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Growth, innovation, scaling and the pace of life in cities

Luís M. A. Bettencourt[1], José Lobo[2], Dirk Helbing[3], Christian Kühnert[3], and Geoffrey B. West[1,4]

[1]Theoretical Division, MS B284, Los Alamos National Laboratory, Los Alamos NM 87545.

[2]Global Institute of Sustainability, Arizona State University,
P.O. Box 873211, Tempe, AZ 85287-3211.

[3]Institute for Transport & Economics, Dresden University of Technology,
Andreas-Schubert-Straße 23, D-01062 Dresden, Germany

[4]Santa Fe Institute, 1399 Hyde Park Road, Santa Fe, NM 87501

Corresponding Author:
Luís M. A. Bettencourt
Theoretical Division
T-7, MS B284
Los Alamos National Laboratory
Los Alamos NM 87545.
Phone: +1 505 667 8453 (office) or +1 505 920 6220 (cell)
Fax: +1 505 665 5757
Email: lmbett@lanl.gov

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Humanity has just crossed a major landmark in its history with the majority of people now living in cities. Cities have long been known to be society's predominant engine of innovation and wealth creation, yet they are also its main source of crime, pollution and disease. The inexorable trend towards urbanization worldwide presents an urgent challenge for developing a predictive, quantitative theory of urban organization and sustainable development. Here we present empirical evidence indicating that the processes relating urbanization to economic development and knowledge creation are very general, being shared by all cities belonging to the same urban system and sustained across different nations and times. Many diverse properties of cities from patent production and personal income to electrical cable length are shown to be power-law functions of population size with scaling exponents, β, which fall into distinct universality classes. Quantities reflecting wealth creation and innovation have $\beta \sim 1.2 > 1$ (increasing returns), whereas those accounting for infrastructure display $\beta \sim 0.8 < 1$ (economies of scale). We predict that the pace of social life in the city increases with population size, in quantitative agreement with data, and discuss how cities are similar to, and differ from, biological organisms, for which $\beta < 1$. Finally we explore possible consequences of these scaling relations by deriving growth equations, which quantify the dramatic difference between growth fueled by innovation *versus* that driven by economies of scale. This suggests that, as population grows, major innovation cycles must be generated at a continually accelerating rate to sustain growth and avoid stagnation or collapse.

Introduction

Humanity has just crossed a major landmark in its history with the majority of people now living in cities (1,2). The present worldwide trend towards urbanization is intimately related to economic development and to profound changes in social organization, land use and patterns of human behavior (1,2). The demographic scale of these changes is unprecedented (2,3) and will lead to important but as of yet poorly understood impacts on the global environment. In 2000 more than 70% of the population in developed countries lived in cities, compared to about 40% in developing countries. Cities occupied a mere 0.3% of the total land area, but some 3% of arable land. By 2030 the urban population of developing countries is expected to more than double to ~4 billion, with an estimated three fold increase in occupancy of land area (3), while in developed countries it may still increase by as much as 20%. Paralleling this global urban expansion, there is the necessity for a sustainability transition (4-6), towards a stable total human population, together with a rise in living standards and the establishment of long term balances between human development needs and the planet's environmental limits (7). Thus, a major challenge worldwide (5,6) is to understand and predict how changes in social organization and dynamics resulting from urbanization will impact the interactions between nature and society (8).

The increasing concentration of people in cities presents both opportunities and challenges (9) towards future scenarios of sustainable development. On the one hand, cities make possible economies of scale in infrastructure (9) and facilitate the optimized delivery of social services, such as education, health care and efficient governance. Other impacts, however, arise due to human adaptation to urban living (9,10-14). These can be direct, resulting from obvious changes in land use (3) (e.g., urban heat island effects (15,16) and increased green house gas emissions (17)) or indirect, following from changes in consumption (18) and human behavior (10-14), already emphasized in classical work by Simmel and Wirth in urban sociology (10-12) and by Milgram in psychology (13). An important result of urbanization is also an increased division of labor (10) and the growth of occupations geared towards innovation and wealth creation (19-22). The features common to this set of impacts are that they are open ended and involve permanent adaptation, while their environmental implications are ambivalent, aggravating stresses on natural environments in some cases and creating the conditions for sustainable solutions in others (9).

These unfolding complex demographic and social trends make it clear that the quantitative understanding of human social organization and dynamics in cities (7,9) is a major piece of the puzzle towards navigating successfully a transition to sustainability. However, despite much historical evidence (19,20) that cities are the principal engines of innovation and economic growth, a quantitative, predictive theory for understanding their dynamics and organization (23,24), and estimating their future trajectory and stability, remains elusive. Significant obstacles towards this goal are the immense diversity of human activity and organization, and an enormous range of geographic factors. Nevertheless, there is strong evidence of quantitative regularities in the increases in economic opportunities (25-29), rates of innovation (21,22) and pace of life (11-14,30) observed between smaller towns and larger cities.

In this paper we show that the social organization and dynamics relating urbanization to economic development and knowledge creation, among other social activities, are very general and appear as non-trivial quantitative regularities common to all cities, across urban systems. We present a new and extensive body of empirical evidence showing that important demographic, socioeconomic and behavioral urban indicators are, on average, scaling functions of city size (31) that are quantitatively consistent across different nations and times. The most thorough evidence at present is for the USA, where extensive reliable data across a wide variety of indicators span many decades. In addition, we show that other nations, including China and European countries, display particular scaling relationships consistent with those in the USA.

Scaling and biological metaphors for the city

Scaling as a tool for revealing underlying dynamics and structure has been instrumental in understanding problems across the entire spectrum of science and technology. This approach has recently been applied to a wide range of biological phenomena leading to a unifying quantitative picture of their organization, structure and dynamics. Organisms as metabolic engines, characterized by energy consumption rates, growth rates, body size, and behavioral times (32-34), have a clear counterpart in social systems (14,35).

Cities as consumers of energy and resources, and producers of artifacts, information and waste have often been compared to biological entities, in both classical studies in urban sociology (14,35) and in recent research concerned with urban ecosystems and sustainable development. Recent analogies include cities as "living systems," (36) or "organisms," (37) and notions of urban "ecosystems" (38) and urban "metabolism" (17,38-40). Are these just qualitative metaphors, or is there quantitative and predictive substance in the implication that social organizations are extensions of biology, satisfying similar principles and constraints? Are the structures and dynamics that evolved with human socialization fundamentally different from those in biology? Answers to these questions provide a framework for the construction of a quantitative theory of the average city, which would incorporate, for example, the roles of innovation and economies of scale, and predictions for growth trajectories, levels of social and economic development and ecological footprints.

To set the stage, consider first some relevant scaling relations characterizing biological organisms. Despite its amazing diversity and complexity, life manifests an extraordinary simplicity and universality in how key structural and dynamical processes scale across a broad spectrum of phenomena and an immense range of energy and mass scales covering over 20 orders of magnitude. Remarkably, almost all physiological characteristics of biological organisms scale with body mass, M, as a power law whose exponent is typically a multiple of 1/4 (which generalizes to 1/(d+1) in d-dimensions). For example, metabolic rate, B, (the power required to sustain the organism) scales as $B \propto M^{3/4}$(32,33). Since metabolic rate per unit mass, $B/M \propto M^{-1/4}$, *decreases* with body size, this implies an *economy of scale* in energy consumption: larger organisms consume less energy per unit time and per unit mass. The predominance and universality of quarter-power scaling has been understood as a manifestation of general underlying principles that constrain the dynamics and geometry of distribution networks within organisms (e.g. the circulatory system). Highly complex, self-sustaining structures, whether cells, organisms or cities require close integration of enormous numbers of constituent units that need efficient servicing. To accomplish this, life at all scales is sustained by optimized, space-filling, hierarchical branching networks (32, 41), which grow with the size of the organism as uniquely specified approximately self-similar structures. Because these networks, e.g. the vascular systems of animals and plants, determine the rates at which energy is delivered to functional terminal units (cells), they set the pace of physiological processes as scaling functions of the size of the organism. Thus, the self-similar nature of resource distribution networks, common to all organisms, provides the basis for a quantitative, predictive theory of biological structure and dynamics, despite much external variation in appearance and form.

