2

A FRAMEWORK FOR THE STUDY OF INDIVIDUAL BEHAVIOR AND SOCIAL INTERACTIONS

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Recent work in economics has begun to integrate sociological ideas into the modeling of individual behavior. In particular, this new approach emphasizes how social context and social interdependences influence the ways in which individuals make choices. This paper provides an overview of an approach to integrating theoretical and empirical analysis of such environments. The analysis is based on a framework due to Brock and Durlauf (2001, forthcoming). Empirical evidence on behalf of this perspective is assessed and some policy implications are explored.

1. INTRODUCTION

> Just as our political life is free and open, so is our day-to-day life in our relations with each other. We do not get into a state with our next-door neighbour if he enjoys himself in his own way, nor do we give him the type of black looks which, though they do no real harm, still do hurt people's feelings. We are free and tolerant in our private lives; but

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> in public affairs we keep to the law. This is because it commands our deep respect. We give our obedience to those whom we put in positions of authority and we obey the laws themselves, especially those which are for the protection of the oppressed, and those unwritten laws which it is an acknowledged shame to break.
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> Pericles' *Funeral Oration to the Athenians*, c. 431–430 B.C.E.
> Thucydides, *History of the Peloponnesian War* (2.37)

> The Athenians owed to the plague the beginnings of a state of unprecedented lawlessness. Seeing how quick and abrupt were the changes of fortune which came to the rich who died and to those who had previously been penniless but now inherited their wealth, people now began openly to venture on acts of self-indulgence which before they used to keep dark. . . . As for what is called honour, no one showed himself willing to abide by its laws, so doubtful was it that one would survive to enjoy the name for it. It was generally agreed that what was both honourable and valuable was the pleasure of the moment.
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> Description of Effects of Plague in Athens, 430 B.C.E.
> Thucydides, *History of the Peloponnesian War* (2.53)[1]

Among the many fascinating features of the Thucydides description of the Peloponnesian War, which may be the first recorded piece of social science in Western history, is his sensitivity to the panoply of sources of individual behavior. One cannot read his description of the interplay of powerful social norms, extreme forms of individualism, and the development of a sense of individual rights of citizenship among Athenians, and not reflect that the complexities required to understand the rise and fall of Athens are echoed in the most modern attempts to understand various social and political groupings. As illustrated by the influence of the plague on Athens, longstanding features of the Athenian "character" could precipitously vanish due to changes in individual incentives. And yet it is the Athenian character that determined both the world-historical achieve-

[1]Taken from the translation by R. Walters (New York: Penguin Books, 1978).

ment of the first democracy and the world-historical folly of the destruction of the Athenian empire through overambition and overcommitment.

This paper is designed to describe an approach for the formal modeling of social interactions. The framework is based upon standard economic models of individual choice, but it expands the determinants of these choices to include social factors that are frequently neglected in economic analyses. While social interactions, broadly defined to include phenomena ranging from societal norms to role models to networks, are a fundamental part of historical studies and other social sciences—especially sociology, of course—they have only recently begun to play a prominent role in economic thinking. This new research is explicitly designed to extend the domain of inquiry by economists into areas that have been the traditional domain of other social scientists. Economists are not making this attempt, however, out of the belief that the substantive ideas in other disciplines should be supplanted by economic models of decision making. Rather, the objective of this research program is to internalize within formal economic models a number of the substantive ideas and perspectives of these other disciplines. This new work therefore has a very ambitious objective: the melding of substantive ideas between economics and sociology in such a way as to produce more powerful models of individual behavior. The current paper describes one class of efforts at such a synthesis.[2]

At some level, virtually all economic models exhibit some form of interactions. However, in standard economic models, these interactions are usually mediated by markets. Once an agent knows the prices for different commodities in the economy, the fact that these prices reflect the supply and demand decisions of others is no longer relevant. The new interest in social interactions among economists stems from an increasing awareness that individual interdependences are far richer than those that are induced by markets. Of course, the preeminence of game theory in modern economic theory reflects a general movement away from market-mediated to direct models of interactions. The new interactions-based models in economics are at one level game-theoretic models. What distinguishes them is the use

[2] To be clear, sociology has generated many analyses that use formal methods to study the sorts of social dynamics I model in this article. Granovetter and Soong (1988) offer a particularly interesting example and one that is closely linked to a number of ideas I try to address.

of particular stochastic processes to uncover interesting properties of interacting populations.

Interactions-based thinking has assumed a particularly prominent role in recent studies of poverty and inequality. Durlauf (1999a) refers to this body of theories as a "memberships theory of inequality" in contrast to the family-based theories of inequality and mobility pioneered by Becker and Tomes (1979) and Loury (1981).[3] In one facet of this new approach, the impact of residential neighborhoods on the future prospects of children has been explored. Bénabou (1993, 1996a, 1996b) and Durlauf (1996a,b) construct models of persistent intergenerational inequality based on the presence of spillover effects from the educational and economic characteristics of a neighborhood on the human capital acquisition of children. In these types of models, children are assumed to be influenced by a range of community characteristics. One source of community influences is institutional—because of local public finance of education, the affluence of a community affects the level of per capita spending on schools.[4] Another source of community effects occurs via role models. If individual aspirations and assessment of educational effort depend on the observed education levels and associated occupations of adults in a community, then stratification of communities by income and education will induce cross-community differences in the educational efforts and attainment of chil-

[3] In fairness, Loury (1977) is a seminal contribution to group-based approaches to the study of inequality.

[4] There is considerable controversy concerning the effect of educational expenditure on educational outcomes. Hanushek (1986) has argued that this type of effect is negligible, when test scores are the outcome variable of interest. In contrast, Card and Krueger (1992) find that predictions of future wages are sensitive to educational quality. I do not take a strong stand on this question, except to say that the empirical literature has typically focused on linear models, whereas the effects may be nonlinear. Certainly Kozol (1991) is consistent with this view, in the sense that he documents how very poor schools are handicapped in the education they provide. One reason for this type of nonlinearity is that while schools may differ widely in the efficiency with which they use revenues, some minimum is needed for each educational quality level. Of course, nonlinearities may also imply that the Card and Krueger results are questionable. For example, Heckman, Layne-Farrar, and Todd (1996) find that the estimated effects of school resources on labor market outcomes vary widely according to what control variables are included and according to educational group. For example, it appears that school quality matters primarily for workers who end up going to college. My own view of this literature is that while some effects of school quality on student outcomes have been identified, little is known about the causal mechanisms or even functional forms.

dren. Empirical evidence which has argued to support the presence of these effects can be found in many studies.[5] A particularly important case is the use of quasi-experiments in which one can compare families that have been given incentives to move to lower poverty neighborhoods with those which have not. While issues of self-selection and interpretation represent serious caveats, analyses based on both the Gautreaux demonstration (Rubinowitz and Rosenbaum 2000) and the Moving to Opportunity demonstration (Katz, Kling, and Liebman 2001) strongly support the focus on group determinants of inequality.

The recent attention on interactions and inequality reflects the strong influence that sociological perspectives on poverty have had on economics. When William Julius Wilson (1987:8) claims

> . . . changes have taken place in ghetto neighborhoods, and the groups that have been left behind are collectively different than those that lived in these neighborhoods in earlier years. It is true that long-term welfare families and street criminals are distinct groups, but they live and interact in the same depressed community and they are part of the population that has, with the exodus of the more stable working- and middle-class segments, become increasingly isolated socially from mainstream patterns and norms of behavior.

or Elijah Anderson (1999:22–23) argues

> The inclination to violence springs from the circumstances of life among the ghetto poor—the lack of jobs that pay a living wage, limited basic public services (police response in emergencies, building maintenance, trash pickup, light-

[5] Important examples in the sociology literature include Brewster (1994a, b); Brooks-Gunn et al. (1993); Crane (1991); Sampson, Morenoff, and Earls (1999); Sampson, Raudenbush, and Earls (1997); South and Crowder (1999); and Sucoff and Upchurch (1998). A nice recent example in economics is Weinberg, Reagan, and Yankow (2000). In addition, much of the massive literature on social capital is in essence attempting to uncover group influences. That being said, as discussed in Brock and Durlauf (2000a, 2000b), Durlauf (2001), and Manski (1993, 2000), there are a host of statistical issues with this literature that call into question exactly what causal relations have been identified in the empirical literature.

> ing . . .), the stigma of race, the fallout from rampant drug use and drug trafficking, and the resulting alienation and absence of hope for the future. Simply living in such an environment places young people at special risk of falling victim to aggressive behavior. Although there are often forces in the community that can counteract the negative influences—by far the most important is a strong loving . . . family that is committed to middle class values—the despair is pervasive enough to have spawned an oppositional culture, that of "the street" as consciously opposed to those of mainstream society.

one finds exactly the sort of substantive ideas that the new theories of inequality are trying to embody. And in fact one sees frequent reference to the sociological literature in justifying memberships-based models of inequality and intergenerational mobility.

Another important source of evidence on social interaction effects is the experimental literature in social psychology; many interesting examples may be found in Aronson (1999). Perhaps the most impressive study in this regard is the celebrated Robbers Cave experiment, described in Brown (1986) as "the most successful experiment ever conducted on intergroup conflict." This experiment is described by Sherif et al. (1961), collaborators who studied the behavior of a group of teenage boys at an isolated retreat in Robbers Cave State Park in Oklahoma. A group of boys were initially placed in a common living quarters and associated social environment. Once friendships and other social relations developed, the experimenters announced that the boys were assigned to two groups, Rattlers and Eagles. The new assignments were essentially random, with the exception that strong friendship pairs were broken up. A set of competitive activities were initiated. Sherif et al. (1961) documents in great detail how the two groups developed strong internal senses of identity along with great animosity toward the other group, animosity that carried over beyond the competitive activities. Previous friendships disappeared, and attribution of negative stereotypes to the other group became commonplace. While the introduction of cooperative activities diminished the hostility, the experiment clearly demonstrated that group identification can strongly influence individual behavior.

