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Pamela Oliver; Gerald Marwell; Ruy Teixeira

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A Theory of the Critical Mass. I. Interdependence, Group Heterogeneity, and the Production of Collective Action[1]

Pamela Oliver and Gerald Marwell
University of Wisconsin—Madison

Ruy Teixeira
University of Notre Dame

Collective action usually depends on a "critical mass" that behaves differently from typical group members. Sometimes the critical mass provides some level of the good for others who do nothing, while at other times the critical mass pays the start-up costs and induces widespread collective action. Formal analysis supplemented by simulations shows that the first scenario is most likely when the production function relating inputs of resource contributions to outputs of a collective good is decelerating (characterized by diminishing marginal returns), whereas the second scenario is most likely when the production function is accelerating (characterized by increasing marginal returns). Decelerating production functions yield either surpluses of contributors or order effects in which contributions are maximized if the *least* interested contribute first, thus generating strategic gaming and competition among potential contributors. The start-up costs in accelerating production functions create severe feasibility problems for collective action, and contractual or conventional resolutions to collective dilemmas are most appropriate when the production function is accelerating.

INTRODUCTION

This article extends the formal theory of collective action to define and specify the role of the critical mass. For a physicist, the "critical mass" is

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the amount of radioactive material that must be present for a nuclear fission explosion to occur. Social movement activists and scholars often use the term in a loose metaphorical way to refer to the idea that some threshold of participants or action has to be crossed before a social movement "explodes" into being.[2] But the critical mass has not before been treated as a serious theoretical concept. In this article we argue that the concept of the critical mass deserves a central place in collective action theory. We show that the critical mass plays very different roles in producing different kinds of collective action.

Recent work on collective action generally begins with Mancur Olson's *The Logic of Collective Action* (1965). This book has been so influential with social scientists primarily because it calls attention to the intrinsic difficulty of mobilizing collective action in pursuit of collective goods, that is, goods that must be provided to all group members if they are provided to any. Olson's analysis is powerful because it is so general, applying to union organizing, business cartels, farmers' lobbies, volunteer fire departments, environmental groups, civil rights demonstrations, and thousands of other kinds of actions and organizations.

But this generality conceals as well as reveals, for collective actions differ. For example, a few actors sometimes provide a collective good enjoyed by many, as when the NAACP legal staff and a few willing complainants won the 1954 Supreme Court decision outlawing segregated schools. At other times, widespread mass action achieves the collective good, as in the Montgomery bus boycott and the civil rights marches. This article provides a theoretical analysis of some of the factors that produce such differences.

Our "dependent variables" are the probability, extent, and effectiveness of group actions in pursuit of collective goods. We have two sets of "independent variables," factors that are particularly crucial to collective action as a social process but that have received scant attention in the literature.

The first, and perhaps most important, of our independent variables is the *form of the production function* relating contributions of resources to changes in the level of the collective good. Different types of production functions create dramatically different dynamics in otherwise similar situations and thus lead to different outcomes. Previous treatments in the literature have obscured these differences.

The second set of factors in our analysis concerns the *heterogeneity* of interests and resources in the population. Olson argues briefly that group

[2] In fact, the newsletter of the ASA's Collective Behavior and Social Movement's section is called the *Critical Mass Bulletin;* the newsletter and its title predated the section.

heterogeneity is favorable for collective action (1965, p. 29). Hardin elaborates this position (1982, pp. 67–89). However, the significance of these arguments has not been widely recognized. Many formal analyses of collective action treat only one actor's decision at a time, extrapolating from the individual to the group with an implicit assumption that collective action is uniformly distributed.[3] In contrast, we assume that collective action usually entails the development of a *critical mass*—a small segment of the population that chooses to make big contributions to the collective action while the majority do little or nothing. These few individuals are precisely those who diverge most from the average. Thus, the heterogeneity of the population—specifically, the number of such deviants and the extremity of their deviance—is one key to predicting the probability, extent, and effectiveness of collective action.

Interdependence: Bringing Social Process Back In

We begin our analysis of these issues by breaking with tradition and assuming *interdependent* decisions. Most writers on collective action, including Olson, start from the usual economists' assumption that each individual makes an isolated, independent decision about contributing.[4] This is a perfectly reasonable assumption for the large markets that economists study, but not for the majority of collective actions. The responses to mass-mailed fund-raising letters may meet the independence assumption, as might certain antipollution efforts, but most phenomena of sociological interest do not.

For a sociologist, interdependence is precisely what is most interesting about collective decisions. Of course, there are many kinds of interdependence. Interdependence is not a "thing": it is definable only as the absence of complete independence. Our analysis focuses on one specific kind of interdependence. We simply assume that individuals take account of how much others have already contributed in making their own decisions about contributing to a collective action. This same assumption is the foundation of Granovetter's threshold models (1978, 1980) and is implicit in some of Oberschall's work (1980).

If people take account of others' previous actions, decisions cannot be

[3] An exception is the article by Oberschall (1980), whose model assumes interest group heterogeneity in the context of the general third-order curve.

[4] In discussing "intermediate" groups, i.e., groups in which members will "notice if any other member is or is not helping to provide the good," Olson notes that the group might be able to "organize" (1965, p. 51). In other words, the actions of group members may influence the actions of others. However, his basic argument, and almost all of his analysis, concerns large or "latent" groups.

made simultaneously. Therefore, we make the further simplifying assumption that decisions are sequential, that is, that individuals take turns, making their decisions one at a time. Clearly, this assumption of complete sequentiality is, in its extreme, as unrealistic as the assumption of instantaneous simultaneity of independent decisions. Nevertheless, it allows us to identify a fundamental property of collective action, namely, whether contributions have positive or negative effects on subsequent contributions. This property underlies our discussion of production functions.

The Independent Variables: Production Functions and Group Heterogeneity

Consider the homeowners in a neighborhood of 1,000 homes who suddenly learn that their highly regarded neighborhood school has been scheduled to close. Even though the residents' most direct concern may be for the children, let us keep the matter simple by focusing on property values and assume that closing the school will produce a total loss in the neighborhood of $20 million, or an average of $20,000 a home.[5] The past behavior of the city school board indicates that political and legal pressure can be effective in reversing decisions to close schools, but the necessary pressure requires professional legal assistance, and that costs money. If the residents spend no money, the school will certainly be closed. If they spend $100,000 to hire a lawyer to develop a legal brief and take the case to court, they will win the case and the school will certainly stay open. Between $0 and $100,000, intermediate contributions produce intermediate probabilities of success. The question confronting each homeowner, then, is how much to contribute to the neighborhood's legal fund. The problem confronting *us* is predicting that response.

Production Functions

If an individual faced with this situation wishes to make a rational decision, the first question that he or she should ask is, How much of a return can be expected from some specified level of contribution? Will a contribution of $100 increase the likelihood of keeping the school open 0.1%, 1%, or 10%? More important, what is the differential effect of a larger

[5] This total value of the public good may seem exceptionally high to the reader who has not worked with numerical instances of public goods. There is a paradox here that is often overlooked even by experienced scholars. If a public good has pure jointness of supply (Head 1974; Samuelson 1954), its "value" increases with the number of people who share in it while its cost does not. Thus, large groups will necessarily have very large total interests in public goods.

contribution? If \$100 raises the probability by 1%, what does \$200 do? Does it simply double the probability change to 2%? Or, does the \$200 change the probability proportionately more (say, 5%) or proportionately less (say, 1.2%)? In short, what is the production function? Obviously, larger contributions are more rational than smaller ones when they have proportionately larger effects on the probability, and vice versa.

Economists usually assume that the production function is a general third-order or S-shaped curve, like that sketched in figure 1*a*, to which we shall return below. In this article, we shall look at two opposite types of production functions that can be viewed as special cases of this general third-order curve: "decelerating" and "accelerating" production functions.

Decreasing marginal returns (decelerating).—The first case involves decreasing marginal returns to contributions, with the first few units of resources contributed having the biggest effect on the collective good, and subsequent contributions progressively less. For ease of exposition, we shall refer to goods with these kinds of production functions as "decelerating." Figure 1*b* presents an example of this kind of curve.

A simple substantive example of deceleration might be calling city hall about a pothole in a middle-class urban area: the first person who takes the time to call makes the probability .4 that the hole will be fixed, the second raises it to .7, the third to .8, the fourth to .85, the fifth to .88, the sixth to .90, and each subsequent call adds only a tiny amount to the probability. Another example might be organizing a picnic. Half the fun of a picnic is assured if someone arranges for a good location and adequately publicizes the time and place. Another good-sized increment comes from making some definite arrangement about food. From then on, each additional contribution of game equipment or food adds to the picnic's likely success, but each of these increments is smaller than the initial ones. Returning to our homeowners fighting a school closing, a decelerating production function would arise if simply paying the first few thousand dollars to the lawyer to begin preparing a legal brief makes the probability quite high that the school board will change their position and keep the school open without the expense of a legal battle (although certain success can be assured only by going to court). This might be the case if the neighborhood has historically been a stronghold of support for the school board and they fear alienating former allies.

