1. Find bases for the four subspaces associated with \( A\), possibly known as Prince Humperdinck:

\[
\begin{bmatrix}
1 & 2 & 4 \\
2 & 4 & 8
\end{bmatrix}
\]

You can do this most easily and most joyfully by finding the reduced row form of both \( A \) and \( A^\top \).

2. True or false (give a reason if true or a counterexample if false):

(a) If \( m = n \) then the row space of \( A \) equals the column space.

(b) The matrices \( A \) and \( -A \) share the same four subspaces.

(c) If \( A \) and \( B \) share the same four subspaces then \( A \) is a multiple of \( B \).

3. Suppose the 3 by 3 matrix \( A \) is invertible (hint: what will \( R_A \) be?). Write down bases for the four subspaces of \( A \), and also for the 3 by 6 matrix \( B = [A \ A] \) (i.e., two copies of \( A \) placed side by side).
4. Draw the ‘big picture’ for the following matrix:

\[
\begin{bmatrix}
1 & 2 \\
3 & 6
\end{bmatrix}
\]

(Hint: first find the rank \( r \), and the dimensions and bases for all four subspaces.)

On your diagram, please indicate subspace name, dimensions, and indicate how a point in row space maps to column space.

Note: this is not the abstract big picture but rather the particular big picture of this \( A \). So please sketch the actual subspaces of \( A \).

5. If \( S \) is a subspace of a vector space \( V \), then we use the notation \( S^\perp \) for its orthogonal complement.

   (a) If \( S \) is the subspace of \( \mathbb{R}^3 \) containing only the zero vector, what is \( S^\perp \)?

   (b) If \( S \) is spanned by \( \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} \) and \( \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix} \), what is \( S^\perp \)?

   (c) If \( S \) is spanned by \( \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \), what is \( S^\perp \)?

6. Construct a matrix with the required property or explain why you can’t:

   (a) Row space contains \( \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix} \) and \( \begin{bmatrix} 5 \\ 3 \\ 5 \end{bmatrix} \), and nullspace contains \( \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} \).

   (b) \( Ax = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \) has a solution and \( A^T \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \).

7. Find \( \vec{p} \), the projection of \( \vec{b} \) onto the vector \( \vec{a} \) given

\[
\vec{b} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix} \quad \text{and} \quad \vec{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.
\]

Also write down the error vector \( \vec{e} \) and check for orthogonality: \( \vec{p}^T\vec{e} = 0 \) (calculus notation: \( \vec{p} \cdot \vec{e} = 0 \)).

8. (Q 3ish, Section 4.2)

For the preceding problem, find the projection 3×3 matrix \( P = \vec{a}\vec{a}^T/((\vec{a}^T\vec{a}) \). Verify that \( P\vec{b} = \vec{p} \) and show that \( P^2 = P \).

(The value in having \( P \) is that we can reuse it to project any \( \vec{b} \). I know this is exciting for you.)
9. Matlab question:

Taking the same matrix from the previous assignment:

\[ A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \]

use Matlab’s rref command to find the following matrices along with a basis for the row space of each:

(a) \( R_A \),

(b) \( R_{AA^T} \),

(c) \( R_{A^TA} \).

Optional: Note any connections between these bases. Can you explain them?

10. Matlab question:

Taking the transpose of the matrix in the preceding question

\[ A^T = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix} \]

use Matlab’s rref command to find a basis for the row space of \( A^T \) by first finding the reduced row echelon form \( R_{A^T} \).

In terms of \( A \)’s four fundamental subspaces, note which one this basis is for, and show its dimensions make sense with your knowledge of \( m, n, \) and \( r \).

Optional: do you see any connection to the reduced row echelon forms in the preceding question?

11. (Bonus, 1 point)

What’s the main ingredient in vegemite?