Mechanisms for Generating Power-Law Size Distributions, Part 2

Principles of Complex Systems | @pocsvox
CSYS/MATH 300, Fall, 2015 | #FallPoCS2015

Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

These slides are brought to you by:

Sealie & Lambie
Productions

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
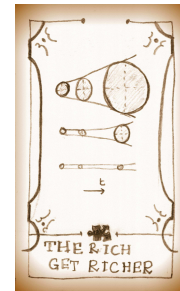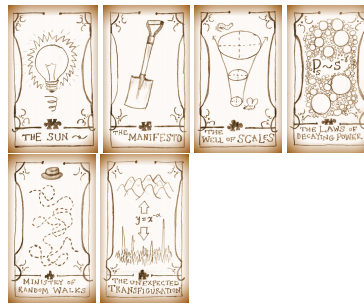And the winner is...?

Nutshell

Extra

References

Outline

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

The deal:

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

Aggregation:

▶ Random walks represent additive aggregation
▶ Mechanism: Random addition and subtraction
▶ Compare across realizations, no competition.
▶ Next: Random Additive/Copying Processes involving Competition.
▶ Widespread: Words, Cities, the Web, Wealth, Productivity (Lotka), Popularity (Books, People, ...)
▶ Competing mechanisms (trickiness)

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

Pre-Zipf's law observations of Zipf's law

▶ 1910s: Word frequency examined re Stenography☍ (or shorthand or brachygraphy or tachygraphy), Jean-Baptiste Estoup☍ [10].
▶ 1910s: Felix Auerbach☍ pointed out the Zipfitude of city sizes in "Das Gesetz der Bevölkerungskonzentration" ("The Law of Population Concentration") [1].
▶ 1924: G. Udny Yule [27]:
\# Species per Genus
▶ 1926: Lotka [14]:
\# Scientific papers per author (Lotka's law)

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

Theoretical Work of Yore:

- 1953: Mandelbrot [16]:
  Optimality argument for Zipf's law; focus on language.
- 1955: Herbert Simon [23, 29]:
  Zipf's law for word frequency, city size, income, publications, and species per genus.
- 1965/1976: Derek de Solla Price [7, 8]:
  Network of Scientific Citations.
- 1999: Barabasi and Albert [2]:
  The World Wide Web, networks-at-large.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
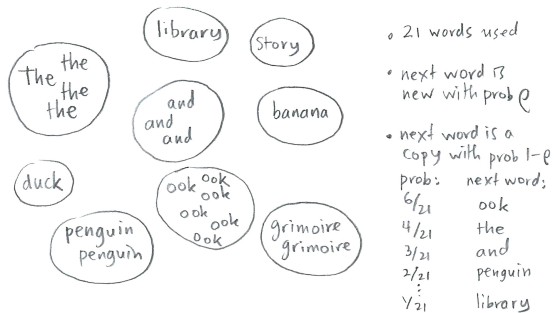Nutshell
Extra
References

Herbert Simon ⧉ (1916–2001):

- Political scientist (and much more)
- Involved in Cognitive Psychology, Computer Science, Public Administration, Economics, Management, Sociology
- Coined 'bounded rationality' and 'satisficing'
- Nearly 1000 publications (see Google Scholar ⧉)
- An early leader in Artificial Intelligence, Information Processing, Decision-Making, Problem-Solving, Attention Economics, Organization Theory, Complex Systems, And Computer Simulation Of Scientific Discovery.
- 1978 Nobel Laureate in Economics (his Nobel bio is here ⧉).

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Essential Extract of a Growth Model:

Random Competitive Replication (RCR):

1. Start with 1 elephant (or element) of a particular flavor at $t = 1$
2. At time $t = 2, 3, 4, ...$, add a new elephant in one of two ways:
   - With probability ρ, create a new elephant with a new flavor
     = Mutation/Innovation

   - With probability $1 - \rho$, randomly choose from all existing elephants, and make a copy.
     = Replication/Imitation

   - Elephants of the same flavor form a group

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Random Competitive Replication:

Example: Words appearing in a language

- Consider words as they appear sequentially.
- With probability ρ, the next word has not previously appeared
  = Mutation/Innovation

- With probability $1 - \rho$, randomly choose one word from all words that have come before, and reuse this word
  = Replication/Imitation

Note: This is a terrible way to write a novel.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

For example:

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Some observations:

- Fundamental Rich-get-Richer story;
- Competition for replication between individual elephants is random;
- Competition for growth between groups of matching elephants is not random;
- Selection on groups is biased by size;
- Random selection sounds easy;
- Possible that no great knowledge of system needed (but more later ...).

Your free set of tofu knives:

- Related to Pólya's Urn Model ⧉, a special case of problems involving urns and colored balls ⧉.
- Sampling with super-duper replacement and sneaky sneaking in of new colors.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Random Competitive Replication:

Some observations:

- ▶ Steady growth of system: +1 elephant per unit time.
- ▶ Steady growth of distinct flavors at rate ρ
- ▶ We can incorporate
  1. Elephant elimination
  2. Elephants moving between groups
  3. Variable innovation rate ρ
  4. Different selection based on group size
     (But mechanism for selection is not as simple...)

