Matrixology (Linear Algebra)—Episode 1/24 MATH 122, Fall, 2016

Introduction

Prof. Peter Dodds

Dept. of Mathematics & Statistics | Vermont Complex Systems Center Vermont Advanced Computing Core | University of Vermont













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Basics:

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Instructor: Prof. Peter Dodds
 Lecture room and meeting times:

Perkins 107, Tuesday and Thursday, 10:05 am to 11:20 am

- Office: Farrell Hall, second floor, Trinity Campus
- E-mail: peter.dodds@uvm.edu
- Course website:

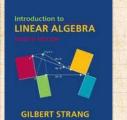
http://www.uvm.edu/ pdodds/teaching/courses/2016-08UVM-122

 Textbook: "Introduction to Linear Algebra" (3rd or 4th or 5th edition) by Gilbert Strang (published by Wellesley-Cambridge Press).

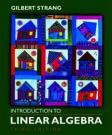




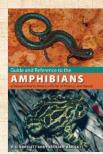
Our Textbook of Excellence:



4th Edition 🛛



3rd Edition



Unhelpful 🗆

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- "Introduction to Linear Algebra" by Gil Strang C;
- Textbook website: http://math.mit.edu/linearalgebra/C
- MIT Open Courseware site for 18.06 (=Linear Algebra):

http://ocw.mit.edu/...linear-algebra-spring-2010/





Yesness:

Money quote from George Cobb's review of Strang's book:

Do you want a book written by a mathematician with a lifetime experience using linear algebra to understand important, authentic, applied problems, a former president of the Society for Industrial and Applied Mathematics, ...

 George Cobb: Robert L. Rooke Professor of Mathematics and Statistics, Mount Holyoke College

Full review here [2] [amazon]

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or do you want a book shaped mainly by the [a]esthetics of pure mathematicians with only a weak, theoretical connection to how linear algebra is used in the natural and social sciences?

- George Cobb: Robert L. Rooke Professor of Mathematics and Statistics, Mount Holyoke College
- Full review here C [amazon]

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Gil Strang, Exalted Friend of the Matrix: Professor of Mathematics at MIT since 1962.



These are 121 cupcakes with my favorite -1, 2, -1 matrix. It was the day before Thanksgiving and two days before my birthday. A happy surprise.

 Many awards including MAA Haimo Award r for Distinguished College or University Teaching of Mathematics

Rhodes Scholar.Legend.

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More on Laplacian matrices, graphs, and other madnesses here .

▶ (Strang's Wikipedia page is here .





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Potential paper products:

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Potential paper products:

1. Outline

Papers to read:

"The Fundamental Theorem of Linear Algebra" ^[2]
 "Too Much Calculus" ^[3]

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Office hours:

 10:00 to 11:55 am Wednesdays, Farrell Hall, second floor, Trinity Campus Episode 1/24: Introduction

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- Place for discussions about all things PoCS including assignments and projects.
 - Once invited; please sign up here: http://team-matrixology.slack.com Very good: Install Slack app on laptops, tablets phone: Everyone will behave wonderfully.

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slack

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1. Levels (40%)

- Ten one-week assignments.
- Lowest assignment score will be dropped.
- The last assignment cannot be dropped!
- Each assignment will have a random bonus point question which has nothing to do with linear algebra.

Three 75 minutes tests distributed throughout the course, all of equal weighting.

 ≤ Three hours of joyful celebration.
 Thursday, December 15, 1:30 pm to 4:15 pm, Perkins 107. Episode 1/24: Introduction

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3. Final Boss Level (20%)

- Structure Str
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4. Mini-levels (10%)

- Most meeting times will end with a 10 to 15 minute mini-level.
- There will be around 20 mini-levels.

Homework (0%)—Problems assigned online from the textbook. Doing these exercises will be most beneficial and will increase happiness. General existence—it is extremely desirable that students attend class, and class presence will be taken into account if a grade is borderline. Episode 1/24: Introduction

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Questions are worth 3 points according to the following scale:

- 3 = correct or very nearly so.
- 2 = acceptable but needs some revisions.
- 1 = needs major revisions.
- 0 = way off.





Schedule: The course will mainly cover chapters 2 through 6 of the textbook. (You should know all about Chapter 1.)