Rich qualitative descriptions of the type found in Anderson (1999) or Mitchell Duneier (1992, 1999) and carefully constructed experiments such as the Robbers Cave study are an important reminder of the limits of the type of formal analysis that is used in this paper. Formal modeling of the type I describe, in many respects only crudely approximates the many subtleties that are associated with social interactions; phenomena related to personal identity that Anderson explores are a good example.[6] On the other hand, to the extent that the objective of a research program is the construction of predictive or evaluative distributions of the effects of alternative policies, then the sort of formalization I describe is essential. To take a classic example, the Coleman report on the determinants of educational outcomes was based upon and was ultimately in many ways discredited because of formal analysis. Alternatively, the most important work on the effects of Head Start and other social programs on individual outcomes has proceeded from the sort of quantitative social science I describe.[7]

This paper is organized as follows. Section 2 develops an abstract model of individual choice in the presence of social interactions. Section 3 shows how this model can be specialized to produce different behavioral rules. Section 4 develops some of the interesting properties of the binary choice model. Section 5 addresses a number of conceptual features of interactions-based models. Section 6 discusses statistical implementation. Section 7 describes some implications of these models for public policy. Section 8 provides conclusions.

2. A BASIC FRAMEWORK

In this section, I develop a baseline description of individual behavior with social interactions for a population of individuals indexed by $i = 1 \ldots I$. I will follow standard economic reasoning in assuming that these behaviors represent purposeful choices subject to some set of preferences, beliefs, and constraints facing each individual. In other words, the choice of each individual i, ω_i, is interpreted as maximizing some payoff (or utility) function V subject to a set Ω_i of possible choices that

[6] Akerlof and Kranton (2000) is an important recent effort at grappling with these issues.

[7] Heckman (2000) is a wonderful overview of the contribution of formal statistical analysis to furthering social science knowledge; Heckman's own research is the exemplar of the uses of quantitative methods in understanding both causal determinants of individual behavior as well as in the evaluation of effects of government policies.

are available to that individual. The goal of the analysis is to develop a probabilistic description of $\underset{\sim}{\omega}$, the vector of choices in the population.

The functional form of V embodies specific features about individual preferences. The primitive modeling assumption underlying the use of such a function is *not* that individuals literally possess these functions and explicitly calculate payoffs from alternative courses of action by using them. Rather, the primitive notion is that each individual possesses preference orderings over the space of possible choices he faces and chooses the one ranked highest. When these preference orderings fulfill certain axioms,[8] they may be mathematically represented by a payoff function; from this perspective an individual's observed choice is the one that maximizes the function among all available choices.

This maximization problem can be given a generic form:

$$\omega_i = argmax_{\omega \in \Omega_i} V(\omega, Z_i, \epsilon_i). \tag{1}$$

The payoff function V is expressed as possessing three distinct arguments. The first is of course the choice; the second is a vector of individual-indexed characteristics, Z_i, which allows for observable heterogeneity in how individuals evaluate choices; the third is a vector of individual-indexed characteristics ϵ_i, which is assumed to be unobservable to a modeler, but is known to individual i, thereby allowing for unobservable heterogeneity across individuals. Introducing unobservable as well as observable heterogeneity is important in developing the model in a direction that permits empirical implementation. This distinction is precisely the same as that between regressors and the disturbance in the specification of a linear regression. In fact, in empirical implementation, the ϵ_i terms will be interpreted in standard statistical fashion. For this reason, I will always assume they are independent across individuals and independent of all Z_i.

Should one start with a utility-based framework in order to develop decision rules? The answer to this question lies at the core of one of the deepest methodological differences between sociologists and economists, and can hardly be addressed, let alone resolved here. For the purposes of this paper, I use the utility language in order to illustrate how a researcher,

[8] These axioms typically impose certain forms of rationality on the individual, such as transitivity of preferences. See Mas-Colell, Whinston, and Green (1995, chap. 3, sec. C) for an introduction to the relationship between preferences and utility functions.

starting with standard economic reasoning, can arrive at a model that embodies substantive notions of social influences on behavior. My own judgment is that choice-based reasoning, at this level of abstraction, is tautological, in the sense that without an explicit description of the determinants of preferences, beliefs, and constraints, any behavior can be interpreted as utility-maximizing in the sense I have described. In turn, the social interactions approach attempts to enrich the choice-framework by introducing a substantive role for social determinants of behavior. Hence, the framework should be judged as to how these social determinants are embodied.

The incorporation of social interactions into the choice framework is, at one level, nothing more than a particular choice as to what variables to include in Z_i. Suppose that the individuals in this population define a group g in which social interactions occur. (It is straightforward to generalize the discussion to the case where individuals are members of distinct groups.) For example, interactions can then be incorporated into individual decisions by including variables that depend on i through variables that are determined at the level of g—i.e., interactions are modeled as the dependence of individual payoffs on variables that depend on characteristics of the group. The average education level among parents in a community or the average rate of cigarette smoking among teenagers in a given ethnic group are examples of such variables.

Of course, the substance of social explanations to behavior will depend on what variables are included and in how the individual payoffs depend on them. In developing this choice-based framework, it is important to distinguish between variables representing the influence that a group's characteristics have on its members and those variables representing the influence that a group's joint behaviors have on its members. Following Manski (1993), variables that measure the former represent *contextual* effects whereas variables that measure the latter are *endogenous* effects. In the context of youth behavior, role models constitute contextual effects whereas the contemporaneous behaviors of friends constitute endogenous effects. This language, of course, closely parallels usage from sociology (cf. Blalock 1984). This consideration means that it is convenient to separate Z_i into three distinct components: X_i, which represents a vector of variables that can vary across individuals within the group, Y_g, often called contextual effects, which represents a vector of variables that are common to all members of the same group and are predetermined with respect to group behavior, and $\mu_i(\underline{\omega}|F_i)$, which captures endogenous effects, representing the beliefs of individual i concerning the choices of members of the group, given some information set F_i.

Contextual and endogenous effects represent group influences. The important difference between them is that contextual effects are usually treated as background variables for the analysis whereas endogenous effects refer to the social consequences of these choices. For example, consider the determination of the level of collegiality chosen by each member of an academic department. The overall level of collegiality in a department is an endogenous effect, if the choice of a level of collegiality by each department member is influenced by the collegiality of others. In contrast, suppose an individual's level of collegiality is influenced by his salary relative to the department average. The effect of his own salary on his behavior is an individual effect and so part of X_i, whereas the effect of others salaries on his behavior is a contextual effect and so is part of Y_g.[9]

The expression $\mu_i(\underset{\sim}{\omega}|F_i)$ refers to the subjective beliefs that individual i has about the choices of members of the population. These subjective beliefs are assumed to have the form of a conditional probability measure, but at this point, nothing is specified as to how these beliefs are formed, except that for individual i the beliefs do not reflect any information concerning ϵ_j for $j \neq i$. Later on, these beliefs will be made endogenous by specifying a relationship between them and the actual determinants of behaviors of the group. At first glance, the use of this expression in the payoff function might appear to be odd, in that each individual presumably knows his own choice and so should not be forming beliefs about it (ω_i is an element of $\underset{\sim}{\omega}$). The underlying idea is that each individual is affected by his beliefs about the choices of others, not himself; I use beliefs over all choices as an argument in the payoff function and implicitly assume that the payoff function for i is unaffected by i's beliefs about his own choice.

The decision to treat endogenous effects as occurring through beliefs concerning behavior rather than their actual behavior (i.e., the choice of $\mu_i(\underset{\sim}{\omega}|F_i)$ instead of the realized choices $\underset{\sim}{\omega}$ as an argument in the payoff function) makes the theoretical analysis of the model substantially simpler.[10] The appropriateness of this assumption, of course, will depend on the particular context in which the model is employed; one would think, for example, if an individual cares about the aggregate characteristics of a large group, then the expectations assumption makes particular sense.

[9] I thank Michael Sobel for suggesting this example.

[10] See Glaeser and Scheinkman (2000, 2001) for analyses that use realized behaviors in the payoff function.

The associated decision problem and choice of individual i can therefore be rewritten as:

$$\omega_i = argmax_{\omega \in \Omega_i} V(\underset{\sim}{\omega}, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i), \epsilon_i) \tag{2}$$

In order to close the model, it is necessary to specify how the subjective beliefs $\mu_i(\underset{\sim}{\omega}|F_i)$ are formed. Within economics, one standard assumption is that expectations are rational, which means that the subjective beliefs of individuals are consistent with the conditional probabilities that actually characterize the variables over which these beliefs are formed.

Operationally, rational expectations may be understood in two steps. First, consider the set of choices by members of the group. Suppose that each individual choice solves an optimization problem as described by equation (2). This means that the choice of each individual can be represented as determined by a choice function m,

$$\omega_i = m(X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i), \epsilon_i) i = 1 \ldots I. \tag{3}$$

Stacking these I choice functions together, one has a vector function M (whose elements correspond to the m-functions for each individual) such that

$$\underset{\sim}{\omega} = M(X_1, \ldots, X_I, Y_g, \mu_1(\underset{\sim}{\omega}|F_1), \ldots, \mu_I(\underset{\sim}{\omega}|F_I), \epsilon_1 \ldots \epsilon_I). \tag{4}$$

What this formulation means is nothing more than that the individual characteristics of each member of the population (X_i's and ϵ_i's), the common group characteristics that affect them (Y_g), and the beliefs each has about the behavior of members of his group ($\mu_i(\underset{\sim}{\omega}|F_i)$'s), determine the set of choices made by members of the population. This is a restatement of the choice-based logic we have assumed for individual behavior. Notice as well that one could start with expressions such as (3) and (4) as the basis for analysis of the model. Empirical work on social interactions in both sociology and economics typically does this. What the choice-based derivation does is establish how such formulations emerge from a particular set of underlying behavioral assumptions.

This formulation is useful in that it allows us to describe the conditional probabilities of actual choices as a function of the individual characteristics, group characteristics, and beliefs of the group members. To see how this may be used to characterize rationality, suppose that each individual possesses an identical information set F from which he forms beliefs about the choices of others in the population. Further, assume that F consists of $X_1 \ldots X_I$ and Y_g. (This is a strong assumption on the amount of information individuals possess; reformulating the model with weaker information assumptions turns out not to add anything except cumbersome notation.) By construction, each individual uses this information to form identical beliefs $\mu^e(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g)$. One can, using equation (4), compute the actual conditional probability of the vector of choices given the model and the information set F. From equation (4), it follows that

$$\mu(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g) = \mu(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g,\mu^e(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g)). \quad (5)$$

If there exists a probability measure $\mu(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g)$ such that

$$\mu(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g) = \mu(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g,\mu(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g)) \quad (6)$$

then this measure $\mu(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g)$ is a rational expectations solution to the model. In words, agents possess rational expectations conditional on the information set F when their beliefs given F, as represented by subjective conditional probabilities, are confirmed by the actual conditional probabilities which arise in the environment under study. For this reason, the concept of rational expectations is synonymous in these models with the ideas that beliefs are self-consistent.

