

Overview of Complex Networks

Principles of Complex Systems | @pocsvox
CSYS/MATH 300, Fall, 2015 | #FallPoCS2015

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Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont



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Etymology
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Graph theory?
Basic definitions

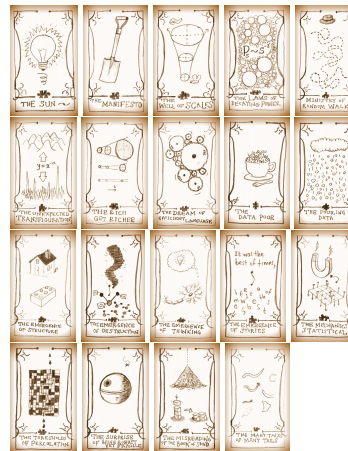
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net·work |'nɛt.wɜːk|

noun

- 1 an arrangement of intersecting horizontal and vertical lines.
 - a complex system of roads, railroads, or other transportation routes : *a network of railroads.*
- 2 a group or system of interconnected people or things : *a trade network.*
 - a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*
 - a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*
 - a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*
 - a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

- link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*
- [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

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Outline

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Thesaurus deliciousness:

network

noun

- 1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.
- 2 *a network of lanes* MAZE, labyrinth, warren, tangle.
- 3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

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Ancestry:

From Keith Briggs's excellent etymological investigation: 

- ▶ Opus reticulatum:
- ▶ A Latin origin?



[<http://serialconsign.com/2007/11/we-put-net-network>]

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Key Observation:

- ▶ Many **complex systems** can be viewed as **complex networks** of physical or abstract interactions.
- ▶ Opens door to mathematical and numerical analysis.
- ▶ Dominant approach of last decade of a **theoretical-physics/stat-mech** flavor.
- ▶ Mindboggling amount of work published on complex networks since 1998...
- ▶ ... largely due to your typical theoretical physicist:



- ▶ *Piranha physicus*
- ▶ Hunt in packs.
- ▶ Feast on new and interesting ideas (see chaos, cellular automata, ...)
- ▶ See also: <https://xkcd.com/793/> 

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Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- ▶ 1658–: reticulate structures in animals
- ▶ 1839–: rivers and canals
- ▶ 1869–: railways
- ▶ 1883–: distribution network of electrical cables
- ▶ 1914–: wireless broadcasting networks

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Popularity (according to Google Scholar)




"Collective dynamics of 'small-world' networks" 

Watts and Strogatz,
Nature, **393**, 440–442, 1998. ^[14]

Times cited: ~ **27,184**  (as of October 8, 2015)



"Emergence of scaling in random networks" 

Barabási and Albert,
Science, **286**, 509–511, 1999. ^[2]

Times cited: ~ **23,532**  (as of October 8, 2015)

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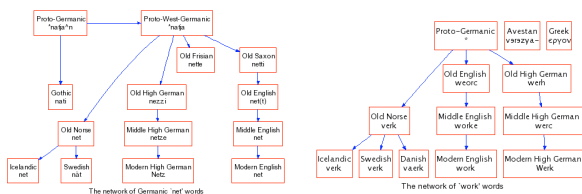


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Ancestry:

Net and Work are venerable old words:

- ▶ '**Net**' first used to mean spider web (King Ælfréd, 888).
- ▶ '**Work**' appear to have long meant purposeful action.



- ▶ 'Network' = something built based on the idea of natural, flexible lattice or web.
- ▶ c.f., ironwork, stonework, fretwork.

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Review articles:




"Complex Networks: Structure and Dynamics" 

Boccaletti et al.,
Physics Reports, **424**, 175–308, 2006. ^[3]

Times cited: ~ **5,791**  (as of October 8, 2015)



"The structure and function of complex networks" 

M. E. J. Newman,
SIAM Rev., **45**, 167–256, 2003. ^[10]

Times cited: ~ **13,156**  (as of October 8, 2015)



"Statistical mechanics of complex networks" 

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Times cited: ~ **15,676**  (as of October 8, 2015)

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Popularity according to textbooks:

Textbooks:

- ▶ Mark Newman (Physics, Michigan) "Networks: An Introduction"
- ▶ David Easley and Jon Kleinberg (Economics and Computer Science, Cornell) "Networks, Crowds, and Markets: Reasoning About a Highly Connected World"

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Numerous others ...

