Dispersed: Wednesday, April 22, 2015.
Due: By start of lecture, Thursday, April 30, 2015.
Sections covered: 6.5, 6.7.

Some useful reminders:
Instructor: Prof. Peter Dodds
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Course website: http://www.uvm.edu/~pdodds/teaching/courses/2015-01UVM-124

- All questions are worth 3 points unless marked otherwise.
- Please use a cover sheet and write your name on the back and the front of your assignment.
- You must show all your work clearly.
- You may use Matlab to check your answers for non-Matlab questions (usually Qs. 1–8).
- Please list the names of other students with whom you collaborated.

Reminder: This assignment cannot be dropped.

1. (Q 4, 6.5) Show that the function $f(x_1, x_2) = x_1^2 + 4x_1x_2 + 3x_2^2$ does not have a minimum at $(0, 0)$ even though it has positive coefficients.

Do this by rewriting $f(x_1, x_2)$ as $[x_1 \ x_2]^T A [x_1 \ x_2]$ and finding the pivots of $A$ and noting their signs (and explaining why the signs of the pivots matter).

Write $f$ as a difference of squares and find a point $(x_1, x_2)$ where $f$ is negative.

Note of caution: All of this signs matching for pivots and eigenvalues falls apart if we have to do row swaps in our reduction.

2. (Q 9, 6.5) Find the 3 by 3 matrix $A$ and its pivots, rank, eigenvalues, and determinant:

$$\begin{bmatrix} x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 \\ x_1 & x_2 & x_3 \end{bmatrix} A \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4(x_1 - x_2 + 2x_3)^2.$$

Is this matrix positive definite, semi-positive definite, or neither?
3. (following set of questions based on Q 7, Section 6.7)

Singular Value Decomposition = Happiness.

Consider

\[ A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}. \]

(a) What are \( m, n, \) and \( r \) for this matrix?

(b) What are the dimensions of \( U, \Sigma, \) and \( V? \)

(c) Calculate \( A^T A \) and \( AA^T. \)

4. For the matrix \( A \) given above, find the eigenvalues and eigenvectors of \( A^T A \), and thereby construct \( V \) and \( \Sigma. \)

See this tweet for some post-it based help:

https://twitter.com/matrixologyvox/status/593540446845947904

5. For the same \( A \), now find the basis \( \{ \hat{u}_i \} \) using the essential connection \( A \hat{v}_i = \sigma_i \hat{u}_i. \)

Construct \( U \) from the basis you find.

Again see this tweet for some post-it based help:

https://twitter.com/matrixologyvox/status/593540446845947904

6. Next find the \( \{ \hat{u}_i \} \) in a different way by finding the eigenvalues and eigenvectors of \( AA^T. \)

7. (a) Put everything together and show that \( A = U \Sigma V^T. \)

(b) Draw the ‘big picture’ for this \( A \) showing which \( \hat{v}_i \)’s are mapped to which \( \hat{u}_i \)’s.

(c) Which basis vectors, if any, belong to the two nullspaces?

8. Finally, for this same \( A \), perform the following calculation:

\[ \sigma_1 \hat{u}_1 \hat{v}_1^T + \sigma_2 \hat{u}_2 \hat{v}_2^T + \ldots + \sigma_r \hat{u}_r \hat{v}_r^T \]

where \( r \) is the rank of \( A. \)

You should obtain \( A... \)


Verify the signs you found for the pivots of \( A \) in question 1 by using Matlab to find \( A \)’s eigenvalues.

10. Matlab question.

Use Matlab to compute the SVD for the matrix \( A \) you explored in questions 3–8.

11. (The bonus one pointer)

Where does the fearsome kiwi rank among among rattites and what’s unusual about the kiwi egg?