Notice that all I have done is define the meaning of rational expectations Nothing has been established about the conditions under which a set of rational expectations exists, or if it exists, whether the set is unique. Properties such as these can only be assessed in the context of particular specifications of an environment.

The rational expectations assumption is controversial, and dissatisfaction with it has led over the last 15 years to a rich literature on bounded rationality (Rubinstein [1998] provides a profound overview of this work). However, the literature on interactions-based models has generally not incorporated this approach in an interesting fashion,[11] and I will assume

[11] In saying that bounded rationality or learning spillovers have not been dealt with in an "interesting" way, I mean that many if not most models that claim to be based on bounded rationality fail to generate insights that differ from models where agents interact through preferences. To be a bit more precise, one can model the effects of the behavior of peers on an individual as due to two distinct factors: (1) a psychological desire to conform to one's peers, which is a claim about interdependent preferences, and (2) an information effect whereby the behavior of others is used by an individual to determine which choice is better for him, which (when not formulated as an optimal extraction of information) is a form of bounded rationality. On the other hand, there is an important related literature on social learning in which the behavior of individuals alters the information sets of others in an environment in which each individual acts rationally conditional on a limited information set; see Bikhchandani, Hirshleifer, and Welch (1992) for a very nice analysis of this type.

rational expectations in what follows, for two reasons. First, the rationality assumption is in certain respects inessential for understanding the qualitative properties of these systems. For example, the rational expectations equilibria of a static model often prove to be the limit points of various learning schemes (see Brock and Durlauf [1998, 2001] and Glaeser and Scheinkman [2000] for specific examples of this). Second, the interesting qualitative properties of interactions-based models do not rely on rational expectations per se but rather are generated by the presence of feedbacks between group and individual behaviors.

Again, this abstract description of individual behaviors in a population under rational expectations incorporates very standard economic reasoning. Individual decisions are explicitly modeled as purposeful choices, and a consistency condition is imposed across the choices, in this case consistency of beliefs with the probabilistic structure of the population's behavior. This combination of individual maximization and self-consistency is no different from what occurs when one specifies a set of individual demand and supply functions for a group of commodities, where each individual takes prices as given, and then requires that prices clear markets.

3. BEHAVIORAL RULES

The choice-based framework described in Section 2 can be specialized in various ways. In this section, two different approaches are outlined.

3.1. *Linear Decision Rules*

Much of the empirical work on interaction effects has assumed that the behavior variable ω_i has continuous support and depends linearly on various individual and neighborhood effects. These assumptions permit a researcher to use standard regression methods for estimation, as will be seen in Section 5. While these regressions have typically not been developed via choice-based reasoning, it is straightforward to do so. Suppose that each individual makes a choice ω_i in order to minimize the squared distance from some ideal point ω_i^*

$$max_{\omega_i \in (-\infty,\infty)} -\tfrac{1}{2} E_i(\omega_i - \omega_i^*)^2 \tag{7}$$

and suppose that this ideal point ω_i^* is defined by

$$\omega_i^* = h_i + Jm_g + \epsilon_i,$$

where m_g is the expected value of the average choice in the population. It is immediate that actual behavior ω_i will follow

$$\omega_i = \omega_i^* = h_i + Jm_g + \epsilon_i. \tag{8}$$

This derivation is, of course, trivial and perhaps is a good example of how a choice-based perspective does not always add insight into an assumed behavioral rule. Nevertheless, one can develop a couple of implications of this type of model that are of interest. Suppose one takes the expected value of both sides of this equation, under the assumption that the values of all h_i are known to group members. Taking the expected value of both sides of (8),

$$m_g = h_g + Jm_g, \tag{9}$$

where $h_g = I^{-1} \sum_{i \in g} h_i$. It is immediately the case that self-consistency imposes the restriction

$$m_g = \frac{h_g}{1 - J}. \tag{10}$$

This solution has two relevant properties. First, the equilibrium expected choice level is unique as each value of h_g maps into a single m_g. Second, J cannot equal 1; in fact one can usually rule out $|J| > 1$ through analyzing dynamic analogs to this model.

3.2. *Binary Decision Rules*

Interesting theoretical models of social interactions have been developed in the context of binary decisions. Standard examples of these decisions include staying in school or dropping out, college attendance versus work, etc. I will assume that the two choices are coded 1 and -1. In this development, I will proceed in two steps, paralleling the derivations in Section 2. First, the model is formulated for an arbitrary set of subjective beliefs $\mu_i(\underset{\sim}{\omega}|F_i)$. Second, rational expectations will be imposed.

For the binary choice model, the individual decision process equation (2) can be expressed as

$$\omega_i = argmax_{\omega \in \{-1,1\}} V(\omega, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i), \epsilon_i) \tag{11}$$

At this point, of course, one still cannot say much about the structure of the group choices that are produced by such a general decision problem. One must place some assumptions on the structure of the $V(\cdot,\cdot,\cdot,\cdot,\cdot)$ function in order to see what insights interactions add to modeling an environment of this type as well as to making the model falsifiable. In this section, I will introduce some assumptions on the form of V that accomplish these goals, following the analysis in Brock and Durlauf (2001, forthcoming).

The first assumption is that the payoff function is additively separable into three distinct components, so that

$$V(\omega_i, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i), \epsilon_i) = u(\omega_i, X_i, Y_g) + S(\omega_i, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i)) + \epsilon_i(\omega_i) \quad (12)$$

Here, $u(\omega_i, X_i, Y_g)$ denotes private deterministic utility, $S(\omega_i, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i))$ denotes social deterministic utility, and $\epsilon_i(\omega_i)$ denotes a private random utility. The distinction between deterministic and random utility is made from the perspective of the modeler—i.e., the model is analyzed under the assumption that only the distribution of the various $\epsilon_i(\omega_i)$'s are known. Notice that this error term is now made an explicit function of ω_i. What this means is that the two choices may have differential effects on the payoff function. So, for example, if one is choosing between a career as a musician ($\omega_i = 1$) or as a painter ($\omega_i = -1$), $\epsilon_i(1)$ represents unobservable musical talent and $\epsilon_i(-1)$ represents unobservable artistic talent.

The additive separability assumption is made for two reasons. First, separating out the random term $\epsilon_i(\omega_i)$ is essential in achieving analytic tractability for the model. Second, this formulation is attractive in terms of empirical implementation. It will turn out that when $S(\omega_i, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i))$ is omitted from the model, the individual behavioral rule reduces to that described by the standard binary choice model, hence one will be able to test for social interactions using relatively standard statistical methods.

The second assumption is that social utility possesses a particular functional form. Let $E_i(\omega_j|F_i)$ denote the subjective expected value that agent i assigns to the choice of agent j given his information set F_i. Social utility is assumed to take the form

$$S(\omega_i, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i)) = -\sum_{j\neq i} \frac{J_{i,j}}{2} (\omega_i - E_i(\omega_j|F_i))^2. \quad (13)$$

The $J_{i,j}$ terms measure the magnitudes of the direct bilateral interactions between members of the population. If $J_{i,j} > 0$, then individual i derives higher utility (other things being equal) from making the same choice as he believes will be made by individual j; $J_{i,j} < 0$ in turn implies a utility benefit from acting differently.

By suitable choices of the $J_{i,j}$'s, one can, in principle, model a wide range of interactions. For example, suppose that there is a single individual k such that $J_{i,k}$ is relatively very large for all i. This person can be thought of as a leader in the group's behavior. Brock and Durlauf (1999) show how this type of formulation can be used to model the development of schools of thought in scientific communities. There is no requirement that the $J_{i,j}$ terms all have the same sign. When these signs differ, incentives to conform and deviate coexist. One example of this may be dialect use, where the choice of nonstandard forms of grammar and syntax appear to stem from a desire for membership in some groups and a rejection of identification with others. For example, as described in Chambers (1995), the dropping of the letter *g* in words ending in *-ing* is strongly associated with being poor and male in the United States, Britain, and Australia. This is generally explained by the need for poorer males to develop an identity that rejects conventional metrics of success. The framework described here would seem to be a natural way of exploring the use of African-American vernacular English versus standard dialects.

Finally, one completes the model by choosing a distribution for the random terms $\epsilon_i(\omega_i)$. This is done by assuming that the difference in the random utility terms is logistically distributed,

$$\mu(\epsilon_i(\omega_i) - \epsilon_i(-\omega_i) \leq z) = \frac{1}{1 + exp(-\beta_i z)}; \beta_i \geq 0. \tag{14}$$

As in the case of the other assumptions, this functional form provides benefits in terms of analytics as well as a way of linking the theoretical model to a statistical one. Notice that β_i indexes the support of the unobserved heterogeneity. Roughly speaking, the larger the value of β_i, the less likely are large draws of $|\epsilon_i(-\omega_i) - \epsilon_i(\omega_i)|$. The logistic error assumption is extremely useful in the development of theoretical models of social interactions as it allows for simple calculations of the equilibrium probability measure for choices, but the qualitative features of the these types of models do not depend on it.

Under these assumptions, it is possible to derive some parsimonious expressions to describe the set of population choices. Before doing so, there is a useful simplification that can be made. One can without loss of generality replace the general utility function $u(\omega_i, X_i, Y_g)$ with a linear function

$$u(\omega_i, X_i, Y_g) = h_i \omega_i + k_i, \tag{15}$$

where the slope term h_i and intercept term k_i are chosen so that

$$h_i + k_i = u(1, X_i, Y_g) \tag{16}$$

and

$$-h_i + k_i = u(-1, X_i, Y_g) \tag{17}$$

This simplification is allowable because choices are binary; so long as the new linear utility function matches the original u function when ω_i equals either -1 or 1—which is what equations (16) and (17) impose via the implied restrictions on h_i and k_i—it is irrelevant that it fails to match the original function for other values of ω_i.