- ▶ Complex Social Networks—F. Vega-Redondo^[12]
- ▶ Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo^[11]
- ▶ Random Graph Dynamics—R. Durrett
- ▶ Scale-Free Networks—Guido Caldarelli
- ▶ Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- ▶ Complex Graphs and Networks—Fan Chung
- ▶ Social Network Analysis—Stanley Wasserman and Kathleen Faust
- ▶ Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster^[5]
- ▶ Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes^[6]

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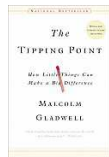
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Popularity according to books:



The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell^[7]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan

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More observations

- ▶ But surely **networks aren't new**...
 - ▶ Graph theory is well established...
 - ▶ Study of social networks started in the 1930's...
 - ▶ So why all this 'new' research on networks?
 - ▶ **Answer:** Oodles of Easily Accessible Data.
 - ▶ We can now inform (alas) our theories with a much more measurable reality.*
 - ▶ A worthy goal: establish **mechanistic explanations**.
- *If this is upsetting, maybe string theory is for you...*

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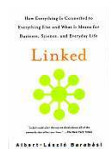
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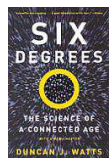


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Popularity according to books:



Linked: How Everything Is Connected to Everything Else and What It Means—Albert-László Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts^[13]

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More observations

- ▶ Web-scale data sets can be overly **exciting**.

Witness:

- ▶ The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired)
- ▶ "The Unreasonable Effectiveness of Data," Halevy et al.^[8]
- ▶ c.f. Wigner's "The Unreasonable Effectiveness of Mathematics in the Natural Sciences"^[15]

But:

- ▶ For scientists, description is only part of the battle.
- ▶ We still need to **understand**.

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Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- ▶ e.g., people, forks in rivers, proteins, webpages, organisms,...

Links = Connections between nodes

- ▶ **Links** may be directed or undirected.
- ▶ **Links** may be binary or weighted.

Other spiffing words: vertices and edges.

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Examples

So what passes for a complex network?

- ▶ Complex networks are **large** (in node number)
- ▶ Complex networks are **sparse** (low edge to node ratio)
- ▶ Complex networks are usually **dynamic** and **evolving**
- ▶ Complex networks can be social, economic, natural, informational, abstract, ...

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Super Basic definitions

Node degree = Number of links per node

- ▶ Notation: Node i 's degree = k_i .
- ▶ $k_i = 0, 1, 2, \dots$
- ▶ Notation: the average degree of a network = $\langle k \rangle$ (and sometimes z)
- ▶ Connection between number of edges m and average degree:

$$\langle k \rangle = \frac{2m}{N}.$$

- ▶ Defn: N_i = the set of i 's k_i neighbors

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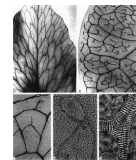
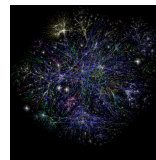


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Examples

Physical networks

- ▶ River networks
- ▶ Neural networks
- ▶ Trees and leaves
- ▶ Blood networks
- ▶ The Internet
- ▶ Road networks
- ▶ Power grids



- ▶ **Distribution** (branching) versus **redistribution** (cyclical)

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Super Basic definitions

Adjacency matrix:

- ▶ We represent a directed network by a matrix A with link weight a_{ij} for nodes i and j in entry (i, j) .
- ▶ e.g.,

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

- ▶ (n.b., for numerical work, we always use sparse matrices.)

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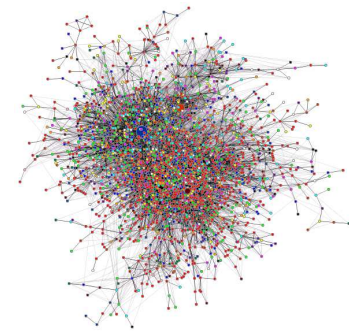


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Examples

Interaction networks

- ▶ The Blogosphere
- ▶ Biochemical networks
- ▶ Gene-protein networks
- ▶ Food webs: who eats whom
- ▶ The World Wide Web (?)
- ▶ Airline networks
- ▶ Call networks (AT&T)
- ▶ The Media



datamining.typepad.com

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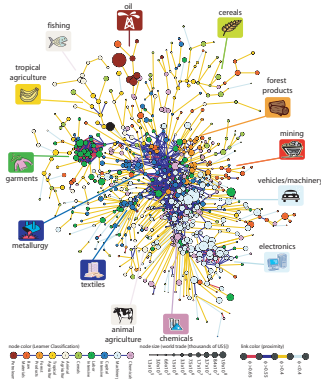
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topics:

- ▶ Hidalgo et al.'s "The Product Space Conditions the Development of Nations" [9]
- ▶ How do products depend on each other, and how does this network evolve?
- ▶ How do countries depend on each other for water, energy, people (immigration), investments?