Specifically, this theory predicts that characteristic physiological times, such as lifespans, turnover times, and times to maturity scale as $M^{1-\beta}\sim M^{1/4}$, whereas associated rates, such as heart rates and evolutionary rates, scale as $M^{\beta-1}\sim M^{-1/4}$. Thus, the pace of biological life slows down with increasing size of the organism.

Conceptually, the existence of such universal scaling laws implies, for example, that in terms of almost all biological rates, times and internal structure an elephant is approximately a blown-up gorilla, which is itself a blown-up mouse, all scaled in an appropriately non-linear, predictable way. This means that dynamically and organizationally all mammals are, on the average, scaled manifestations of a single idealized mammal, whose properties are determined as a function of its size.

From this perspective it is natural to ask whether social organizations also display universal power law scaling for variables reflecting key structural and dynamical characteristics. In what sense, if any, are small, medium and large cities scaled versions of one another, thereby implying that they are manifestations of the same average idealized city? In this way urban scaling laws, to exist, may provide fundamental quantitative insights and predictability into underlying social processes, responsible for flows of resources, information and innovation.

Results

Scaling Relations for Urban Indicators

To explore scaling relations for cities we gathered an extensive body of data, much of it never before published, across national urban systems, addressing a wide range of characteristics, including energy consumption, economic activity, demographics, infrastructure, innovation, employment and patterns of human behavior. While much data are available for specific cities, scaling analysis requires coverage of entire urban systems. We have obtained datasets at this level of detail mostly for the USA, where typically more data are available, and in more particular cases for European countries and China.

As we show below, the data assembled and examined here can be grouped into three categories: material infrastructure, individual human needs, and patterns of social activity. We adopted a definition of cities that is as much as possible devoid of arbitrary political or geographic boundaries, as integrated economic and social units, usually referred to as *unified labor markets,* comprising of urban cores and including all administrative subdivisions with substantial fractions of their population commuting to work within its boundaries. In the USA these correspond to Metropolitan Statistical Areas (MSAs), in the European Union, Larger Urban Zones (LUZs) and in China, Urban Administrative Units (UAUs). More detailed definitions of city boundaries are desirable and an active topic of research in urban geography (3).

Using population, $N(t)$, as the measure of city size at time t, power law scaling takes the form

$$Y(t) = Y_0 N(t)^{\beta}. \tag{1}$$

Y can denote material resources (such as energy or infrastructure) or measures of social activity (such as wealth, patents, and pollution); Y_0 is a normalization constant. The exponent, β, reflects general dynamical rules at play across the urban system. Summary results for selected exponents are shown in Table 1 and typical scaling curves are shown in Figure 1. These results indicate that scaling is indeed a pervasive property of urban organization. We find robust and commensurate scaling exponents across different nations, economic systems, levels of development and recent time periods for a wide variety of indicators. This implies that, in terms of these quantities, cities that are superficially quite different in form, location, etc are in fact, on the average, scaled versions of one another, in a very specific but universal fashion prescribed by the scaling laws of Table 1.

Despite the ubiquity of approximate power law scaling, there is no simple analogue to the universal quarter-powers observed in biology. Nevertheless, Table 1 reveals a taxonomic universality whereby exponents fall into three categories defined by β=1 (linear), β<1 (sublinear), and β>1 (superlinear), with β in each category clustering around similar values: (i) $\beta \approx 1$ is usually associated with individual human needs (job, house, household water consumption). (ii) β~0.8<1 characterizes material quantities displaying economies of scale associated with infrastructure, analogous to similar quantities in biology. (iii) β~1.1-1.3 >1 signifies *increasing returns* with population size and is manifested by quantities related to social currencies, such as information, innovation or wealth, associated with the intrinsically social nature of cities.

The most striking feature of the data is perhaps the many urban indicators that scale superlinearly (β>1). These reflect unique social characteristics with no equivalent in biology and are the quantitative expression that knowledge spillovers drive growth (25, 26), that such spillovers in turn drive urban agglomeration (26, 27) and that larger cities are associated with higher levels of productivity (28,29). Wages, income, GDP, bank deposits, as well as rates of invention, measured by new patents and employment in creative sectors (21,22) all scale superlinearly with city size, over different years and nations with exponents that, although differing in detail, are statistically consistent. Costs, such as housing, similarly scale superlinearly, approximately mirroring increases in average wealth.

One of the most intriguing outcomes of the analysis is that the value of the exponents in each class clusters around the same number for a plethora of phenomena that are superficially quite different and seemingly unrelated, ranging from wages and patent production to the speed of walking (see below). This strongly suggests that there is a universal social dynamic at play that underlies all these phenomena, inextricably linking them in an integrated dynamical network. This implies, for instance, that an increase in productive social opportunities, both in number and quality, leads to quantifiable changes in individual behavior across the full complexity of human expression (10-14), including those with negative consequences, such as costs, crime rates and disease incidence (19,42).

For systems exhibiting scaling in rates of resource consumption, characteristic times are predicted to scale as $N^{1-\beta}$, while rates scale as their inverse, $N^{\beta-1}$. Thus, if β<1, as in biology, the pace of life decreases with increasing size, as observed. However, for processes driven by innovation and wealth creation, β>1 as in urban systems, the situation is reversed: *thus, the pace of urban life is predicted to increase with size* (Fig. 2a,b). Anecdotally, this is a widely recognized feature of urban life, pointed out long ago by Simmel, Wirth, Milgram and others (11-14). Quantitative confirmation is provided by urban crime rates (42), rates of spread of infectious diseases such as AIDS, and even pedestrian walking speeds (30), which, when plotted logarithmically, exhibit power law scaling with an exponent of 0.09±0.02 (R^2=0.80), Figure 2a, consistent with our prediction.

There are therefore two distinct characteristics of cities revealed by their very different scaling behaviors. These result from fundamentally different, and even competing, underlying dynamics (9): material economies of scale characteristic of infrastructure networks, vs. social interactions, responsible for innovation and wealth creation. The tension between these is illustrated by the ambivalent behavior of energy-related variables: while household consumption scales approximately linearly and economies of scale are realized in electrical cable lengths, total consumption scales superlinearly. This can only be reconciled if the distribution network is sub-optimal, as observed in the scaling of resistive losses, where β=1.11±0.06 (R^2=0.79). Which, then, of these two dynamics, efficiency or wealth creation, is the primary determinant of urbanization and how does each impact urban growth?