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

"The Self-Organizing Economy" 🗗
by Paul Krugman (1996). [13]

Ch. 3: An Urban Mystery, p. 46

"...Simon showed—in a completely impenetrable exposition!—that the exponent of the power law distribution should be ..."[1, 2]

---

[1] Krugman's book was handed to the Deliverator by a certain Álvaro Cartea 🗗 many years ago at the Santa Fe Institute Summer School.

[2] Let's use π for probability because π's not special, right guys?

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

Random Competitive Replication:

Definitions:

- ▶ k_i = size of a group i
- ▶ $N_{k,t}$ = # groups containing k elephants at time t.

Basic question: How does $N_{k,t}$ evolve with time?

First: $\sum_k k N_{k,t} = t =$ number of elephants at time t

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

Random Competitive Replication:

$P_k(t)$ = Probability of choosing an elephant that belongs to a group of size k:

- ▶ $N_{k,t}$ size k groups
- ▶ $\Rightarrow k N_{k,t}$ elephants in size k groups
- ▶ t elephants overall

$$P_k(t) = \frac{k N_{k,t}}{t}.$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

Random Competitive Replication:

$N_{k,t}$, the number of groups with k elephants, changes at time t if

1. An elephant belonging to a group with k elephants is replicated:
   $N_{k,t+1} = N_{k,t} - 1$
   Happens with probability $(1-\rho)k N_{k,t}/t$

2. An elephant belonging to a group with $k-1$ elephants is replicated:
   $N_{k,t+1} = N_{k,t} + 1$
   Happens with probability $(1-\rho)(k-1)N_{k-1,t}/t$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

Random Competitive Replication:

Special case for $N_{1,t}$:

1. The new elephant is a new flavor:
   $N_{1,t+1} = N_{1,t} + 1$
   Happens with probability ρ

2. A unique elephant is replicated:
   $N_{1,t+1} = N_{1,t} - 1$
   Happens with probability $(1-\rho)N_1/t$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

Random Competitive Replication:

Putting everything together:

For $k > 1$:

$$\langle N_{k,t+1} - N_{k,t}\rangle = (1-\rho)\left((+1)(k-1)\frac{N_{k-1,t}}{t} + (-1)k\frac{N_{k,t}}{t}\right)$$

For $k = 1$:

$$\langle N_{1,t+1} - N_{1,t}\rangle = (+1)\rho + (-1)(1-\rho)1 \cdot \frac{N_{1,t}}{t}$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Random Competitive Replication:

Assume distribution stabilizes: $N_{k,t} = n_k t$
(Reasonable for t large)

▶ Drop expectations
▶ Numbers of elephants now fractional
▶ Okay over large time scales

▶ For later: the fraction of groups that have size k is n_k/ρ since

$$\frac{N_{k,t}}{\rho t} = \frac{n_k \not{t}}{\rho \not{t}} = \frac{n_k}{\rho}.$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Random Competitive Replication:

Stochastic difference equation:

$$\langle N_{k,t+1} - N_{k,t}\rangle = (1-\rho)\left((k-1)\frac{N_{k-1,t}}{t} - k\frac{N_{k,t}}{t}\right)$$

becomes

$$n_k(t+1) - n_k t = (1-\rho)\left((k-1)\frac{n_{k-1}t}{t} - k\frac{n_k t}{t}\right)$$

$$n_k(\not{t}+1-\not{t}) = (1-\rho)\left((k-1)\frac{n_{k-1}\not{t}}{\not{t}} - k\frac{n_k\not{t}}{\not{t}}\right)$$

$$\Rightarrow n_k = (1-\rho)\left((k-1)n_{k-1} - kn_k\right)$$

$$\Rightarrow n_k\left(1 + (1-\rho)k\right) = (1-\rho)(k-1)n_{k-1}$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Random Competitive Replication:

We have a simple recursion:

$$\frac{n_k}{n_{k-1}} = \frac{(k-1)(1-\rho)}{1 + (1-\rho)k}$$

▶ Interested in k large (the tail of the distribution)
▶ Can be solved exactly.
  Insert question from assignment 4 ⬀
▶ For just the tail: Expand as a series of powers of $1/k$
  Insert question from assignment 4 ⬀
  We (okay, you) find

$$n_k \propto k^{-\frac{(2-\rho)}{(1-\rho)}} = k^{-\gamma}$$

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

▶ Micro-to-Macro story with ρ and γ measurable.

