System Robustness

Principles of Complex Systems | @pocsvox CSYS/MATH 300, Fall, 2015 | #FallPoCS2015

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System Robustness

Robustness

HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness







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Robustness

HOT theory Narrative causality Self-Organized Criticality Network robustness







Outline

Robustness

HOT theory
Narrative causality
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Self-Organized Criticality
COLD theory
Network robustness

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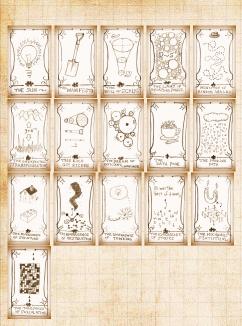
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Random forests
Self-Organized Criticality
COLD theory

Network robustness
References









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COLD theory
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THE THRESHOLDS OF PERCOLATION



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Outline

Robustness **HOT** theory

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System Robustness

Robustness

HOT theory

Narrative causality Self-Organized Criticality Network robustness







- Many complex systems are prone to cascading catastrophic failure:
 - Blackouts
 - ► Disease outbreak
 - ▶ Wildfires
 - ▶ Earthquakes
- But complex systems also show persisten robustness
- Robustness and Failure may be a power-law story...

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System Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness





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System Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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System Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory
Network robustness





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Robustness HOT theory

Narrative causality Random forests Self-Organized Criticality COLD theory

Network robustness





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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness





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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory

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Robustness HOT theory Narrative causality

Random forests
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COLD theory
Network robustness

References

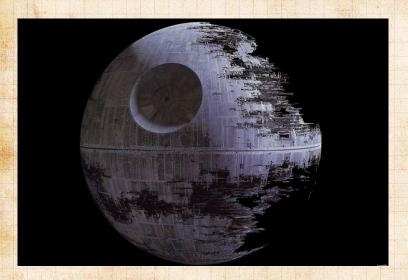
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Our emblem of Robust-Yet-Fragile:



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"Trouble ..."

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Narrative causality
Random forests
Self-Organized Criticality
COLD theory
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System robustness may result from

HOT theory

Narrative causality

Random forests

Self-Organized Criticality

COLD theory
Network robustness
References

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- Idea: Explore systems optimized to perform under uncertain conditions.
- ► The handle: 'Highly Optimized Tolerance' (HOT)
- ► The catchphrase: Robust yet Fragile
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- Great abstracts of the world #73: "There aren't any."





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1. Evolutionary processes

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Self-Organized Criticality
COLD theory
Network robustness

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Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Self-Organized Criticality
COLD theory
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COLD theory
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COLD theory
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Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness

References





20 9 of 43

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COLD theory
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Features of HOT systems: [5, 6]

- High performance and robustness
- Designed/evolved to handle known stochastic environmental variability
- Fragile in the face of unpredicted environmental signals
- Highly specialized, low entropy configurations
- Power-law distributions appear (of course...)

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HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness







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Random forests
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COLD theory
Network robustness







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Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Random forests
Self-Organized Criticality
COLD theory
Network robustness





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HOT combines things we've seen:

- ▶ Variable transformation
- Constrained optimization
- Need power law transformation between variables: (Y = X + Y)
- ▶ Recall PLIPLO is bad...
- ► MIWO is good
- X has a characteristic size but Y does no

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Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness







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Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness







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Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Narrative causality
Random forests
Self-Organized Critical
COLD theory
Network robustness





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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Critical
COLD theory
Network robustness





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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Critical
COLD theory
Network robustness





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Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





Forest fire example: [5]

- ightharpoonup Square $N \times N$ grid
- ▶ Sites contain a tree with probability ρ = density
- Sites are empty with probability
- ▶ Fires start at location (i, j) according to some distribution P_i;
- Fires spread from tree to tree (nearest neighbor only)
- Connected clusters of trees burn completely
- Empty sites block fire
- Best case scenario:
 Build firebreaks to maximize average # trees left intact give none spark

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Robustness

HOT theory Narrative causality

Random forests
Self-Organized Criticality
COLD theory
Network robustness





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System Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Robustness HOT theory

Narrative causality
Random forests
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COLD theory
Network robustness