The Urban Growth Equation

Growth is constrained by the availability of resources and their rates of consumption. Resources, *Y,* are utilized for both maintenance and growth. If, on average, it requires a quantity *R* per unit time to maintain an individual, and a quantity *E* to add a new one to the population, then this is expressed as $Y = R\,N + E\ (dN/dt)$, where *dN/dt* is the population growth rate. This leads to the general growth equation:

$$\frac{dN(t)}{dt} = \left(\frac{Y_0}{E}\right)N(t)^{\beta} - \left(\frac{R}{E}\right)N(t). \qquad (2)$$

Its generic structure captures the essential features contributing to growth. Although additional contributions can be made, they can be incorporated by a suitable interpretation of the parameters Y_0, *R* and *E* (see Supporting Information, published on the PNAS web site, www.pnas.org, for generalization). The solution of Eq. 2 is given by

$$N(t) = \left[\frac{Y_0}{R} + \left(N^{1-\beta}(0) - \frac{Y_0}{R}\right)\exp[-\frac{R}{E}(1-\beta)t]\right]^{\frac{1}{1-\beta}} \qquad (3)$$

This exhibits strikingly different behaviors depending on whether β<1,>1,or =1: When β=1, the solution reduces to an exponential: $N(t) = N(0)e^{(Y_0 - R)t/E}$ (Fig. 3b), while for β<1 it leads to a sigmoidal growth curve, in which growth ceases at large times (*dN*/*dt*=0), as the population approaches a finite carrying capacity $N(\infty) = (Y_0 / R)^{1/(1-\beta)}$ (Fig. 3a). This is characteristic of biological systems where the predictions of Eq. 2 are in excellent agreement with data (41). Thus, cities and, more generally, social organizations driven by economies of scale are destined to eventually stop growing (43-45).

The character of the solution changes dramatically when growth is driven by innovation and wealth creation, β>1. If $N(0) < (R/Y_0)^{1/(\beta-1)}$, Eq. 2 leads to *unbounded growth* for *N(t)* (Fig.3c). Growth becomes faster than exponential eventually leading to an *infinite* population in a *finite* amount of time given by

$$t_c = -\frac{E}{(\beta-1)R}\ln\left[1-\frac{R}{Y_0}N^{1-\beta}(0)\right] \approx \left[\frac{E}{(\beta-1)R}\right]\frac{1}{N^{\beta-1}(0)}. \qquad (4)$$

This has powerful consequences since, in practice, the resources driving Eq. 2 are ultimately limited so the singularity is never reached; thus, if conditions remain unchanged, unlimited growth is unsustainable. Left unchecked, this triggers a transition to a phase where $N(0) > (R/Y_0)^{1/(\beta-1)}$, leading to stagnation and ultimate collapse; (Fig. 3d).

To avoid this crisis and subsequent collapse, major qualitative changes must occur which effectively reset the initial conditions and parameters of Eq. 3. Thus to maintain growth, the response must be "innovative" to ensure that the predominant dynamic of the city remains in the "wealth and knowledge creation" phase where $\beta>1$ and $N(0) > (R/Y_0)^{1/(\beta-1)}$. In that case, a new cycle is initiated and the city continues to grow following Eq. 2 and Fig.3c, but with new parameters and initial conditions, $N_i(0)$, the population at the transition time between adjacent cycles. This process can be continually repeated leading to multiple cycles, thereby pushing potential collapse into the future, Fig. 4a.

Unfortunately, however, the solution that innovation and corresponding wealth creation are stimulated responses to ensure continued growth has further consequences with potentially deleterious effects. Eq. 4 predicts that the time between cycles, t_i, necessarily decreases as population grows: $t_i \propto t_c \sim 1/N_i(0)^{\beta-1}$. Thus, to sustain continued growth, major innovations or adaptations must arise at an accelerated rate. Not only does the pace of life increase with city size, but so also must the rate at which new major adaptations and innovations need to be introduced to sustain the city. These predicted successive accelerating cycles of faster than exponential growth are consistent with observations for the population of cities, Fig. 4b, waves of technological change (46) and the world population (47,48).

It is worth noting that the ratio E_i/R_i has a simple interpretation as the time needed for an average individual to reach productive maturity. Expressing this as $E_i/R_i \approx \tau \times 20$ years, where τ is of order unity, and the population at the beginning of a cycle as $N_i(0) \approx \nu \times 10^6$, gives $t_c \approx 50\tau\nu^{1-\beta}$ years. For a large city, this is typically a few decades, slowly decreasing with population. Actual cycle times must be shorter than t_c.

Discussion

Despite the enormous complexity and diversity of human behavior and extraordinary geographic variability, we have shown that cities belonging to the same urban system obey pervasive scaling relations with population size, characterizing rates of innovation, wealth creation, patterns of consumption and human behavior as well as properties of urban infrastructure. Most of these indicators deal with temporal processes associated with the social dimension of cities as spaces for intense interaction across the spectrum of human activities. It is remarkable that it is principally in terms of these rhythms that cities are self-similar organizations, indicating a universality of human social *dynamics*, despite enormous variability in urban *form*. These findings provide new quantitative underpinnings for social theories of "urbanism as a way of life" (12).

Our primary analytical focus here was concerned with the consequences of population size on a variety of urban metrics. In this sense we have not addressed the issue of location (49-51) as a determinant of form and size of human settlements. We can however shed some light on associated ideas of urban hierarchy and urban dominance (14,51): increasing rates of innovation, wealth creation, crime, *etc,* per capita imply flows of these quantities from places where they are created faster (sources), to those where they are produced more slowly (sinks), along an urban hierarchy of cities dictated, on average, by population size.

A related point deals with limits to urban population growth. Although population increases are ultimately limited by impacts on the natural environment, we have shown that growth driven by innovation implies, in principle, no limit to the size of a city, providing a quantitative argument against classical ideas in urban economics (43-45).

The tension between economies of scale and wealth creation, summarized in Table 2, represents a new phenomenon where innovation occurs on timescales that are now shorter than individual lifespans and are predicted to become even shorter as populations increase and become more connected, in contrast to biology where the innovation timescales of natural selection greatly exceed individual lifespans. Our analysis suggests a uniquely human social dynamics that transcends biology and redefines metaphors of urban “metabolism”. Open-ended wealth and knowledge creation require the pace of life to increase with organization size and for individuals and institutions to adapt at a continually accelerating rate to avoid stagnation or potential crises. These conclusions very likely generalize to other social organizations, such as corporations and businesses, potentially explaining why continuous growth necessitates an accelerating treadmill of dynamical cycles of innovation.

The practical implications of these findings highlight the importance of measuring and understanding the drivers of economic and population growth in cities across entire urban systems. Scaling relations predict many of the characteristics that a city is expected to assume, on average, as it gains or looses population. The realization that most urban indicators scale with city size nontrivially, implying increases per capita in crime or innovation rates and decreases on the demand for certain infrastructure, is essential to set realistic targets for local policy. New indices of urban rank according to deviations from the predictions of scaling laws also provide more accurate measures of the successes and failures of local factors (including policy) in shaping specific cities.

In closing we note that much more remains to be explored in generalizing the empirical observations made here to other quantities, especially those connected to environmental impacts, as well as to other urban systems and in clarifying the detailed social organizational structures that give rise to observed scaling exponents. We believe that the further extension and quantification of urban scaling relations will provide a unique window into the spontaneous social organization and dynamics that underlie much of human creativity, prosperity and resource demands on the environment. This knowledge will suggest paths along which social forces can be harnessed to create a future where open-ended innovation and improvements in human living standards are compatible with the preservation of the planet's life support systems.

Materials and Methods

Extensive datasets covering metropolitan infrastructure, individual needs and social indicators were collected for entire urban systems, from a variety of sources worldwide (e.g. US Census Bureau, Eurostat Urban Audit, China's National Bureau of Statistics). Details about these sources, web links, acknowledgements and additional comments are provided as Supporting Information, published on the PNAS web site www.pnas.org).

