The Amusing Law of Benford Principles of Complex Systems | @pocsvox CSYS/MATH 300, Fall, 2015 | #FallPoCS2015 PoCS | @pocsvox Benford's law

Benford's Law References

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# Outline

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#### Benford's Law

#### References





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Benford's Law

 $P(\text{first digit} = d) \propto \log_b \left(1 + \frac{1}{d}\right)$ 

for certain sets of 'naturally' occurring numbers in base *b* 

Around 30.1% of first digits are '1', compared to only 4.6% for '9'. First observed by Simon Newcomb' in 188' 'Note on the Frequency of Use of the Offere Digits in Natural Numbers' Independently discovered in 1938 by

Newcomb almost always noted but Benford gets the stamp, according to suglers, alwind





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#### Observed for

- Fundamental constants (electron mass, charge, etc.)
- Utility bills
- Numbers on tax returns (ha!)
- Death rates
- Street addresses
- Numbers in newspapers

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Cited as evidence of fraud <sup>C</sup> in the 2009 Iranian elections.





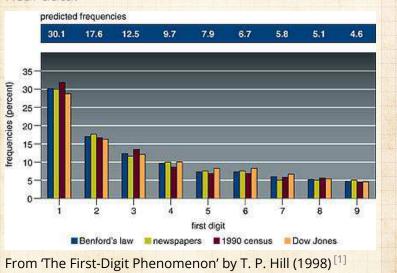
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#### Real data:



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#### Physical constants of the universe:

0.35 Benford's Law Physical Constants -0.3 025 0.2 Frequency 0.15 0.1 0.05 0 2 3 7 1 1 5 6 8 a

First Digit

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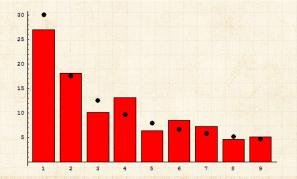
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#### Population of countries:

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 $P(\text{first digit} = d) \propto \log_b \left(1 + \frac{1}{d}\right)$ 

Observe this distribution if numbers are distributed uniformly in log-space:

 $P(\ln x) d(\ln x) \propto 1 \cdot d(\ln x)$ 

Power law distributions at work again Extreme case of  $\gamma \simeq 1$ .





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$$\begin{split} P(\text{first digit} = d) \propto \log_b \left(1 + \frac{1}{d}\right) \\ \propto \log_b \left(\frac{d+1}{d}\right) \end{split}$$

$$\propto \log_{h}\left(d+1\right) - \log_{h}\left(d\right)$$

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 $\propto \log_b\left(\frac{a+1}{d}\right)$ 

 Observe this distribution if numbers are distributed uniformly in log-space:

 $P(\ln x) \operatorname{d}(\ln x) \propto 1 \cdot \operatorname{d}(\ln x) = x^{-1} \operatorname{d} x$ 

Power law distributions at work again.. Extreme case of  $\gamma \simeq 1$ .

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Power law distributions at work again...
 Extreme case of γ ≃ 1.

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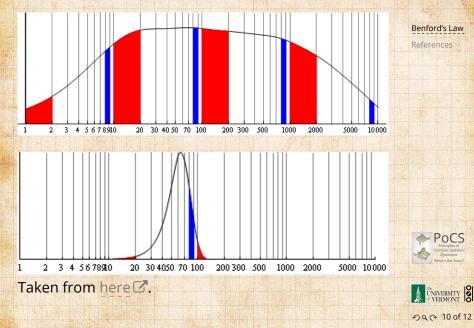


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#### On counting and logarithms:



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 Now: Benford's Law C.





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### **References** I

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 [2] S. Newcomb.
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