The model now has enough detail to allow a parametric description of the conditional probabilities of the vector of choices $\underset{\sim}{\omega}$. One does this first by calculating

$$\mu(\omega_i | X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i)), \tag{18}$$

the conditional probability of individual i's choice given his observable characteristics and beliefs. Since the choice ω_i is made only when the payoff from the choice exceeds that which would be generated by the choice $-\omega_i$, for any information set the conditional probability of ω_i is equal to the probability that the payoff at ω_i is greater than the payoff at $-\omega_i$—that is,

$$\begin{aligned} &\mu(\omega_i | X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i)) \\ &\quad = \mu(V(\omega_i, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i), \epsilon_i(\omega_i)) \\ &\qquad > V(-\omega_i, X_i, Y_g, \mu_i(\underset{\sim}{\omega}|F_i), \epsilon_i(-\omega_i))). \end{aligned} \tag{19}$$

Substituting the social utility function (13) and the linearized deterministic private utility function (15) into equation (12), this inequality may be rewritten as

$$\mu\left(h_i\omega_i - \sum_{j\neq i}\frac{J_{i,j}}{2}(\omega_i - E_i(\omega_j|F_i)\right)^2 + \epsilon_i(\omega_i)$$

$$> -h_i\omega_i - \sum_{j\neq i}\frac{J_{i,j}}{2}(-\omega_i - E_i(\omega_j|F_i))^2 + \epsilon_i(-\omega_i))$$

$$= \mu(\epsilon_i(\omega_i) - \epsilon_i(-\omega_i) > -2h_i\omega_i - \sum_{j\neq i}2J_{i,j}\omega_i E_i(\omega_j)). \qquad (20)$$

Using equation (14), the logistic assumption for the errors, it is straightforward to manipulate this expression and conclude that

$$\mu(\omega_i|X_iY_g,\mu_i(\underset{\sim}{\omega})) \propto exp(\beta_i h_i\omega_i + \sum_{j\neq i}\beta_i J_{i,j}\omega_i E_i(\omega_j)), \qquad (21)$$

where "∝" means "is proportional to."

Moving from individual to joint conditional probabilities is now trivial, since the random utility terms are independent across individuals. The joint probability measure for the population choices is

$$\mu(\underset{\sim}{\omega}|X_1,\ldots,X_I,Y_g,\mu_1(\underset{\sim}{\omega}|F_1),\ldots,\mu_I(\underset{\sim}{\omega}|F_I))$$

$$\propto \prod_i exp\left(\beta_i h_i\omega_i + \sum_{j\neq i}\beta_i J_{i,j}\omega_i E_i(\omega_j)\right). \qquad (22)$$

Once one specifies the distributions across the population of private incentives, h_i—as derived through equations (16) and (17)—the probability distribution of unobserved heterogeneity, β_i, the interaction terms, $J_{i,j}$, and beliefs about the behaviors of others, $E_i(\omega_j)$, one has a complete characterization of the joint probability measure for observed behaviors $\underset{\sim}{\omega}$. This explicit mapping of the distributions of individual characteristics and interdependence terms $J_{i,j}$ into a distribution of individual behaviors is the hallmark of interactions-based models.

In the context of binary choice, rational expectation requires that the beliefs $E_i(\omega_j)$ coincide with the mathematical expectations $E(\omega_j)$ following the sort of self-consistency argument given by equation (18). Recalling the definition of the hyperbolic tangent function, $tanh(x) = (e^x - e^{-x})/(e^x + e^{-x})$, we can use equation (21) to verify that the expected value of each choice ω_i obeys

$$E(\omega_i|X_i,Y_g,\mu_i(\underset{\sim}{\omega}|F_i)) = tanh\left(\beta_i h_i + \sum_{j\neq i}\beta_i J_{i,j}E_i(\omega_j)\right). \qquad (23)$$

A rational expectations equilibrium for this model requires that there exists a set of numbers $E(\omega_i)$ such that for all i and j

$$E(\omega_i) = tanh\Big(\beta_i h_i + \sum_{j \neq i} \beta_i J_{i,j} E(\omega_j)\Big), \ i = 1 \ldots I \tag{24}$$

The question of the existence of a rational expectations equilibrium is thus a fixed point problem for the set of I equations described by equation (24). Fortunately, this is an easy case to analyze. Since the $tanh(\cdot)$ function is continuous with range $[-1,1]$, Brouwer's fixed-point theorem may be immediately invoked (see Mas-Colell, Whinston, and Green [1995:952] for a statement of the theorem) to establish that at least one rational expectations solution exists.

4. PROPERTIES OF A BINARY CHOICE MODEL WITH SOCIAL INTERACTIONS

The binary choice model described in Section 3 can be specialized in many ways to incorporate different types of decisions, interactions environments, and the like. In order to elucidate the general properties of models of this type, it is useful to consider a baseline case that has been extensively analyzed in Brock and Durlauf (2001). This case assumes that for all i and j (1) $h_i = h$, (2) $\beta_i = \beta$, and (3) $J_{i,j} = J/I - 1 \geq 0$. Substantively, these assumptions do two things. First, the three assumptions eliminate all heterogeneity in individual behavior except that which is generated by the errors $\epsilon_i(\omega_i)$. Second, the social interactions have the property that each individual weighs the decisions of all others equally. This is obviously a strong restriction on the nature of conformity effects.

Under these assumptions, the system of equations described by (24) reduces to

$$E(\omega_i) = tanh\Big(\beta h + \frac{\beta J}{I-1} \sum_{j \neq i} E(\omega_j)\Big) \ \forall \ i, j. \tag{25}$$

Since each agent is associated with the same parameters β, h, and J, one can show that all expectations $E(\omega_i)$ are equal. This in turn implies that the group $m = I^{-1} \sum_j E(\omega_j)$ equals $E(\omega_i)$ and that the average choices of others relative to i, $m_{-i} = (I-1)^{-1} \sum_{j \neq i} E(\omega_j)$ must equal this same number. Therefore m will obey the functional relationship

$$m = tanh(\beta h + \beta J m). \tag{26}$$

Any m that is consistent with this equation is a possible equilibrium for expected average group-level behavior. At least one solution m solves this equation, as was discussed in the analysis of the binary choice model in Section 3.

4.1. *Multiple Equilibria*

Once the existence of an equilibrium expected choice level m has been established, a second question is to evaluate whether equation (26) possesses a unique solution. When multiple solutions exist, then the individual-level or micro-level structure of the model does not uniquely determine its macro-level characteristics. Why should one think that, in expectation, the average choice level is not uniquely determined? The answer lies in the assumption that $J \geq 0$. The magnitude of J influences the extent to which each individual makes a choice based on his or her beliefs concerning the choices of others. When this conformity effect is strong enough, it means that for many population members, the desire to conform to others dominates the other factors which influence choice. But when individual behavior is driven by a desire to be similar to others, this does not provide any information on what they actually do; rather it merely implies that whatever behaviors occur, there will be substantial within-group correlation due to conformity effects. Of course, the role of the conformity effect is determined by its strength relative to the private incentives agents face.

These considerations suggest that the number of equilibria should reflect an interplay of the various parameters of the model. Brock and Durlauf (2001) contains the following theorem that characterizes the number of equilibria in this model.

Theorem 1: Relationship between individual behavioral parameters and number of self-consistent equilibria in the binary choice model with social interactions

i. If $\beta J < 1$, then there exists a single solution to equation (26).
ii. If $\beta J > 1$ and $h = 0$, there exist three solutions to equation (26). One of these solutions is positive, one is zero and one is negative.
iii. If $\beta J > 1$ and $h \neq 0$, there exists a threshold H (which depends on βJ) such that
   a. for $|\beta h| < H$, there exist three solutions to equation (26) one of which has the same sign as h, and the others possessing opposite sign.

b. for $|\beta h| > H$, there exists a unique solution to equation (26) with the same sign as h.

This theorem provides a description of the ways in which private incentives, h, unobserved heterogeneity, β, and social incentives, J, combine to determine the number of self-consistent equilibria in this population. Notice that it is the interplay of private and social incentives that determines the multiplicity versus uniqueness of the equilibrium. Suppose that one fixes β and J so that $\beta J > 1$. In this case, different values of h will induce different numbers of equilibria. This is qualitatively illustrated in Figure 1.

A critical role is played by the βJ, in that large (in a sense specified in the theorem) values of this composite parameter are required for multiplicity. The role of J is easy to understand. Small values of J mean that the strength of endogenous interactions is weak, which mitigates against

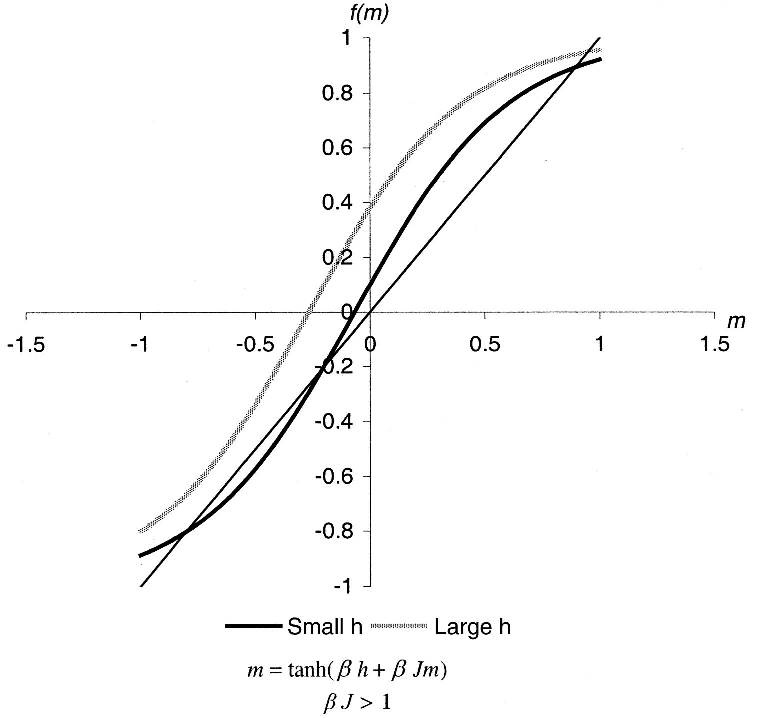

FIGURE 1. Equilibria for expected average choice.

self-consistent bunching of behavior. To understand the role of β, recall that small values of β imply that the likelihood of a large value of $|\epsilon_i(1) - \epsilon_i(-1)|$ is large. Hence for small β's, a relatively large percentage of the population will have draws of $\epsilon_i(1) - \epsilon_i(-1)$ which, roughly speaking, dominate the deterministic parts of their payoffs. Furthermore, the expected value of the percentage who are led to choose 1 due to the random utility draws will, in expectation, be the same as the expected value of the percentage of the population led to choose -1. In other words, small β's imply that a relatively small percentage of the population is susceptible to self-consistent bunching, in the sense that their decisions are likely to be dominated by the social utility component of their payoff functions. Now, consider what is needed for multiple self-consistent equilibria. Intuitively, what is needed is that enough of the population is susceptible to being influenced by the expected choices of others. But this requires, for fixed J, relatively large values of m. When the percentage of individuals whose behavior is dominated by the unobserved heterogeneity is high enough, then the range of possible values of m is restricted, which precludes the self-consistent bunching at multiple levels.