Increasing marginal returns (accelerating).—In contrast, in cases with increasing marginal returns, successive contributions generate progressively larger payoffs; therefore, each contribution makes the next one more likely. Initial contributions of resources have only negligible effects on the collective good, and only after long start-up costs have been borne do subsequent contributions start to make a big difference in the collective good. Figure 1*c* presents an "accelerating" curve.

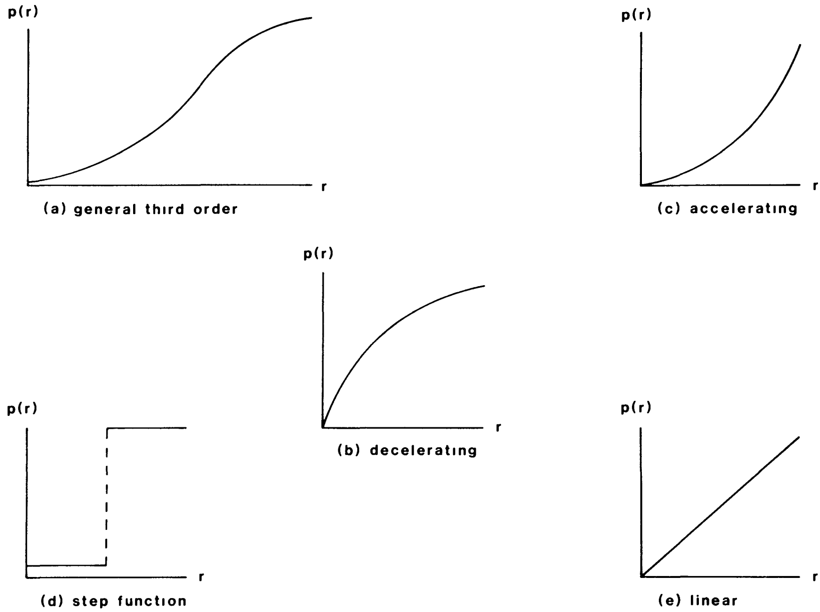

FIG. 1.—Different shapes of production functions

An example of an accelerating production function might be calling about a pothole from a poor minority urban area with little political clout: it takes 20 calls before the probability reaches even .01 and another 20 to reach .1, but the next 20 calls worry city hall and make the probability .9. A second example might be creating a community center: hours and dollars have to be spent buying the land and materials and building the structure before the last few hours of painting it and furnishing it produce big payoffs in having a place to meet. Our homeowners would face an accelerating production function if paying for the lawyer's initial work has only a negligible effect on the school board, whereas raising the money to cover the preliminary work and take the case to court makes the probability of victory accelerate to certainty. This case might arise if the neighborhood were politically unimportant.

Assumptions in the literature.—Neither of these cases has received much attention in the literature on collective action. Political scientists interested in voting have studied step functions such as the one shown in figure 1*d,* in which the one vote changing a minority to a majority is critical (see Hardin 1982, pp. 52–61, for an extended discussion). Many nontechnical discussions assume that each unit of resource contributed "buys" a constant amount of the collective good, implying that the production function is linear (see figure 1*e*). We show below that this deceptively simple assumption yields rather extreme consequences.

The more usual assumption of economists (and of some political scientists and sociologists, such as Oberschall [1980]) is that the production function is an S-shaped, third-order curve, something like figure 1*a*. It begins with a period of start-up costs or other sources of low but increasing marginal returns, which leads to a period of higher returns; then satiation sets in and produces diminishing marginal returns. This general curve is applicable to a wide variety of situations, but its very generality obscures the importance of the relative sizes and slopes of the periods of increasing and decreasing marginal returns. The decelerating curve central to our analysis may be seen as a general third-order curve whose period of initial low returns to investment has been reduced to zero, and the accelerating curve is interpretable as a third-order curve with an especially long initial period of low returns and a period of high returns extending to the edge of the range of feasible contributions.

Group Heterogeneity

Within most interest groups there is a range of interest in (or desire for) the collective good. Although we all want clean air, those among us suffering from emphysema want it more. For those with homes of equal value, a potential school closing is more important to those homeowners with school-age children than to those without. Similarly, group members may differ substantially in the resources available to them. Some homeowners might have cash available to contribute to the neighborhood war chest, while others may be "strapped" by other cash demands. Both of these forms of heterogeneity affect the level of collective action that might be expected from a given group.

Of these two factors only interest heterogeneity has received much attention from previous scholars. For example, Olson describes the "exploitation of the great by the small" (1965, p. 29), by which he means the difference in participation in collective action by those individuals who have a very large interest in the good and those whose interest, though positive, is relatively small. Since the former are so interested, he argues, they will provide the good themselves, regardless of the actions of the less interested parties. The latter exploit the "great" by not contributing at all: they know they will get the good anyway, because the "great" will provide it. Hardin (1982) argues that interest in nonfungible goods is more heterogeneous than interest in fungible goods (p. 71), that collective action is more likely when individuals with high interest in a collective good lack private alternatives (pp. 72–75), and that the same "good" may have widely different consequences for the interests of different people (pp. 76–83).

Heterogeneity of resources has received much less attention in the literature (exceptions are Marwell and Ames 1979; Marwell and Oliver 1984, pp. 15–16; and arguments such as that of Hardin [1982, pp. 83–89] that political processes tend to produce goods that mainly benefit wealthier people). In large measure, this is because economists generally assume that money is always available at a high enough interest rate and that time, expertise, energy, and even political influence may be bought from others with that money. Thus, if interest is high enough, resources can always be had, and the cost of those resources is part of the cost of the good. Those of us who have been involved in non-business-oriented collective actions, though, know how often it is the scarcity of resources that seems to bound our options. It is very hard to get financial institutions to lend us money to contribute toward political agitation, no matter how much loss to the value of our homes we hope to avoid. Borrowing funds to give to the United Way is a rare event as well. Often we are left with those resources of strength, skill, energy, time, and spirit with which we are personally endowed and nothing else. Since these kinds of collective action are of widespread sociological interest, we prefer to maintain the variables of resource availability and heterogeneity in this analysis.

Our analysis of heterogeneity focuses on the *form* of the distribution. If an interest group is heterogeneous, there may be some highly interested or highly resourceful people available for a critical mass even when the mean interest or resource level is rather low. Greater variance and positive skew are the statistical properties of distributions that favor the presence of such persons. Hardin tells the true story of an industrialist willing to pay $27,000 in campaign contributions to lobby for a tax change worth $15 million to him, even though many others would gain a total of $150 million from that change (1982, pp. 78–79). In terms of our homeowner example, a similar situation would arise if 50 of the 1,000 houses in the neighborhood were new homes about to be sold by a developer. If every other homeowner were going to lose $20,000 when the school closed, the developer would lose $1 million. We do not need formal analysis to conclude that the developer would find it profitable to support hiring the neighborhood's lawyer: even if he had to pay the entire $100,000 cost, the net loss avoided would be $900,000. In contrast, in a homogeneous neighborhood where every owner stands to lose exactly $20,000, no homeowner would find it profitable to contribute $100,000, and it is entirely possible that no one would contribute anything. Interesting and realistic cases are more complex than this, but it is always misleading to treat a heterogeneous interest group as if it were homogeneous by examining only the aggregate group interest in the collective good.

We show below that the specific effects of interest and resource heterogeneity depend on the form of the production function. In particular, interest heterogeneity is always significant, whereas resource heterogeneity is much more important when the production function is accelerating. It should be obvious, however, that regardless of the form of the production function, a positive correlation between interest and resources is highly favorable for collective action, as it increases the probability of there being a few highly interested and highly resourceful people who are willing and able to provide the good for everyone. Conversely, of course, it has often been argued that the negative correlation that obtains for poor people's interests makes collective action difficult (e.g., O'Brien 1975; Piven and Cloward 1977).

Additional Assumptions

Our analysis requires several assumptions beyond those noted above (such as sequential interdependence). Several of these concern the decision processes of individuals. Particularly important is the assumption that all group members work with full information. More specifically, each person is assumed to know the production function and how much has been contributed by all other group members at any given time. This assumption is often unrealistic, but the fundamental dynamics we outline below underlie responses to imperfect information as well. Our analysis suggests that imperfect information will have different consequences in different cases. A second assumption is that there is some common metric or standard of comparison for the cost of contributing a unit of resource on the one hand and the value of the collective good on the other hand. This does not mean that everything can be translated into dollars, but it does mean that people have some standard of comparison. A third assumption is that the criterion of maximizing expected value characterizes people's decisions, rather than risk aversion or some other criterion.

