Dispersed: Thursday, January 22, 2015.
Due: By start of lecture, Thursday, January 29, 2015.
Sections covered: 2.3, 2.4.

Some useful reminders:
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Course website: http://www.uvm.edu/~pdodds/teaching/courses/2015-01UVM-124

- All questions are worth 3 points unless marked otherwise.
- Please use a cover sheet and write your name on the back and the front of your assignment.
- You must show all your work clearly.
- You may use Matlab to check your answers for non-Matlab questions (usually Qs. 1–8).
- Please list the names of other students with whom you collaborated.

1. (similar to Q 24, Section 2.3) Apply elimination to the 2 by 3 augmented matrix \([ A \ b]\) for the equation given below. Do this using elimination matrix \(E_{21}\). What is the triangular system \(U\vec{x} = \vec{c}\)? What is the solution \(\vec{x}\)?

\[
\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}
\]

2. Write down the 3 by 3 matrices that produce the following elimination or permutation steps:
   (a) \(E_{21}\) subtracts 4 times row 1 from row 2.
   (b) \(E_{32}\) subtracts -3 times row 2 from row 3.
   (c) \(P_{23}\) swaps rows 2 and 3.

3. (modified version of Q 3, Section 2.3) Which three matrices \(E_{21}, E_{31},\) and \(E_{32}\) put \(A\) into triangular form \(U\)? What is \(U\) here? \(A = \begin{bmatrix} 1 & 1 & 0 \\ -2 & 2 & 0 \\ 4 & 6 & 1 \end{bmatrix}\) and \(E_{32}E_{31}E_{21}A = U\).
4. (Q 6, Section 2.4) Show that \((A + B)^2\) is different from \(A^2 + 2AB + B^2\), when
\[
A = \begin{bmatrix} 1 & 2 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 0 \\ 3 & 0 \end{bmatrix}.
\]
Write down the correct rule for \((A + B)(A + B)\).

5. (Q 14, Section 2.4) True or false (briefly explain why):
   
   (a) If \(A^2\) is defined then \(A\) is necessarily square.
   
   (b) if \(AB\) and \(BA\) are defined then \(A\) and \(B\) are square.
   
   (c) if \(AB\) and \(BA\) are defined then \(AB\) and \(BA\) are square.
   
   (d) if \(AB = B\) then \(A = I\) (where \(I\) is the identity matrix and the matrix \(B\) is not filled with zilches (0s)).

6. (Q 26, Section 2.4) Multiply \(AB\) using columns times rows:
\[
AB = \begin{bmatrix} 1 & 0 \\ 2 & 4 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 3 & 3 & 0 \\ 1 & 2 & 1 \end{bmatrix}.
\]
(You are calculating 'outer products' instead of inner products as we did for an example in Episode 4.)

7. Find all the powers \(A^2, A^3, \ldots\) and \(AB, (AB)^2, \ldots\) for
\[
A = \begin{bmatrix} .5 & .5 \\ .5 & .5 \end{bmatrix}
\]
and
\[
B = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.
\]

8. (Q 36, Section 2.4) Find all matrices \(A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\) that satisfy
\[
A \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} A.
\]

9. (2 pts)
Matlab action: Compute the following
\[
A = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \times \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.
\]
Incredibly, the above product of three matrices will be one very useful way to view \(A\).
More later.
10. (4 pts)

Matlab action:

For the $A$ you found in the previous question, compute (a) $A^2$, (b) $A^5$, (c) $A^{10}$, and (d) $A^{100}$.

What appears to be happening? We'll fully understand what's going on in about 10 more weeks.

11. (Bonus, 1 point)

List five of the numerous peculiarities of the very curious species ornithorhynchus anatinus.