Under standard dynamic analogs to this static model, it turns out that the equilibria with the largest and smallest values of m are locally stable, whereas the equilibrium whose associated m lies between them is not.[12] One can ignore this "interior" equilibrium as it cannot be expected to arise in practice. Thus, when multiple equilibria are present, we can restrict attention to the possible equilibrium m_+^*, in which the average choice is expected to be positive, and m_-^*, in which the average choice is expected to be negative.

4.2. *Social Welfare*

The presence of multiple equilibria leads to the question of the relationship between a particular equilibrium and aggregate social welfare. In the baseline case of common h, β, and J values, it is natural to ask which equilibrium maximizes the expected average payoff in the population. Brock and Durlauf (2001) prove the following theorem

Theorem 2: Welfare rankings of equilibria

[12] These dynamic analogs typically assume that agents at t react to the expected average choice for $t - 1$.

i. If $h > 0$ (< 0) then the equilibrium associated with m_+^* (m_-^*) provides a higher level of expected utility for each agent than the equilibrium associated with m_-^* (m_+^*).
ii. If $h = 0$, then the equilibrium associated with m_+^* and the equilibrium associated with m_-^* provide equal levels of expected utility for each agent.

In words, when $h \neq 0$, individuals are better off when the average choice is of the same sign as h. Intuitively, when this holds, the social incentives to conform work in the same direction as the private incentives. On the other hand, when $h > 0$ and $m = m_-^*$, these incentives clash. In this case, the average member of the population would be better off if the other equilibrium prevailed. The existence of an inferior equilibrium in the sense described illustrates how individually rational decisions can be collectively undesirable.

5. CONCEPTUAL ISSUES OF INTERACTIONS-BASED MODELS

5.1. *Methodological Individualism*

Interactions-based models represent an effort to introduce richer sociological structure into economic theory. The absence of such structures has been the source of severe criticism of economic theory by social scientists who are not economists as well as among heterodox economists. Granovetter (1985) gives a typical critique:

> Classical and neoclassical economics operates, in contrast, with an atomized and *under*socialized conception of human action . . . The theoretical arguments disallow by hypothesis any impact of social structure and social relations on production, distribution, or consumption. (P. 55)

The interactions-based approach represents one way of answering this criticism without sacrificing any of the basic microeconomic behavioral assumptions of economics. I make this claim in two respects: one superficial and the other somewhat deeper.

First, the modeling exercise described in Sections 2, 3, and 4 illustrates how one can formally integrate group-level influences into individual decisions so long as these influences can be modeled as variables, as is done with Y_g and m_g. Self-consistency conditions of the type we have modeled impose whatever feedbacks exist between members of a group, in ways exactly analogous to the relationship between the modeling of individual demand schedules as functions of prices and the requirement that these prices clear markets.

This perspective on methodological individualism is quite similar to that taken by Elster (1989) in the context of studying social norms:

> I believe one can define, discuss, and defend a theory of social norms in a wholly individualistic framework. *A norm in this perspective is the propensity to feel shame and to anticipate sanctions by others at the thought of behaving in a certain forbidden way* . . . this propensity becomes a *social* norm when and to the extent that it is shared with other people . . . the social character of the norm is also manifest in the existence of higher-order norms that enjoin us to punish violators of the first-order norm. To repeat, this conception of a network of shared beliefs and common emotional reactions does not commit us to thinking of norms as supraindividual entities that somehow exist independently of their supports. (Pp. 105–106)

Second, the modeling framework I have described illustrates how social interactions lead to features of group behavior that are qualitatively different from those that arise in environments without such interactions. This is an example of the property of “emergence” that is a common feature of environments of this type. Emergence leads to the question of the relationship between these models and statistical mechanics models in physics, which is discussed next.

5.2. *Statistical Mechanics and Social Science*

The binary model of social interactions that I have described lies in a class of mathematical models originating in physics, specifically in the area of statistical mechanics.[13] These models were originally developed to explain how magnets arise in nature. A magnet occurs when, for a piece

[13] Yeomans (1992) is an accessible introduction to statistical mechanics.

of iron, a majority of the atoms spin up or spin down (spin being a binary property of atoms). In the early twentieth century, the existence of natural magnets was a major puzzle, as there are physical reasons why the ex ante probability that a given atom spins up or down (once external temperature considerations are removed) is $\frac{1}{2}$. The resolution of this puzzle, first instantiated in the celebrated Ising-Lenz model, is to assume that the probability that a given atom possesses a certain spin depends on the spins of the nearest neighbors to that atom. The subsequent statistical mechanics literature has extended analyses of this type to a wide range of alternative interaction structures that correspond to different choices of $J_{i,j}$.

For the purposes of social science, of course, the physical interpretation of these models is of no interest. What is of enormous interest are the mathematical properties of these systems, which have made the same mathematical models valuable in areas ranging from computer science (where neural networks have this mathematical structure) to biology (models of molecular evolution); Anderson and Stein (1984) is an accessible discussion of this. Among the many interesting properties of these systems are emergence, symmetry breaking, nonergodicity, phase transition, and universality. I provide a brief description of each property to clarify how it arises in the model that has been developed. Additional discussion may be found in Blume and Durlauf (2001) and Durlauf (1999b).

5.2.1. *Emergence*[14]

In a system of interacting agents, emergent properties are those that cannot be reduced to statements about the individual elements when studied in isolation. In physics, magnetism is an emergent phenomenon as it is a collective property of many iron atoms whose atomic spins are aligned; similarly, ice is a property of the way in which many water molecules are arrayed, not one molecule in isolation. The multiple equilibria described in Theorem 1 are examples of emergence in a socioeconomic context, as they constitute a property that arises only with respect to a group rather than for a single individual. Another example of emergence is Schelling's (1971) celebrated demonstration of how complete segregation is produced from mildly discriminatory preferences.

One important aspect of emergence is that it breaks any logical relationship between methodological individualism and reductionism. What I mean is that emergent properties cannot be understood through

[14] See Anderson (1972) for a physical perspective on emergence.

the individual elements of a system, as they are intrinsically collective. This is so even though the behaviors of these elements determine whether or not emergent properties are present.

5.2.2. *Symmetry Breaking*

Symmetry breaking occurs when, for a specification of symmetrically specified agents, asymmetric outcomes occur. In other words, suppose one starts with a set of identically specified agents. Do conditions exist under which their outcomes will differ? To see how this model exhibits symmetry-breaking, consider the case $h = 0$. Each agent is ex ante privately indifferent between the two choices—i.e., in the absence of any social interaction effect, the probability that agent chooses 1 (and of course -1) is $\frac{1}{2}$. However, when $J > 1$, the choices will bunch (in expected value) around one of the choices. Suppose that one has two groups of identically specified agents. It would be possible for one group to be associated with an equilibrium where most choices center on 1 whereas the other group centers on -1.

Symmetry breaking is important in modeling spatial agglomeration of agents (see Arthur [1987] and Krugman [1996] for a stochastic process/statistical mechanics perspective) into regions and cities. In such models, agents face identical incentives and possess identical characteristics, yet distinct bunching of the agents into subgroups will occur.

5.2.3. *Nonergodicity*

A probabilistic system is nonergodic if the conditional probabilities that describe the behavior of each element of the system conditional on the other elements fail to uniquely characterize the behavior of the system as a whole. The simplest example of a nonergodic system is a Markov chain whose transition probabilities are

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

This conditional probability structure does not tell us which state of the system will be observed.

The cases where (26) exhibits multiple solutions are thus nonergodic. In these cases, one specifies the conditional probability choices of each member of the population and imposes self-consistency; however, this does not uniquely determine the aggregate behavior of the population. Multiple equilibria are a common feature of coordination games in economics, which are noncooperative environments in which individuals conform to one another; see Cooper (1997) for an overview. Such models typically embody conformity effects of the type that have been described.

5.2.4. *Phase Transition*

A model exhibits a phase transition when a small change in a model parameter induces a qualitative shift in the model's properties. The binary choice with social interactions model exhibits phase transitions along two dimensions. Recalling Theorem 1, holding h constant, there will be a threshold value H (which depends on h) such that if βJ moves from less than H to greater than H, the number of equilibria shifts from 1 to 3. On the other hand, for every $\beta J > 1$, there is a threshold K, depending on βJ, such that as h moves from less than K to greater than K, the number of equilibria shifts from 3 to 1.

5.2.5. *Universality*

A universal property of a system is one that does not depend on details of the system's micro-level specification. Such properties are found in many physical contexts; for example, magnetization of the type captured in the Ising-Lenz model does not depend on the nearest neighbor interaction structure and in fact occurs for a wide range of alternative interaction structures. Universality is extremely appealing from the perspective of social science modeling, since we often do not have any real justification for choosing interactions structures, forms of interaction effects, etc. outside of analytical convenience.

To see how this model exhibits some types of universality, suppose that each individual is associated with a neighborhood $g(i)$ that characterizes the set of individuals in the population with whom he wishes to conform; $\#(g(i))$ denotes the neighborhood's population size. Assume that all members of the neighborhood weight the expected choices of the others equally, so that $J_{i,j} = J/(\#(g(i)) - 1)$. A self-consistent equilibrium for this system is any set of solutions $E(\omega_1) \ldots E(\omega_I)$ to the set of I equations

$$E(\omega_i) = tanh\left(\beta h + \frac{\beta J}{\#(g(i)) - 1} \sum_{j \in g(i)} E(\omega_j)\right). \tag{27}$$

This mapping must possess at least one fixed point and hence at least one self-consistent equilibrium exists. Notice that any solution of the model with global interactions must also represent a solution to this model, so that

$$E(\omega_1) = E(\omega_2) = \ldots = E(\omega_I) = m = tanh(\beta h + \beta Jm). \tag{28}$$

Hence the properties we have found for the global interactions model occur for a wide variety of alternative interaction structures. This being said, universality has been relatively unexplored in social science applications of statistical mechanics. My judgment is that this is an important area for future work. One obvious possibility concerns Zipf's Law or the rank-size rule for city populations, which appears to occur for countries with very different socioeconomic structures. A number of interesting ideas along these lines appear in Krugman (1996).