In addition to these psychological assumptions, we need to make several simplifying assumptions about the situation. We will consider only a dichotomous collective good (keeping the school open) that has probability P of being provided.[6] Furthermore, we assume that P is entirely a

[6] Assuming a dichotomous good and continuous probability makes the exposition easier without restricting the substance of the analysis. Continuous goods provided determinately have comparable production functions. Continuous goods provided probabilistically cannot be as neatly discussed, but they are subject to the same general principles.

function of r, the total number of units of some resource that has been contributed and that it is bounded by zero and one. If no resources are contributed, P is zero. If R or more units of resource are contributed, P is one and the collective good is provided with certainty.[7] In our example, R is $100,000. The resource in our example is dollars, but it could be hours or something else. We also assume that the production function increases monotonically and is continuous and twice-differentiable. In the example, this means that each dollar contributed has a positive effect on the probability of keeping the school open, although the effect may be very small. Contributing a unit of resource has a constant cost k that is the same for all individuals.[8] In our example, we assume that the cost of contributing a dollar is $1. The variable V stands for an individual's value or interest in the collective good; we assume that V is nonnegative for everyone in the interest group.[9] By definition, the sum of V across all group members is greater than the product kR, otherwise the good would not be a collective good. The sum of V in our example is $20 million and kR is $100,000.

We shall investigate the dynamics of accelerating and decelerating production functions by contrasting two extreme cases, which we may call *uniformly accelerating* and *uniformly decelerating*. The slope of a uniformly accelerating production function is everywhere increasing; that is, its second derivative is always positive. The slope of a uniformly decelerating production function is everywhere decreasing; that is, its second derivative is always negative. In both cases, the production function

[7] The assumption that the production function is bounded has mathematical implications; in particular, our definitions of uniform deceleration and uniform acceleration below require it. The lower bound is obvious for collective action since there is some baseline level of the collective good that will occur if nothing is done; it does not matter for the analysis whether this lower bound is zero or any other number. One can imagine production functions with no upper bound; i.e., each additional unit of contribution makes *some* difference, but there is always some point above which this difference is negligible.

[8] It is common to assume that costs are not constant, in particular that the distribution of marginal costs is U-shaped. Our general argument about decelerating and accelerating production functions developed below could be recast in terms of cost and would yield the same general conclusions. Accelerating production functions would be those with high start-up costs and declining marginal costs thereafter, whereas decelerating production functions would arise when the cost of initial contributions is low but marginal costs increase thereafter.

[9] In keeping with the usual definitions of "group" in the literature, we assume that group membership is defined by a positive interest in the good. If, for some reason, a neighborhood resident feels that he would *profit* by the school's being closed, he is not considered a member of the group and is not part of the present analysis. Polarized interests may lead to interesting conflicts and social processes, but, as will become clear, models for the extent of a single collective action have enough complexity for one article.

itself is everywhere increasing—greater contributions always produce greater probabilities of obtaining the collective good. It is whether the *marginal* returns are increasing or decreasing that differentiates the two cases. All the mathematical results below refer to uniformly accelerating or uniformly decelerating production functions. We often omit the qualifier "uniformly" in an attempt to aid the flow of the discussion.

RESULTS I: DIFFERENCES AMONG PRODUCTION FUNCTIONS

In this section we begin by considering very simple sets of conditions, in which we focus primarily on the effects of varying production functions.

The One-Actor Case

Let us begin with a radical simplification and assume that the good is really a private good, that is, a good whose benefits accrue to only one actor. In our example, imagine that all the homes in the neighborhood are rental units owned by a large corporation that stands to lose $20 million if the school is closed.

We may represent the individual's (corporation's) decision by letting N stand for the net payoff to an individual from a contribution of a given size r, in which case we may write this simple decision equation:

$$N(r) = VP(r) - kr. \tag{1}$$

This equation says that the net payoff $N(r)$ from contributing r units of resource equals the expected payoff of the contribution minus its cost. The expected payoff is the product of the individual's interest level V (which is constant for an individual) and the probability $P(r)$ produced by a contribution of size r. The cost is the product of k, the constant cost per unit contribution, and r, the number of units contributed. Since k and V are constants, the net payoff N varies with $P(r)$, which in turn varies with r.

We wish to find the value for r that maximizes the individual's profit. To find this optimum contribution, we take the derivative of equation (1) with respect to r, set the derivative denoted by $N'(r)$ equal to 0, and solve for the derivative of the production function, $P'(r)$, yielding:

$$P'(r) = \frac{k}{V}. \tag{2}$$

The term $P'(r)$ is the derivative or slope of the production function, that is, the change in the probability of obtaining the collective good produced by a contribution of one resource unit at the point r. In general, the slope $P'(r)$ varies with r. The steeper the slope, the greater the effect of a unit contribution. The term k/V is the ratio of the cost of one unit of resource to the actor's (total) value for the collective good. Since k and V are assumed to be constants for an individual, this ratio is constant. When $P'(r)$ is greater than k/V, each unit contributed produces a profit; when $P'(r)$ is less than k/V, each contribution produces a loss. When $P'(r)$ equals k/V, we are at the turning point between profit and loss. The maximum net payoff, if any, will occur at such a turning point.

A Homogeneous Group

Collective action is about groups of actors, of course, not wealthy individuals or corporations. Therefore, we next consider a slightly more complex case, a completely homogeneous group in which all individuals attach the same value V to the collective good (e.g., \$20,000 per house if the school remains open) and have the same fixed but small quantity Q of resources (e.g., \$2,000) that they can contribute or not. To find the optimum contribution, we need only to recognize that the $P(r)$ curve is the same one that we were considering for the private good. The only difference is that here the probability of the collective good depends not only on an individual's own contribution (and the form of the production function) but also on C, the amount that has already been contributed by others, and our attention is restricted to that portion of the curve between C and $C + Q$. At this point in the analysis we are not concerned with the effects that individuals' expectations regarding the future behavior of other group members have on their own behavior. In keeping with our "sequential" decision model, each individual is brought up to the decision point in turn and decides whether or not to contribute wholly in terms of whether or not such a contribution will be immediately profitable to him or her. In the next "Results" section this assumption will be relaxed.

Solutions Differ for Different Production Functions

If we were economists, this might be the end of our inquiry, as the general third-order production function economists usually assume guarantees that equation (2) has a point solution that is a maximum. However, in the realm of collective action, many production functions do not meet these assumptions, so we need to pursue the matter further.

Linear functions and other dichotomous situations.—We begin by dismissing cases that are not of great interest in the present work, although they can be substantively important. If the slope of the production function $P'(r)$ is everywhere greater or less than the cost/value ratio k/V, the equality is never satisfied and the shape of the production function is irrelevant. Everyone will contribute either everything possible or nothing. If $P'(r)$ is always greater than k/V, every resource expenditure brings that much more profit, whereas if $P'(r)$ is always less than k/V, every expenditure brings that much more loss. The corporation buying a private good and the 1,000 small contributors produce the same result in either case—no action or maximum action.

What may not be immediately obvious is that we are always in this situation if the production function $P(r)$ is *linear*. Since each unit of resources spent produces a *constant* increase in the probability of the collective good, this constant must always be greater than, less than, or equal to k/V. If it is greater than k/V, everyone should contribute the maximum possible; if it is less than or equal to k/V, everyone should contribute nothing. Therefore, a linear production function *produces* dichotomous collective action in which each individual's choice is independent of anyone else's choice.

If every telephone call raises the probability of fixing the pothole by .02, or if every \$2 buys another dinner for the city's poor, a person's decision to act does not depend on what anybody else does. Linear production functions have no start-up costs and (until some maximum is abruptly reached) no satiation effects. They are not very common in collective action.

Decelerating functions.—Since contributions never lower the probability of the collective good, the slope $P'(r)$ is always positive. In the uniformly decelerative case, this positive slope begins at its maximum and then consistently decreases toward zero. This means that the point where $P'(r) = k/V$ is a maximum. The rational decision maker should invest to this point, thereby investing an optimum amount of resources to produce a high (but not certain) probability of obtaining the collective good. For example, the corporation in our "one-actor" case might find \$15,000 to be the optimal amount to spend on the lawyer, making success quite probable without "wasting" the additional \$85,000 to buy certainty. This basic pattern is very similar to the situation that economists usually discuss.

When the production function is decelerating, a homogeneous group of small contributors produces the same analytic result as does one large corporate actor. Individuals should contribute sequentially until the point is reached where $P'(r) = k/V$, then no individual should contribute any more. If everyone cares equally about a picnic, we will probably find that once one person spends an hour arranging for a place and another spends two hours arranging for food, it is unlikely that anyone else will volunteer to spend comparable amounts of time on things that are less central to the success of the picnic. Once a core of citizens initiates an anticrime Neighborhood Watch program, it will be difficult to persuade others to do their share. Once someone has called about a pothole in a middle-class neighborhood, no one else is likely to call. Given our assumptions about sequential decisions, we would predict that if the production function is decelerating *some* collective action will occur, but provision of the good with certainty (the maximum that *could* occur) is quite unlikely.