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Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Robustness HOT theory Narrative causality

Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Robustness

Robustness HOT theory Narrative causality

Random forests
Self-Organized Criticality
COLD theory
Network robustness





Forest fire example: [5]

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Forest fire example: [5]

- ▶ Build a forest by adding one tree at a time

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Forest fire example: [5]

- Build a forest by adding one tree at a time
- ► Test *D* ways of adding one tree
- ▶ D = design parameter
- \triangleright Average over P_{ij} = spark probability
- $\rightarrow D = 1$; random addition
- $D = N^2$: test all possibilities

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Robustness
HOT theory
Narrative causality

Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Robustness

HOT theory Narrative causality

Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Network robustness





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Measure average area of forest left untouched

- $\blacktriangleright f(c)$ = distribution of fire sizes c (= cost)
- Yield = $Y = \rho + \langle c \rangle$

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COLD theory

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Specifics:

•

$$P_{ij} = P_{i;a_x,b_x} P_{j;a_y,b_y}$$

where

$$P_{i;a,b} \propto e^{-[(i+a)/b]^2}$$

- In the original work, $b_y > b_x$
- ▶ Distribution has more width in *y* direction.

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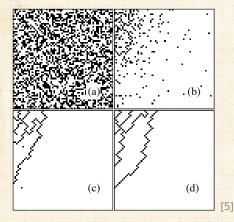
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$$N = 64$$

- (a) D = 1
- (b) D = 2
- (c) D=N
- (d) $D = N^2$

 P_{ij} has a Gaussian decay

- Optimized forests do well on average
- ▶ But rare extreme events occur

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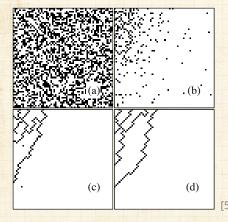
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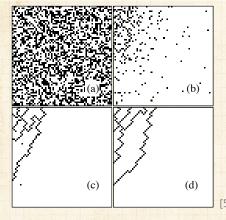
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Random forests
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- (b) D = 2
- (c) D = N
- $(d) D = N^2$

 P_{ij} has a Gaussian decay

- Optimized forests do well on average
- But rare extreme events occur

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Robustness HOT theory

Narrative causality

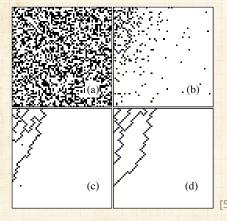
Random forests
Self-Organized Criticality

Network robustness









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Robustness HOT theory

Narrative causality

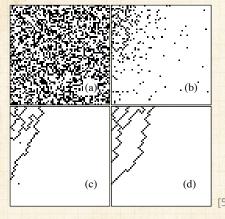
Random forests Self-Organized Criticality COLD theory

Network robustness









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Robustness **HOT** theory

Narrative causality Self-Organized Criticality COLD theory Network robustness







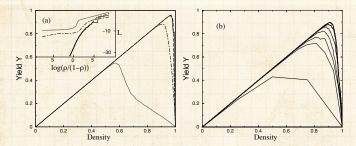


FIG. 2. Yield vs density $Y(\rho)$: (a) for design parameters D =1 (dotted curve), 2 (dot-dashed), N (long dashed), and N^2 (solid) with N = 64, and (b) for D = 2 and $N = 2, 2^2, ..., 2^7$ running from the bottom to top curve. The results have been averaged over 100 runs. The inset to (a) illustrates corresponding loss functions $L = \log[\langle f \rangle/(1 - \langle f \rangle)]$, on a scale which more clearly differentiates between the curves.

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System Robustness

Robustness **HOT** theory

Narrative causality Self-Organized Criticality COLD theory

Network robustness References







Y = 'the average density of trees left unburned in a configuration after a single spark hits.' [5]

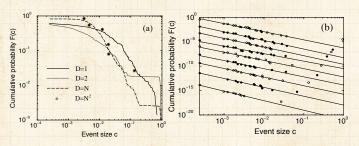


FIG. 3. Cumulative distributions of events F(c): (a) at peak yield for D = 1, 2, N, and N^2 with N = 64, and (b) for D = N^2 , and N = 64 at equal density increments of 0.1, ranging at $\rho = 0.1$ (bottom curve) to $\rho = 0.9$ (top curve).