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Examples

Relational networks

- ▶ Consumer purchases (Wal-Mart, Target, Amazon, ...)
- ▶ Thesauri: Networks of words generated by meanings
- ▶ Knowledge/Databases/Ideas
- ▶ Metadata—Tagging: bit.ly/1fXkRqG [flickr](https://www.flickr.com/photos/pocsvox/)

common tags cloud | [list](#)

community daily dictionary education [encyclopedia](#)
english free imported info information internet knowledge
learning news [reference](#) [research](#) resource
resources search tools useful web web2.0 [wiki](#)
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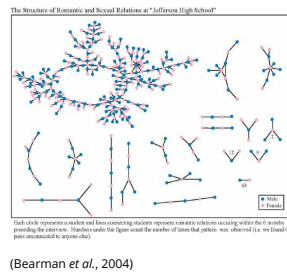


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Examples

Interaction networks: social networks

- ▶ Snogging
- ▶ Friendships
- ▶ Acquaintances
- ▶ Boards and directors
- ▶ Organizations
- ▶ [facebook](#) [twitter](#)
- ▶ 'Remotely sensed' by: email activity, instant messaging, phone logs (*cough*).



(Bearman et al., 2004)

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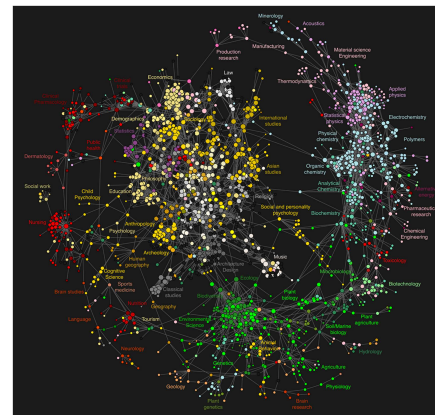
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Clickworthy Science:



"Clickstream Data Yields High-Resolution Maps of Science",
Bollen et al. [4], 2009.

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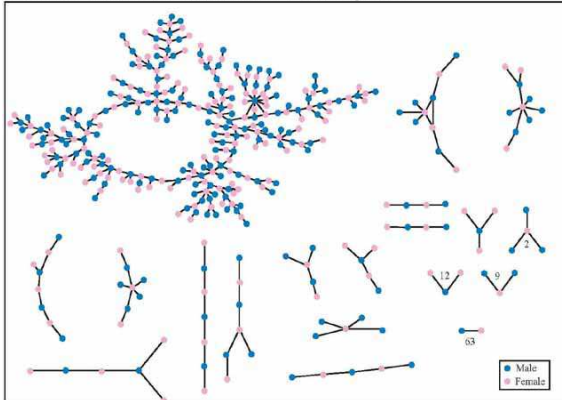
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Examples

The Structure of Romantic and Sexual Relations at "Jefferson High School"



Each circle represents a student and lines connecting students represent romantic relations occurring within the 6 months preceding the interview. Numbers under the figure count the number of times that pattern was observed (i.e. we found 63 pairs unconnected to anyone else).

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- [2] A.-L. Barabási and R. Albert. Emergence of scaling in random networks. [Science](#), 286:509–511, 1999. [pdf](#)
- [3] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez, and D.-U. Hwang. Complex networks: Structure and dynamics. [Physics Reports](#), 424:175–308, 2006. [pdf](#)

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Clickstream data yields high-resolution maps of science.
[PLoS ONE](#), 4:e4803, 2009. pdf

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[Handbook of Graphs and Networks](#).
Wiley-VCH, Berlin, 2003.

[6] S. N. Dorogovtsev and J. F. F. Mendes.
[Evolution of Networks](#).
Oxford University Press, Oxford, UK, 2003.

[7] M. Gladwell.
[The Tipping Point](#).
Little, Brown and Company, New York, 2000.



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The unreasonable effectiveness of mathematics in the natural sciences.
[Communications on Pure and Applied Mathematics](#), 13:1–14, 1960. pdf



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[8] A. Halevy, P. Norvig, and F. Pereira.
The unreasonable effectiveness of data.
[IEEE Intelligent Systems](#), 24:8–12, 2009. pdf

[9] C. A. Hidalgo, B. Klinger, A.-L. Barabási, and R. Hausman.
The product space conditions the development of nations.
[Science](#), 317:482–487, 2007. pdf

[10] M. E. J. Newman.
The structure and function of complex networks.
[SIAM Rev.](#), 45(2):167–256, 2003. pdf



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[11] I. Rodríguez-Iturbe and A. Rinaldo.
[Fractal River Basins: Chance and Self-Organization](#).
Cambridge University Press, Cambridge, UK, 1997.

[12] F. Vega-Redondo.
[Complex Social Networks](#).
Cambridge University Press, 2007.

[13] D. J. Watts.
[Six Degrees](#).
Norton, New York, 2003.

[14] D. J. Watts and S. J. Strogatz.
Collective dynamics of ‘small-world’ networks.
[Nature](#), 393:440–442, 1998. pdf

