# Is language evolution grinding to a halt? The scaling of lexical turbulence in English fiction suggests it is not 

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#### Abstract

Of basic interest is the quantification of the long term growth of a language's lexicon as it develops to more completely cover both a culture's communication requirements and knowledge space. Here, we explore the usage dynamics of words in the English language as reflected by the Google Books 2012 English Fiction corpus. We critique an earlier method that found decreasing birth and increasing death rates of words over the second half of the 20th Century, showing death rates to be strongly affected by the imposed time cutoff of the arbitrary present and not increasing dramatically. We provide a robust, principled approach to examining lexical evolution by tracking the volume of word flux across various relative frequency thresholds. We show that while the overall statistical structure of the English language remains stable over time in terms of its raw Zipf distribution, we find evidence of an enduring 'lexical turbulence': The flux of words across frequency thresholds from decade to decade scales superlinearly with word rank and exhibits a scaling break we connect to that of Zipf's law. To better understand the changing lexicon, we examine the contributions to the Jensen-Shannon divergence of individual words crossing frequency thresholds. We also find indications that scholarly works about fiction are strongly represented in the 2012 English Fiction corpus, and suggest that a future revision of the corpus should attempt to separate critical works from fiction itself.


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## 1. Introduction

In studying any entity or system, a fundamental scientific goal is the satisfactory characterization of temporal dynamics, whether empirically observed, simulated, or theoretically predicted. For language, there are many kinds and scales of temporal dynamics to consider such as the introduction and usage decline of specific words [1], the evolution of accents, the long term development of individual languages [2], and the changes in the overall ecology of human languages which has now moved well into an era of die off [3].

Here, we are concerned with the dynamics of the English language's lexicon. Primarily, we want to know how the usage of words has changed in time, and how this is reflected in the English lexicon's evolution. This focus leads us to several core questions: (1) What are the rates at which words are born and at which they

[^0]die? (2) How do we reasonably identify word births and deaths in the first place? (3) As the English lexicon has expanded, how have overall statistical patterns such as Zipf's law [4] changed, if at all? We are especially interested with revisiting work on word "birth" and "death" rates as performed in [1]. As we will show, the methods employed in [1] suffer from boundary effects, and we propose and investigate an alternative approach insensitive to time range choice. We also investigate lexical changes at a range of usage frequency levels.

We will perform our analyses using the Google Books corpus [5,6] whose incredible volume generated from an extensive coverage of all written works would seemingly make it an ideal candidate for linguistic research. However, there are two major caveats that limit its potency and we will lay them out before proceeding.

In previous research [7], we broadly explored the characteristics and dynamics of the unfiltered English and English Fiction data sets from both the 2009 and 2012 versions of the Google Books corpus. We showed that the 2009 and 2012 unfiltered English data sets and, surprisingly, the 2009 English Fiction data set, all become increasingly influenced by scientific texts throughout the 1900s, with medical research language being especially prevalent. We


Fig. 1. Total 1-gram counts for the Google Books corpus 2012 English Fiction data set as a function of publication year. More than simple words, 1-grams include any sequence of unbroken non-space characters as well as punctuation marks, such as commas and brackets, which are broken away from words. The number of 1 -grams is proportional to the number of books and pages in the corpus but does not account for readership popularity [7]. A roughly exponential increase in 1-gram volume is apparent over time with several periods of stasis and decline. Volume decreases are particularly apparent during the American Civil War, World War I, and World War II as indicated by the three vertical gray shadings. Both World Wars follow a decade of stagnancy and decline for the corpus.
concluded that, without sophisticated processing or the provision of extensive metadata, only the 2012 English Fiction data set is suitable for any kind of analysis and deduction as it stands.

We also described the confounding problem of the library-like nature of the Google Books corpus. Each book is, in principle, represented only once (re-editions are one exception). Word frequency is thus a deceptive aspect of the Google Books corpus as book popularity is not encoded in any way. Word counts are in no way reflective of how often these words are read-as might be informed by book sales and library borrowing data-much less spoken by the general public. Nevertheless, the Google Books corpus registers an imprint of a language's lexicon and remains worthy of study, as long as we remain mindful of its nature.

In this paper, we therefore focus only on the 2012 version of the English Fiction data set. To provide a sense of scale for this corpus, we show in Fig. 1 the total number of 1-grams for this data set between 1800 and 2000 (1-grams are defined to be contiguous text elements and are more general than words including, for example, punctuation; for ease of expression, we will use word and 1-gram interchangeably). An exponential increase in volume is apparent over time with notable exceptions during major conflicts when the total volume decreases. There is effectively zero growth in volume over first half of the 20th Century.

A number of researchers have carried out studies of the Google Books corpus with the aim of examining properties and dynamics of entire languages. These include analyses of Zipf's and Heaps' laws as applied to the corpus [8], the rates of verb regularization [5], rates of word "birth" and "death" and durations of cultural memory [1], as well as an observed decrease in the need for new words in several languages [2]. However, most of the studies were performed before the release of the second version, and, to our knowledge, none have taken into account the substantial effects of scientific literature on the data sets.