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Narrative causality Self-Organized Criticality COLD theory







Outline

Robustness

Narrative causality

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System Robustness

Robustness HOT theory

Narrative causality Self-Organized Criticality COLD theory Network robustness







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System Robustness

Robustness HOT theory

Narrative causality
Random forests

Self-Organized Criticality
COLD theory
Network robustness







Outline

Robustness

Random forests

PoCS | @pocsvox System

Robustness

Robustness HOT theory

Narrative causality Random forests

Self-Organized Criticality COLD theory

Network robustness







D=1: Random forests = Percolation [11]

- Randomly add trees.
- \blacktriangleright Below critical density ρ_c , no fires take off.
- Above critical density ρ_c , percolating cluster of trees burns.
- Only at ρ_a, the critical density, is there a power-law distribution of tree cluster sizes.
- Forest is random and featureless.

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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness







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Robustness

Narrative causality
Random forests

Self-Organized Criticality COLD theory Network robustness





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Robustness

HOT theory

Narrative causality

Random forests

Self-Organized Criticality

COLD theory

Network robustness





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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness





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System
Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness





Highly structured

- Power law distribution of tree cluster sizes for $\rho > \rho_e$
- \triangleright No specialness of ρ_c
- ▶ Forest states are tolerant
- Uncertainty is okay if well characterized
- ▶ If P_{ij} is characterized poorly, failure becomes highly likely

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Robustness

HOT theory
Narrative causality

Random forests
Self-Organized Criticality

COLD theory

Network robustness





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PoCS | @pocsvox System Robustness

Robustness

HOT theory

Narrative causality Random forests

Self-Organized Criticality COLD theory

Network robustness





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System
Robustness

Robustness

HOT theory

Narrative causality

Random forests

Self-Organized Criticality

COLD theory

Network robustness





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System
Robustness

Robustness

HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness





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Robustness HOT theory

Narrative causality
Random forests

Self-Organized Criticality COLD theory

Network robustness





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System
Robustness

Robustness HOT theory

Narrative causality
Random forests

Self-Organized Criticality COLD theory Network robustness





HOT forests—Real data:

"Complexity and Robustness," Carlson & Dolye [6]

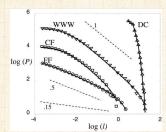


Fig. 1. Log-log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for $\beta = 0.0.9, 0.9, 1.85$, or $\alpha = 1/\beta = \infty, 1.1, 1.1, 0.054$ respectively) and the SOCFF model ($\alpha = 0.15$, dashed). Reference lines of $\alpha = 0.5$, 1 (dashed) are included. The cumulative distributions of frequencies (P(I ≥ I)) vs. Ii describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the >10,000 largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km2 (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.

- ► PLR = probability-lossresource.
- Minimize cost subject to resource (barrier) constraints:

$$\begin{split} C &= \sum_i p_i l_i \\ \text{given} \\ l_i &= f(r_i) \text{ and } \sum r_i \leq R. \end{split}$$

▶ DC = Data Compression.

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System Robustness

Robustness HOT theory

Narrative causality Random forests Self-Organized Criticality

COLD theory Network robustness References







HOT forests—Real data:

"Complexity and Robustness," Carlson & Dolye [6]

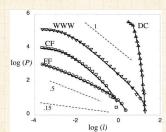


Fig. 1. Log-log (base 10) comparison of DC, WWW, Cf, and Ff data (symbol) with PR. Models (odd line) (for g = 0, 0.9, 0.18, 5c, or a = 1/B = 9, 1.11, 1.026, respectively) and the SOC FF model (a = 0.15, dashed). Reference lines of a = 0.5, I clashed are included. The cumulatte distributions of Fepuerics 97 $f \ell = 1/M$, describe the areas burned in the largest 4,284 frees from 1386 to 1995 on all of the U.S. Fish and Wildlife Service Lands (Ff) (T), the a = 1.000 Largest California brushfrees from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW), and bytes (CoT) and the logarithmic describation of the data are chosen for visualization of the data are chosen for visualization.