Week # (dates)	Tuesday	Thursday
1 (8/30 and 9/01)	$\mathbf{A}\vec{x} = \vec{b}$	$\mathbf{A}\vec{x} = \vec{b}$ + Level 1
2 (9/06 and 9/08)	$\mathbf{A}\vec{x} = \vec{b}$	$\mathbf{A}\vec{x} = \vec{b}$ + Level 2
3 (9/13 and 9/15)	$\mathbf{A}\vec{x} = \vec{b}$	$\mathbf{A}\vec{x} = \vec{b}$ + Level 3
4 (9/20 and 9/22)	$\mathbf{A}\vec{x} = \vec{b}$ and review	Challenge Level 1
5 (9/27 and 9/29)	Big picture	Big picture + Level 4
6 (10/04 and 10/06)	Big picture	Big picture + Level 5
7 (10/11 and 10/13)	Big picture	Big picture + Level 6
8 (10/18 and 10/20)	Big picture	Challenge Level 2
9 (10/25 and 10/27)	Normal equation	Gram-Schmidt Process +
		Level 7
10 (11/01 and 11/03)	Eigenstuff	Eigenstuff + Level 8
11 (11/08 and 11/10)	Determinants	Determinants + Level 9
12 (11/15 and 11/17)	Eigenstuff	textitChallenge Level 3
13 (11/22 and 11/24)	Thanksgiving	Thanksgiving
14 (11/29 and 12/01)	Positive Definite Matrices	SVD
	+ Level 10	
15 (12/06)	SVD	

Important dates:

- 1. Classes run from Tuesday, August 30 to Tuesday, December 6.
- 2. Add/Drop, Audit, Pass/No Pass deadline—Monday, September 12.
- Last day to withdraw—Monday, October 31 (Sadness!).
- Reading and Exam period—Saturday, December 10 to Friday, December 16.

More stuff:

Do check your zoo account for updates regarding the course.

Academic assistance: Anyone who requires assistance in any way (as per the ACCESS program or due to athletic endeavors), please see or contact me as soon as possible. Episode 1/24: Introduction

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More stuff:

Being good people:

- In class there will be no electronic gadgetry, no cell phones, no beeping, no text messaging, etc. You really just need your brain, some paper, and a writing implement here (okay, and Matlab).
 - Second, Lencourage you to email me questions, ideas, comments, etc., about the class but request that you please do so in a respectful fashion. Finally, as in all UVM classes, Academic honesty will be expected and departures will be dealt with appropriately. See http://www.uvm.edu/cses/for

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Even more stuff:

Late policy: Unless in the case of an emergency (a real one) or if an absence has been predeclared and a make-up version sorted out, assignments that are not turned in on time or tests that are not attended will be given 0%.

Computing: Approximately 2 out of 10 questions per assignment will be Matlab based.

Note: for assignment problems, written details of calculations will be required.

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Why are we doing this?

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If real data is continuous, we almost always discretize it (0's and 1's)



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If real data is continuous, we almost always discretize it (0's and 1's)



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Why are we doing this? Big deal: Linear Algebra is a body of mathematics that deals with discrete problems. Many things are discrete:

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Many things are discrete:

Information (0's & 1's, letters, words)
 People (sociology)
 Networks (the Web, people again, food well
 Sounds (musical notes)

If real data is continuous, we almost always discretize it (0's and 1's) Episode 1/24: Introduction

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VERMONT S

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Even more:



If real data is continuous, we almost always discretize it (0's and 1's) Episode 1/24: Introduction

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 $I \heartsuit$ $N(A^{T})$



Why are we doing this? Linear Algebra is used in many fields to solve problems:

- Engineering
- Computer Science
- Physics

- Biology
- Ecology
- Economics
- Science of the Sociotechnocene



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Big example: Google's Pagerank





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Big example: Google's Pagerank

Some truth:

Linear Algebra is as important as Calculus...





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Big example: Google's Pagerank 🗹

Some truth:

- Linear Algebra is as important as Calculus...
- Calculus \equiv the blue pill...





Why are we doing this?

TOOK EVERY CALCULUS CLASS

EVERYTHING

DISCRETE

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You are now choosing the red pill:

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... or the blue pill?

The red pill...





The Truth:

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Calculus is the Serpent's Mathematics.

