# Overview of Complex Networks

Principles of Complex Systems | @pocsvox CSYS/MATH 300, Fall, 2015 | #FallPoCS2015

# Prof. Peter Dodds | @peterdodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont























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Overview of Complex Networks

Complex Networks Basics

Popularity
Graph theory?
Basic definitions

Examples of Complex Networks

> Physical networks nteraction network Relational networks







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Overview of Complex Networks

Complex **Networks Basics** 

Etymology Graph theory? Basic definitions

Examples of Complex Networks

Physical networks Interaction networks







# Outline

Complex Networks Basics

Etymology
Popularity
Graph theory?
Basic definitions

**Examples of Complex Networks** 

Physical networks Interaction networks Relational networks

References

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Overview of
Complex

Networks

Complex Networks Basics

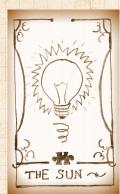
Etymology
Popularity
Graph theory?
Basic definitions

Examples of Complex Networks Physical networks

Physical networks Interaction networks Relational networks







Overview of Complex Networks

### Complex Networks Basics

Etymology Graph theory? Basic definitions

### Examples of Complex Networks

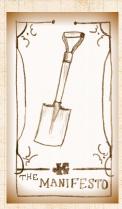
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Overview of Complex Networks

### Complex Networks Basics

Etymology Graph theory? Basic definitions

### Examples of Complex Networks

Physical networks Interaction networks

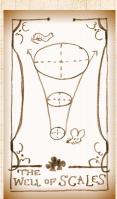












Overview of Complex Networks

### Complex Networks Basics

Etymology Popularity Graph theory? Basic definitions

### Examples of Complex Networks

Physical networks Interaction networks Relational networks













Overview of Complex Networks

### Complex Networks Basics

Etymology Graph theory? Basic definitions

#### Examples of Complex Networks

Physical networks Interaction networks











Overview of Complex Networks

#### Complex Networks Basics Etymology

Graph theory? Basic definitions

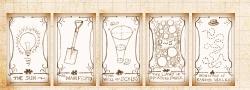
### Examples of Complex Networks

Physical networks Interaction networks











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### Complex Networks Basics

Etymology Graph theory? Basic definitions

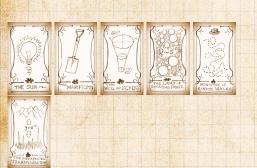
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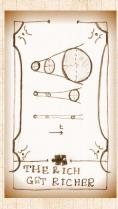
Physical networks Interaction networks











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### Complex Networks Basics

Etymology Graph theory? Basic definitions

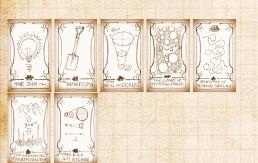
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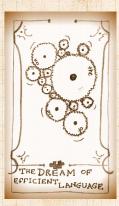
Physical networks Interaction networks











Overview of Complex Networks

### Complex Networks Basics

Etymology Graph theory? Basic definitions

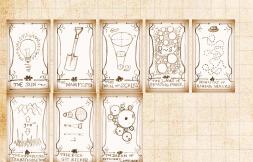
#### Examples of Complex Networks

Physical networks Interaction networks











Overview of Complex Networks

### Complex Networks Basics

Etymology Graph theory? Basic definitions

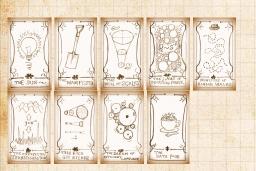
#### Examples of Complex Networks

Physical networks Interaction networks











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### Complex Networks Basics

Etymology Popularity Graph theory? Basic definitions

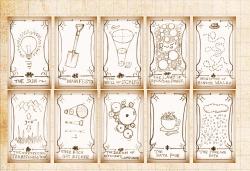
#### Examples of Complex Networks

Physical networks Interaction networks Relational networks











Overview of Complex Networks

#### Complex Networks Basics Etymology

Graph theory? Basic definitions

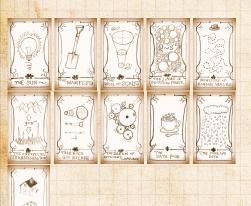
### Examples of Complex Networks

Physical networks Interaction networks









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Overview of Complex Networks