$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$

▶ Observe $2 < \gamma < \infty$ for $0 < \rho < 1$.
▶ For $\rho \simeq 0$ (low innovation rate):

$$\gamma \simeq 2$$

▶ 'Wild' power-law size distribution of group sizes, bordering on 'infinite' mean.
▶ For $\rho \simeq 1$ (high innovation rate):

$$\gamma \simeq \infty$$

▶ All elephants have different flavors.
▶ Upshot: Tunable mechanism producing a family of universality classes.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

▶ Recall Zipf's law: $s_r \sim r^{-\alpha}$
  (s_r = size of the rth largest group of elephants)
▶ We found $\alpha = 1/(\gamma - 1)$ so:

$$\boxed{\alpha = \frac{1}{\gamma - 1} = \frac{1}{\not{1} + \frac{1}{(1-\rho)} - \not{1}} = 1 - \rho.}$$

▶ $\gamma = 2$ corresponds to $\alpha = 1$
▶ We (roughly) see Zipfian exponent [29] of $\alpha = 1$ for many real systems: city sizes, word distributions, ...
▶ Corresponds to $\rho \to 0$, low innovation.
▶ Krugman doesn't like it) [13] but it's all good.
▶ Still, other quite different mechanisms are possible...
▶ Must look at the details to see if mechanism makes sense... more later.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

What about small k?:

We had one other equation:

-
$$\langle N_{1,t+1} - N_{1,t}\rangle = \rho - (1-\rho)1 \cdot \frac{N_{1,t}}{t}$$

- As before, set $N_{1,t} = n_1 t$ and drop expectations

-
$$n_1(t+1) - n_1 t = \rho - (1-\rho)1 \cdot \frac{n_1 t}{t}$$

-
$$n_1 = \rho - (1-\rho)n_1$$

- Rearrange:
$$n_1 + (1-\rho)n_1 = \rho$$

-
$$\boxed{n_1 = \frac{\rho}{2-\rho}}$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
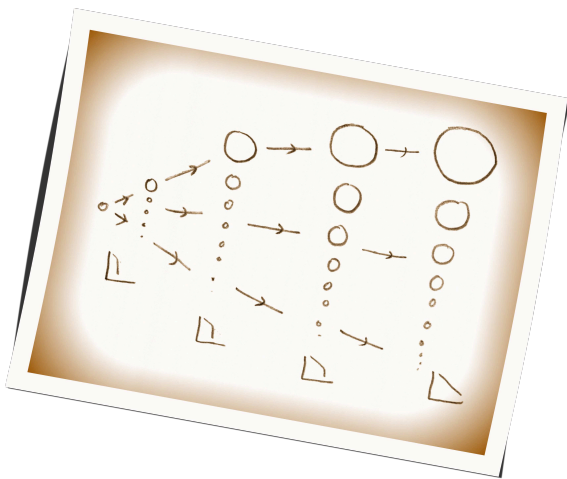Nutshell
Extra
References

So...
$$N_{1,t} = n_1 t = \frac{\rho t}{2-\rho}$$

- Recall number of distinct elephants = ρt.
- Fraction of distinct elephants that are unique (belong to groups of size 1):
$$\frac{1}{\rho t}N_{1,t} = \frac{1}{\rho t}\frac{\rho t}{2-\rho} = \frac{1}{2-\rho}$$
 (also = fraction of groups of size 1)
- For ρ small, fraction of unique elephants $\sim 1/2$
- Roughly observed for real distributions
- ρ increases, fraction increases
- Can show fraction of groups with two elephants $\sim 1/6$
- Model works well for large and small k #awesome

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Words:

From Simon [23]:

Estimate ρ_{est} = # unique words/# all words

For Joyce's Ulysses: $\rho_{est} \simeq 0.115$

N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Evolution of catch phrases:

- Yule's paper (1924) [27]:
  "A mathematical theory of evolution, based on the conclusions of Dr J. C. Willis, F.R.S."
- Simon's paper (1955) [23]:
  "On a class of skew distribution functions" (snore)

From Simon's introduction:

It is the purpose of this paper to analyse a class of distribution functions that appear in a wide range of empirical data—particularly data describing sociological, biological and economic phenomena.

Its appearance is so frequent, and the phenomena so diverse, that one is led to conjecture that if these phenomena have any property in common it can only be a similarity in the structure of the underlying probability mechanisms.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Evolution of catch phrases:

Derek de Solla Price:

- First to study network evolution with these kinds of models.
- Citation network of scientific papers
- Price's term: Cumulative Advantage
- Idea: papers receive new citations with probability proportional to their existing # of citations
- Directed network
- Two (surmountable) problems:
  1. New papers have no citations
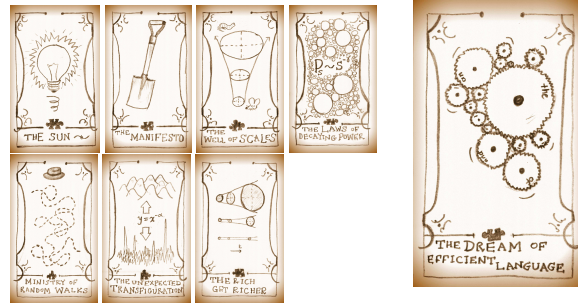 2. Selection mechanism is more complicated

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Evolution of catch phrases:

Robert K. Merton: the Matthew Effect⬚

▶ Studied careers of scientists and found credit flowed disproportionately to the already famous

From the Gospel of Matthew:
"For to every one that hath shall be given...
(Wait! There's more....)
but from him that hath not, that also which he seemeth to have shall be taken away.
And cast the worthless servant into the outer darkness; there men will weep and gnash their teeth."