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The Platypus of Truth:



Platypuses are masters of Linear Algebra.

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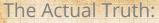
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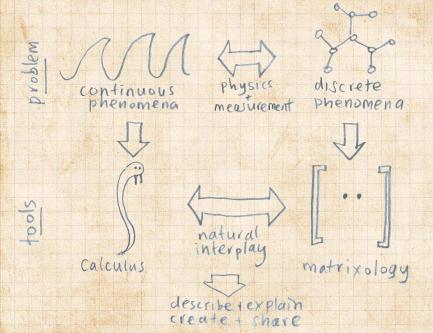
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A matrix A transforms a vector \vec{x} into a new vector \vec{x}' through matrix multiplication (whatever that is):

$$\vec{x}' = A \vec{x}$$

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A matrix A transforms a vector \vec{x} into a new vector \vec{x}' through matrix multiplication (whatever that is):

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We can use matrices to:

Shrink vectors Rotate vectors

Do all these things in different directions

Reveal the true ur-dystopian reality.

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We can use matrices to:

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 - Shrink vectors Rotate vectors Flip vectors Do all these things in different direction

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Digital photographs are matrices:

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Digital photographs are matrices:

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Usually three matrices: RGB color model .



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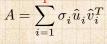
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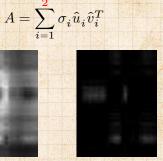
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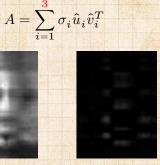
Key problems

Three ways of looking...

Colbert on Equations

References









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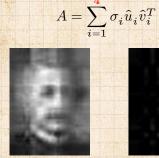
Usages

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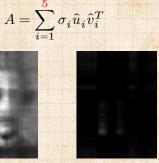
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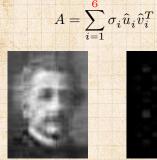
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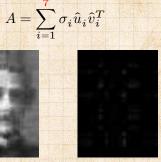
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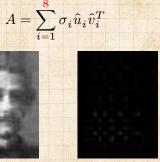
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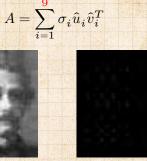
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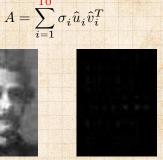
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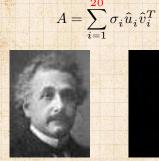
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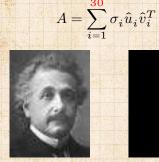
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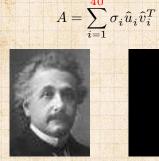
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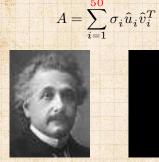
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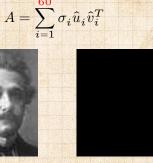
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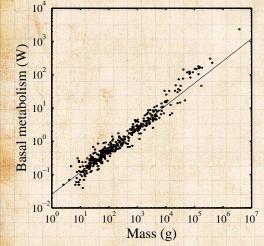








Best fit line (least squares):



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Linear

this

Calculus

algebra does

beautifully;

version is clunky.

Key problems

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Colbert on Equations

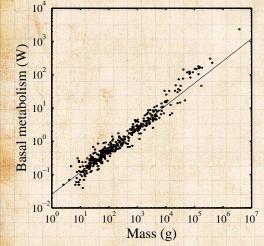
References

From "Re-examination of the '3/4' law of metabolism" ^[1] Dodds, Rothman, and Weitz, Journal of Theoretical Biology, 209, 9–27, 2001





Best fit line (least squares):



Episode 1/24: Introduction

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And evil.

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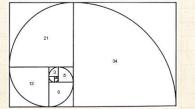
VERMONT

The many delights of Eigenthings:

Using Linear Algebra we'll somehow connect:







 Fibonacci Numbers,

- Golden Ratio,
- Spirals,
- Sunflowers, pine cones,

Harvard Square.

...

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 $\begin{bmatrix} \mathbf{I} \heartsuit \\ \mathcal{N}(\mathbb{A}^{\mathsf{T}}) \end{bmatrix}$



This is a math course:

WAX OFF WAX ON

http://www.pimpartworks.com/artwork/randomsteveo/Wax-On-Wax-Off

It's all connected. "More later."