#### Complex Networks Basics Etymology

Graph theory? Basic definitions

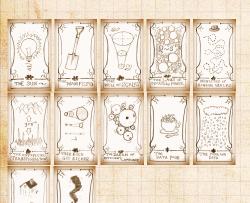
#### Examples of Complex Networks

Physical networks Interaction networks









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Overview of Complex Networks

#### Complex Networks Basics Etymology

Graph theory? Basic definitions

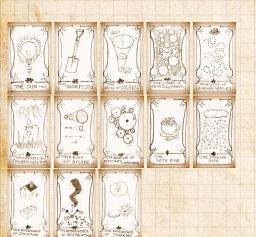
#### Examples of Complex Networks

Physical networks Interaction networks











Overview of Complex Networks

Complex Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks Interaction networks











Overview of Complex Networks

### Complex Networks Basics

Etymology Graph theory? Basic definitions

#### Examples of Complex Networks

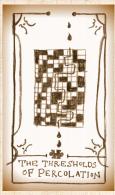
Physical networks Interaction networks











Overview of Complex Networks

#### Complex Networks Basics Etymology

Graph theory? Basic definitions

#### Examples of Complex Networks

Physical networks Interaction networks









THE THRESHOLDS OF PERCOLATION



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Overview of Complex Networks

### Complex Networks Basics

Etymology

Graph theory? Basic definitions

#### Examples of Complex Networks

Physical networks

Interaction networks











Overview of Complex Networks

### Complex Networks Basics

Etymology

Graph theory? Basic definitions

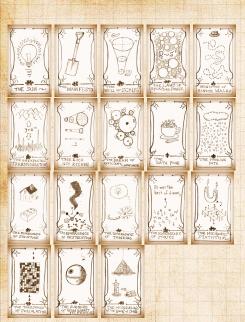
#### Examples of Complex Networks

Physical networks Interaction networks











Overview of Complex Networks

#### Complex Networks Basics Etymology

Graph theory? Basic definitions

#### Examples of Complex Networks

Physical networks Interaction networks











Overview of Complex Networks

#### Complex Networks Basics Etymology

Graph theory? Basic definitions

#### Examples of Complex Networks

Physical networks Interaction networks







# Outline

# Complex Networks Basics Etymology

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Overview of Complex Networks

Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks Interaction networks Relational networks







# net-work | 'net, work |

noun

1 an arrangement of intersecting horizontal and vertical lines.

• a complex system of roads, railroads, or other transportation routes: a network of railroads.

2 a group or system of interconnected people or things: a trade network.

- a group of people who exchange information, contacts, and experience for professional or social purposes : a support network.
- a group of broadcasting stations that connect for the simultaneous broadcast of a program: the introduction of a second TV network | [as adj.] netroork television
- a number of interconnected computers, machines, or operations: specialized computers that manage multiple outside connections to a network | a local cellular phone network.
- a system of connected electrical conductors.

# verb [trans.]

connect as or operate with a network: the stock exchanges have proven to be resourceful in networking these deals.

- link (machines, esp. computers) to operate interactively : [as adj. ] ( networked) networked workstations.
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career: the skills of networking, bargaining, and negotiation.

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Overview of Complex Networks

Complex Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks

Interaction networks







# Thesaurus deliciousness:

# network

noun

- 1 a network of arteries WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2 a network of lanes MAZE, labyrinth, warren, tangle.
- 3 a network of friends SYSTEM, complex, nexus, web, webwork.

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Overview of Complex Networks

Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks







From Keith Briggs's excellent etymological investigation:

- ▶ Opus reticulatum:
- ► A Latin origin?



[http://serialconsign.com/2007/11/we-put-netnetwork]

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Overview of Complex Networks

# Networks Basics

### Etymology

Graph theory? Basic definitions

#### Examples of Complex Networks

Physical networks







# First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

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Complex Networks

Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks Physical networks







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Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks







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Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks







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Networks Basics Etymology

Graph theory? Basic definitions

Examples of Complex Networks







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Overview of Complex Networks

Networks Basics Etymology

Graph theory? Basic definitions

Examples of Complex Networks





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- ▶ 1839-: rivers and canals
- ▶ 1869–: railways
- ▶ 1883-: distribution network of electrical cables
- ▶ 1914–: wireless broadcasting networks

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Overview of
Complex
Networks

Complex Networks Basics Etymology

Popularity

Graph theory? Basic definitions

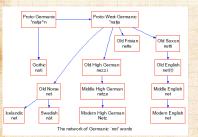
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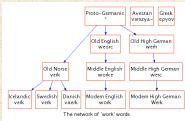




# Net and Work are venerable old words:

- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appear to have long meant purposeful action.