▶ (Hath = suggested unit of purchasing power.)

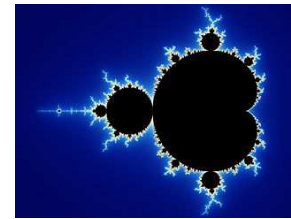▶ Matilda effect:⬚ women's scientific achievements are often overlooked

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Evolution of catch phrases:

Merton was a catchphrase machine:

1. Self-fulfilling prophecy
2. Role model
3. Unintended (or unanticipated) consequences
4. Focused interview → focus group

And just to be clear...

Merton's son, Robert C. Merton, won the Nobel Prize for Economics in 1997.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Evolution of catch phrases:

▶ Barabasi and Albert [2]—thinking about the Web

▶ Independent reinvention of a version of Simon and Price's theory for networks

▶ Another term: "Preferential Attachment"

▶ Considered undirected networks (not realistic but avoids 0 citation problem)

▶ Still have selection problem based on size (non-random)

▶ Solution: Randomly connect to a node (easy) ...

▶ ...and then randomly connect to the node's friends (also easy)

▶ "Scale-free networks" = food on the table for physicists

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

The deal:

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Benoît Mandelbrot⬚

▶ Mandelbrot = father of fractals
▶ Mandelbrot = almond bread
▶ Bonus Mandelbrot set action: here⬚.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Another approach:

Benoît Mandelbrot

▶ Derived Zipf's law through optimization [16]

▶ Idea: Language is efficient

▶ Communicate as much information as possible for as little cost

▶ Need measures of information (H) and average cost (C)...

▶ Language evolves to maximize H/C, the amount of information per average cost.

▶ Equivalently: minimize C/H.

▶ Recurring theme: what role does optimization play in complex systems?

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

The Quickening⧉—Mandelbrot v. Simon:
There Can Be Only One:⧉

- Things there should be only one of: Theory, Highlander Films.
- Feel free to play Queen's It's a Kind of Magic⧉ in your head (funding remains tight).

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

The UNIVERSITY of VERMONT
↺ ⚲ ↻ 43 of 88

We were born to be Princes of the Universe

Mandelbrot vs. Simon:
- Mandelbrot (1953): "An Informational Theory of the Statistical Structure of Languages" [16]
- Simon (1955): "On a class of skew distribution functions" [23]
- Mandelbrot (1959): "A note on a class of skew distribution functions: analysis and critique of a paper by H.A. Simon" [17]
- Simon (1960): "Some further notes on a class of skew distribution functions" [24]

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

The UNIVERSITY of VERMONT
↺ ⚲ ↻ 45 of 88

I have no rival, No man can be my equal

Mandelbrot vs. Simon:
- Mandelbrot (1961): "Final note on a class of skew distribution functions: analysis and critique of a model due to H.A. Simon" [19]
- Simon (1961): "Reply to 'final note' by Benoit Mandelbrot" [26]
- Mandelbrot (1961): "Post scriptum to 'final note'" [19]
- Simon (1961): "Reply to Dr. Mandelbrot's post scriptum" [25]

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

The UNIVERSITY of VERMONT
↺ ⚲ ↻ 46 of 88

I am immortal, I have inside me blood of kings

Mandelbrot:
"We shall restate in detail our 1959 objections to Simon's 1955 model for the Pareto-Yule-Zipf distribution. Our objections are valid quite irrespectively of the sign of p-1, so that most of Simon's (1960) reply was irrelevant." [18]

Simon:
"Dr. Mandelbrot has proposed a new set of objections to my 1955 models of the Yule distribution. Like his earlier objections, these are invalid." [26]

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

The UNIVERSITY of VERMONT
↺ ⚲ ↻ 47 of 88

Zipfarama via Optimization:

Mandelbrot's Assumptions:
- Language contains n words: $w_1, w_2, \ldots, w_n$.
- ith word appears with probability p_i
- Words appear randomly according to this distribution (obviously not true...)
- Words = composition of letters is important
- Alphabet contains m letters
- Words are ordered by length (shortest first)

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

The UNIVERSITY of VERMONT
↺ ⚲ ↻ 50 of 88

Zipfarama via Optimization:

Word Cost
- Length of word (plus a space)
- Word length was irrelevant for Simon's method

Objection
- Real words don't use all letter sequences

Objections to Objection
- Maybe real words roughly follow this pattern (?)
- Words can be encoded this way
- Na na na-na naaaaa...