Fits to data were performed using ordinary least squares (OLS) with a correction for heteroskedasticity using the Stata software package. We performed additional tests on the data, fitting its cumulative distribution and using logarithmic binning, to assess the robustness of the exponents β.

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31. Note that the much studied "Zipf's law" [see e.g Gabaix, X. (1999) *Q. J. Econ.* **114**: 739] for the rank-size distribution of urban populations is just one example of the many scaling relationships presented in this paper.

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Table and Figure Captions:

Table 1 – Scaling exponents for urban indicators vs. city size. (Data sources supplied in Supporting Information, published on the PNAS web site, www.pnas.org).

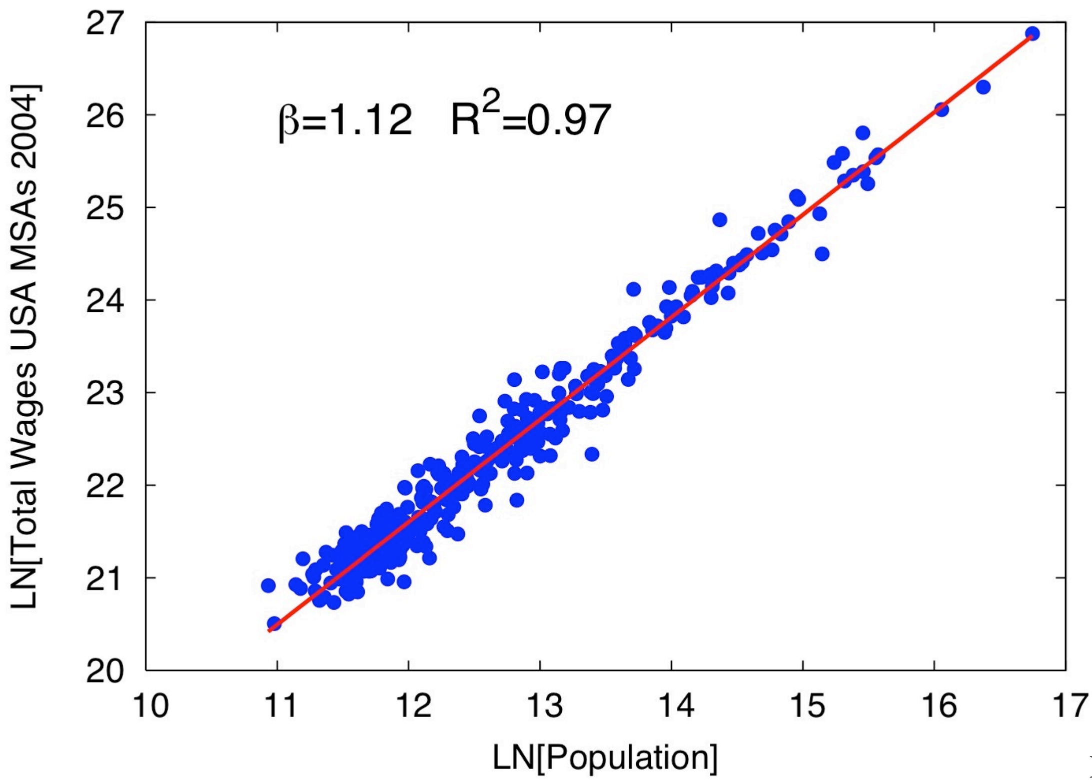
Table 2 – Classification of scaling exponents for urban properties and their implications for growth.

Figure 1 **Example of scaling relationships** a) Total wages per MSA in 2004 for the USA (blue points), *vs*. metropolitan population b) Supercreative employment per MSA in 2003, for the USA (blue points), *vs*. metropolitan population. Best fit scaling relations are shown as solid lines.

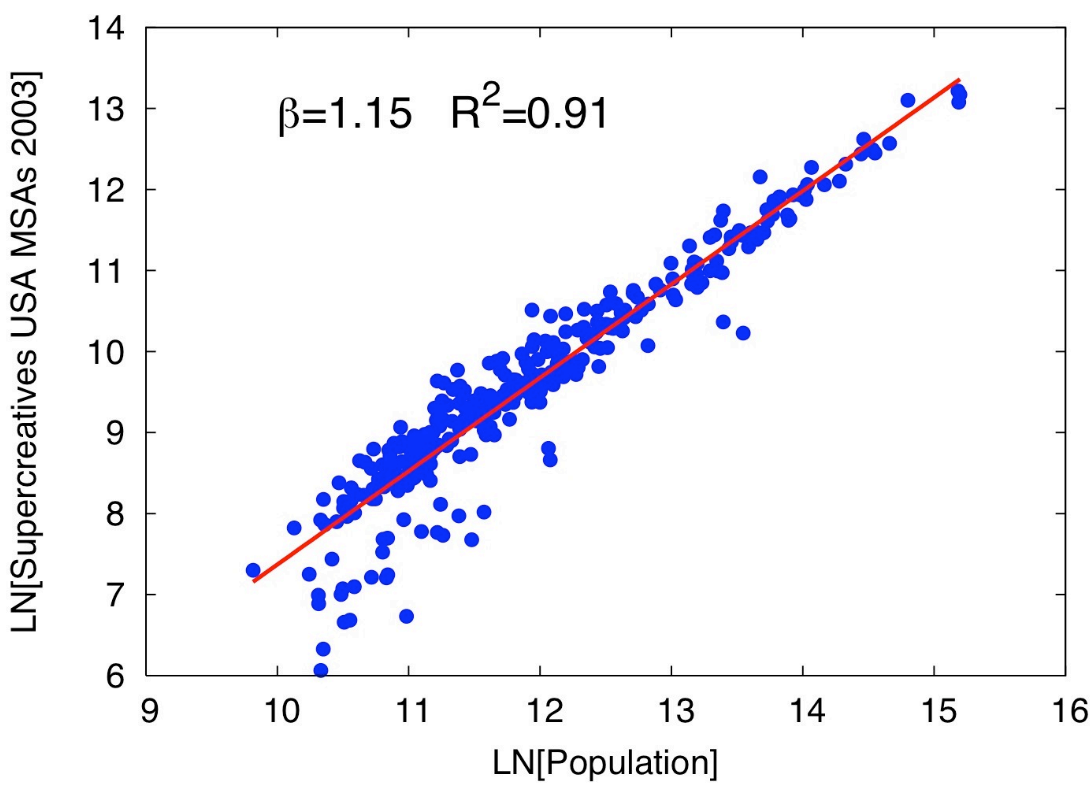
Figure 2 **The pace of urban life increases with city size in contrast to the pace of biological life which decreases with organism size:** a) The scaling of walking speed vs. population for cities around the world b) Heart rate vs. the size (mass) of organisms.