6. STATISTICAL IMPLEMENTATION

In bringing social interactions models to data, a number of difficult statistical issues arise. In particular, under the assumption of rational expectations, there exist relationships between the various regressors that comprise the model. These relationships in turn influence identification. This possibility was first recognized by Wallis (1980) in the context of time series models. A seminal paper by Manski (1993) has developed this idea in the context of interactions-based models. Brock and Durlauf (forthcoming) provide both a survey of the relevant literature and many new results. See also Moffitt (2001) for a number of valuable insights into the relevant statistical issues.

6.1. *Linear-in-Means Models*

To see how an identification problem arises in interactions-based models, it is useful to start with a linear regression that is analogous to the theoretical model I have described in Section 3.1. For this empirical model, which is a generalization of the one studied by Manski (1993),[15] each agent is assumed to be a member of a group $g(i)$. An individual is assumed to be affected by contextual and endogenous characteristics that are specific to his group. In a typical data set one would expect that the observations represent individuals from different groups. Hence, each individual

[15] In Manski (1993), $Y_{g(i)}$ is assumed to equal $X_{g(i)}$, the average of X_i across all members of $g(i)$.

in the data set will be associated with an individually-indexed set of contextual effects $Y_{g(i)}$ and a distinct expected choice level among members of group $g(i)$, $m_{g(i)}$. As before, I assume that each individual possesses rational expectations and that the information set on which these expectations are formed includes all individual and contextual effects.

Relative to the linear model (8) the main modeling question is how to render (8) empirically operational while allowing for heterogeneity in the individual incentive terms h_i. Standard empirical practice assumes that these terms are linearly determined by the individual and contextual effects experienced by each individual

$$h_i = k + c'X_i + d'Y_{g(i)}. \tag{29}$$

Thus in the linear version of a social interactions model the behavior of a given individual is described by

$$\omega_i = k + c'X_i + d'Y_{g(i)} + Jm_{g(i)} + \epsilon_i \tag{30}$$

where ϵ_i is a regression error. The dependence of individual choices on averages of expected behaviors and contextual effects has led this structure to be called the linear-in-means model. The parameters of interest are k, c (an r-length vector), d (an s-length vector), and J. I assume that the available data include ω_i, X_i, $Y_{g(i)}$, and $X_{g(i)}$, where $X_{g(i)}$ equals the average of X's among members of neighborhood $g(i)$.[16]

To see how an identification problem arises, it is useful to work with the reduced form model for individual choices. Following the simple example in Section 3.2, one first computes the expected value of both

[16] This formulation differs from the hierarchical linear model (HLM) approach. well described in Bryk and Raudenbush (1992) that is popular in the sociology and education literatures. Relative to equation (29),

$$\omega_i = k + c'_{g(i)}X_i + \epsilon_i$$

$$c_{g(i)} = Jm_{g(i)} + DY_{g(i)} + \eta_i,$$

where J and D are $r \times 1$ and $r \times s$ matrices of coefficients and η_i is an r-length vector of errors. Relative to the linear-in-means model, there are two main differences. First, randomness in the coefficients is allowed. Second, when one substitutes the $c_{g(i)}$ equation into the ω_i equation described above, it is clear that one is in essence estimating a regression for ω_i where the regressors are the products of X_i with the various endogenous and contextual variables. Hence the two approaches estimate rather different behavioral models. I plan to explore the comparative merits of the approaches in future work.

sides of equation (30) for a given neighborhood $g(i)$. This leads to the expression

$$m_{g(i)} = k + c'X_{g(i)} + d'Y_{g(i)} + Jm_{g(i)}. \tag{31}$$

The expected average choice in the neighborhood can be solved for

$$m_{g(i)} = \frac{k + c'X_{g(i)} + d'Y_{g(i)}}{1 - J}. \tag{32}$$

Substituting this into (30), one can see that

$$\omega_i = \frac{k}{1 - J} + c'X_i + \frac{1}{1 - J}\, d'Y_{g(i)} + \frac{J}{1 - J}\, c'X_{g(i)} + \epsilon_i. \tag{33}$$

This regression is in principle estimable by ordinary least squares. A potential identification problem occurs because of the possible linear dependence between $Y_{g(i)}$ and $X_{g(i)}$. For example, in the model studied by Manski (1993), it is assumed that $Y_{g(i)} = X_{g(i)}$—i.e., that the contextual effects that affect individuals are their expectations of the neighborhood averages of the same variables that affect them on an individual level. This is the source of the nonidentification or reflection problem that Manski (1993) examined.

On the other hand, this formulation makes clear that there are two paths by which identification may be achieved. First, in the presence of prior information on which individual and contextual variables influence individual behavior, it is possible that the spaces spanned by elements of $Y_{g(i)}$ and $X_{g(i)}$ are not identical, which may allow for identification. This follows from an analysis of the reduced form equation (33). This regression has $2r + s + 1$ regressors and $r + s + 2$ unknowns; it is easy to verify that if the regressors are linearly independent then the system is identified if $r > 0$ and overidentified if $r > 1$.

Furthermore, suppose we rewrite $X_{g(i)}$ as

$$X_{g(i)} = \Pi_0 + \Pi_1 X_i + \Pi_2 Y_{g(i)} + \eta_i \tag{34}$$

In this formulation, η_i is the part of $X_{g(i)}$ that cannot be predicted given a constant, X_i and $Y_{g(i)}$. Put differently, it is the part of $X_{g(i)}$ that cannot be predicted using those variables that are assumed in equation (30) to predict individual behavior. Notice that η_i can always be constructed by computing the regression (34). This allows us to rewrite the individual reduced form as

$$\omega_i = \frac{k}{1-J} + \frac{Jc'}{1-J}\Pi_0 + \left(c' + \frac{Jc'}{1-J}\Pi_1\right)X_i + \left(\frac{J}{1-J}d' + \frac{Jc'}{1-J}\Pi_2\right)Y_{g(i)} + \frac{J}{1-J}c'\eta_i + \epsilon_i. \quad (35)$$

Equation (34) identifies Π_0, Π_1, and Π_2. Since η_i is orthogonal to the other regressors in (35), the terms $k/(1-J) + Jc'/(1-J)\Pi_0$, $c' + Jc'/(1-J)\Pi_1$, and $J/(1-J)d' + Jc'/(1-J)\Pi_2$ respectively are identified from a regression of ω_i onto a constant, X_i, and $Y_{g(i)}$. Hence, this regression will have $r + s + 1$ coefficients for $r + s + 2$ unknowns. In order to identify the structural coefficients, it is necessary that the regressors η_i provide an additional estimate of some component of the vector $J/(1-J)c'$. This will give as many coefficients as there are unknowns. This in turn requires that η_i is not null—i.e., that there is some part of the neighborhood averages of the individual controls that do not lie in the space spanned by a constant, X_i, and $Y_{g(i)}$. In turn, a necessary condition for η_i to be non-null is that there is at least one regressor x_j such that its neighborhood average $x_{j,g(i)}$ is excluded from $Y_{g(i)}$ prior to the exercise. These arguments are the basis for the following theorem, which is taken from Brock and Durlauf (forthcoming).

Theorem 3: Necessary conditions for identification in the linear-in-means model with social interactions and rational expectations

In the linear-in-means model it is necessary for identification of the model's parameters that

i. The dimension of the linear space spanned by elements of 1, X_i and $Y_{g(i)}$ is $r + s + 1$.[17]
ii. The dimension of the linear space spanned by the elements of 1, X_i, $Y_{g(i)}$ and $X_{g(i)}$ is at least $r + s + 2$.

[17] The element 1 should be interpreted as a random variable whose value is 1 with probability 1.

6.2. *Nonlinear Models*

Notice that linearity plays a critical role in creating the potential for non-identification. Suppose that instead of (30), the individual-level behavioral equation is

$$\omega_i = k + c'X_i + d'Y_{g(i)} + J\phi(m_{g(i)}) + \epsilon_i. \tag{36}$$

for some invertible function $\phi(\cdot)$. The rational expectations condition for this model is

$$m_{g(i)} = \psi(k + c'X_{g(i)} + d'Y_{g(i)}), \tag{37}$$

where $\psi(r) = (r - J\phi(r))^{-1}$. The associated reduced form equation equals

$$\omega_i = k + c'X_i + d'Y_{g(i)} + J\phi(\psi(k + c'X_{g(i)} + d'Y_{g(i)})) + \epsilon_i. \tag{38}$$

This is an example of a partially linear model (cf. Horowitz 1998). If $\phi(\cdot)$—and by implication $\psi(\cdot)$—is known, this equation is a standard nonlinear regression. Brock and Durlauf (forthcoming) verify that the parameters of this model are locally identified under weak assumptions on the variables in the system. Interestingly, the conditions for identification are weaker than the linear model when considered from the perspective of what relationship must exist between $X_{g(i)}$ and $Y_{g(i)}$. The reason for this is that the nonlinear function $\phi(\cdot)$ ensures that $m_{g(i)}$ and $Y_{g(i)}$ cannot be collinear, as is the potential source of nonidentification in the linear case.

6.3. *Binary Choice*

Using the linear model as background, I now consider identification in the binary choice model. This model will possess a likelihood function

$$\begin{aligned} L(\underset{\sim}{\omega}_I \mid X_i, Y_{g(i)}, m^e_{g(i)}\ \forall\ i) \\ &= \prod_i \mu(\omega_i = 1 \mid X_i, Y_{g(i)}, m^e_{g(i)})^{(1+\omega_i)/2} \\ &\quad \cdot \mu(\omega_i = -1 \mid X_i, Y_{g(i)}, m^e_{g(i)})^{(1-\omega_i)/2} \\ &\propto \prod_i (exp(\beta k + \beta c'X_i + \beta d'Y_{g(i)} + \beta J m^e_{g(i)})^{(1+\omega_i)/2} \\ &\quad \cdot exp(-\beta k - \beta c'X_i - \beta d'Y_{g(i)} - \beta J m^e_{g(i)})^{(1-\omega_i)/2}), \end{aligned} \tag{39}$$

where the assumption of rational expectations imposes the restriction

$$m^e_{g(i)} = m_{g(i)} = \int tanh(\beta k + \beta c'X + \beta d'Y_{g(i)} + \beta J m_{g(i)})dF_{X|Y_{g(i)}} \qquad (40)$$

under the assumption that agents only know $Y_{g(i)}$ within a neighborhood.[18] Notice the multiplicative structure of the parameters of the model; this makes it necessary to normalize the parameters in order to achieve identification, which can be done by setting $\beta = 1$.