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

The UNIVERSITY of VERMONT
↺ ⚲ ↻ 51 of 88

Zipfarama via Optimization:

Binary alphabet plus a space symbol

i	1	2	3	4	5	6	7	8
word	1	10	11	100	101	110	111	1000
length	1	2	2	3	3	3	3	4
$1 + \log_2 i$	1	2	2.58	3	3.32	3.58	3.81	4

- ▶ Word length of 2^kth word: $= k + 1 = 1 + \log_2 2^k$
- ▶ Word length of ith word $\simeq 1 + \log_2 i$
- ▶ For an alphabet with m letters,
  word length of ith word $\simeq 1 + \log_m i$.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Zipfarama via Optimization:

Total Cost C
- ▶ Cost of the ith word: $C_i \simeq 1 + \log_m i$
- ▶ Cost of the ith word plus space:
  $C_i \simeq 1 + \log_m(i + 1)$
- ▶ Subtract fixed cost: $C_i' = C_i - 1 \simeq \log_m(i + 1)$
- ▶ Simplify base of logarithm:

$$C_i' \simeq \log_m(i+1) = \frac{\log_e(i+1)}{\log_e m} \propto \ln(i+1)$$

- ▶ Total Cost:

$$C \sim \sum_{i=1}^{n} p_i C_i' \propto \sum_{i=1}^{n} p_i \ln(i+1)$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Zipfarama via Optimization:

Information Measure
- ▶ Use Shannon's Entropy (or Uncertainty):

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i$$

- ▶ (allegedly) von Neumann suggested 'entropy'...
- ▶ Proportional to average number of bits needed to encode each 'word' based on frequency of occurrence
- ▶ $-\log_2 p_i = \log_2 1/p_i$ = minimum number of bits needed to distinguish event i from all others
- ▶ If $p_i = 1/2$, need only 1 bit ($\log_2 1/p_i = 1$)
- ▶ If $p_i = 1/64$, need 6 bits ($\log_2 1/p_i = 6$)

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Zipfarama via Optimization:

Information Measure
- ▶ Use a slightly simpler form:

$$H = -\sum_{i=1}^{n} p_i \log_e p_i / \log_e 2 = -g \sum_{i=1}^{n} p_i \ln p_i$$

$$\text{where } g = 1/\ln 2$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Zipfarama via Optimization:

- ▶ Minimize

$$F(p_1, p_2, \ldots, p_n) = C/H$$

subject to constraint

$$\sum_{i=1}^{n} p_i = 1$$

- ▶ Tension:
  (1) Shorter words are cheaper
  (2) Longer words are more informative (rarer)

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Zipfarama via Optimization:

Time for Lagrange Multipliers:
- ▶ Minimize

$$\Psi(p_1, p_2, \ldots, p_n) =$$
$$F(p_1, p_2, \ldots, p_n) + \lambda G(p_1, p_2, \ldots, p_n)$$

where

$$F(p_1, p_2, \ldots, p_n) = \frac{C}{H} = \frac{\sum_{i=1}^{n} p_i \ln(i+1)}{-g \sum_{i=1}^{n} p_i \ln p_i}$$

and the constraint function is

$$G(p_1, p_2, \ldots, p_n) = \sum_{i=1}^{n} p_i - 1 (= 0)$$

Insert question from assignment 3 ⧉

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
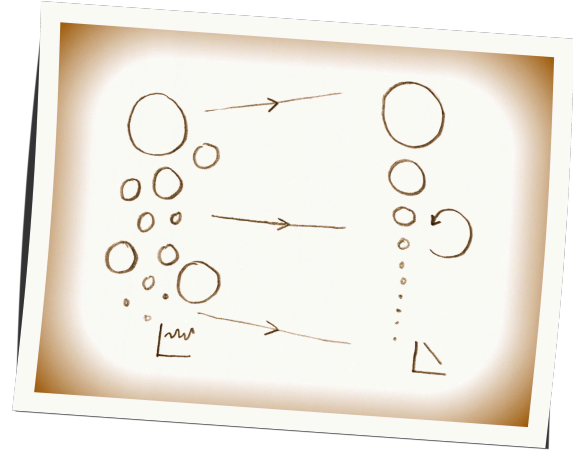 Analysis
  And the winner is...?
Nutshell
Extra
References

Zipfarama via Optimization:

Some mild suffering leads to:

▶

$$p_j = e^{-1 - \lambda H^2/gC}(j+1)^{-H/gC} \propto (j+1)^{-H/gC}$$

▶ A power law appears [applause]: $\boxed{\alpha = H/gC}$

▶ Next: sneakily deduce λ in terms of g, C, and H.

▶ Find

$$p_j = (j+1)^{-H/gC}$$

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Zipfarama via Optimization:

Finding the exponent

▶ Now use the normalization constraint:

$$1 = \sum_{j=1}^{n} p_j = \sum_{j=1}^{n} (j+1)^{-H/gC} = \sum_{j=1}^{n} (j+1)^{-\alpha}$$

▶ As $n \to \infty$, we end up with $\zeta(H/gC) = 2$ where ζ is the Riemann Zeta Function

▶ Gives $\alpha \simeq 1.73$ (> 1, too high) or $\gamma = 1 + \frac{1}{\alpha} \simeq 1.58$ (very wild)

▶ If cost function changes ($j+1 \to j+a$) then exponent is tunable

▶ Increase a, decrease α

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

Zipfarama via Optimization:

All told:

▶ Reasonable approach: Optimization is at work in evolutionary processes

▶ But optimization can involve many incommensurate elephants: monetary cost, robustness, happiness,...