Accelerating functions.—The equivalence between the homogeneous group and the corporate actor does not hold true for the accelerative case. Consider the private good for the corporate actor first. Although the slope of a uniformly accelerating production function is positive, it starts at its minimum (near zero) and consistently increases. This means that the point where $P'(r) = k/V$ is a *minimum,* not a maximum. As long as r is below this point, each additional unit contributed produces another unit of loss for the corporation. For larger r's, each additional unit invested produces a profit, but the initial losses still have to be compensated for, so r must be much greater than the turning point before the net payoff is greater than zero. From that point, things get better and better, and the actor should, rationally, contribute everything available to the collective action. This is a somewhat paradoxical result: if the production function is accelerating, the "optimum" is to provide the good with certainty, if one has the resources to do so. If one is interested enough to be willing to make some investment in the good and possesses enough resources to "buy" the whole thing, one should buy all of it, and any less would be irrational.

For a homogeneous group of small investors, though, an accelerating production function has radically opposite effects—the collective good generally will not be provided. This is because at the initial levels of previous contributions, $P'(r)$ is less than $k/V,$ so no individual should contribute. If no individual contributes, the point where $P'(r)$ is greater than k/V will never be reached. If, somehow, contributions happen to be made so that C is at the point where $P'(r)$ equals $k/V,$ each additional contributor would obtain a positive payoff from his contribution. But under the assumption of a homogeneous group making small contributions, this will never happen.

If, somehow, the initial contributions get made, later contributions can become profitable and collective action will tend to snowball, drawing in more and more people until the maximum is reached. It is accelerating production functions that tend to produce widespread mass action once something starts. In homogeneous groups, though, this should not happen. Instead, what we expect to see in most relatively homogeneous groups of not very resourceful individuals facing accelerating production functions is a lot of nothing going on. Residents of poor areas do not call about local problems, community centers do not get built, lawsuits are not brought, and protests are not organized.

General third-order curves.—The general third-order curve has two solutions to equation (2): the first is the minimum net payoff, a net loss, reached at the end of the initial period of low returns; the second is the desired maximum payoff, which occurs at the end of the period of high returns. The key question in this situation is therefore the length of the initial, accelerating segment. Third-order curves with short periods of initial low returns foster the dynamics of optimization characteristic of accelerating curves and common in economic analysis. But when the initial period of low returns is significant, as it is in general, homogeneous groups of small individuals do *not* exhibit the dynamics of optimization. In such a case the result from our investigation of the accelerative case holds: no contributions should be made, and no collective action should occur.

As the foregoing discussion indicates, decelerating production functions are always more favorable for some (rather than no) collective action than are comparable accelerating production functions, in the sense that it takes a much smaller interest in the collective good to make one willing to contribute in the decelerative case than in the accelerative case. This comparison is developed technically in Section 1.3 of the Appendix. Another way of saying this is that a person with a given level of interest in the collective good will obtain a much higher profit from an initial contribution to a decelerating production function than to an accelerating production function.

If we look at the universe of actual contributions to real collective goods, we find that the vast majority are made when the production function approximates the decelerating curve. In contrast, the vast majority of *opportunities* for collective action have significant start-up costs that give rise to the fundamental dynamic of the accelerative case—nothing happens.

This fundamental contrast holds, but the dynamics become more complex when we add heterogeneous interests and resources to the analysis in the next section.

RESULTS II: INTERACTIONS AMONG GROUP HETEROGENEITY, INTERDEPENDENCE, AND PRODUCTION FUNCTIONS

This section explores the interactions among the form of the production function, group heterogeneity, and strategic interdependence. In the process of developing the theory in this section, we relied heavily on the computer simulations described in Section 4 of the technical Appendix. The simulations allowed us to identify complex interactions that were not readily apparent to us. Once they were identified, we were often able to provide formal statements of these relationships that do not depend on any particular simulation. The simulations also provided information about the relative magnitudes of various effects. All our numerical examples are drawn from these simulations.

Decelerating Production Functions

Our discussion so far allows us to say two important things about decelerative collective goods, that is, those with diminishing marginal returns. First, compared with accelerative collective goods, the prospects for *initiating* collective action are quite favorable, since comparatively low levels of individual interest in the collective good are required to motivate initial contributions. Second, the general dynamic of collective action is negative interdependence, since each contribution to a decelerating production function reduces the marginal return of subsequent contributions.

In this section we go beyond our initial discussion and show that decelerative collective goods exhibit either of two less obvious phenomena: either there will be an "order effect," in which maximum contributions to the good are obtained when the *least* interested group members contribute first, or there will be a surplus of contributors for some region of the curve. Either situation opens the door to problems of strategic gaming that may disrupt the otherwise favorable environment that decelerating production functions create for collective action.

Willing subsets.—The dynamics of decelerative collective goods arise from the fact that individuals' payoffs are higher from initial contributions than from later ones, so more individuals are willing to contribute initially than later. As described in more detail in Section 2.1 of the Appendix, we may speak formally about this fact by defining the "willing subset" for a region of the production function as the set of individuals whose interest in the collective good is high enough relative to the slope of the production function that they are willing to contribute in that region. It should be obvious that, for a decelerating production function, the largest willing subset occurs initially, when no contributions have yet been made and that willing subsets get smaller as more and more resources are contributed.[10]

Order effects.—Section 2.2 of the Appendix technically describes a way of considering both interests and resources to define what may be called the "likely subset" for a region, with each individual assigned to the one region in which he should contribute. The likely total group contribution

[10] The extreme cases in which no group member is willing to contribute anywhere on the curve or in which all group members are willing to contribute everywhere on the curve will not be considered in this analysis, as they are not subject to the processes described in this section.

is the maximum contribution level for which there is a non-null likely subset. Because progressively higher interest levels are required for individuals to be willing to contribute once others have, total contributions by the group are maximized if people are asked to make their contributions in order, from least to most interested.[11] That is, individuals whose interest level is such that they will contribute initially but not later are assumed to make the initial contributions, while those with higher interest levels are "saved" for the later regions where a higher interest is required to be part of the willing subset.

There is obviously a problem here. Although total contributions toward a decelerative good are maximized if the more interested contribute after the less interested, it is clearly unlikely that this would actually happen. We would generally expect the opposite, that the most interested would contribute first, followed by the next most interested, and so forth. Sometimes contributions could be in random order with respect to interest. In any case, we certainly would never expect people with the least to gain to be the first to contribute.

This implies that interest groups facing a decelerating production function would not generally be expected to make contributions in an order that would maximize their aggregate contribution. When the amount of resources available is roughly equal to the amount required, the order of contributions can have a big effect. In our simulations, having individuals make contributions in the order from most interested to least interested reduced the aggregate contribution of the group by as much as 20%–30% over the optimal least-to-most order. Random orders fall between these extremes.

We can give some everyday examples of order effects. Consider a sociology department with its own computer, whose software must be contributed from research grants. All researchers need basic software such as word processing and SPSS, while only those with more intensive computer needs would be willing to contribute more exotic items such as GLIM or SIMSCRIPT, so it would be rational for the least interested members to buy the basics, freeing resources for the more interested to buy the other items. But the heavy computer users are likely to make the first

[11] In contrast to the decelerating case, we may quickly dismiss the order of contributions as a factor for accelerating production functions. If a less interested person contributes on an earlier, flatter part of the curve before a more interested person, the more interested person will be even happier to contribute on a subsequent portion of the curve because it is steeper. The only exception is the unrealistic special case in which each person has exactly one chance to decide whether or not to contribute. In this case, it is desirable to have the most interested persons decide first so that the chances of a less interested person encountering a steep-enough portion of the curve to be willing to contribute are maximized.

purchases of common software items, and the researchers with less interest in the computer are likely to spend their grant money on something else, such as secretarial services, rather than contribute to additional software. Local voluntary action often exhibits these same dynamics. Many people want the neighborhood association or the Girl Scout troop or the computer users' group to exist, but few care about planting flowers in a median strip, or teaching the girls astronomy, or communicating with a national network. The people who would be interested enough to do the additional jobs usually are the ones to take on the basic tasks of calling meetings, taking minutes, and keeping books, and because the others are not interested enough to do anything except maintain the organization, nothing else gets done.

In short, unless the most interested people can engage in some kind of strategic gaming, restrain their enthusiasm, and withhold their contributions until the less interested make theirs, contributions under decelerating production functions can be expected to be suboptimal. Such strategic interaction is often difficult, since one's greater interest in the collective good is usually obvious. One response to this problem can be seen in fund raising, when a highly interested donor promises to match smaller contributions, thus "forcing" the less interested to contribute first.