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References





1. Given a matrix A and a vector \vec{b} , find \vec{x} such that

 $A\vec{x} = \vec{b}.$

Eigenvalue problem: Given A, find λ and \vec{v} suthat $A\vec{v} = \lambda \vec{v}$. Coupled linear differential equations:

Our focus will be largely on #1, partly on #2

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Coupled linear differential equations

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Key problems

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2. Eigenvalue problem: Given A, find λ and \vec{v} such that

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3. Coupled linear differential equations:

$$\frac{\mathsf{d}}{\mathsf{d}t}y(t) = A\,y(t)$$

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Three ways of looking...

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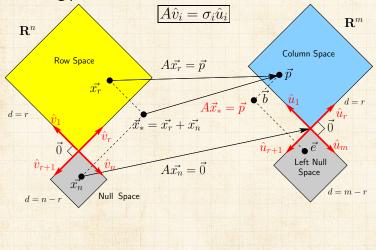
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References



Major course objective:

To deeply understand the equation $A\vec{x} = \vec{b}$, the Fundamental Theorem of Linear Algebra, and the following picture:



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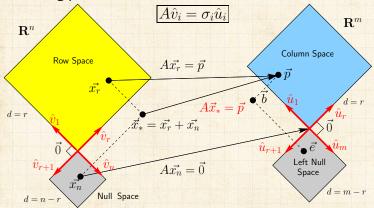
References



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Major course objective:

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What is going on here? We have 24 episodes to find out...

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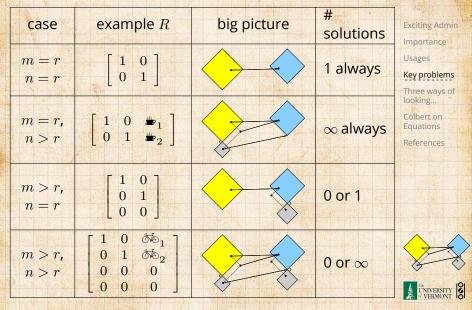
References



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The fourfold ways of $\mathbf{A}\vec{x} = \vec{b}$:

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Our new BFF: $A\vec{x} = \vec{b}$ Broadly speaking, $A\vec{x} = \vec{b}$ translates as follows:

b represents reality (e.g., music, structure) A contains building blocks (e.g., notes, shapes) \vec{a} specifies how we combine our building block make \vec{b} (as best we can). Episode 1/24: Introduction

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Our new BFF: Ax = b
Broadly speaking, Ax = b translates as follows:
b represents reality (e.g., music, structure)
contains building blocks (e.g., notes, shapes)
c specifies how we combine our building blocks make b (as best we can).

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How can we disentangle an orchestra's sound?

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How can we disentangle an orchestra's sound?



► Radiolab ^C's amazing piece: A 4-Track Mind ^C Episode 1/24: Introduction

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Our new BFF: $A\vec{x} = \vec{b}$ Broadly speaking, $A\vec{x} = \vec{b}$ translates as follows: • \vec{b} represents reality (e.g., music, structure) • A contains building blocks (e.g., notes, shapes)

▶ \vec{x} specifies how we combine our building blocks to make \vec{b} (as best we can).

How can we disentangle an orchestra's sound?



► Radiolab G's amazing piece: A 4-Track Mind G

What about pictures, waves, signals, ...?

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Is this your left nullspace?:

及領文写で感が給しオ会観美イナ

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UNIVERSITY

Linear Algebra compliments/putdowns:

Wow, you have such a tiny/huge [delete as applicable] left nullspace!

I S TINY LEFT NULL SPACES Episode 1/24: Introduction

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Linear Algebra compliments/putdowns:

Wow, you have such a tiny/huge [delete as applicable] left nullspace!

LEFT NULL SPACES

▶ See also: The Dunning-Kruger effect. C

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What does knowing \vec{x} give us?

Compress information See how we can alter information (filtering) Find a system's simplest representation Find a system's most important elements See how to adjust a system in a principled v Episode 1/24: Introduction

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What does knowing \vec{x} give us?