▶ Mandelbrot's argument is not super convincing

▶ Exponent depends too much on a loose definition of cost

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

From the discussion at the end of Mandelbrot's paper:

▶ A. S. C. Ross: "M. Mandelbrot states that 'the actual direction of evolution (sc. of language) is, in fact, towards fuller and fuller utilization of places'. We are, in fact, completely without evidence as to the existence of any 'direction of evolution' in language, and it is axiomatic that we shall remain so. Many philologists would deny that a 'direction of evolution' could be theoretically possible; thus I myself take the view that a language develops in what is essentially a purely random manner."

▶ Mandelbrot: "As to the 'fundamental linguistic units being the least possible differences between pairs of utterances' this is a logical consequence of the fact that two is the least integer greater than one."

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

More:

Reconciling Mandelbrot and Simon

▶ Mixture of local optimization and randomness

▶ Numerous efforts...

1. Carlson and Doyle, 1999:
   Highly Optimized Tolerance
   (HOT)—Evolved/Engineered Robustness [5, 6]

2. Ferrer i Cancho and Solé, 2002:
   Zipf's Principle of Least Effort [12]

3. D'Souza et al., 2007:
   Scale-free networks [9]

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases
Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?
Nutshell
Extra
References

More

Other mechanisms:

- Much argument about whether or not monkeys typing could produce Zipf's law... (Miller, 1957) [20]
- Miller gets to slap Zipf rather rudely in an introduction to a 1965 reprint of Zipf's "Psycho-biology of Language" [21, 28]
- Let us now slap Miller around by simply reading his words out (see next slides):

- Side note: Miller mentions "Genes of Language."
- Still fighting: "Random Texts Do Not Exhibit the Real Zipf's Law-Like Rank Distribution" [11] by Ferrer-i-Cancho and Elevåg, 2010.

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

What Shannon said about meaning in his 1948 paper "A mathematical theory of communication": [22]

The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have *meaning*; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one *selected from a set* of possible messages. The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design.

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

INTRODUCTION

The Psycho-Biology of Language is not calculated to please every taste. Zipf was the kind of man who would take roses apart to count their petals; if it violates your sense of values to tabulate the different words in a Shakespearean sonnet, this is not a book for you. Zipf took a scientist's view of language — and for him that meant the statistical analysis of language as a biological, psychological, social process. If such analysis repels you, then leave your language alone and avoid George Kingsley Zipf like the plague. You will be much happier reading Mark Twain: "There are liars, damned liars, and statisticians." Or W. H. Auden: "Thou shalt not sit with statisticians nor commit a social science."

However, for those who do not flinch to see beauty murdered in a good cause, Zipf's scientific exertions yielded some wonderfully unexpected results to boggle the mind and tease the imagination. Language *is* — among other things — a biological, psychological, social process; to apply statistics to it merely acknowledges its essential unpredictability, without which it would be useless. But who would have thought that in the very heart of all the freedom language allows us Zipf would find an invariant as solid and reliable as the law of gravitation?

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Put it this way. Suppose that we acquired a dozen monkeys and chained them to typewriters until they had produced some very long and random sequence of characters. Suppose further that we defined a "word" in this monkey-text as any sequence of letters occurring between successive spaces. And suppose finally that we counted the occurrences of these "words" in just the way Zipf and others counted the occurrences of real words in meaningful texts. When we plot our results in the same manner, we will find exactly the same "Zipf curves" for the monkeys as for the human authors. Since we are not likely to argue that the poor monkeys were searching for some equilibrium between uniformity and diversity in expressing their ideas, such explanations seem equally inappropriate for human authors.

A mathematical rationalization for this result has been provided by Benoit Mandelbrot. The crux of it is that if we assume that word-boundary markers (spaces) are scattered randomly through a text, then there will necessarily be more occurrences of short than long words. Add to this fact the further observation that the variety of different words available increases exponentially with their length and the phenomenon Zipf reported becomes inescapable: a few short words will be used an enormous number of times while a vast number of longer words will occur infrequently or not at all.

So Zipf was wrong. His facts were right enough, but not his explanations. In a broader sense he was right, however, for he called attention to a stochastic process that is frequently seen in the social sciences, and by accumulating statistical data that cried out for some better explanation he challenged his colleagues and his successors to explore an important new type of probability distribution. Zipf belongs among those rare but stimulating men whose failures are more profitable than most men's successes.

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

So who's right?

Bornholdt and Ebel (PRE), 2001:
"World Wide Web scaling exponent from Simon's 1955 model" [4].

- Show Simon's model fares well.
- Recall ρ = probability new flavor appears.
- Alta Vista ⬀ crawls in approximately 6 month period in 1999 give $\rho \simeq 0.10$
- Leads to $\gamma = 1 + \frac{1}{1-\rho} \simeq 2.1$ for in-link distribution.
- Cite direct measurement of γ at the time: 2.1 ± 0.1 and 2.09 in two studies.