The order effect is a supplementary explanation for the "exploitation of the great by the small" described by Olson (1965, p. 29). Again, the less interested group members free ride on the initial contributions of the most interested, and total group contributions are suboptimal. The significance of our analysis is to stress that this phenomenon arises when the production function is decelerating and there is not a great surplus of potential contributions.

Surplus.—Rapidly decelerating production functions may produce surpluses instead of order effects.[12] If a production function starts with a very steep slope, it must become relatively flat later. The difference in slopes may be great enough to "wash out" variation in group members' interest levels, making many willing to contribute initially whereas none are willing later. This condition eliminates the "order effect," but it has its own consequences.

To provide a numerical example of surplus, suppose our homeowners face a production function like the steepest one in table A1 (in the Appendix): Paying $10,000 to retain a lawyer would make the probability of keeping the school open .9, whereas paying an additional $10,000 raises the probability only .02 more, to .92. The interest level necessary to be in the willing subset for the first region is 11,111, whereas an interest level of 500,000 is necessary for the second region. Suppose interest (prevented

[12] Surplus is defined technically in Section 2.3 of the Appendix.

loss) in having the school remain open is distributed normally with mean $20,000 and standard deviation $27,000. Then 630 of the 1,000 homeowners would be expected to have an interest level greater than 11,111, but no one would have an interest level greater than 500,000. If each homeowner has $2,000 in resources, the expected pool of potential contributions for the first $10,000 is $1,260,000, yielding a surplus of $1,250,000. But none of this is available for subsequent contributions. Even if each of these homeowners has only $100 that he or she could contribute, there is a potential of $63,000 that might be tapped by the fund, yielding a surplus of $53,000. Obviously, very steep production functions tend to yield very large surpluses. Under the assumptions we have been using in this article, homogeneous groups facing a decelerating production function always generate a surplus, for they have enough resources to provide the good with certainty but are only willing to contribute up to some intermediate level.

Examples of surplus potential contributions to a collective good abound, especially since the phenomenon applies also to the decelerating segments of general third-order production functions. Upper-middle-class neighborhoods usually respond quickly and effectively to proposed threats or to lapses in municipal services. Even though many residents free ride, the few who do not have sufficient interest and resources to protect the entire neighborhood. Surpluses are often associated with collective goods that are inexpensive relative to their worth, such as office coffeepots, shoveling snow off a privately maintained street, or telephone calls about problems such as potholes.

It is important to stress that the problem of surplus is different from the general dilemma of collective action, although economists often intertwine their discussions of the two. The general dilemma (as formulated by Olson) arises when the payoff to an individual from a contribution to a collective good is lower than the cost of that contribution, even though every individual would be better off making the contribution and having the good than making no contribution and lacking it. In these circumstances, predictions about others' behavior are irrelevant, for contributions are irrational no matter what other people do. In contrast, surplus arises when many individuals find that their own individual payoffs from a contribution *do* exceed the cost of those contributions, and the production function is such that there is no positive payoff from contributions after a certain level is achieved. It is only under conditions of surplus that individuals may rationally consider the possibility that the good will be provided by others. When there is a surplus, an individual who is convinced that everyone else will refuse to contribute should rationally make a contribution, for his own individual benefit from this action will exceed its cost, and he should be unconcerned that others will also benefit. Oliver

(1984) reports that active members of neighborhood organizations are *less* likely than token members to believe that their neighbors would engage in collective action in response to a neighborhood problem. Of course, individuals in this situation have a strategic interest in persuading others that they will refuse to contribute.

Idiosyncratic factors or random events may decide who contributes when there is a surplus. Our simple sequential model implies that the first ones who happen to be faced with the decision are "stuck." They will contribute because they find it profitable to do so, while those whose turn to decide comes later will free ride. Although this is extreme, it is probably not far from the reality of many situations.

Surplus creates the conditions for strategic gaming, since individuals might reasonably expect to be provided the optimum level of the good through the efforts of others. There is the ironic possibility that the very surfeit of resources could stymie collective action. One's empirical predictions in this situation depend on the assumptions one brings to bear. Economists generally predict that no one will contribute anything because there is no equilibrium solution; that is, everyone can hope to get the good without paying for it. Social psychologists aware of the diffusion of responsibility literature (Piliavin et al. 1981, pp. 120–32; Latane and Nida 1981) may generalize from small group experiments and predict that the higher the surplus, the lower the provision level.

Our own predictions are less pessimistic. The question is whether surplus lowers individual probabilities of contributing so much as to counter the positive effect of having so many more contributors. We think not. The fact that there is a huge aggregate profit to be made opens the door for some resolution. If the value of the good is high enough, relative to the cost of an individual's contribution, the individual may not be concerned about the risk of making a redundant contribution. This is why many people call when the lights go out or a pothole develops. Obviously, the most interested group members are most likely to decide that it is worth it to pay for the good alone, and it is likely that others expect them to. This would resolve the question of who will bear the cost, again through the "exploitation of the great by the small." Surplus may also create a potential profit for political entrepreneurs (Frohlich, Oppenheimer, and Young 1971) who provide the good for a price, although they must have some sort of incentive available (Oliver 1980) as part of the enterprise.

Heterogeneity of resources.—The total contribution from a group is determined by the amount of resources controlled by those interested enough to contribute. Our discussion so far has focused on interest heterogeneity while treating resources as homogeneous. This is reasonable because in most decelerative situations the total or mean level of available resources matters much more than their dispersion. Our simulations indicate that heterogeneity of resources around a given mean does not alter the average total contribution, although it can increase the variance of the total contribution (depending on sequencing, i.e., the order of contributions) by as much as 100%. Of course, a positive correlation between interests and resources may raise the expected contribution level dramatically: in our simulations, a perfect correlation produced total contributions two to three times larger than those produced under the same conditions with a homogeneous resource distribution.

Accelerating Production Functions

Accelerating production functions yield entirely different dynamics in heterogeneous groups than do decelerating functions. Mathematically, this is because their increasing marginal returns make optimization and instantaneous slopes inappropriate analytic tools. Instead, we have to compare average rates of change, that is, the relation between a contribution of a given size and the total difference it makes in the probability of obtaining the collective good. The concepts we have developed for decelerating curves can be defined for accelerating curves, but they are not very useful for understanding them. Conversely, the ideas developed below may be defined for decelerating curves but are of little value in understanding their dynamics.

We have previously noted two key features of accelerating production functions. On the negative side, feasibility is a central problem because collective action must start at the flattest part of the curve. Therefore, collective action rarely even begins. On the positive side, each contribution moves subsequent decisions to a more favorable part of the curve. Thus, if somehow contributions begin, collective action tends to snowball, involving more and more contributors until the good is provided with certainty. We believe that accelerating production functions underlie the mass actions popularly associated with the term "collective action," such as political demonstrations or revolutions. They are rare events relative to the grievances that might give rise to them, but they tend to accelerate once they start.

In the absence of contracts or considerations of indirect production (discussed below), the resolution of an accelerative collective dilemma is highly problematic, depending on the rare circumstance of there being a critical mass of persons whose combination of interests and resources is high enough to overcome the feasibility problem. Groups fortunate enough to have a critical mass can enjoy the collective good; less fortunate groups cannot. Resource and interest heterogeneity are essential to the resolution of accelerative collective dilemmas, for a homogeneous group cannot contain a critical mass. A positive correlation between interest and resources obviously improves the chances of there being a critical mass. The more usual situation of zero or even negative correlation obviously makes the existence of a critical mass much less likely.

In contrast to the decelerating case, resource heterogeneity as well as interest heterogeneity may have significant effects on the prospect for a critical mass when the production function is accelerating. As may be recalled from our earlier discussion, an individual with a given interest V in an accelerative good finds that, after the minimum point where $P'(r) = k/V$ as passed, his or her net payoff increases with each unit contributed until the good is provided with certainty. Thus, *individuals whose potential contributions are larger, that is, who have more resources, are more likely to find it profitable to contribute*. This is shown more formally in Section 3.1 of the Appendix. This result has surprising consequences. It leads to the prediction that if two people have the same interest in an accelerative collective good, but one is much "richer" in resources than the other, the richer person is more likely to find contributing rational. This is not because his opportunity costs are lower (we assume this is not true) but because his larger possible contribution can buy a greater proportionate return. In fact, our computer simulations suggest that often a "rich" individual with a much *lower* interest will find contribution rational while a "poor" person with a higher interest will not. Thus, interested wealthy benefactors who provide the good single-handedly may represent one resolution to accelerative collective dilemmas. Such a resolution is not of great theoretical interest, and its practical importance is limited.