The key question in terms of identification of the linear-in-means model is whether $m_{g(i)}$ is collinear with the other regressors in the individual behavioral equation due to self consistency. This same issue arises in the binary choice case. However, in the binary choice model, the expected value of a neighborhood choice is a nonlinear function of the other variables in the model. The technical appendix at the end of this paper gives a formal statement of the conditions under which the binary choice model with social interactions is identified, but the key intuition for identification follows from the nonlinearity built into (40). As the theorem indicates, there is no need for an exclusion restriction on the contextual variables in order to achieve identification, as was true in the linear-in-means model. This is an example of a more general phenomenon noted by McManus (1992)—namely, that lack of identification in parametric systems is typically associated with linearity.

This being said, the standard errors in estimating the binary choice model may be extremely large if the distributions of individual and contextual effects are such that (40) is "close" to linear. Exclusion restrictions of the type that generate identification in the linear model—i.e., the presence of elements of X_i in (39) whose group levels analogs do not appear in $Y_{g(i)}$—can facilitate accurate estimation for this case.

Finally, while identification has been established for the binary choice model, as well as for longitudinal analogs (Brock and Durlauf forthcoming), there has yet to be any investigation of issues that arise in the implementation of the models. The presence of latent expectations vari-

[18] It is technically convenient, in working with the binary choice model, to assume that the information sets for individuals take this form, rather than to assume each individual knows the distribution of X_i within his group, as was done in the linear case. This is so because, as seen in Manski (1988), identification arguments in the binary choice model are more subtle than for linear regressions; see Brock and Durlauf (forthcoming) for details.

ables (m_g's) that may not be uniquely determined by observables (due to multiple equilibria) suggests that computational issues, for example, will be far from trivial. Thus there is much additional research needed in order to fully understand how to apply interactions-based models to data.

7. IMPLICATIONS FOR PUBLIC POLICY

An interactions-based perspective on socioeconomic outcomes has important implications for the design and evaluation of public policy. How do interactions-based models affect the assessment of public policies? There are at least three respects in which one may explore this question.

From the perspective of policy assessment, interactions-based models make clear the importance of accounting for nonlinearities. To see this, suppose that a policymaker is assessing the effects of altering h, the private incentive of each individual within a population. Suppose as well that the equilibrium choice level for the population is described by equation (26). In the vicinity of a given equilibrium, the derivative of the equilibrium expected average choice m with respect to h is

$$\frac{dm}{dh} = \frac{\beta(1 - tanh^2(\beta h + \beta Jm))}{(1 - \beta J(1 - tanh^2(\beta h + \beta Jm)))}. \tag{41}$$

This is obviously highly nonlinear. However, the derivative is monotonically decreasing in h when $h > 0$, so one can at least infer that the marginal effect on a group will be higher the weaker the group's fundamentals, so long as the fundamentals have the same sign. However, because of the relationship between fundamentals and the number of equilibria, one must additionally ask whether nonmarginal changes in h will alter the number of equilibria. This creates the possibility that it is cost efficient to raise the private incentives for the relatively better off. For example, in Figure 1, when one moves from low to high h, the welfare inferior equilibria disappear.

A second implication for policy is implied by this equation. Suppose that a policymaker is considering whether to implement a system of subsidies to raise incentives for I individuals out of a population of I^2. Suppose each member of the population is described by equation (21) and these individuals do not form a single group but instead form I separate groups; as before, each group's equilibrium is described by (26). The policymaker is assumed to have two options: (1) raise incentives from h to

$h + dh$ for I individuals scattered across I different groups, or (2) raise incentives from h to $h + dh$ for all I members of a given group. Let m_1 denote the expected value of the average choice for I persons sampled across the I groups and m_2 denote the expected value of the average choice for the I members of a given group. Assuming that the groups are large, so that one can ignore the effect of the behavior of one individual on the group, then the total effect on behavior under the first policy option is

$$\frac{dm_1}{dh} = \beta(1 - tanh^2(\beta h + \beta Jm)). \tag{42}$$

Equation (41) gives the effect of the second policy. Therefore, the relative impact of the two policies is

$$\frac{dm_2}{dh} \bigg/ \frac{dm_1}{dh} = \frac{1}{(1 - \beta J(1 - tanh^2(\beta h + \beta Jm)))} > 1. \tag{43}$$

These expressions differ because when the private incentives are affected for all members of a given group, the influences on individual behavior are magnified as the changes in the behavior of each person will simultaneously affect others in the population. The net effect of the change in private incentives on aggregate behavior will therefore be increased. This is known as a social multiplier in the literature. The presence of a social multiplier means that a cost benefit analysis would suggest concentrating expenditures in order to take advantage of the social multiplier that amplifies the effects of a higher h within a given group.

Third, interactions-based models suggest the importance of exploring alternatives to forms of redistribution that are designed to raise private incentives. One way to interpret welfare and other cash and/or in-kind aid programs is that they are forms of income redistribution. Such programs typically transfer (through taxes paid either contemporaneously or over time to retire government debt that funded the initial program) income from one group to another. An alternative form of equality-enhancing policies falls in the category of “associational redistribution” (Durlauf 1996c). These policies treat group memberships as potential objects of redistribution.

A number of past and current public policies are interpretable as promoting associational redistribution. For example, many education policies are attempts to engage in associational redistribution. Affirmative action in college admissions is nothing more than a choice of what criteria are used to construct student bodies. School busing for racial integration, while substantially less important now than 20 years ago, had exactly the same effect. Recent efforts to promote integration through magnet schools may be interpreted the same way.

Associational redistribution is a far more controversial class of policies than standard tax/transfer policies, as the visceral public hostility to affirmative action makes clear. Further, it seems clear that the development of a rigorous ethical defense of associational redistribution is more difficult than for income redistribution; even as egalitarian a thinker as Walzer (1983) finds various forms of quotas to be ethically problematic. While the presence of interactions in determining socioeconomic outcomes cannot, of course, resolve these complexities, their presence is nevertheless important in assessing whether particular forms of associational redistribution are just. For example, suppose one follows Roemer (1998) and concludes that society ought to indemnify individuals against adverse outcomes in life to the extent the outcomes are caused by factors outside their control. Clearly, ethnicity, residential neighborhood of youth, and the like are not variables that one chooses. Hence, the pursuit of equality of opportunity along the lines outlined by Roemer would require interventions to render these groupings irrelevant in predicting socioeconomic outcomes. One obvious and perhaps necessary way to achieve this is to alter those group memberships that are not immutable.

8. CONCLUSIONS

This paper has described a general model of social interactions that attempts to combine the rigorous choice-based modeling of economics with the richer social structures, interdependences, and contexts, which are the hallmark of sociology. The theoretical framework embodies methodological individualism, yet illustrates how social context means that it is impossible to reduce the analysis of aggregate behavior to individual level descriptions. In terms of conceptualizing behaviors, the approach allows one to integrate private incentives and social influences in a common structure. This framework is compatible with structural econometric analysis and so can be falsified using standard statistical methods.

In terms of future research, my own view is that the most important contributions can be made in the areas of statistical methodology and empirical work. As the survey of evidence suggests, the strongest evidence in favor of social interactions lies in those contexts most removed from the substantive phenomena that this new literature tries to address. Advances in this regard will probably require much more attention to data collection. For example, virtually no attention has been paid to the question of identifying which groups influence individuals as opposed to which groups are currently measured; as Manski (1993) argues, identification of relevant groups from data is probably impossible. Census tracts may have been chosen to approximate homogeneous neighborhoods, but this does not imply that they actually define reference groups. Detailed survey information may be needed to elicit information on what groups actually matter to individuals in a sample. An important effort in this respect is the Project on Human Development in Chicago Neighborhoods. Sampson, Morenoff, and Earls (1999), for example, show how the detailed survey data from this project help clarify some of the amorphous aspects of social capital discussions.

At a minimum, the framework helps to make clear, I believe, that the disciplinary barriers between sociology and economics are in many respects artificial. For phenomena such as inner city poverty, social pathologies, and the like, each field contains important and fundamental theoretical ideas that are not only compatible but in substantive ways complementary to one another. My own belief is that the continuing synthesis of choice-based reasoning with social interactions will prove to be one of the most promising areas of *socioeconomic* theory and empirical work.

TECHNICAL APPENDIX

Theorem 4: Identification in the binary choice model with social interactions and rational expectations

For the binary choice model with probability structure

$$\mu(\underset{\sim}{\omega}|X_i, Y_{g(i)}, m^e_{g(i)} \; \forall \; i) \propto \prod_i exp(\beta k + \beta c' X_i \omega_i + \beta d' Y_{g(i)} \omega_i + \beta J m^e_{g(i)} \omega_i) \qquad \text{(A.1)}$$

and

$$m^e_{g(i)} = m_{g(i)} = \int tanh(\beta k + \beta c' X + \beta d' Y_{g(i)} + \beta J m_{g(i)}) dF_{X|Y_{g(i)}}, \qquad \text{(A.2)}$$

assuming β is normalized to 1, if

i. The support of the vector consisting of the elements of X_i and $Y_{g(i)}$ is not contained in a proper linear subspace of R^{r+s}.
ii. The support of the vector consisting of the elements of $Y_{g(i)}$ is not contained in a proper linear subspace of R^s.
iii. No element of X_i or $Y_{g(i)}$ is constant.
iv. There exists at least one group g_0 such that conditional on Y_{g_0}, X_i is not contained in a proper linear subspace of R_r.
v. None of the regressors in $Y_{g(i)}$ possesses bounded support.
vi. $m_{g(i)}$ is not constant across all groups g.

then, the parameters of the model (k, c, d, J) are identified relative to any distinct alternative $(\bar{k}, \bar{c}, \bar{d}, \bar{J})$.

Proof. See Brock and Durlauf (forthcoming).

