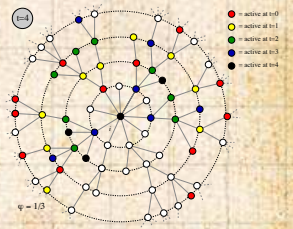
References II

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[4] J. M. Carlson and J. Doyle.
Highly optimized tolerance: A mechanism for power laws in designed systems.
Phys. Rev. E, 60(2):1412–1427, 1999. pdf

[5] J. M. Carlson and J. Doyle.
Highly optimized tolerance: Robustness and design in complex systems.
Phys. Rev. Lett., 84(11):2529–2532, 2000. pdf

[6] N. A. Christakis and J. H. Fowler.
The spread of obesity in a large social network over 32 years.
New England Journal of Medicine, 357:370–379, 2007. pdf



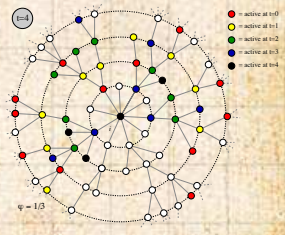
References III

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[7] N. A. Christakis and J. H. Fowler.
The collective dynamics of smoking in a large social network.
New England Journal of Medicine, 358:2249–2258, 2008. pdf

[8] R. B. Cialdini.
Influence: Science and Practice.
Allyn and Bacon, Boston, MA, 4th edition, 2000.

[9] P. S. Dodds, K. D. Harris, and C. M. Danforth.
Limited Imitation Contagion on random networks: Chaos, universality, and unpredictability.
Phys. Rev. Lett., 110:158701, 2013. pdf



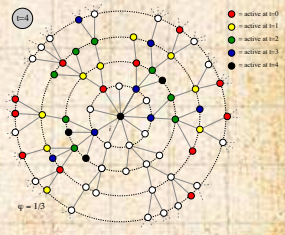
References IV

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[10] P. S. Dodds, K. D. Harris, and J. L. Payne.
Direct, phyiscally motivated derivation of the contagion condition for spreading processes on generalized random networks.
Phys. Rev. E, 83:056122, 2011. pdf

[11] J. H. Fowler and N. A. Christakis.
Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.
BMJ, 337:article #2338, 2008. pdf

[12] M. Gladwell.
The Tipping Point.
Little, Brown and Company, New York, 2000.



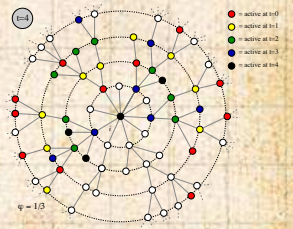
References V

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[13] J. P. Gleeson.
Cascades on correlated and modular random networks.
Phys. Rev. E, 77:046117, 2008. pdf

[14] J. P. Gleeson and D. J. Cahalane.
Seed size strongly affects cascades on random networks.
Phys. Rev. E, 75:056103, 2007. pdf

[15] M. Granovetter.
Threshold models of collective behavior.
Am. J. Sociol., 83(6):1420–1443, 1978. pdf



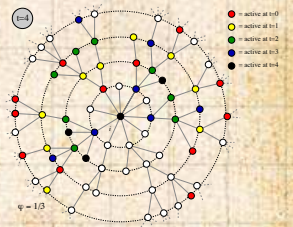
References VI

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[16] M. Granovetter and R. Soong.
Threshold models of diversity: Chinese restaurants, residential segregation, and the spiral of silence.
Sociological Methodology, 18:69–104, 1988. pdf

[17] M. S. Granovetter and R. Soong.
Threshold models of interpersonal effects in consumer demand.
J. Econ. Behav. Organ., 7:83–99, 1986. pdf

[18] K. D. Harris, C. M. Danforth, and P. S. Dodds.
Dynamical influence processes on networks: General theory and applications to social contagion.
Phys. Rev. E, 88:022816, 2013. pdf



References VII

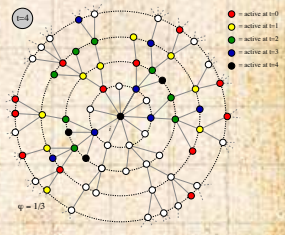
Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[19] E. Katz and P. F. Lazarsfeld.
Personal Influence.
The Free Press, New York, 1955.

[20] T. Kuran.
Now out of never: The element of surprise in the east european revolution of 1989.
World Politics, 44:7–48, 1991. pdf

[21] T. Kuran.
Private Truths, Public Lies: The Social Consequences of Preference Falsification.
Harvard University Press, Cambridge, MA, Reprint edition, 1997.

[22] T. C. Schelling.
Dynamic models of segregation.
J. Math. Sociol., 1:143–186, 1971. pdf



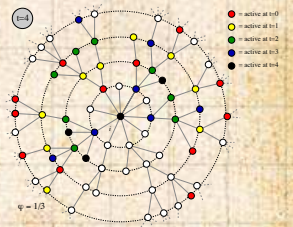
References VIII

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[23] T. C. Schelling.
Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities.
J. Conflict Resolut., 17:381–428, 1973. pdf

[24] T. C. Schelling.
Micromotives and Macrobehavior.
Norton, New York, 1978.

[25] D. Sornette.
Critical Phenomena in Natural Sciences.
Springer-Verlag, Berlin, 1st edition, 2003.



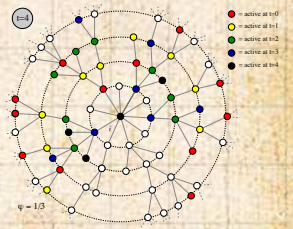
References IX

Social Contagion Models
Background
Granovetter's model
Network version
Final size
Spreading success
Groups
References

[26] D. J. Watts.
A simple model of global cascades on random networks.
Proc. Natl. Acad. Sci., 99(9):5766–5771, 2002. pdf

[27] D. J. Watts and P. S. Dodds.
Influentials, networks, and public opinion formation.
Journal of Consumer Research, 34:441–458, 2007. pdf

[28] D. J. Watts and P. S. Dodds.
Threshold models of social influence.
In P. Hedström and P. Bearman, editors, The Oxford Handbook of Analytical Sociology, chapter 20, pages 475–497. Oxford University Press, Oxford, UK, 2009. pdf



The UNIVERSITY of VERMONT

References X

[29] U. Wilensky.
Netlogo segregation model.
http://ccl.northwestern.edu/netlogo/models/Segregation.
Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL., 1998.