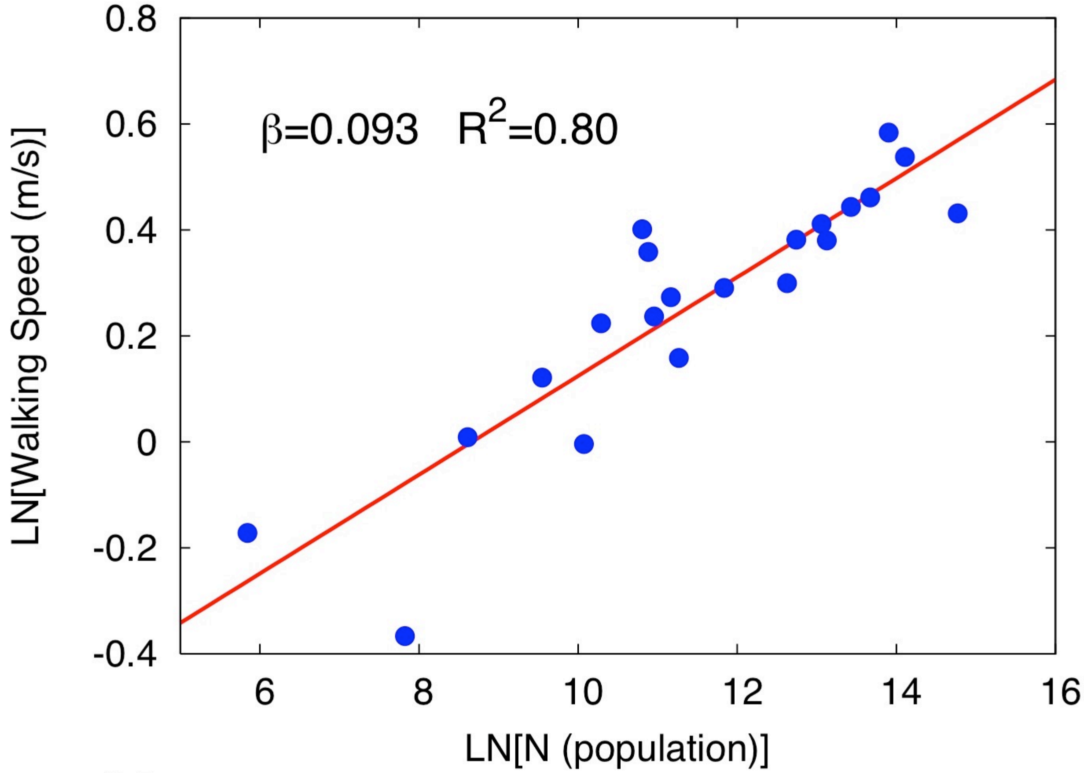
Figure 3 **Regimes of urban growth. Plots of size *N* vs. time *t*:** a) Growth driven by sublinear scaling eventually converges to the carrying capacity N_∞. b) Growth driven by linear scaling is exponential. c) Growth driven by superlinear scaling diverges within a finite time t_c (dashed vertical line) d) Collapse characterizes superlinear dynamics when resources are scarce.

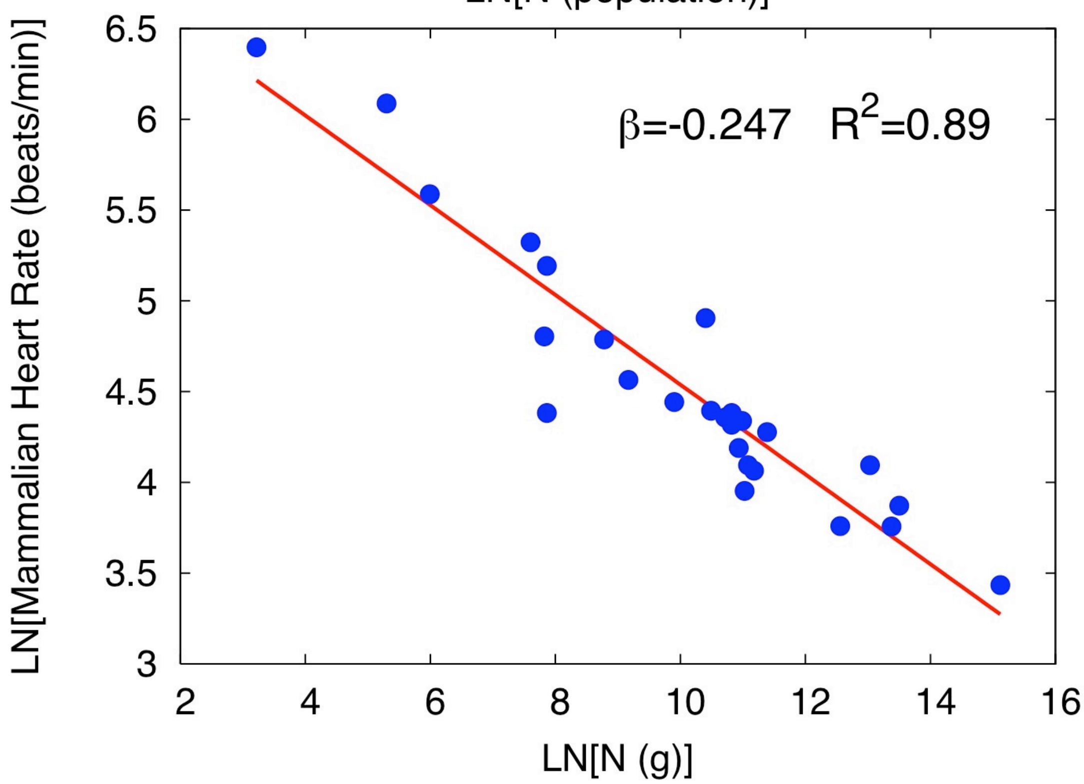
Figure 4 **Successive cycles of superlinear innovation reset the singularity and postpone instability and subsequent collapse:** a) Vertical dash lines indicate the sequence of potential singularities. Eq. 4, with $N \sim 10^6$ predicts t_c in decades. b) The relative population growth rate of New York City over time reveals periods of accelerated (super-exponential) growth. Successive shorter periods of super exponential growth appear, separated by brief periods of deceleration. Inset shows t_c for each of these periods *vs.* population at the onset of the cycle. Observations are well fit by expression (3), with β=1.09 (green line).