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- DC = Data Compression.
- Horror: log. Screaming: "The base! What is the base!? You monsters!"

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System Robustness

Robustness
HOT theory
Narrative causality

Random forests
Self-Organized Criticality
COLD theory
Network robustness







The abstract story, using figurative forest fires:

- Given some measure of failure size y_i and correlated resource size x_i , with relationship $y_i = x_i^{-\alpha}, i = 1, ..., N_{\text{sites}}$.
- Design system to minimize $\langle y \rangle$ subject to a constraint on the x_i .
- ▶ Minimize cost.

$$C = \sum_{i=1}^{N_{\text{sites}}} Pr(y_i) y$$

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Robustness
HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness
References





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System
Robustness

Robustness
HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness References





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$$C = \sum_{i=1}^{N_{\rm sites}} Pr(y_i) y_i$$

Subject to $\sum_{i=1}^{N_{\text{sites}}} w_i = \text{constant}$

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Robustness
HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness







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Robustness
HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness







$$C_{ ext{fire}} \propto \sum_{i=1}^{N_{ ext{sites}}} p_i a_i.$$

 a_i = area of ith site's region, and p_i = avg. prob. of fire at ith site over some time frame.

Constraint: building and maintaining firewall

$$C_{ ext{firewalls}} imes \sum_{i=1}^{N_{ ext{sites}}} a_i^{1/2} a_i^{-1/2}$$

assignment at to find:

 $\Pr(a_i) \propto a_i^{\gamma_i}$

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System

System Robustness

Robustness

HOT theory

Narrative causality
Random forests

Self-Organized Criticality
COLD theory

Network robustness





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2. Constraint: building and maintaining firewalls. Per unit area, and over same time frame:

$$C_{ ext{firewalls}} \propto \sum_{i=1}^{N_{ ext{sites}}} a_i^{1/2} a_i^{-1}.$$

- ▶ We are assuming isometry.
- ▶ In d dimensions, 1/2 is replaced by (d-1)/d

R. Wishtigue Son from assignments & to find

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System

Robustness

Robustness

HOT theory
Narrative causality

Random forests

Self-Organized Criticality
COLD theory

Network robustness





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System

Robustness

Robustness

HOT theory
Narrative causality

Random forests

Self-Organized Criticality
COLD theory

Network robustness





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PoCS | @pocsvox System

System Robustness

Robustness

HOT theory

Narrative causality

Random forests

Self-Organized Criticality

COLD theory

Network robustness





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System Robustness

Robustness
HOT theory
Narrative causality

Random forests
Self-Organized Criticality
COLD theory

Network robustness





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- ▶ We are assuming isometry.
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System
Robustness

Robustness
HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness





1. Cost function:

$$\langle C \rangle = \int C(\vec{x}) p(\vec{x}) \mathrm{d}\vec{x}$$

where C is some cost to be evaluated at each point in space \vec{x} (e.g., $V(\vec{x})^{\alpha}$), and $p(\vec{x})$ is the probability an Ewok jabs position \vec{x} with a sharpened stick (or equivalent).

2. Constraint:

$$\int R(\vec{x})\mathsf{d}(\vec{x}) = \mathsf{d}(\vec{x})$$

where c is a constant

► Claim/observation in is that typically

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System

Robustness

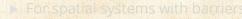
Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory
Network robustness









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System
Robustness

Robustness

HOT theory

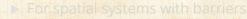
Narrative causality

Random forests
Self-Organized Criticality

COLD theory
Network robustness









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System

Robustness

Robustness

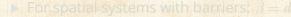
Narrative causality
Random forests

Self-Organized Criticality

Network robustness









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System
Robustness

Robustness

HOT theory
Narrative causality
Random forests

Self-Organized Criticality COLD theory Network robustness





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▶ For spatial systems with barriers: $\beta = d$.