- 'Network' = something built based on the idea of natural, flexible lattice or web.
- c.f., ironwork, stonework, fretwork.

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Overview of
Complex

Complex Networks Basics

Etymology Etymology

conularity

Networks

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks Interaction network Relational networks

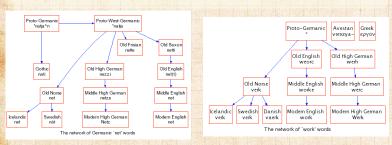






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Overview of Complex
Networks

Complex Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks Physical networks

Interaction network Relational network

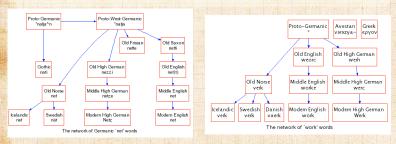






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Overview of Complex Networks

Networks Basics

Etymology

Graph theory?

Examples of Complex Networks







- Many complex systems can be viewed as complex networks of physical or abstract interactions.

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Networks Basics

Etymology

Networks

Graph theory? Basic definitions Examples of

Complex Networks Physical networks

Interaction networks







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Networks Basics

Etymology

Networks

Graph theory? Basic definitions

Examples of Complex Networks







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Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks







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Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks

Interaction networks







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Overview of
Complex
Networks

Complex Networks Basics

Etymology Popularity

Graph theory?
Basic definitions
Examples of

Complex Networks Physical networks

Interaction networks Relational networks







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Piranha physicus



Feast on new and interesting ideas (see chaos, cellular automata, ...)

See also:

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Complex
Networks

Complex Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks Physical networks

Interaction networks Relational networks

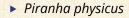






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- Hunt in packs.
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Overview of Complex Networks

Complex Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks Physical networks

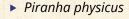
Interaction networks Relational networks







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Overview of
Complex
Networks

Complex Networks Basics

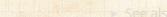
Etymology

Graph theory? Basic definitions

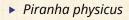
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- ➤ See also: https://xkcd.com/793/ ☑

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Overview of
Complex
Networks

Complex Networks Basics

Etymology Popularity

Graph theory? Basic definitions

Examples of Complex Networks Physical networks

nteraction networks









## Outline

Complex Networks Basics

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Overview of Complex Networks

Networks Basics

Etymology Popularity

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks Interaction networks Relational networks







# Popularity (according to Google Scholar)



"Collective dynamics of 'small-world' networks"

Watts and Strogatz, Nature, **393**, 440–442, 1998. [14]

Times cited: ~ 27 184 🗹 (as of October 8, 2015)



"Emergence of scaling in random networks" Barabási and Albert, Science, **286**, 509–511, 1999. [2]

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Overview of Complex
Networks

Complex Networks Basics Etymology Popularity

Graph theory? Basic definitions

Examples of Complex Networks
Physical networks
Interaction networks
Relational networks





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Overview of Complex

Networks

Complex
Networks Basics
Etymology
Popularity

Graph theory? Basic definitions

Examples of Complex Networks
Physical networks Interaction networks Relational networks





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Overview of Complex

Networks

Complex Networks Basics Etymology Popularity

Graph theory? Basic definitions

Examples of Complex Networks
Physical networks Interaction networks





#### Review articles:



"Complex Networks: Structure and Dynamics" Dynamics" Boccaletti et al., Physics Reports, **424**, 175–308, 2006. [3]

Times cited:  $\sim 5,791$  (as of October 8, 2015)



"The structure and function of complex networks" 
M. E. J. Newman,
SIAM Rev., 45, 167–256, 2003. [10]

Times cited:  $\sim$  13,156  $\Box$  (as of October 8, 2015)



"Statistical mechanics of complex networks"

Albert and Barabási, Rev. Mod. Phys., **74**, 47–97, 2002. [1]

Times cited:  $\sim 15,676$  (as of October 8, 2015)

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Overview of
Complex
Networks

Complex Networks Basics

Popularity
Graph theory?
Basic definitions

Examples of Complex Networks Physical networks Interaction networks

References





20 0 14 of 42

# Popularity according to textbooks:

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Overview of Complex Networks

Networks Basics

Etymology Popularity Graph theory? Basic definitions

Examples of Complex Networks Physical networks

Interaction networks







# Popularity according to textbooks:

#### Textbooks:

- Mark Newman (Physics, Michigan) "Networks: An Introduction"
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell) "Networks, Crowds, and Markets: Reasoning About a Highly Connected World"

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Overview of Complex Networks

Networks Basics Etymology

Popularity Graph theory? Basic definitions

Examples of Complex Networks







# Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [7]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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Overview of Complex Networks

Complex Networks Basics Etymology

Popularity
Graph theory?