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Way 1: The Row Picture
Way 2: The Column Picture
Way 3: The Matrix Picture

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Way 1: The Row Picture
Way 2: The Column Picture
Way 3: The Matrix Picture

Example:

Call this a 2 by 2 system of equations. 2 equations with 2 unknowns. Standard method of simultaneous equations: solve above by adding and subtracting multip of equations to each other Episode 1/24: Introduction

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Example:

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Standard method of simultaneous equations: solve above by adding and subtracting multiples of equations to each other = Row Picture. Episode 1/24: Introduction

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Row Picture—what we are doing:
 (a) Finding intersection of two lines
 (b) Finding the values of x₁ and x₂ for which equations are satisfied (true/happy)
 A splendid and deep connection:

 (a) Geometry = (b) Algebra

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Three possible kinds of solution:
1. Lines intersect at one point
2. Lines are parallel and disjoint
3. Lines are the same

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Three possible kinds of solution:

- 1. Lines intersect at one point —One, unique solution
- 2. Lines are parallel and disjoint
- 3. Lines are the same

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Three possible kinds of solution:

- 1. Lines intersect at one point —One, unique solution
- 2. Lines are parallel and disjoint —No solutions
- 3. Lines are the same

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Row Picture—what we are doing:

- (a) Finding intersection of two lines
- (b) Finding the values of x₁ and x₂ for which both equations are satisfied (true/happy)
- A splendid and deep connection:
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The column picture:

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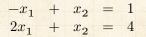
Column vectors are our 'building blocks' Key idea: try to 'reach' \vec{b} by combining (summir multiples of column vectors \vec{a}_1 and \vec{a}_2 .



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The column picture:

See



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The column picture:

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General problem

$$x_1\vec{a}_1+x_2\vec{a}_2=\vec{b}$$

Column vectors are our 'building blocks'

Key idea: try to 'reach' by combining (summing multiples of column vectors a₁ and a₂. Episode 1/24: Introduction

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We love the column picture:

Generalizes easily to many dimensions

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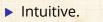
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(assuming neither \vec{a}_1 or \vec{a}_1 are $\vec{0}$)

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Difficulties:

- Do we give up if $A\vec{x} = \vec{b}$ has no solution?
 - No! We can still find the \vec{x} that gets us as close to as possible.
 - Method of approximation—very important! We may not have the right building blocks but we can do bur best.

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The Matrix Picture:

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The Matrix Picture:

Now see

$$x_1 \begin{bmatrix} -1 \\ 2 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}.$$

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The Matrix Picture:

Now see

$$x_{1} \begin{bmatrix} -1\\2 \end{bmatrix} + x_{2} \begin{bmatrix} 1\\1 \end{bmatrix} = \begin{bmatrix} 1\\4 \end{bmatrix}.$$
$$\vec{x} = \vec{b} : \begin{bmatrix} -1 & 1\\2 & 1 \end{bmatrix} \begin{bmatrix} x_{1}\\x_{2} \end{bmatrix} = \begin{bmatrix} 1\\4 \end{bmatrix}$$

as

A 2 $1 \mid \lfloor x_2 \mid$

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The Matrix Picture:

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A is now an operator:

• A transforms \vec{x} into \vec{b} .

Roughly speaking, *A* does two things to . Rotation/Flipping Dilation (stretching/contraction) Episode 1/24: Introduction

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The Matrix Picture:

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$$x_{1} \begin{bmatrix} -1\\ 2 \end{bmatrix} + x_{2} \begin{bmatrix} 1\\ 1 \end{bmatrix} = \begin{bmatrix} 1\\ 4 \end{bmatrix}.$$
$$\vec{x}_{-} \vec{h}_{-} \begin{bmatrix} -1 & 1 \end{bmatrix} \begin{bmatrix} x_{1} \end{bmatrix} - \begin{bmatrix} 1 \end{bmatrix}$$

as

$$A\vec{x} = \vec{b} : \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

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 - 1. Rotation/Flipping
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Key idea in linear algebra:

Decomposition or factorization of matrices.

Matrices can often be written as products or s of simpler matrices

 $A = LU, A = QR, A = U\Sigma V^T, A = \sum_i$

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Key idea in linear algebra:

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Key idea in linear algebra:

- Decomposition or factorization of matrices.
- Matrices can often be written as products or sums of simpler matrices

•
$$A = LU, A = QR, A = U\Sigma V^T, A = \sum_i \lambda_i \vec{v} \vec{v}^T, \dots$$



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More Truth about Mathematics:

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