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System
Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness







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System Robustness

Robustness

HOT theory Narrative causality

Random forests

Self-Organized Criticality Network robustness







Outline

Robustness

HOT theory Narrative causalit Random forests

Self-Organized Criticality

COLD theory

Network robustnes

References

PoCS | @pocsvox

System Robustness

Robustness

HOT theory

Narrative causality

Random forests
Self-Organized Criticality

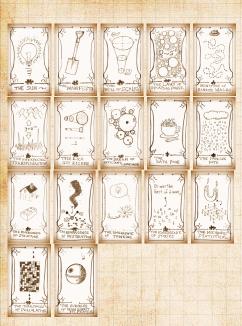
COLD theory

Network robustness









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System Robustness

Robustness

HOT theory
Narrative causality
Random forests

Self-Organized Criticality COLD theory

Network robustness











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System Robustness

Robustness HOT theory

Narrative causality Random forests

Self-Organized Criticality

Network robustness

References







9 9 0 29 of 43

SOC = Self-Organized Criticality

- Idea: natural dissipative systems exist at 'critical states';
- Analogy: Ising model with temperature somehow self-tuning;
- Power-law distributions of sizes and frequencies arise 'for free';
- Introduced in 1987 by Bak, Tang, and
 Weisenfeld
 "Self-organized criticality an explanation of 1/1 noise" (PRL, 1987);
- Problem: Critical state is a very specific point,
- Self-tuning not always possible;
- Much criticism and arguing...

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System

System Robustness

Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality

COLD theory
Network robustness

State of the state





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Robustness

Robustness

HOT theory Narrative causality Self-Organized Criticality

COLD theory Network robustness







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System
Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness







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System

Robustness

Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness







SOC = Self-Organized Criticality

- Idea: natural dissipative systems exist at 'critical states';
- Analogy: Ising model with temperature somehow self-tuning;
- Power-law distributions of sizes and frequencies arise 'for free';
- Introduced in 1987 by Bak, Tang, and Weisenfeld ^[3, 2, 8]: "Self-organized criticality - an explanation of 1/f noise" (PRL, 1987);
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System

Robustness

Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness







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System
Robustness

Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality

COLD theory
Network robustness







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System
Robustness

Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness









"How Nature Works: the Science of Self-Organized Criticality"
by Per Bak (1997). [2]

Avalanches of Sand and Rice ...



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Robustness

Robustness

HOT theory

Narrative causality
Random forests

Self-Organized Criticality

Network robustness









"Complexity and Robustness" Carlson and Doyle, Proc. Natl. Acad. Sci., **99**, 2538–2545, 2002. [6]

HOT versus SOC

- ▶ Both produce power laws
- Optimization versus self-tuning
- HOT systems viable over a wide range of high densities
- SOC systems have one special density
- HOT systems produce specialized structures
- SOC systems produce generic structure

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System
Robustness

Robustness HOT theory

Narrative causality
Random forests

Self-Organized Criticality COLD theory Network robustness







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System

Robustness

Robustness

HOT theory
Narrative causality

Self-Organized Criticality
COLD theory
Network robustness







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System
Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory
Network robustness







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System

Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness







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System
Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness







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System

Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality

COLD theory

Network robustness





HOT theory—Summary of designed tolerance [6]

Table 1. Characteristics of SOC, HOT, and data

	Property	SOC	HOT and Data
1	Internal	Generic,	Structured,
	configuration	homogeneous,	heterogeneous,
		self-similar	self-dissimilar
2	Robustness	Generic	Robust, yet
			fragile
3	Density and yield	Low	High
4	Max event size	Infinitesimal	Large
5	Large event shape	Fractal	Compact
6	Mechanism for	Critical internal	Robust
	power laws	fluctuations	performance
7	Exponent α	Small	Large
8	α vs. dimension d	$\alpha \approx (d-1)/10$	$\alpha \approx 1/d$
9	DDOFs	Small (1)	Large (∞)
10	Increase model	No change	New structures,
	resolution		new sensitivities
11	Response to	Homogeneous	Variable
	forcing		

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System Robustness

Robustness

HOT theory Narrative causality

Self-Organized Criticality COLD theory

Network robustness

References







9 9 € 33 of 43

Outline

Robustness

COLD theory

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Robustness

Robustness HOT theory Narrative causality Self-Organized Criticality

COLD theory Network robustness







Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations [9]
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe
- Power law distribution of fire sizes is truncated

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System

Robustness

Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness





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Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness





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System
Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness
References





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Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





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Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness





Cutoffs

Observed:

Power law distributions often have an exponential cutoff

$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where x_c is the approximate cutoff scale.