However, a related characteristic of accelerating functions is more interesting and important: as we show formally in Section 3.2 of the Appendix, initial contributions lower the interest necessary for subsequent contributions. A pool of highly interested and resourceful individuals willing to contribute in the initial region of low returns may therefore become a "critical mass" creating the conditions for more widespread contributions. If even one such person exists, he or she may begin a process in which continuously increasing numbers of group members find that the contributions of others have changed the situation to one in which they, too, wish to contribute. The bandwagon may roll, started by a single person. For the process to start, however, this initiator must have an extraordinarily high interest in the collective good, perhaps several hundred times greater than that necessary to initiate action for a decelerative good (see App. table A1 below).

Anticipating indirect production.—The prospects for starting collective action for accelerative goods are bleak, but not quite as bleak as our discussion so far implies. If actors have "full information" about the form of the production function and about everyone else's interests and resources, they know what we know—that they can affect others' contributions by making a contribution of their own—and adjust their own payoff calculations accordingly. We may think of this as "indirect production" of the collective good, in contrast to the "direct production" we have been considering so far. Consider the following numerical example. Jones may stand to lose $30,000 if the school is closed and has $10,000 he could invest in the fight. If the production function is like curve 4 in Appendix table A1, the $10,000 "buys" a .0156 probability increase for an expected payoff of $468, yielding a net loss of $9,532. But suppose Jones knows that if he contributes $10,000 toward the legal fund, he will set in motion a chain of events resulting in others contributing the $90,000 necessary to guarantee that the school will stay open. If Jones considers this "indirect" production, he calculates a $20,000 profit from his $10,000 investment. This process is similar to the macroeconomic concept of anticipating the multiplier effect of specific investments.

A complete accounting of indirect production can have many complicated steps, since part of the individual's calculation may be that the next person in line will also calculate the indirect effects of a contribution, and so on. These projections do not require any probabilistic inference about other homeowners' decisions; they are determinate calculations based on knowledge of the interests and resources of each individual in the group. Nevertheless, they can be so complex that we cannot model them, and even our ideal rational actors probably would not compute them.

Indirect production may resolve the collective dilemma when circumstances permit simplifying assumptions that eliminate the problem of computational complexity. Because each contribution makes subsequent ones more profitable, individuals might reasonably conclude that "starting the ball rolling" with a good example would produce widespread enough participation to justify the investment, even though they could not predict the exact chain of events.[13] This is, of course, exactly what happens in all sorts of real-life circumstances. Mass fund-raising drives begin with a core of organizers who assume that if they coordinate the candy sale or door-to-door solicitation, many people will contribute small amounts of money. Campaigns for political office have this character, when the candidate starts as an unknown and gradually adds supporters

[13] It is important to stress that indirect production is a resolution of the collective dilemma in the accelerative case but not in the decelerative case. In the accelerative case, each individual knows that his or her contribution increases the probability that others will contribute because it lowers the value that others need to attach to the collective good to be willing to contribute. But under deceleration, initial contributions move subsequent decisions to *less* productive portions of the curve where the slope is less steep and contributions require higher interest levels.

to the bandwagon. (Of course, once the candidate looks like a sure winner, a decelerating portion of the production function obtains, and the dynamics are different.)

What we call "indirect production" is a common occurrence, but it is not universal, as many budding activists have learned the hard way. It is a structurally rational way to provide collective goods only if the production function is accelerating. Agreeing to be president of the PTA, for example, rarely starts anything rolling because most parents of schoolchildren are sufficiently satisfied if the PTA simply exists; that is, the production function is decelerating. Although production functions are not the only factor that determines the trajectory of various types of collective action, they are certainly an important one.

Contracts or conventions.—Individuals rarely have the kind of full information required to calculate the indirect production from a contribution. However, the same effect can be obtained from explicit or implicit "all or none" contracts. Consider a very simple contract: a group of homeowners agree that they will all contribute specified amounts to the legal fund and that if anyone fails to contribute by a specified deadline, the fund will be dissolved and all the contributors will get their money back. This "all or none" agreement provides the same calculation of return as the more complex process of anticipating indirect production. Jones will receive not only the increase in probability directly produced by his own contribution but also the increase produced indirectly through the contributions of the other parties to the contract. Since the others will not contribute unless an agreement is reached and everyone contributes, we may base each homeowner's calculation of payoff on the total change in probability produced by the contracted contributions. One way to think of this is that failing to live up to the contract will make the payoff zero instead of what it would have been under the contract. This is formalized in Section 3.3 of the Appendix. An event such as a wildcat strike often involves an implicit "all or none" contract, since individuals can turn around and go back to work if too few others are walking out with them.

When contracts may be reached, the necessary interest required to make a contribution rational is the ratio of the *individual's* contribution size to the change in probability of obtaining the collective good produced by the *whole group's* total contributions. Obviously, this is a much more favorable ratio than we have encountered before. Continuing the example from the section on indirect production, Jones would need an interest level of $641,026 to be willing to invest his $10,000 without a contract. But if $100,000 in resources are committed to the contract, any interest level greater than $10,000 will make Jones's investment in the contract rational. If only $80,000 were committed, which would raise the probability of the collective good by .56, Jones would need an interest of $17,857 to be willing to participate.

"All or none" rules are clearly one solution to an accelerative collective dilemma for a group whose members have only moderate interests or resources. Substantively, these rules might take the form of revocable contributions, the invocation of third parties to enforce contracts, or simply very high levels of trust among the group members. Hardin (1982, pp. 155–230) stresses the importance of what he calls "conventions," norms or agreements that everyone will act in the same way because it is to everyone's benefit to do so. These are the informal or implicit equivalent of "all or none" contracts. Although conventions or contracts are useful resolutions to any collective dilemma, they are especially crucial when an accelerating production function makes any other resolution almost impossible. They should be less important in decelerative situations where the basic problem is declining returns to later contributions. This analysis raises issues that are related to those raised by Marxist scholars of collective action such as Offe and Wiesenthal (1980), who stress that workers face acute collective dilemmas in the conflict between individual and collective rationality and require ideological solidarity (conventions) and union organization (contracts) to overcome these dilemmas.

Since many real-life production functions involve the high start-up costs that characterize the accelerating curves, we suspect that Hardin is right to stress the importance of conventions in collective action. To the extent that this is true, an understanding of collective action in the face of accelerative collective dilemmas requires attention to the problems and costs of organizing and enforcing contracts, or to the cultural forces that shape cooperative norms. These rather substantial issues are beyond the scope of this article.

SUMMARY AND CONCLUSIONS

In general, the problem of collective action is one of getting some relatively small subset of a group interested in the provision of a public good to make contributions of time, money, or other resources toward the production of that good. This subset is the critical mass needed to begin any collective action. Whether it emerges and what role it plays if it does depend on a variety of factors. In this article we have attended to two fundamental conditions: the shape of the production function and the distribution of interest and resources across the group of potential contributors.

Our analysis has stressed the contrast between production functions dominated by increasing or decreasing marginal returns, including third-order S-shaped curves that approximate one or the other extreme. But the contrast is also relevant to the more general S-shaped production function considered over time. In such cases, collective action begins with the feasibility problem of the initial, flat period of the production function, when it is ruled by the dynamics of acceleration. Most potential collective actions never get off the ground because of this problem, but if any action is forthcoming, it is the most interested group members who will contribute. By the time the group has gathered sufficient contributions to be on the steep portion of the curve, it is mostly the less interested who remain. During this period of high returns, these less interested actors jump on the bandwagon. However, once the later, flatter portion of the curve is reached, it is ruled by the dynamics of deceleration, and the less interested cease contributing. Since the more interested individuals have already contributed on the very first part of the curve, the collective action will tend to "top out" at some level well below the maximum.

Although by definition any public good involves a "free-rider problem," free riding is not the crucial dynamic for the accelerating regions of production functions because others' contributions increase one's willingness to contribute and reduce the propensity to free ride. The usual outcome of the accelerative collective dilemma is that nobody rides free because nobody contributes and there is no ride. However, an "irrational" contributor may well find that, instead of being a "patsy," he or she is a role model or organizer whose action sets off others' actions and, in the end, vindicates the original contribution.

In this sense, it is in accelerative cases that the critical mass is likely to fill the role associated with the nuclear metaphor. A small core of interested and resourceful people can begin contributions toward an action that will tend to "explode," to draw in the other, less interested or less resourceful members of the population and to carry the event toward its maximum potential.

Clearly, contractual solutions to the collective dilemma are most important in accelerating cases. Often the resolution of this dilemma depends on the possibilities for organizing, communicating, and coordinating an explicit or implicit "all or none" contract. Organization and communication costs then become central issues. It is in this case that the idealistic organizer who sees his or her role as "bringing people together" and "showing people their true interests" is most likely to fulfill his or her goal of fostering mass action.