Basic definitions

Examples of Complex Networks Physical networks

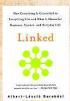
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# Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [13]

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Overview of Complex Networks

Complex Networks Basics Etymology

Popularity
Graph theory?

Basic definitions

Examples of Complex Networks Physical networks

Interaction networks Relational networks







#### Numerous others ...

- ► Complex Social Networks—F. Vega-Redondo [12]
- ► Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [11]
- ► Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- ► Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [5]
- ► Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [6]

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Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity

Graph theory? Basic definitions

Examples of Complex Networks Physical networks





## Outline

Complex Networks Basics

Graph theory?

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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks Interaction networks Relational networks







- ▶ But surely networks aren't new...
- Graph theory is well established.
- Study of social networks started in the 1930's.
- So why all this 'new' research on networks?
- ► Answer: Oodles of Easily Accessible Data
- We can now inform (alas) our theories with a much more measurable reality.
- ► A worthy goal: establish mechanistic explanations

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Overview of
Complex

Networks

Complex Networks Basics

Popularity
Graph theory?

Graph theory?

Basic definitions

Examples of

Complex
Networks
Physical networks

Relational networks







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Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks







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Networks Basics

Networks

Graph theory? Basic definitions

Examples of Complex Networks







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Networks Basics

Graph theory?

Basic definitions Examples of

Complex Networks







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Overview of the Complex
Networks

Networks Basics

Popularity
Graph theory?

Basic definitions

Examples of

Networks
Physical networks

Interaction networks Relational networks





- ▶ But surely networks aren't new...
- Graph theory is well established...
- ▶ Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- ► Answer: Oodles of Easily Accessible Data.
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Overview of +
Complex
Networks

Complex Networks Basics

Popularity
Graph theory?

Basic definitions

Examples of

Complex
Networks
Physical networks
Interaction networks





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Overview of the Complex
Networks

Complex Networks Basics

Popularity
Graph theory?
Basic definitions

Examples of Complex Networks Physical networks





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  \*If this is upsetting, maybe string theory is for you...

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Overview of Complex
Networks

Complex Networks Basics Etymology

Popularity
Graph theory?
Basic definitions

Examples of Complex Networks Physical networks Interaction networks





▶ Web-scale data sets can be overly exciting.

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Overview of Complex Networks

Complex Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks Physical networks Interaction networks







▶ Web-scale data sets can be overly exciting.

#### Witness:

- ▶ The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)

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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks









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Overview of Complex

Networks

Networks Basics

Graph theory? Basic definitions Examples of

Complex Networks







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- We still need to understand.

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Overview of the Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?

Examples of Complex Networks Physical networks Interaction networks

Basic definitions







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Overview of
Complex
Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?

Examples of Complex Networks Physical networks Interaction networks







## Outline

Complex Networks Basics

**Basic definitions** 

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Overview of Complex Networks

Networks Basics

Etymology Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks Interaction networks Relational networks







# Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

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Overview of Complex Networks

Networks Basics

Etymology Graph theory? Basic definitions

Examples of Complex Networks Physical networks

Interaction networks







# Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

e.g., people, forks in rivers, proteins, webpages, organisms,...

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Overview of Complex Networks

Networks Basics Etymology

Graph theory?

Basic definitions

Examples of Complex Networks Physical networks







Nodes = A collection of entities which have properties that are somehow related to each other

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#### Links = Connections between nodes

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Overview of Complex Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks







Nodes = A collection of entities which have properties that are somehow related to each other

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#### Links = Connections between nodes

- Links may be directed or undirected.

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Overview of Complex Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks







Nodes = A collection of entities which have properties that are somehow related to each other

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- Links may be directed or undirected.
- Links may be binary or weighted.

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Complex Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks







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- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.

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Networks Basics

Graph theory? Basic definitions

Examples of

Complex Networks







# Node degree = Number of links per node

$$|k\rangle = \frac{2m}{N}$$

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Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks Interaction networks







## Node degree = Number of links per node

- Notation: Node i's degree =  $k_i$ .