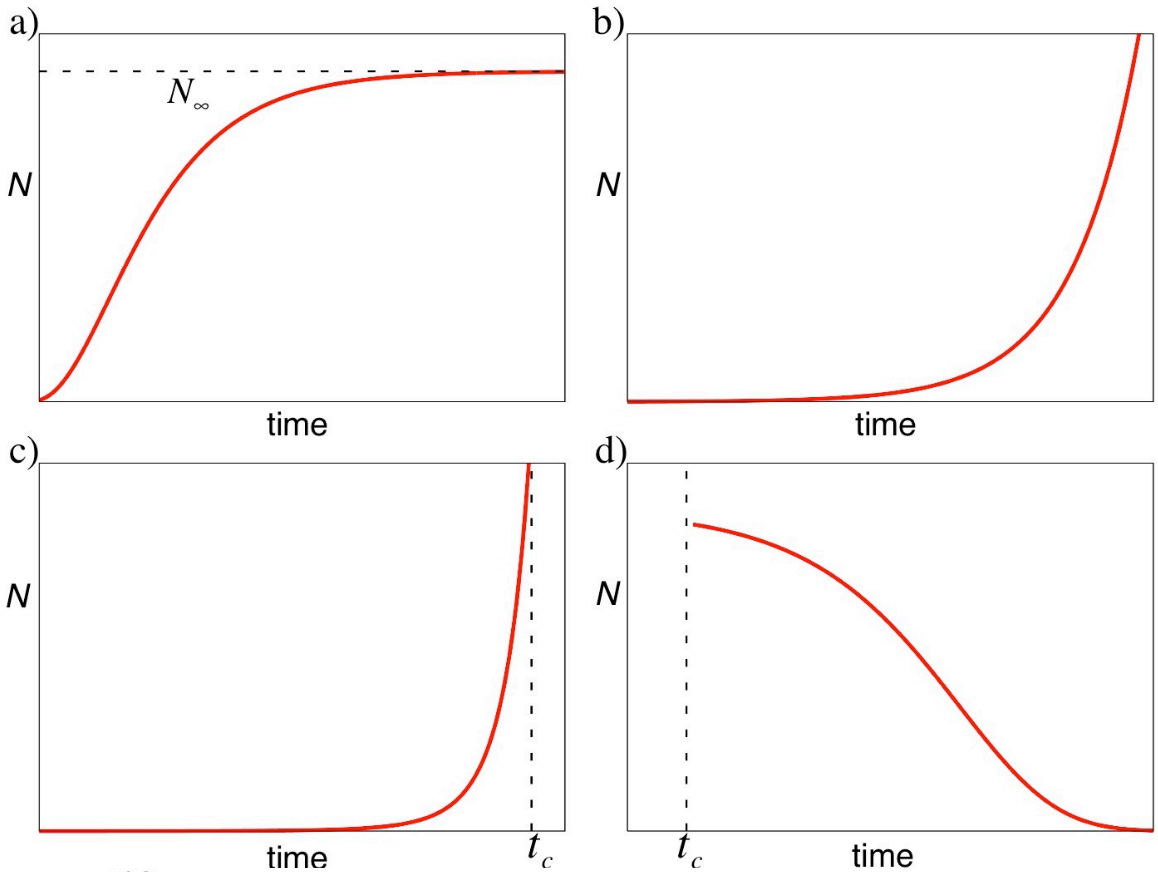

β=1.12 R2=0.97
LN[Total Wages USA MSAs 2004]
LN[Population]
20
21
22
23
24
25
26
27
10
11
12
13
14
15
16
17

Fi


β=1.15 R2=0.91
LN[Supercreatives USA MSAs 2003]
LN[Population]
6
7
8
9
10
11
12
13
14
9
10
11
12
13
14
15
16


β=0.093 R^2=0.80
LN[Walking Speed (m/s)]
LN[N (population)]
0.8
0.6
0.4
0.2
0
-0.2
-0.4
6
8
10
12
14
16


β=-0.247 R^2=0.89
LN[Mammalian Heart Rate (beats/min)]
LN[N (g)]
6.5
6
5.5
5
4.5
4
3.5
3
2
4
6
8
10
12
14
16


a)
N_∞
N
time
b)
N
time
c)
N
time
t_c
d)
N
t_c
time

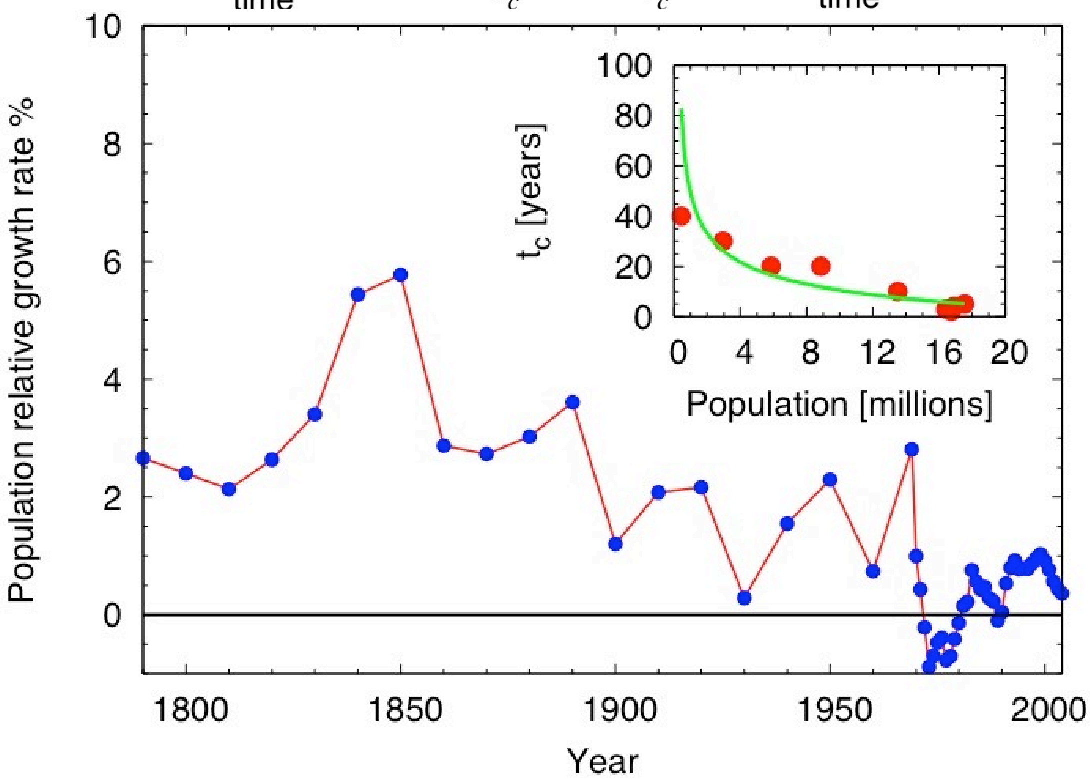

Population relative growth rate %
Year
10
8
6
4
2
0
1800
1850
1900
1950
2000
t_c [years]
Population [millions]
100
80
60
40
20
0
0
4
8
12
16
20

Table 1

Y	β	95% CI	Adj-R^2	Observations	Country-Year
new patents	1.27	[1.25,1.29]	0.72	331	USA 2001
Inventors	1.25	[1.22,1.27]	0.76	331	USA 2001
private R&D employment	1.34	[1.29,1.39]	0.92	266	USA 2002
"supercreative" employment	1.15	[1.11,1.18]	0.89	287	USA 2003
R&D establishments	1.19	[1.14,1.22]	0.77	287	USA 1997
R&D employment	1.26	[1.18,1.43]	0.93	295	China 2002
total wages	1.12	[1.09,1.13]	0.96	361	USA 2002
total bank deposits	1.08	[1.03,1.11]	0.91	267	USA 1996
GDP	1.15	[1.06,1.23]	0.96	295	China 2002
GDP	1.26	[1.09,1.46]	0.64	196	EU 1999-2003
GDP	1.13	[1.03,1.23]	0.94	37	Germany 2003
total electrical consumption	1.07	[1.03,1.11]	0.88	392	Germany 2002
new AIDS cases	1.23	[1.18,1.29]	0.76	93	USA 2002-2003
total housing	1.00	[0.99,1.01]	0.99	316	USA 1990
Total employment	1.01	[0.99,1.02]	0.98	331	USA 2001
household electrical consumption	1.00	[0.94,1.06]	0.88	377	Germany 2002
household electrical consumption	1.05	[0.89,1.22]	0.91	295	China 2002
household water consumption	1.01	[0.89,1.11]	0.96	295	China 2002
Gasoline stations	0.77	[0.74,0.81]	0.93	318	USA 2001
Gasoline sales	0.79	[0.73,0.80]	0.94	318	USA 2001
length of electrical cables	0.87	[0.82,0.92]	0.75	380	Germany 2002
road surface	0.83	[0.74,0.92]	0.87	29	Germany 2002

Table 2

Scaling Exponent	Driving Force	Organization	Growth
β<1	Optimization, Efficiency	Biological	Sigmoidal long-term population limit
β>1	Creation of Information, Wealth and Resources	Sociological	Boom / Collapse finite-time singularity/unbounded growth accelerating growth rates / discontinuities
β=1	Individual Maintenance	Individual	Exponential

Supporting Information

A. Data sources for specific indicators and additional remarks:

-New Patents refers to number of new patents granted by the U.S. Patent and Trademark Office over the period of one year to authors residing in a given MSA. Inventors refers to number of patent authors over a given year, inferred from the same source. Data courtesy of Deborah Strumsky, see also *(1)*. We studied the scaling relations for Patents and Inventors over the period 1980-2001, and have found systematic scaling with metropolitan size and exponents that are commensurate within 95% confidence intervals, over this time period.