If, by some mechanism, a group manages to get to the decelerating part of the general third-order curve, or if the initial region of low returns is small (relative to group members' resources), a different set of issues appears to determine the dynamics of collective action. In the decelerating case, the critical mass is likely to be a relatively small subset of a larger pool of interested group members who provide some of the good for the benefit of all. If we may expand Olson's metaphor, free riding is likely in the decelerative case, but the ride is short. That is, there will likely be a ride, and some will ride free (not have to contribute), but the ride will be cut short at the optimum, and no one will pay to finish the trip. It is hard to imagine any resolution, social or otherwise, that would lead to maximum contributions in the decelerative case.

We may speculate about which group members are likely to pay for the ride. Again, it seems most plausible that those with the highest interest in the collective good would pay and that they would be faster to act than those with lower interests. It is also likely that other actors would assume that the most interested persons would pay. Thus, considerations of inherent motivation to act and of projections of others' behavior converge on the prediction that the most interested will be most likely to provide the good, even if others might also be willing to do so. For this reason, decelerating production functions seem especially likely to lead to the "exploitation of the great by the small" that Olson describes.

Interestingly, our analysis of order effects indicates that, if there is not a surplus, having the most interested contribute first can lead to a suboptimal outcome. Our analysis is very different from Olson's, but it leads to the same conclusion, that exploitation of the great by the small tends to produce suboptimal results. However, we go beyond Olson and argue that suboptimality occurs when the production function is decelerating and there is no surplus.

If there is a surplus, lots of people may ride free, but it is not entirely clear who will pay for the ride. It seems most likely that the most interested will pay, even though less interested people would also be willing to pay. In this context, of course, people are motivated to appear less interested than they really are, and strategic gaming and misleading statements about one's interests may result. Economists have long been fascinated by the potential complexities that such gaming can introduce, although strategic behavior does not seem to us to be very common in collective action in the social or political sphere. A common resolution to surplus that we have observed is normative reciprocity. People expect that particular actions will have a few contributors and many free riders but that the contributors should rotate across actions. In neighborhood organizations, it is common for people to feel that they should "take a turn" at serving on the board. Academicians often feel obliged to take a turn as department chair.

In sum, an understanding of the differences among production functions, particularly when groups are heterogeneous, is the first step toward recognizing the processes underlying the ebb and flow of interdependent collective actions and toward identifying important differences among them.

APPENDIX

This appendix provides a more formal treatment of several topics than is possible in the text.

1. Basic Concepts

1.1 *Metric.*—To simplify the analysis here, we may, without loss of generality, choose a metric so that $k = 1$; this makes the cost/value ratio $1/V$. Obviously, the more an individual values the collective good, the smaller $1/V$ will be.

1.2 *Relating slope and value.*—We may tie regions of the production function to individuals with particular levels of interest in the collective good. For each region of the production function with a unique slope, there is a level of value for the collective good which we will denote V^* such that $P'(r) = 1/V^*$. Among a group of individuals, those whose values V for the collective good are greater than V^* will find it profitable to contribute at point r on the production function, whereas those whose values V are less than V^* will not find it profitable to contribute at this point. Thus, we may identify which members of a group would be willing to contribute at each point on a production function. Given a portion of the curve, we can identify the group members who would be willing to contribute, or given a set of individuals, we may identify the portions of the curve that would attract their contributions. We use this relation extensively in subsequent developments.

1.3 *Favorability.*—To see the relation among linear, uniformly decelerating and uniformly accelerating production functions, we compare families of production functions with the same lower limit ($r = 0$, $P = 0.0$) and the same upper limit ($r = R$, $P = 1.0$). There is only one linear production function in such a family, and it has slope $1/R$. The average slope of all production functions in the same family must be $1/R$. Since the slopes of a uniformly decelerating production function decline, their initial slopes must be greater than $1/R$, and their final slopes must be less than $1/R$. Conversely, the initial slopes of a uniformly accelerating production function must be less than $1/R$, with final slopes greater than $1/R$. Uniformly decelerating and accelerating production functions approach the linear production function as a limit, as the differences among their slopes approach zero.

Since $V^* = 1/P'(r)$, the V^* for the initial segments of uniformly decelerating production functions must be less than R (since these slopes must be greater than $1/R$), whereas for uniformly accelerating production functions, V^* for the initial segments must be greater than R (since these slopes must be less than $1/R$). This means that it is always "harder," in the sense of requiring a higher interest level V, to start collective action in a uniformly accelerative case than in a uniformly decelerative case from the same family. The steeper the curves, the greater the disparity.

2. Decelerating Production Functions

2.1 *Willing subsets.*—The analysis of the decelerative case hinges on optimization and the relation between the slope $P'(r)$ and its associated $V^*(r)$. As indicated above, for each point on the production function there is a $V^*(r)$, the level of interest in the collective good an individual must have to be willing to contribute at that point. At each point r, we may sort the members of a heterogeneous interest group into two subsets: (*a*) the willing subset, $W(r)$, those who would be willing to contribute at that point on the production function, that is, those group members for whom V is greater than $V^*(r)$; (*b*) the unwilling subset, those group members who would not be willing to contribute, that is, for whom V is less than or equal to $V^*(r)$. In the uniformly decelerative case, the smallest V^* for a particular production function must be $V^*(0)$; furthermore, for any $i < j$, $V^*(i) < V^*(j)$, and therefore $W(j)$ is a subset of $W(i)$.

Obviously, if the largest V in the population is less than $V^*(0)$, no one will contribute anything; the willing subset is thus null for the entire curve. Similarly, if the smallest V in the population is greater than the maximum $V^*(R)$, all group members are willing to contribute across the whole production function, and the willing subset is the total population for all segments of the curve. The rest of the discussion will assume that neither of these extremes holds.

In general, regions of the production function may be described according to the size of the willing subset associated with them. It should be obvious that $W(0)$ is the largest willing subset and that willing subsets get smaller as r gets larger.

2.2 *Allocating resources.*—The problem is to determine how many resource units might be contributed by an interest group. We know that for each region of the curve there is a willing subset of actors, but we need a way of determining whether they have sufficient resources to do what they are willing to do. The procedure for making this assessment needs to be general enough to allow for unusual shapes of production functions or distributions of interest. We may divide the production function into J regions, each with a different willing subset. Within each region, the same individuals are willing to contribute. Obviously, the number of regions (J) depends on the variability of the slope of the production function (and, therefore, V^*) as compared with the variability of V in the interest group, but the logic of the analysis is the same whether J is 2 or 2,000. We index the regions by j, which runs from 1 to J, letting $r(j)$ denote the resource level dividing region j from region $j + 1$, and letting $B(j)$ stand for the region of the curve between $r(j - 1)$ and $r(j)$. Each of these regions has a willing subset that we now denote by $W(j)$, where $W(j) = W[r(j)]$. Of course, for $m < n$, $W(n)$ is a subset of $W(m)$. We then define nonoverlapping sets of individuals $L(j)$ so that each individual is assigned to the rightmost region to which he is willing to contribute.

Now we are ready to take account of resources. Let $a(j)$ be the amount of resources available for region j, where $a(j)$ is the sum of resources across individuals who are members of $L(j)$. We need to compare these available resources to needed resources. Let $d(j) = r(j) - r(j - 1)$; this is the amount of resources used in this region of the production function. If for every region, $a(j)$ is greater than or equal to $d(j)$, there are enough resources so that individuals can contribute what they are willing to contribute. But if this is not true, we need an adjustment process to determine the expected level of provision of the collective good.

We make this adjustment in an iterative procedure, working with each region from 1 to J in turn. If $a(j)$ is greater than or equal to $d(j)$, move on to the next region. Otherwise, move individuals from $L(j + 1)$ to $L(j)$, then from $L(j + 2)$ to $L(j)$, and so forth until $a(j)$, the sum of resources across individuals in the revised set $L(j)$, is equal to $d(j)$. (If we are moving an individual who has more resources than are needed for the lower region, we will notationally split the individual into two, placing each in the appropriate subset $L(j)$ and allocating resources to the two regions appropriately.) We then repeat this procedure for the next region, moving individuals from right to left until we run out of individuals and resources.

We may refer to the revised subset $L(j)$ as the "likely subset" for that region, that is, the individuals who would be expected to make their contributions at that point on the production function.

This procedure finds the amount of resources the interest group can feasibly contribute given its distribution of interest and resources: it is the point on the curve at which we "run out" of resources.

2.3 *Surplus.*—The surplus in a region is defined as $a(j) - d(j)$. The total surplus is the sum of these differences.

3. Accelerating Production Functions

3.1 *Minimum necessary interest.*—Let us define the "minimum necessary interest," $M(r|C)$, as the value an individual must have to experience a net payoff of zero from a contribution of r units when C units have previously been contributed. Unless it would be ambiguous, we will usually denote the minimum necessary interest from the origin, $M(r|0)$, by the simpler $M(r)$.