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Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Physical networks Interaction networks







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Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Physical networks Interaction networks







## Node degree = Number of links per node

- Notation: Node i's degree =  $k_i$ .
- $k_i = 0,1,2,...$
- ▶ Notation: the average degree of a network =  $\langle k \rangle$

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Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks Physical networks







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- Notation: Node i's degree =  $k_i$ .
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- Notation: the average degree of a network =  $\langle k \rangle$ (and sometimes z)

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Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks Physical networks







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- Connection between number of edges m and average degree:

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Networks Basics

Graph theory? Basic definitions

Networks

Examples of Complex Networks







# Node degree = Number of links per node

- Notation: Node i's degree =  $k_i$ .
- $k_i = 0,1,2,...$
- Notation: the average degree of a network =  $\langle k \rangle$  (and sometimes z)
- ► Connection between number of edges *m* and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

▶ Defn:  $N_i$  = the set of i's  $k_i$  neighbors

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Overview of
Complex

Networks

Complex Networks Basics

Popularity
Graph theory?
Basic definitions

Examples of Complex Networks

hysical networks nteraction networks elational networks







#### Adjacency matrix:

- ▶ We represent a directed network by a matrix A with link weight  $a_{ij}$  for nodes i and j in entry (i, j).

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Overview of Complex Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks







#### Adjacency matrix:

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- e.g.,

$$A = \left[ \begin{array}{ccccc} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{array} \right]$$

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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks







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(n.b., for numerical work, we always use sparse matrices.) PoCS | @pocsvox
Overview of

Overview of Complex Networks

Complex Networks Basics

Popularity
Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks Interaction networks Relational networks







## So what passes for a complex network?

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Overview of Complex Networks

## Networks Basics

Graph theory? Basic definitions

#### Examples of Complex Networks

Physical networks Interaction networks







#### So what passes for a complex network?

- Complex networks are large (in node number)

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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks







#### So what passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)

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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks







#### So what passes for a complex network?

- Complex networks are large (in node number)
- ► Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving

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Overview of Complex Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks







#### So what passes for a complex network?

- Complex networks are large (in node number)
- Complex networks are sparse (low edge to node ratio)
- Complex networks are usually dynamic and evolving
- Complex networks can be social, economic, natural, informational, abstract, ...

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Overview of Complex Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks







## Outline

#### **Examples of Complex Networks** Physical networks

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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Physical networks

Interaction networks

Relational networks







#### Physical networks

- River networks



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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks

Interaction networks







#### Physical networks

- River networks
- Neural networks



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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex

Networks Physical networks

Interaction networks







#### Physical networks

- River networks
- Neural networks
- Trees and leaves





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Overview of Complex Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks Interaction networks

Relational networks







#### Physical networks

- River networks
- Neural networks
- Trees and leaves
- Blood networks





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Overview of Complex Networks

Networks Basics Etymology

Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks

Interaction networks







#### Physical networks

- River networks
- Neural networks
- ▶ Trees and leaves
- Blood networks

- ► The Internet
- Road networks
- Power grids







Distribution (branching) versus (cyclical).

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Overview of Complex Networks

Complex Networks Basics

Etymology Popularity Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks

nteraction networks







#### Physical networks

- River networks
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Overview of Complex Networks

Networks Basics

Etymology Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks









#### Physical networks

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- Neural networks
- Trees and leaves
- Blood networks

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Overview of Complex Networks

Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex

Networks Physical networks

References







28 of 42

#### Physical networks

- River networks
- Neural networks
- ► Trees and leaves
- ▶ Blood networks

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 Distribution (branching) versus redistribution (cyclical) PoCS | @pocsvox

Overview of Complex Networks

Complex Networks Basics

Popularity Graph theory?

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks

nteraction network Relational networks







## Outline

#### **Examples of Complex Networks**

Interaction networks

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Overview of Complex Networks

Networks Basics

Etymology Graph theory? Basic definitions

Examples of Complex Networks Physical networks Interaction networks

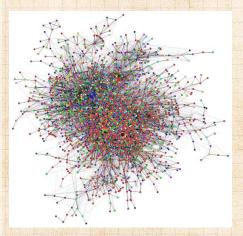






#### Interaction networks

- ► The Blogosphere



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Networks Basics

Etymology

Graph theory? Basic definitions

Examples of Complex Networks Physical networks Interaction networks



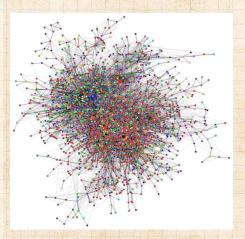






#### Interaction networks

- ▶ The Blogosphere
- ▶ Biochemical networks



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Complex Networks

Networks Basics Etymology

Graph theory?