- Private R&D Employment (USA) and private R&D establishments (USA) data is taken from the U.S. Economic Census which provides information on the employment levels and number of establishments engaged in conducting original investigation undertaken on a systematic basis to gain new knowledge (research) and/or the application of research findings or other scientific knowledge for the creation of new or significantly improved products or processes (experimental development). Data available from the Economic Census, U.S. Census Bureau (http://www.census.gov/econ/census02).

- R&D employment (China) is analogous to Private R&D Employment USA, but also includes public establishments, which in China comprise a substantial fraction of employment in the sector. Data from the National Bureau of Statistics (NBS), China (http://www.stats.gov.cn), courtesy of Shannon Larsen.

- Total wages (USA) refers to total income from wages over the period of a year. Data from Bureau of Economic Analysis, Regional Economic Accounts (http://www.bea.gov/bea/regional/data.htm).

- GDP (China) refers to Gross Domestic Product of metropolitan areas in China (Urban Administrative Units). Data from the National Bureau of Statistics (NBS), China (http://www.stats.gov.cn), courtesy of Shannon Larsen.

- GDP (Europe and EU countries) analogous to metropolitan GDP (China). Data from Urban Audit, Eurostat (http://epp.eurostat.cec.eu.int/). German metropolitan Road Surface obtained from same source.

- Total housing (USA) refers to total dwellings per Metropolitan Statistical Area. Data from County and City Data Book, US Census Bureau (http://www.census.gov/statab/www/ccdb.html).

- Total employment (USA) refers to full-time and part-time wage positions. Data from Bureau of Economic Analysis, Regional Economic Accounts (http://www.bea.gov/bea/regional/data.htm).

- Total household electricity consumption (China) refers to total energy consumed in households in Urban Administrative Units in 2002. Data from the National Bureau of Statistics (NBS), China (http://www.stats.gov.cn), courtesy of Shannon Larsen.

- Total household water consumption (China) is the total volume of water consumed by metropolitan households in Urban Administrative Units in 2002. Data from the National Bureau of Statistics (NBS), China (http://www.stats.gov.cn), courtesy of Shannon Larsen.

- Fuel sales by gasoline station (USA) refers to the amount of fuel sales in dollars per year sold by gasoline stations located in metropolitan areas. Data available from the Economic Census, U.S. Census Bureau (http://www.census.gov/econ/census02).

- New AIDS cases in selected American cities were obtained from the Center for Disease Control (USA), Divisions of AIDS/HIV Prevention (http://www.cdc.gov/hiv/surveillance.htm).

- According to the definition put forward by Florida in "The Rise of the Creative Class" (Basic Books, Cambridge MA, 2002) pp.327-329, "supercreative" professions are "Computer and Mathematical, Architecture and Engineering, Life, Physical and Social Science Occupations, Education, Training and Library, Arts, Design, Entertainment, Sports and Media Occupations." The occupational classifications were derived from the Standard Occupational Classification System (SOC) introduced by U.S. Bureau of Labor Statistics in 1998. The SOC classification data is constructed using the North American

Industrial Classification System (NAICS). Data Courtesy of Richard Florida and Kevin Storalick.

- German electricity data refers to consumption, generation and distribution for German cities. Data compiled by the Verband der Elektrizitätswirtschaft (VDEW) and published by VWEW (Verlags- und Wirtschaftsgesellschaft der Energiewirtschaft) Energieverlag (http:www.vwew.de). The data contain variables collected from German electricity producers, most of which are local electricity suppliers to a single cities. The electricity market was opened in 1998, so that that local power plants no longer have not to be owned by their city authority, and also that a city can now buy its electricity from other nonlocal producers. In practice, however, according to information from the VDEW, suppliers still deliver today nearly 100% of their generated power to their local cities and meet their needs almost completely.

B. Estimation of the magnitude of superlinear scaling exponents

The generality of superlinear exponents associated with social indicators leads to the question of the magnitude of their values. Here we present an argument that leads to a semi-quantitative estimate of the observed numerical ranges. We emphasize that a detailed predictive theory for the scaling exponents β, integrating interactions between people and institutions, is the ultimate long term goal of any picture of urban scaling. The argument below simply integrates some of its necessary ingredients to produce a rough estimate of the exponent's plausible ranges.

The exponents β are commensurate for many social quantities, but there is no strong indication that they must be identical for different urban systems. Can we determine β from the formulation of a maximization or minimization principle, as was done in biology for the properties of networks of resource distribution? There is little doubt that human interactions in a city may be represented in terms of networks, it is difficult to foresee their general structural properties. What we do know is that if a city provides an enlarged space of opportunities for effective interactions between people but also that the number and intensity of such interactions is constrained by time, effort and by limits on individual cognition. This is what Milgram, writing on the experience of living in cities *(2),* referred to as information saturation. This observation can be used to produce an estimate of the values of β.

First consider the total number of effective contacts C between individuals in a population of size N. The maximal value that C can take is $C=N(N-1)/2$, implying a bound on $\beta \leq 2$. This upper bound corresponds to every individual in a city knowing everyone else, which is clearly not realistic as cities grow large. Instead consider that the quantities Y of Table I are proportional to the number of effective contacts so that $C(N) = C_0 N^{\beta}$. Let's now define P as the ratio of productive contacts *per capita* between the largest city with population N_{max} and the smallest city with population N_{min}, so that

$$P = \left(\frac{N_{max}}{N_{min}}\right)^{\beta-1} \to \beta = 1 + \frac{\log(P)}{\log(\frac{N_{max}}{N_{min}})}$$

P expresses by how much an individual's time, effort and cognitive ability can be expanded in response to the greater demands of the largest city, relative to those of the smallest town. If we assume *P*=10-100, and $N_{max}/N_{min}=10^7$, we obtain β=1.14–1.28, which is in qualitative agreement with the observations.

C. Additional Figures for Super-Exponential Growth periods of New York City population

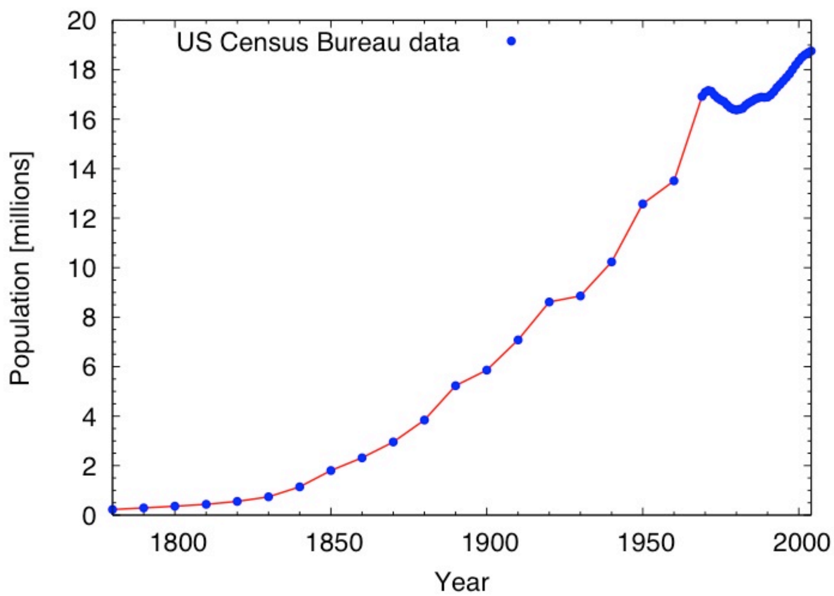


Figure S1 – The population of New York City's MSA over time 1780-2004 (data from US Census Bureau). Population evolution is punctuated by periods of super-exponential growth, separated by brief periods of deceleration, see also Figure S4.