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

So who's right?
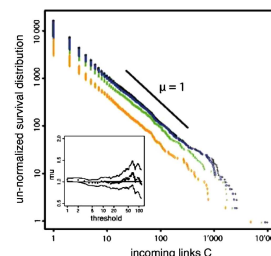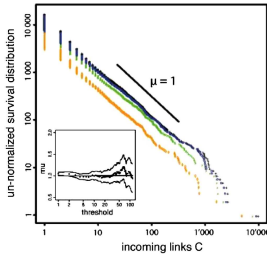
Recent evidence for Zipf's law...

FIG. 1 (color online). (Color Online) Log-log plot of the number of packages in four Debian Linux Distributions with more than C in-directed links. The four Debian Linux Distributions are Woody (19.07.2002) (orange diamonds), Sarge (06.06.2005) (green crosses), Etch (15.08.2007) (blue circles), Lenny (15.12.2007) (black+'s). The inset shows the maximum likelihood estimate (MLE) of the exponent μ together with two boundaries defining its 95% confidence interval (approximately given by $1 \pm 2/\sqrt{n}$, where n is the number of data points using in the MLE), as a function of the lower threshold. The MLE has been modified from the standard Hill estimator to take into account the discreteness of C.

Maillart et al., PRL, 2008:
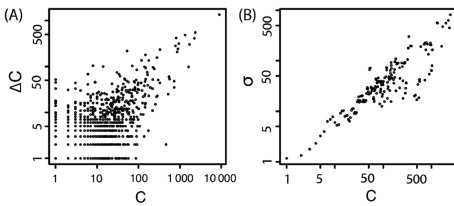"Empirical Tests of Zipf's Law Mechanism in Open Source Linux Distribution" [15]

PoCS | @pocsvox
Power-Law Mechanisms, Pt. 2

Rich-Get-Richer Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

So who's right?

FIG. 1 (color online). (Color Online) Log-log plot of the number of packages in four Debian Linux Distributions with more than C in-directed links. The four Debian Linux Distributions are Woody (19.07.2002) (orange diamonds), Sarge (06.06.2005) (green crosses), Etch (15.08.2007) (blue circles), Lenny (15.12.2007) (black+'s). The inset shows the maximum likelihood estimate (MLE) of the exponent μ together with two boundaries defining its 95% confidence interval (approximately given by $1 \pm 2/\sqrt{n}$, where n is the number of data points using in the MLE), as a function of the lower threshold. The MLE has been modified from the standard Hill estimator to take into account the discreteness of C.

Maillart et al., PRL, 2008:
"Empirical Tests of Zipf's Law Mechanism in Open Source Linux Distribution" [15]

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

So who's right?

FIG. 2. Left panel: Plots of ΔC versus C from the Etch release (15.08.2007) to the latest Lenny version (05.05.2008) in double logarithmic scale. Only positive values are displayed. The linear regression $\Delta C = R \times C + C_0$ is significant at the 95% confidence level, with a small value $C_0 = 0.3$ at the origin and $R = 0.09$. Right panel: same as left panel for the standard deviation of ΔC.

▶ Rough, approximately linear relationship between C number of in-links and ΔC.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

So who's right?

Nutshell:
▶ Simonish random 'rich-get-richer' models agree in detail with empirical observations.
▶ Power-lawfulness: Mandelbrot's optimality is still apparent.
▶ Optimality arises for free in Random Competitive Replication models.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Others are also not happy:

Krugman and Simon
▶ "The Self-Organizing Economy" (Paul Krugman, 1996) [13]
▶ Krugman touts Zipf's law for cities, Simon's model
▶ "Déjà vu, Mr. Krugman" (Berry, 1999)
▶ Substantial work done by Urban Geographers

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Who needs a hug?

From Berry [3]
▶ Déjà vu, Mr. Krugman. Been there, done that. The Simon-Ijiri model was introduced to geographers in 1958 as an explanation of city size distributions, the first of many such contributions dealing with the steady states of random growth processes, ...
▶ But then, I suppose, even if Krugman had known about these studies, they would have been discounted because they were not written by professional economists or published in one of the top five journals in economics!

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

Who needs a hug?

From Berry [3]
▶ ... [Krugman] needs to exercise some humility, for his world view is circumscribed by folkways that militate against recognition and acknowledgment of scholarship beyond his disciplinary frontier.
▶ Urban geographers, thank heavens, are not so afflicted.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
  Simon's Model
  Analysis
  Words
  Catchphrases
Optimization
  Minimal Cost
  Mandelbrot vs. Simon
  Assumptions
  Model
  Analysis
  And the winner is...?
Nutshell
Extra
References

References I

[1] F. Auerbach.
Das gesetz der bevölkerungskonzentration.
Petermanns Geogr. Mitteilungen, 59:73–76, 1913.

[2] A.-L. Barabási and R. Albert.
Emergence of scaling in random networks.
Science, 286:509–511, 1999. pdf ☑

[3] B. J. L. Berry.
Déjà vu, Mr. Krugman.
Urban Geography, 20:1–2, 1999. pdf ☑

[4] S. Bornholdt and H. Ebel.
World Wide Web scaling exponent from Simon's
1955 model.
Phys. Rev. E, 64:035104(R), 2001. pdf ☑

References II

[5] J. M. Carlson and J. Doyle.
Highly optimized tolerance: A mechanism for
power laws in designed systems.
Phys. Rev. E, 60(2):1412–1427, 1999. pdf ☑

[6] J. M. Carlson and J. Doyle.
Complexity and robustness.
Proc. Natl. Acad. Sci., 99:2538–2545, 2002. pdf ☑

[7] D. J. de Solla Price.
Networks of scientific papers.
Science, 149:510–515, 1965. pdf ☑

[8] D. J. de Solla Price.
A general theory of bibliometric and other
cumulative advantage processes.
J. Amer. Soc. Inform. Sci., 27:292–306, 1976.