Basic definitions

Examples of Complex Networks Physical networks

Interaction networks

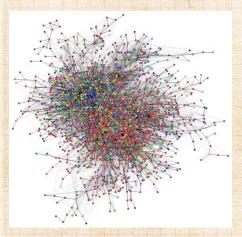






# Interaction networks

- ▶ The Blogosphere
- Biochemical networks
- Gene-protein networks
- Food webs; who eats whom
- Web (?)
- ► Airline network:
- Call networks
  (AT&T)
- ▶ The Media



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Overview of Complex Networks

Complex Networks Basics

Etymology Popularity

Graph theory?

Basic definitions

Examples of Complex Networks Physical networks

Physical networks Interaction networks

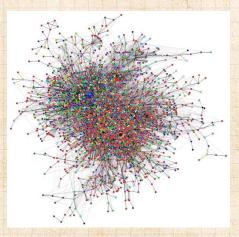






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Overview of Complex Networks

Complex Networks Basics

Etymology

Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks Interaction networks

Relational networ

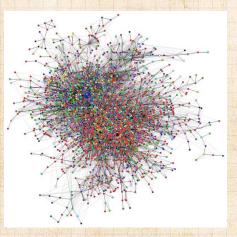






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Overview of Complex Networks

Networks Basics

Etymology

Graph theory?

Basic definitions

Examples of Complex

Networks

Interaction networks



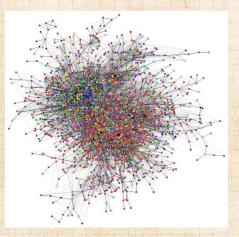






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Complex Networks

Networks Basics

Etymology

Graph theory?

Basic definitions

Examples of Complex Networks

Interaction networks



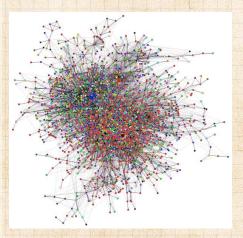






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- Airline networks
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Complex Networks

Networks Basics

Etymology

Graph theory? Basic definitions

Examples of

Complex Networks

Interaction networks



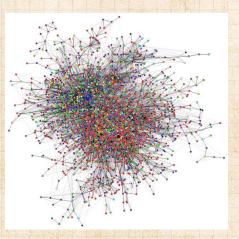






#### Interaction networks

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- Airline networks
- Call networks (AT&T)
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Complex Networks

Networks Basics Etymology

Graph theory?

Basic definitions

Examples of Complex Networks

Interaction networks





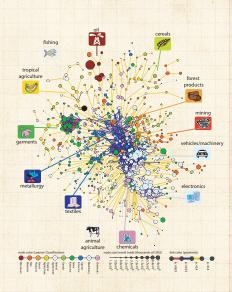




# topics:

- ► Hidalgo et al.'s

  "The Product
  Space Conditions
  the Development
  of Nations" [9]
- How do products depend on each other, and how does this network evolve?
- How do countries depend on each other for water, energy, people (immigration), investments?



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Overview of Complex Networks

Complex
Networks Basics
Etymology
Popularity
Graph theory?

Examples of Complex Networks Physical networks Interaction networks

Basic definitions

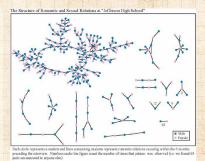






#### Interaction networks: social networks

- Snogging



(Bearman et al., 2004)

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Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Interaction networks

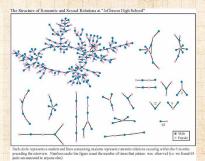






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- Snogging
- Friendships



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Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Interaction networks

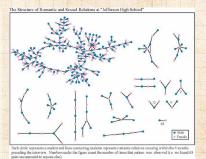






# Interaction networks: social networks

- Snogging
- ▶ Friendships
- ▶ Acquaintances
- Boards and directors
- Organizations
- ▶ facebook to twitter



(Bearman et al., 2004)

▶ 'Remotely sensed' by: email activity, instant messaging, phone logs PoCS | @pocsvox
Overview of
Complex

