Review

Matrixology (Linear Algebra)—Episode 7/26
MATH 124, Spring, 2015

Prof. Peter Dodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center
Vermont Advanced Computing Core | University of Vermont

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Outline

Review for Challenge Level 1

Basics:

Material covered on first midterm:

- Episodes 1–7, online here.
- Chapter 1 and Chapter 2 (Sections 2.1–2.7).
- Chapter 2 is our focus.
- Knowledge of Chapter 1 as needed for Chapter 2 = solving $A\vec{x} = \vec{b}$.
- Want 'understanding' and 'doing' abilities.

Stuff to know:

Row, Column, & Matrix Pictures of Linear Systems ($A\vec{x} = \vec{b}$)

- What dimensions of A mean:
  - m = number of equations.
  - n = number of unknowns (x_1, x_2, ...).
- How to draw the row and column pictures.
- Be able to identify row picture (e.g., as representing 2 planes in 3-d).
- How to convert between the three pictures.

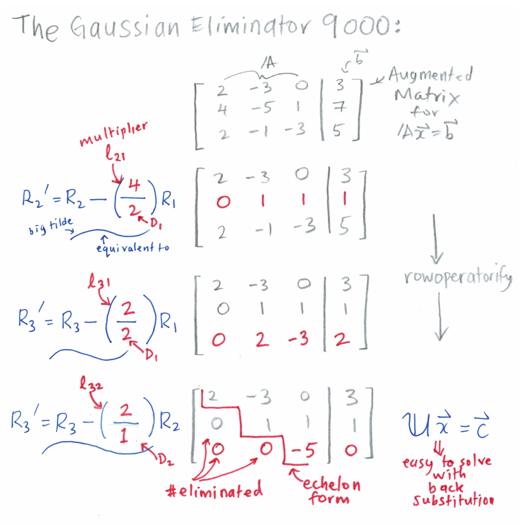
Solving $A\vec{x} = \vec{b}$ by elimination

Solve four equivalent ways:

1. Simultaneous equations (snore).
2. Row operations on augmented matrix:
   - Systematically transform $A\vec{x} = \vec{b}$ into $U\vec{x} = \vec{c}$ (and soon into $R\vec{x} = \vec{d}$).
   - Solve by back substitution.
3. Row operations with E_{ij} and P_{ij} matrices.
4. Factor A as $A = LU$:
   - Solve two triangular systems by forward and back substitution
   - First $L\vec{c} = \vec{b}$ then $U\vec{x} = \vec{c}$.
   - More generally, $PA = LU$ and sometimes $PA = LDU$ where D is the pivot matrix.

Understand number of solutions business:

- 0, 1, or ∞: why, when, ...



Stuff to know:

More on $PA = LU$ and $PA = LDU$:

- If A is m by n then P is m by m, L is m by m, and U is m by n.
- Be able to find the pivots of A (they live in U and are hidden in A).
- Understand how elimination matrices (E_{ij}'s). are constructed from multipliers (l_{ij}'s).
- Understand how L is made up of inverses of elimination matrices
  - e.g.: $L = E_{21}^{-1} E_{31}^{-1} E_{32}^{-1} A$.
- Understand how L is made up of the l_{ij} multipliers.
- Understand how inverses of elimination matrices are simply related to elimination matrices.

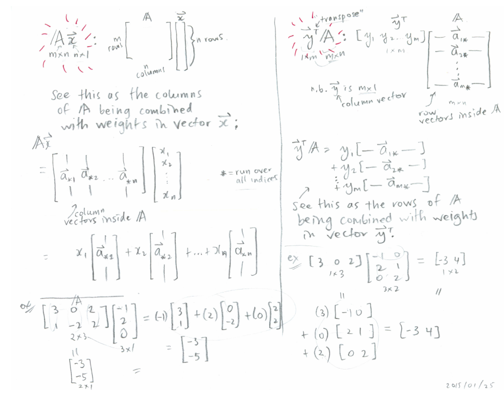
Matrix algebra:

- Understand basic matrix algebra (wrangling).
- Understand matrix multiplication.
- Understand multiplication order matters.
- Understand $AB = BA$ is rarely true.

Inverses:

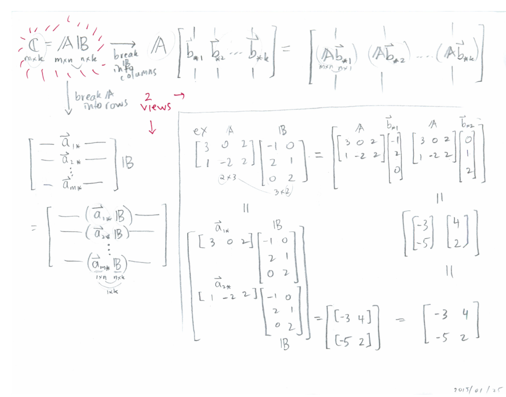
- Understand identity matrix I.
- Understand $A\,A^{-1} = A^{-1}A = I$.
- Find A^{-1} with Gauss-Jordan elimination.
- Perform row reduction on augmented matrix $[\,A\,|\,I\,]$.
- Understand that that finding A^{-1} solves $A\vec{x} = \vec{b}$ but is often prohibitively expensive to do.
- $(AB)^{-1} = B^{-1}A^{-1}$.

Advanced matrix wrangling 1/2:



Video explanation here.

Advanced matrix wrangling 2/2:



Video explanation here.

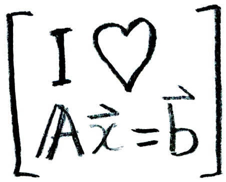
Stuff to know:

Transposes

- Definition: flip entries across main diagonal.
- $A = A^T$: A is symmetric.
- Important property: $(AB)^T = B^T A^T$.

Extra pieces:

- If $A\vec{x} = \vec{0}$ has a non-zero solution, A has no inverse.
- If $A\vec{x} = \vec{0}$ has a non-zero solution, then $A\vec{x} = \vec{b}$ always has infinitely many solutions.
- $(A^{-1})^T = (A^T)^{-1}$.



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