May be Weibull distributions

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System

Robustness

Robustness HOT theory

Narrative causality Random forests Self-Organized Criticality COLD theory

Network robustness
References







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$$P(x) \sim x^{-\gamma} e^{-ax^{-\gamma+1}}$$

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System

Robustness

Robustness HOT theory

Narrative causality Random forests Self-Organized Criticality COLD theory

Network robustness





Outline

Robustness

Network robustness

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System Robustness

Robustness

HOT theory Narrative causality Self-Organized Criticality COLD theory Network robustness







Robustness

We'll return to this later on:

- ▶ Network robustness.
- ► Albert et al., Nature, 2000: "Error and attack tolerance of complex networks" [1]
- General contagion processes acting on complex networks. [13, 12]
- ► Similar robust-yet-fragile stories ...

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System Robustness

Robustness HOT theory

Narrative causality
Random forests
Self-Organized Criticality
COLD theory

Network robustness







The Emperor's Robust-Yet-Fragileness:

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System Robustness

Robustness

HOT theory

Narrative causality

Random forests

Self-Organized Criticality

COLD theory

Network robustness







References I

[1] R. Albert, H. Jeong, and A.-L. Barabási. Error and attack tolerance of complex networks. Nature, 406:378-382, 2000. pdf

P. Bak. [2] How Nature Works: the Science of Self-Organized Criticality. Springer-Verlag, New York, 1997.

P. Bak, C. Tang, and K. Wiesenfeld. [3] Self-organized criticality - an explanation of 1/f noise.

Phys. Rev. Lett., 59(4):381-384, 1987. pdf

J. M. Carlson and J. Doyle. [4] Highly optimized tolerance: A mechanism for power laws in designed systems. Phys. Rev. E, 60(2):1412-1427, 1999. pdf

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Robustness

HOT theory Narrative causality Self-Organized Criticality COLD theory Network robustness







References II

[6]

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[5] J. M. Carlson and J. Doyle. Highly optimized tolerance: Robustness and design in complex systems. Phys. Rev. Lett., 84(11):2529–2532, 2000. pdf HOT theory
Narrative causality
Random forests
Self-Organized Criticality
COLD theory
Network robustness

J. M. Carlson and J. Doyle.

Complexity and robustness.

Proc. Natl. Acad. Sci., 99:2538–2545, 2002. pdf

References

Robustness





References III

- [8] H. J. Jensen.
 Self-Organized Criticality: Emergent Complex
 Behavior in Physical and Biological Systems.
 Cambridge Lecture Notes in Physics. Cambridge
 University Press, Cambridge, UK, 1998.
- [9] M. E. J. Newman, M. Girvan, and J. D. Farmer. Optimal design, robustness, and risk aversion. Phys. Rev. Lett., 89:028301, 2002.
- [10] D. Sornette.

 <u>Critical Phenomena in Natural Sciences.</u>

 <u>Springer-Verlag, Berlin, 1st edition, 2003.</u>
- [11] D. Stauffer and A. Aharony.
 Introduction to Percolation Theory.
 Taylor & Francis, Washington, D.C., Second edition, 1992.

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System
Robustness

Robustness
HOT theory
Narrative causality
Random forests
Self-Organized Criticalit
COLD theory

Network robustness References







References IV

pdf

[12] D. J. Watts and P. S. Dodds. Influentials, networks, and public opinion formation. Journal of Consumer Research, 34:441–458, 2007.

[13] D. J. Watts, P. S. Dodds, and M. E. J. Newman. Identity and search in social networks. Science, 296:1302–1305, 2002. pdf

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System
Robustness

Robustness

HOT theory
Narrative causality
Random forests
Self-Organized Criticali
COLD theory
Network robustness