We structure the paper as follows. In Section 2, we critique the method from [1] which examines the birth and death rates of 1grams for several languages using the first Google Books corpus. Through a number of different analyses, we show that while 1-gram
birth rate has slowed, death rates have not increased substantially. In Section 3, we describe information theoretic methods for examining lexical evolution using the Jensen-Shannon divergence, and then present our observations in the form of word shift graphs. We first recall and confirm an apparent bias toward increased usage rates of 1 -grams over time [7]. We then measure the flux of 1 -grams across various relative frequency boundaries in both directions for the 2012 English Fiction data set. We describe the use of the largest contributions to the Jensen-Shannon divergence between successive decades from among the 1 -grams crossing each boundary as signals to highlight the specific dynamics of 1 -gram growth and decay over time. We display ranked examples of these 1-gram usage changes and explore the factors contributing to the observed disparities between growth and decay. In releasing the original data set, Michel et al. [5] noted that English Fiction contained scholarly articles about fictional works (but not scholarly works in general), and we also investigate this mixing of texts. We offer concluding remarks in Section 5 Supporting material can be found at our paper's Online Appendices [9].

To compare across years, we will work with relative frequency, $f$. For a 1 -gram $w, f_{w ; y}$ is the usage frequency of $w$ normalized by the total number of 1 -grams in the year $y$ (the total number of 1 -grams is to be distinguished form the number of unique 1 -grams). We will also use the average relative frequency for a 1 -gram over the time scale of a decade.

## 2. On quantifying the birth and death of words

In this section, we aim to measure the births and deaths of words over time. As we will show, this will turn out to be a delicate and arguably ill-defined task. We will arrive at this conclusion by considering and attempting to reproduce the work of Petersen et al. [1] on word life spans, and, by doing so, show how word death rates are strongly affected by our vantage point in history.

Petersen et al. [1] examined the birth and death rates of words over time for various data sets in the 2009 version of the Google Books corpus including unfiltered English, English Fiction, Spanish, and Hebrew.

Their quantification of birth and death is nuanced and requires some examination. They define the birth year and death year of an individual word as the first and last year, respectively, that the given word's relative frequency $f_{w ; y}$ is found to be equal to or greater than a cutoff frequency $f_{w ; y_{1}, y_{2}}^{c u t}$ equal to one twentieth its median relative frequency $f_{w ; y_{1}, y_{2}}^{\mathrm{med}}$, i.e., $f_{w ; y} \geq f_{w ; y_{1}, y_{2}}^{\text {cuu }}=0.05 f_{w ; y_{1}, y_{2}}^{\mathrm{med}}$. The subscripts $y_{1}$ and $y_{2}$ indicate the first and last year of the overall time period. They exclude words appearing in only one year (we will show this is problematic) and words appearing for the first time before $y_{1}=1700$. The rates of word birth and death, respectively, are then found by normalizing the numbers of word births and deaths by the total number of unique words in a given year.

For all four data sets, Petersen et al. found strongly decreased birth rates and increased death rates over time, a variation for both of two to three orders of magnitude occurring most rapidly between 1950 and 2000. They noted that they obtained qualitatively similar results when one tenth the median frequency is used as the cutoff threshold.

The very specific nature of the analysis raise questions as to the robustness of the method. Three major concerns:

- Use of the median relative frequency for a threshold of birth and death. This quantity depends on the word and the year range chosen. Hypothetically, a rare word $w_{1}$ that has a constant relative frequency over time will never be identified as being born or dying, while a word $w_{2}$ with much higher relative frequencies may die out yet still never fall to the relative frequency of $w_{1}$. In
short, the standards for a word's birth and death vary from word to word and from time range to time range. As we will see in particular, shifting the end year $y_{2}$ using this analysis strongly affects death rates.
- The problematic use of the median for very rare words. Rare words that have a zero relative frequency in more than half of the years examined will have a median relative frequency $f_{w ; y_{1}, y_{2}}^{m e d}$ $=0$. The strict definition in [1] means such a word will never be born or die as its relative frequency will always be greater than or equal to 0 (such a state could be possibly termed 'unalive' [10]). If we simply ignore such words, then we will at the same time be including words with lower overall abundance, e.g., words that appear only in two consecutive years. To overcome this issue, we adjust Petersen et al.'s criterion to involve an inequality: $f_{w ; y}>f_{w ; y_{1}, y_{2}}^{\text {cut }}=0.05 f_{w ; y_{1}, y_{2}}^{\text {med }}$. We note that we presume the computation of the median in [1] was carried out for the range of years covering the first and last appearance of a word.
- By necessity, the Google Books corpus was constructed with a frequency threshold for a word to be included or not (a word must appear at least 200 times). Thus, a word having a relative frequency of 0 in the data set does not mean it was entirely absent. We do not attempt to incorporate this issue of censusing here but note that it becomes problematic for rare words (which are collectively legion).