References III

[9] R. M. D'Souza, C. Borgs, J. T. Chayes, N. Berger,
and R. D. Kleinberg.
Emergence of tempered preferential attachment
from optimization.
Proc. Natl. Acad. Sci., 104:6112–6117, 2007. pdf ☑

[10] J.-B. Estoup.
Gammes sténographiques: méthode et exercices
pour l'acquisition de la vitesse.
Institut Sténographique, 1916.

[11] R. Ferrer-i Cancho and B. Elvevåg.
Random texts do not exhibit the real Zipf's
law-like rank distribution.
PLoS ONE, 5:e9411, 03 2010.

References IV

[12] R. Ferrer-i Cancho and R. V. Solé.
Zipf's law and random texts.
Advances in Complex Systems, 5(1):1–6, 2002.

[13] P. Krugman.
The Self-Organizing Economy.
Blackwell Publishers, Cambridge, Massachusetts,
1996.

[14] A. J. Lotka.
The frequency distribution of scientific
productivity.
Journal of the Washington Academy of Science,
16:317–323, 1926.

References V

[15] T. Maillart, D. Sornette, S. Spaeth, and G. von
Krogh.
Empirical tests of Zipf's law mechanism in open
source Linux distribution.
Phys. Rev. Lett., 101(21):218701, 2008. pdf ☑

[16] B. B. Mandelbrot.
An informational theory of the statistical
structure of languages.
In W. Jackson, editor, Communication Theory,
pages 486–502. Butterworth, Woburn, MA, 1953.
pdf ☑

[17] B. B. Mandelbrot.
A note on a class of skew distribution function.
Analysis and critique of a paper by H. A. Simon.
Information and Control, 2:90–99, 1959.

References VI

[18] B. B. Mandelbrot.
Final note on a class of skew distribution
functions: analysis and critique of a model due to
H. A. Simon.
Information and Control, 4:198–216, 1961.

[19] B. B. Mandelbrot.
Post scriptum to 'final note'.
Information and Control, 4:300–304, 1961.

[20] G. A. Miller.
Some effects of intermittent silence.
American Journal of Psychology, 70:311–314,
1957. pdf ☑

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

PoCS
Principles of
Complex Systems
@pocsvox
What's the Story?

The UNIVERSITY of VERMONT

↺ ⚲ ↻ 80 of 88
81 of 88
82 of 88
83 of 88
84 of 88
85 of 88

References VII

[21] G. A. Miller.
Introduction to reprint of G. K. Zipf's "The Psycho-Biology of Language." MIT Press, Cambridge MA, 1965. pdf ⬀

[22] C. E. Shannon.
A mathematical theory of communication.
The Bell System Tech. J., 27:379–423,623–656, 1948. pdf ⬀

[23] H. A. Simon.
On a class of skew distribution functions.
Biometrika, 42:425–440, 1955. pdf ⬀

[24] H. A. Simon.
Some further notes on a class of skew distribution functions.
Information and Control, 3:80–88, 1960.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

PoCS
Principles of
Complex Systems
@pocsvox
What's the Story?

The UNIVERSITY of VERMONT

↺ ⚲ ↻   86 of 88

References VIII

[25] H. A. Simon.
Reply to Dr. Mandelbrot's post scriptum.
Information and Control, 4:305–308, 1961.

[26] H. A. Simon.
Reply to 'final note' by Benoît Mandelbrot.
Information and Control, 4:217–223, 1961.

[27] G. U. Yule.
A mathematical theory of evolution, based on the conclusions of Dr J. C. Willis, F.R.S.
Phil. Trans. B, 213:21–, 1924.

[28] G. K. Zipf.
The Psycho-Biology of Language.
Houghton-Mifflin, New York, NY, 1935.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

PoCS
Principles of
Complex Systems
@pocsvox
What's the Story?

The UNIVERSITY of VERMONT

↺ ⚲ ↻   87 of 88

References IX

[29] G. K. Zipf.
Human Behaviour and the Principle of Least-Effort.
Addison-Wesley, Cambridge, MA, 1949.

PoCS | @pocsvox
Power-Law
Mechanisms, Pt. 2

Rich-Get-Richer
Mechanism
Simon's Model
Analysis
Words
Catchphrases

Optimization
Minimal Cost
Mandelbrot vs. Simon
Assumptions
Model
Analysis
And the winner is...?

Nutshell

Extra

References

PoCS
Principles of
Complex Systems
@pocsvox
What's the Story?

The UNIVERSITY of VERMONT

↺ ⚲ ↻   88 of 88