REFERENCES

Akerlof, George. 1997. "Social Distance and Social Decisions." *Econometrica* 65: 1005–28.
Akerlof, George, and Rachel Kranton. 2000. "Identity and Economics." *Quarterly Journal of Economics* CVX, 719–53.
Anderson, Elijah. 1999. *Code of the Street*. New York: Norton.
Anderson, Philip. 1972. "More Is Different." *Science* 177, 393–96.
Anderson, Philip, and Daniel Stein. 1984. "Broken Symmetry, Emergent Properties, Dissipative Structures, Life: Are They All Related." In *Basic Notions of Condensed Matter Physics*, edited by P. Anderson. Menlo Park, CA: Addison Wesley.
Aronson, Elliot. 1999. *The Social Animal*, 8th ed., New York: W. H. Freeman.
Arthur, W. Brian. 1987. "Urban Systems and Historical Path Dependence." Pp. 85–97 in *Urban Systems and Infrastructure*, edited by R. Herman and J. Ausubel, Washington, D.C.: National Academy of Sciences/National Academy of Engineering.
Becker, Gary, and Nigel Tomes. 1979. "An Equilibrium Theory of the Distribution of Income and Intergenerational Mobility." *Journal of Political Economy* 87:1163–89.
Bénabou, Roland. 1993. "Workings of a City: Location, Education and Production." *Quarterly Journal of Economics*, 108:619–52.
———. 1996a. "Equity and Efficiency in Human Capital Investment: The Local Connection." *Review of Economic Studies* 62, 237–64.
———. 1996b. "Heterogeneity, Stratification, and Growth: Macroeconomic Effects of Community Structure." *American Economic Review* 86:584–609.

Bikhchandani, Sushil, David Hirshleifer, and Ivo Welch. 1992. "A Theory of Fads, Fashion, Custom, and Cultural Exchange as Information Cascades." *Journal of Political Economy* 100:992–1026.

Blalock, Herbert. 1984. "Contextual-Effects Models: Theoretical and Methodological Issues." *Annual Review of Sociology* 10: 353–72.

Blume, Lawrence, and Steven Durlauf. 2001. "The Interactions-Based Approach to Socioeconomic Behavior." Pp. 15–44 in *Social Dynamics*, edited by H. P Young and S. Durlauf. Cambridge, MA: MIT Press.

Brewster, Karen. 1994a. "Race Differences in Sexual Activity Among Adolescent Women: The Role of Neighborhood Characteristics." *American Sociological Review* 59: 408–24.

———. 1994b. "Neighborhood Context and the Transition to Sexual Activity Among Black Women." *Demography* 31: 603–14.

Brock, William, and Steven Durlauf, 1999. "A Formal Model of Theory Choice in Science." *Economic Theory* 14:113–30.

———. 2001. "Discrete Choice with Social Interactions." *Review of Economic Studies* 68, 2:235–60.

———. forthcoming. "Interactions-Based Models." *Handbook of Econometrics*, vol. 5, edited by J. Heckman and E. Leamer. Amsterdam: North-Holland.

Brooks-Gunn, Jeanne, Greg Duncan, P. Klebanov, and N. Sealand. 1993. "Do Neighborhoods Affect Child and Adolescent Development?" *American Journal of Sociology* 99:353–95.

Brown, Roger. 1986. *Social Psychology*. Cambridge, MA: Harvard University Press.

Bryk, Anthony, and Steven Raudenbush. 1992. *Hierarchical Linear Models: Applications and Data Analysis Methods*. New York: Sage.

Card, David, and Alan Krueger. 1992. "Does School Quality Matter? Returns to Education and the Characteristics of Public Schools in the United States." *Journal of Political Economy*: 1–40.

Chambers, Jack K. 1995. *Sociolinguistic Theory*. Oxford, England: Blackwell Publishers.

Coleman, James. 1990. *Foundations of Social Theory*. Cambridge, MA: Harvard University Press.

Cooper, Russell. 1997. *Coordination Games*, Cambridge, England: Cambridge University Press.

Crane, Jonathan. 1991. "The Epidemic Theory of Ghettos and Neighborhood Effects on Dropping Out and Teenage Childbearing." *American Journal of Sociology* 96: 1226–59.

Duneier, Mitchell. 1992. *Slim's Table*. Chicago: University of Chicago Press.

———. 1999. *Sidewalk*. New York: Farrar, Straus, and Giroux.

Durlauf, Steven. 1996a. "A Theory of Persistent Income Inequality." *Journal of Economic Growth* 1: 75–93

———. 1996b. "Neighborhood Feedbacks, Endogenous Stratification, and Income Inequality." Pp. 505–34 in *Dynamic Disequilibrium Modelling*, edited by W. Barnett, G. Gandolfo, and C. Hillinger. Cambridge, England: Cambridge University Press.

———. 1996c. “Associational Redistribution: A Defense.” *Politics and Society* 24: 391–410.

———. 1997a. “Statistical Mechanics Approaches to Socioeconomic Behavior.” Pp. 81–104 in *The Economy as a Complex Evolving System II*, edited by W. B. Arthur, S. Durlauf, and D. Lane. Redwood City: Addison-Wesley, forthcoming.

———. 1999a. “The Memberships Theory of Inequality: Ideas and Implications.” Pp. 161–78 in *Elites, Minorities, and Economic Growth*, edited by E. Brezis and P. Temin. Amsterdam: North Holland.

———. 1999b. “How Can Statistical Mechanics Contribute to the Study of Science?” *Proceedings of the National Academy of Sciences* 96; 10582–84.

———. 2001. “On the Empirics of Social Capital.” University of Wisconsin. Unpublished manuscript.

Elster, Jon. 1989. *The Cement of Society*. Cambridge, England: Cambridge University Press.

Glaeser, Edward, Bruce Sacerdote, and José Scheinkman. 1996. “Crime and Social Interactions.” *Quarterly Journal of Economics*, 111, 507–48.

Glaeser, Edward, and José Scheinkman, 2000. “Nonmarket Interactions.” Princeton University. Unpublished manuscript.

———. 2001. “Measuring Social Interactions.” Pp. 83–132 in *Social Dynamics*, edited by S. Durlauf and H. P. Young. Cambridge, MA: MIT Press.

Granovetter, Mark. 1985. “Economic Action and Social Structure: The Problem of Embeddedness.” *American Journal of Sociology* 91: 481–510.

Granovetter, Mark, and Roland Soong. 1988. “Threshold Models of Diversity: Chinese Restaurants, Residential Segregation, and the Spiral of Silence.” Pp. 69–104 in *Sociological Methodology*, edited by C. Clogg. Cambridge, MA: Blackwell Publishers.

Hanushek, Eric. 1988. “The Economics of Schooling: Production and Efficiency in Public Schools.” *Journal of Economic Literature* 24: 1141–77.

Hauser, Robert. 1970. “Context and Consex: A Cautionary Tale.” *American Journal of Sociology* 75: 645–64.

Heckman, James. 2000. “Causal Parameters and Policy Analysis in Economics: A Twentieth Century Retrospective.” *Quarterly Journal of Economics* 115: 45–97.

Heckman, James, Anne Layne-Farrar, and Petra Todd. 1996. “Does Measured School Quality Really Matter? An Examination of the Earnings-Quality Relationship.” Pp. 192–289 in *Does Money Matter?* edited by G. Burtless. Washington, DC: Brookings Institution Press.

Horowitz, Joel. 1998. *Semiparametric Methods in Econometrics*. New York: Springer-Verlag.

Katz, Lawrence, Jeffrey Kling, and Jeffrey Liebman. 2001. “Moving to Opportunity in Boston: Early Impacts of a Randomized Mobility Experiment.” *Quarterly Journal of Economics* 116: 607–54.

Krugman, Paul. 1996. *The Self-Organizing Economy*, Oxford, England: Blackwell Publishers.

Kozol, Jonathan. 1991. *Savage Inequalities*. New York: Crown.

Loury Glenn. 1977. “A Dynamic Theory of Racial Income Differences.” Pp. 153–86 in *Women Minorities and Employment Discrimination,* edited by P. Wallace and A. LaMond. Lexington, MA: Lexington Books.

———. 1981. "Intergenerational Transfers and the Distribution of Earnings." *Econometrica* 49: 843–67.

Manski, Charles. 1988. "Identification of Binary Response Models." *Journal of the American Statistical Association* 83: 729–38.

———. 1993. "Identification of Endogenous Social Effects: The Reflection Problem." *Review of Economic Studies* 60: 531–42.

———. 2000. "Economic Analysis of Social Interactions", *Journal of Economic Perspectives* 14.3: 115–36.

Mas-Colell, Andrea, Michael Whinston, and Jerry Green. 1995. *Microeconomic Theory*. New York: Oxford University Press.

McManus, Douglas. 1992. "How Common Is Identification in Parametric Models?" *Journal of Econometrics* 53: 5–23.

Moffitt, Robert. 2001. "Policy Interventions, Low-Level Equilibria, and Social Interactions." Pp. 45–82 in *Social Dynamics*, edited by S. Durlauf and H. P. Young. Cambridge, MA: MIT Press.

Roemer, John. 1998. *Equality of Opportunity*. Cambridge, MA: Harvard University Press.

Rubinstein, Ariel. 1998. *Modeling Bounded Rationality*. Cambridge, MA: MIT Press.

Rubinowitz, Leonard, and James Rosenbaum. 2000. *Crossing the Class and Color Lines*, Chicago: University of Chicago Press.

Sampson, Robert, Jeffrey Morenoff, and Felton Earls. 1999. "Beyond Social Capital: Collective Efficacy for Children." *American Sociological Review* 64: 633–60.

Sampson, Robert, Steven Raudenbush, and Felton Earls. 1997. "Neighborhoods and Violent Crime: A Study of Collective Efficacy." *Science* 277: 918–24.

Schelling, Thomas. 1971. "Dynamic Models of Segregation." *Journal of Mathematical Sociology* 1: 143–86.

Sherif, Muzafer, O. Harvey, B. White, W. Hood, and C. Sherif. 1961. *Intergroup Conflict and Cooperation: The Robbers Cave Experiment*. Norman: Institute of Group Relations, University of Oklahoma; reprinted by Wesleyan University Press, 1988.

South, Scott, and Kyle Crowder. 1999. "Neighborhood Effects on Family Formation: Concentrated Poverty and Beyond." *American Sociological Review* 64: 113–32.

Sucoff, Clea, and Dawn Upchurch. 1998. "Neighborhood Context and the Risk of Childbearing Among Metropolitan-Area Black Adolescents." *American Sociological Review* 63: 571–85.

Wallis, Kenneth. 1980. "Econometric Implications of the Rational Expectations Hypothesis." *Econometrica* 48: 49–73.

Walzer, Michael. 1983. *Spheres of Justice*. New York: Basic Books.

Weinberg, Bruce, Patricia Reagan, and Jeffrey Yankow. 2000. "Do Neighborhoods Affect Work Behavior? Evidence from the NLSY79." Department of Economics, Ohio State University. Unpublished manuscript.

Wilson, William J. 1987. *The Truly Disadvantaged*. Chicago: University of Chicago Press.

Yeomans, Julia. 1992. *Statistical Mechanics of Phase Transitions*. Oxford, England: Oxford University Press.