# MATH 124: Matrixology (Linear Algebra)—Assignment 10
## University of Vermont, Fall 2011

**Dispersed:** Monday, November 28, 2011.
**Due:** 4 pm, end of office hours, Farrell Hall, Wednesday, December 7, 2011.
**Sections covered:** 6.5, 6.7.

*Some useful reminders:*
**Instructor:** Prof. Peter Dodds
**Office:** Farrell Hall, second floor, Trinity Campus
**E-mail:** peter.dodds@uvm.edu
**Office hours:** 12:50 pm to 3:50 pm, Wednesday
**Course website:** http://www.uvm.edu/∼pdodds/teaching/courses/2011-08UVM-124
**Textbook:** "Introduction to Linear Algebra" (3rd or 4th editions) by Gilbert Strang (published by Wellesley-Cambridge Press).

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All questions are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of other students with whom you collaborated.

**Reminder:** This assignment cannot be dropped.

1. (Q 4, 6.5) Show that the function $f(x, y) = x^2 + 4xy + 3y^2$ does not have a minimum at $(0, 0)$ even though it has positive coefficients.

   Do this by rewriting $f(x, y)$ as $\begin{bmatrix} x & y \end{bmatrix} A \begin{bmatrix} x \\ y \end{bmatrix}$ and finding the pivots of $A$ and noting their signs (and explaining why the signs of the pivots matter).

   Write $f$ as a difference of squares and find a point $(x, y)$ where $f$ is negative.

2. (Q 9, 6.5) Find the 3 by 3 matrix $A$ and its pivots, rank, eigenvalues, and determinant:

$$\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 4(x_1 - x_2 + 2x_3)^2.$$

   Is this matrix positive definite, semi-positive definite, or neither?

3. (following set of questions based on Q 7, Section 6.7)

   Singular Value Decomposition = Happiness.

   Consider

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}.$$

   (a) What are $m$, $n$, and $r$ for this matrix?

(b) What are the dimensions of $U$, $\Sigma$, and $V$?

(c) Calculate $A^{\mathrm{T}}A$ and $AA^{\mathrm{T}}$.

4. For the matrix $A$ given above, find the eigenvalues and eigenvectors of $A^{\mathrm{T}}A$, and thereby construct $V$ and $\Sigma$.

5. For the same $A$, now find the basis $\{\hat{u}_i\}$ using the essential connection $A\hat{v}_i = \sigma_i \hat{u}_i$.

   Construct $U$ from the basis you find.

6. Next find the $\{\hat{u}_i\}$ in a different way by finding the eigenvalues and eigenvectors of $AA^{\mathrm{T}}$.

7. (a) Put everything together and show that $A = U\Sigma V^{\mathrm{T}}$.

   (b) Draw the 'big picture' for this $A$ showing which $\hat{v}_i$'s are mapped to which $\hat{u}_i$'s.

   (c) Which basis vectors, if any, belong to the two nullspaces?

8. Finally, for this same $A$, perform the following calculation:

$$\sigma_1 \hat{u}_1 \hat{v}_1^{\mathrm{T}} + \sigma_2 \hat{u}_2 \hat{v}_2^{\mathrm{T}} + \ldots + \sigma_r \hat{u}_r \hat{v}_r^{\mathrm{T}}$$

   where $r$ is the rank of $A$.

   You should obtain $A$...

9. (The bonus one pointer)

   Where does the fearsome kiwi rank among among rattites and what's unusual about the kiwi egg?