Networks

Complex Networks Basics

Etymology Popularity

Graph theory? Basic definitions

Examples of Complex Networks Physical networks Interaction networks

References

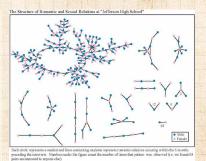
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#### Interaction networks: social networks

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Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks

Interaction networks

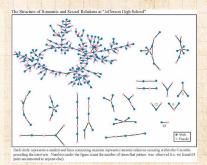






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Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Interaction networks

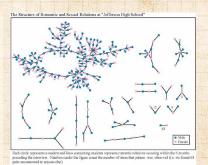






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- Acquaintances
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PoCS | @pocsvox Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Interaction networks

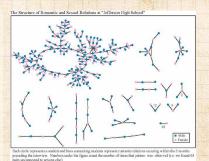






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 'Remotely sensed' by: email activity, instant messaging, phone logs ("cought"). PoCS | @pocsvox
Overview of Complex
Networks

Complex Networks Basics Etymology

Popularity
Graph theory?
Basic definitions

Examples of Complex Networks Physical networks Interaction networks

Relational network

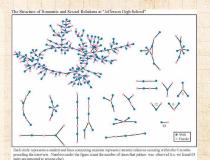






#### Interaction networks: social networks

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- Friendships
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(Bearman et al., 2004)

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Networks Basics

Graph theory?

Basic definitions Examples of

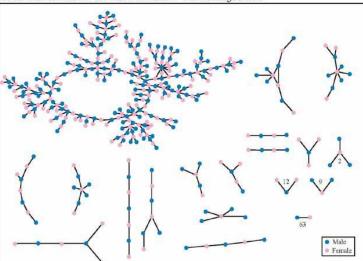
Complex Networks Interaction networks







The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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Overview of Complex Networks

Complex Networks Basics

Etymology Popularity

Graph theory?

Basic definitions

Examples of Complex Networks

Physical networks
Interaction networks

Relational networks







# Outline

#### **Examples of Complex Networks**

Relational networks

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Overview of Complex Networks

Networks Basics

Etymology Graph theory? Basic definitions

Examples of Complex Networks Physical networks

Interaction networks Relational networks







#### Relational networks

- Consumer purchases

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Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Physical networks

Interaction networks Relational networks









#### Relational networks

- Consumer purchases (Wal-Mart, Target, Amazon, ...)

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Overview of Complex Networks

Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks Physical networks

Interaction networks Relational networks







#### Relational networks

- Consumer purchases (Wal-Mart, Target, Amazon, ...)
- ▶ Thesauri: Networks of words generated by meanings

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Overview of Complex Networks

Networks Basics

Graph theory?

Basic definitions

Examples of Complex Networks Physical networks

Relational networks







#### Relational networks

- Consumer purchases (Wal-Mart, Target, Amazon, ...)
- Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- Metadata—Tagging:

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Overview of Complex Networks

Complex Networks Basics

Popularity
Graph theory?

Basic definitions

Examples of Complex Networks

Interaction networks Relational networks







#### Relational networks

- Consumer purchases (Wal-Mart, Target, Amazon, ...)
- ▶ Thesauri: Networks of words generated by meanings
- Knowledge/Databases/Ideas
- ▶ Metadata—Tagging: bit.ly flickr

#### common tags cloud | list

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Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks

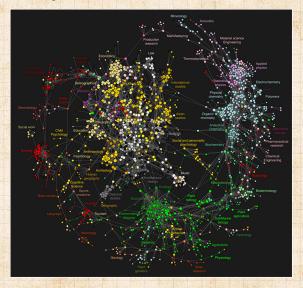
Relational networks







# Clickworthy Science:



"Clickstream Data Yields High-Resolution Maps of Science", Bollen et al. [4], 2009.

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Relational networks







# Neural reboot (NR):

Dog has fun.

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Complex Networks Basics

Etymology Graph theory? Basic definitions

Examples of Complex Networks Physical networks

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Complex Networks Basics Etymology

Graph theory?

Basic definitions

Examples of Complex Networks Physical networks Interaction networks







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Complex Networks Basics

Popularity
Graph theory?

Basic definitions

Examples of Complex Networks Physical networks Interaction networks







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Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks







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Networks Basics

Graph theory? Basic definitions

Examples of Complex Networks

Physical networks