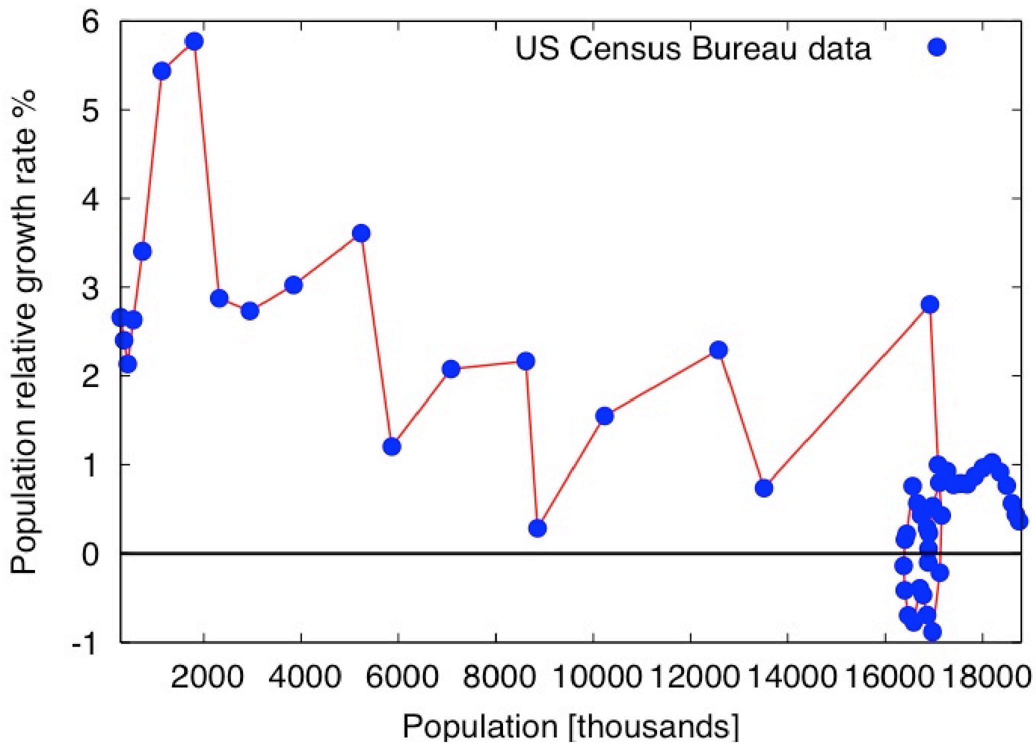


Figure S2 – The relative growth rate of New York City's population experiences successive super-exponential growth periods, punctuated by brief decelerations. During these periods the relative rate grows with the population as predicted under super linear scaling (see also Figure S4). Note that exponential growth would result in a horizontal line, independent of population, which is not a feature of the observed dynamics.

Note also that periods of super-exponential growth have been identified for the total human population of the world in *(3,4,5)*.

D. Generalized growth equation

Here we dispense with the assumption that the costs of maintenance are linear in *N*. As such we write the growth equation as

$$\frac{dN(t)}{dt} = \left(\frac{Y_0}{E}\right) N^{\beta}(t) - \left(\frac{R}{E}\right) N^{\alpha}(t). \qquad \text{(A1)}$$

Qualitatively the solution has the same properties discusses in the main text. If both β, α <1 the solution is always a sigmoid. If at least one α, β >1 there are always two distinct possibilities 1) $\beta>1$, $\beta>\alpha$, and 2) $\alpha>1$, $\alpha>\beta$.

Regime 1), which includes the interesting possibility that $\beta > \alpha > 1$, has two possible solutions:

- $N(0) > (R/Y_0)^{1/(\beta-\alpha)}$: The rate of growth is dominated by available resources and the solution is a growing super-exponential, which reaches infinity in the finite time.

- $N(0) < (R/Y_0)^{1/(\beta-\alpha)}$: Costs dominate the rate and the population collapses.

Consequently, in this regime, the fixed point $N=(R/Y_0)^{1/(\beta-\alpha)}$ is unstable and small perturbations around it grow either towards a finite time singularity or collapse.

Regime 2), analogously, has two possible solutions

- $N(0) > (R/Y_0)^{1/(\beta-\alpha)}$ costs dominate the growth rate and the solution decreases towards the carrying capacity $N_\infty=(R/Y_0)^{1/(\beta-\alpha)}$.

- $N(0) < (R/Y_0)^{1/(\beta-\alpha)}$ available resources dominate the rate and the solution grows towards N_∞.

Thus in Regime 2) the carrying capacity $N=N_\infty$ is a stable fixed point to which population trajectories in both regimes converge to at long times.

For completeness we give an analytical method to solve the generalized growth equation. Define $A= Y_0/E$ and $B=Y_0/E$. The growth equation can be simplified by a change of variables $y = N^{1-\beta}$, which results in

$$\frac{dy}{dt} = (1-\beta)A - (1-\beta)By^{\lambda}$$

where $\lambda=(\alpha-\beta)/(1-\beta)$. This equation can be solved easily given the function f such that

$$\frac{df}{dy} = \frac{1}{A - By^{\lambda}}$$

The solution then obeys $f(y)=f(N^{1-\beta})=(1-\beta)t$.

In particular we see that if $\lambda=1$ (i.e. $\alpha=1$) then $f(y) = - B^{-1} \log (A-B\, y)/K$, where K is an integration constant, and a little algebra reveals the solution of the main text.

In general we obtain

$$f(y) = \frac{y}{A}\, {}_2F_1(\lambda^{-1}, 1, 1+\lambda^{-1}, -\tfrac{B}{A} y^{\lambda}) - K'$$

where ${}_2F_1$ is Gauss' hypergeometric function and K' is an integration constant. The function ${}_2F_1$ does not have a simple closed form in terms of elementary functions. However it does have an interesting transformation property due to Euler

$$ {}_2F_1(a,b,c,z) = (1-z)^{-b}\, {}_2F_1(c-a,b,c,\frac{z}{z-1}) $$

With this transformation we can write the solution as

$$ \frac{N^{1-\beta}}{A+BN^{\alpha-\beta}}\, {}_2F_1(1,1,1+\lambda^{-1},\frac{\frac{B}{A}N^{\alpha-\beta}}{1+\frac{B}{A}N^{\alpha-\beta}}) = K + (1-\beta)t $$

where K equals the left hand side of the equation at time t=0. Although non-trivial in general the argument of the hypergeometric function goes to a pure number in the limit of large N. We are then left with a transcendental equation for N, which depends both on the sign of 1-β and of α–β.

References:

1. Bettencourt, L. M. A., Lobo, J., Strumsky, D., (2006) Santa Fe Institute Working Paper 04-12-038, in print in *Res. Policy*.

2. Milgram, S. (1970) *Science* **167**: 1461.

3. Cohen, J. E. (1995) *Science* **269:** 341 (1995).
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4. Kremer, M. (1993) *Q. J. Econ.* **108:** 681.

5. Kapitza, S. P. (1996) *PHYS-USP* **39**: 57.