Solving for the minimum necessary interest requires a different mathematical strategy from the more usual problem of optimizing. If we set $k = 1$, the net payoff from a contribution of r units from the origin is $VP(r) - r$. To find $M(r)$, we substitute it for V in this equation and set the payoff equal to zero to obtain

$$0 = M(r)P(r) - r. \tag{A1}$$

Solving for $M(r)$, we obtain

$$M(r) = r/P(r), \text{ for } P(r) > 0. \tag{A2}$$

$M(r)$ is undefined at $P(r) = 0$; the substantive meaning of this is that if the contribution does not change the probability of obtaining the collective good, there is no interest level that will make the net payoff zero.

In the general case of a contribution of r units beginning with C, we would obtain:

$$M(r|C) = r/[P(C + r) - P(C)],\ P(C + r) > P(C). \tag{A3}$$

The denominator in equation (A3) is simply the *difference* the contribution makes in the probability of obtaining the collective good. So, in general, the minimum necessary interest is the ratio of the contribution size to the change in probability produced by that contribution.

Under the assumptions that define the uniformly accelerating production function, $M(r)$ *decreases* (or, more strictly, is nonincreasing) as r increases. That is, the amount of interest in the collective good necessary to make a given contribution rational decreases as the size of that contribution increases. (In contrast, the assumptions of uniform deceleration imply that $M(r)$ increases with r in that case.)

3.2 *Interest and previous contributions.*—This means that the interest level necessary to make a contribution declines with the amount already contributed by others. We may see this by calculating the derivative of $M(r|C)$ with respect to C:

$$\frac{dM(r|C)}{dC} = \frac{-\,r[P'(C + r) - P'(C)]}{[P(C + r) - P(C)]^2}. \tag{A4}$$

Since $P''(r) > 0$ in the accelerative case, $P'(C + r) > P'(C)$, making the difference $[P'(C + r) - P'(C)]$ positive. Therefore, $M'(r|C)$ is always negative in the accelerative case, meaning that $M(r|C)$ decreases as C increases. Obviously, $M(r|C)$ increases in the decelerative case with

$P''(r) < 0$ (i.e., decreasing slopes); in that case, the interest necessary to make a contribution *increases* as others' contributions increase.

3.3 *"All or none" contracts.*—Let G represent the total resources to be contributed by a group of actors acting according to an "all or none" rule, and let $t(G)$ represent the total change in probability associated with G. Then an individual's payoff decision equation may be written as

$$N = Vt(G) - r. \tag{A5}$$

To find the minimum necessary interest for a contribution of size r, we substitute $M(r)$ for V in equation (A5), set $N = 0$, and solve for $M(r)$:

$$M(r) = r/t(G). \tag{A6}$$

4. Simulations

4.1 *Parameterization.*—Both our formal analysis and simulations use a family of production functions in which the group size is 1,000 and the collective good is provided with certainty when 100,000 units of resource (e.g., dollars) have been contributed. Provision of the collective good therefore benefits the group as a whole if the average interest is greater than 100; provision of the good with certainty is possible if the group has an average of 100 units per person. We use such large numbers because we want to have heterogeneous distributions in which decimal places are not significant. In this standard parameterization, the slope of the linear production function is .00001, and V^* for this linear function is 100,000.

4.2 *Production functions.*—We employ 10 production functions, five uniformly decelerating and five uniformly accelerating. The functions are paired, in that each accelerating function has the same series of slopes as one decelerating function, but in reverse order. Since we are not interested in the minute peculiarities of particular functions, we follow standard practice and approximate our curves of interest with spline fits of 10 line segments each. These spline fits have "edges" where the slopes change discontinuously; this affects the particular results obtained, but we are aware of this and never attach substantive importance to these point solutions. Smoother curves would show the same basic patterns but would have more "scattered" results. The specific curves employed are shown in table A1 and are sketched in figure 2.

4.3 *Algorithm.*—The program we used was written in FORTRAN and consisted of several relatively simple steps. First, each member of a population of 1,000 individuals was assigned an interest and resource level. These distributions were constant, normal, or skewed with a given mean and variance.

TABLE A1

SPLINE FITS FOR DECELERATING AND ACCELERATING CURVES: SLOPES, PROBABILITY CHANGES, AND CRITICAL VALUES FOR LINE SEGMENTS

	SEGMENTS									
	1(10)	2(9)	3(8)	4(7)	5(6)	6(5)	7(4)	8(3)	9(2)	10(1)
Curve 1:										
Slope	.9E-4	.2E-5	.178E-5	.156E-5	.133E-5	.111E-5	.89E-6	.67E-6	.44E-6	.22E-6
Δ *P*	.9	.02	.0178	.0156	.0133	.0111	.0089	.0067	.0044	.0022
Critical value	11111	500000	561797	641026	751880	900901	1123596	1492537	2272727	4545454
Curve 2:										
Slope	.7E-4	.6E-5	.533E-5	.467E-5	.4E-5	.333E-5	.267E-5	.2E-5	.133E-5	.67E-6
Δ *P*	.7	.06	.0533	.0467	.04	.0333	.0267	.02	.0133	.0067
Critical value	14286	166667	187617	214133	250000	300300	374532	500000	751880	1492537
Curve 3:										
Slope	.5E-4	.1E-4	.889E-5	.778E-5	.667E-5	.556E-5	.444E-5	.333E-5	.222E-5	.111E-5
Δ *P*	.5	.1	.0889	.0778	.0667	.0556	.0444	.0333	.0222	.0111
Critical value	20000	100000	112486	128535	149925	179856	225225	300300	450450	900901
Curve 4:										
Slope	.3E-4	.14E-4	.1244E-4	.1089E-4	.933E-5	.778E-5	.622E-5	.467E-5	.311E-5	.156E-5
Δ *P*	.3	.14	.1244	.1089	.0933	.0778	.0622	.0467	.0311	.0156
Critical value	33333	71429	80386	91827	107181	128535	160772	214133	321543	641026
Curve 5:										
Slope	.15E-4	.14E-4	.13E-4	.12E-4	.11E-4	.9E-5	.8E-5	.7E-5	.6E-5	.5E-5
Δ *P*	.15	.14	.13	.12	.11	.09	.08	.07	.06	.05
Critical value	66667	71429	76923	83333	90909	111111	125000	142857	166667	200000

NOTE.—The rows are the different curves, going from steepest (1) to flattest (5); the columns are the different line segments, with the first number representing the position of that segment in the decelerating curve and the number in parentheses giving the corresponding position on the accelerating curve.

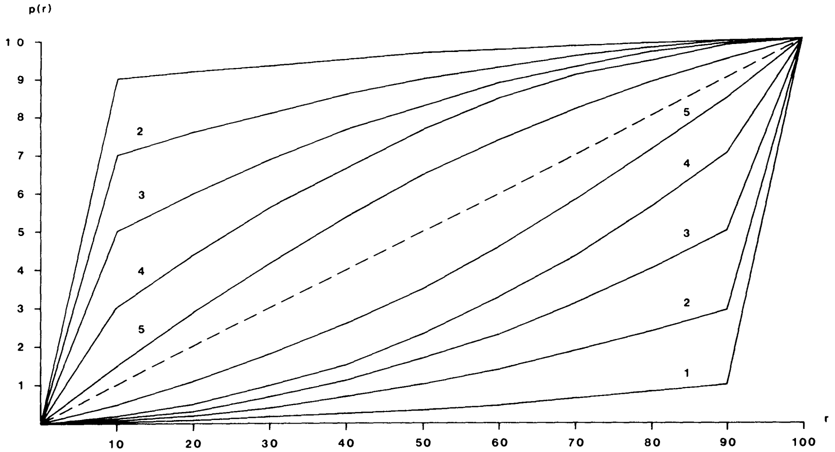

FIG. 2.—Production functions used in analysis and simulations. Dotted line is linear production function. The five above the line are decelerating; the five below are accelerating. Curves with the same number have the same series of slopes, but in reverse order. Note: r is in thousands.

Second, if necessary, subroutines processed the interest and resource arrays to give them a particular order, such as sequenced from highest value to lowest value, or to make the interest and resource arrays correlated with one another. Third, a subroutine selected the particular production function desired for the simulation. The functions used were the spline fits described in table A1.

The next set of subroutines had each individual decide how much (if anything) to contribute to the collective good. The decision rules were those described in the text: a comparison of interest and instantaneous slopes for decelerating production functions and a comparison of total contribution and total payoff for the accelerating production functions.

This decision process was repeated 10 times, each time feeding back through the program a resource array adjusted for the resources previously contributed. This allowed individuals the opportunity to respond to others' previous contributions. We found that 10 iterations were plenty to allow everyone to contribute who would ever contribute.

Each "case" (combination of parameters and options) was repeated at least 10 times to allow estimation of the effects of random variation in probabilistically generated distributions. Results were written to an output file for examination.

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