With these points in mind, we recreated the described analysis of [1] for the 2012 version of English Fiction at the level of individual years. Per [1], we initially exclude words appearing in only one year. We also limit our analysis to 1800- (rather than 1700-) and our findings will help us address this choice. We believe these differences with [1] should not be substantive, and allow us to re-examine their work and build out our own in meaningful ways.

We compare the birth and death rates as observed at different end points of history by performing the analysis with $y_{2}=1860$ through to $y_{2}=2000$ in increments of 20 years. We present the resulting birth and death rates in Fig. 2 (cf. Fig. 2 in [1]). Subsequent modifications which we will explain below will give us the comparison plots shown in Fig. 2B and D.

We observe in Fig. 2A that birth rates decline approximately exponentially overall, and here we find general agreement with [1] and [2]. However, we also see sharp departures to much lower birth rates near the end point of each history. We are able to entirely attribute these drops to the decision in [1] to ignore all 1-grams that have a non-zero relative frequency in a single year. By including these 1 -grams, Fig. 2A becomes Fig. 2B and we see that birth rate is no longer affected by the choice of $y_{2}$. We therefore see that words that appeared in only one year before a selected end year $y_{2}$ may well be just sputtering into existence. Such words will be retrospectively declared born when the end of history moves forward.

We can also now see that the apparent speeding up of the drop in birth rate after 1980 in Fig. 2B appears to be real, consistent with [1]. We note that this is a complicated time with massive growth and change in information technology and publishing, and we will see that literary criticism starts to populate the corpus during this time period as well.

We now turn to word death rates in Fig. 2B. In contrast to birth rates, there is no overlap between death rates at any point in time as a function of the end of history $y_{2}$. For example, death rates in the late 1800 s are estimated at $10 \%$ if $y_{2}=1900$ but $<10^{-3} \%$ if $y_{2}=2000$. It appears that words are not in fact dying out.

So why is the word death rate used in [1] affected so profoundly by boundaries? Including words appearing in only one year as we did for birth rates, does not resolve this issue: Fig. 2D is essentially the same as Fig. 2C.

The problem lies instead in that the relative frequency threshold for a word "existing" in a given year $y$ is determined by range of years being considered. We argue that a number of example relative frequency trajectories are problematic for a range dependent definition.

Consider two different ranges of years, $\left[y_{1}, y_{2}\right]$ and $\left[y_{1}, y_{2}^{\prime}\right]$ with $y_{2}<y_{2}^{\prime}$ and a year $y$ internal to both ranges. The median relative frequency for the same 1 -gram will very likely differ for the two ranges, and a word which is alive in year $y$ for the $\left[y_{1}, y_{2}\right]$ range may be either not yet born or dead for the same year $y$ in the $\left[y_{1}^{\prime}, y_{2}^{\prime}\right]$ range.

This complication allows for unintuitive results such as a 1-gram appearing to have died out by $y_{2}$ but over a longer period of time ending at $y_{2}^{\prime}$, it qualifies as having being alive, or possibly, "undead."

We provide two examples of dead-undead behavior in Fig. 3. First, in Fig. 3A, we show the word "CHAP" (all capitals, likely short for Chapter). We chose this word as one with a reasonably high median relative frequency but otherwise fairly randomly from all words with oscillating dead-undead states. The main curve is the relative frequency for 'CHAP' over time showing a gradual decline over around three orders of magnitude. In both Figs. 3A and 3 B, the blue region outlines the lowest possible relative frequency for each year (i.e., 1 divided by the total number of 1 -grams recorded).

We measure median relative frequency over a series of time ranges with $y_{1}=1800$ and ends-of-history at $y_{2}=1850$ through to $y_{2}=2000$ in decade steps. The circles mark the cutoff frequency $f_{W: y_{1}, y_{2}}^{\text {cut }}$ for each time range. Open circles indicate the relative frequency of 'CHAP' has exceeded the cutoff at that $y_{2}$-'CHAP' is alive-while filled circles show that 'CHAP' has died.

In 1850, the word 'CHAP' would have appeared to have snuffed it in 1848; then viewed as having only temporarily been stunned and revived for the following 8 decadal end points; been declared an ex-word again in 1940, nailed to the perch as it were; and finally seen again to be only resting and not at all ready to push up the daises through to 2000 [11].

In Fig. 3B, we show the relative frequency for a much less common word which displays a different kind of dead-undead cycling: "Coryphaeus" (the head of a Greek chorus). The time series includes numerous zeroes (which we must remember pertain only to the sample behind the Google Books 2012 English Fiction corpus). This example shows a decadal-scale swapping between being dead and undead from 1850 on, and demonstrates how zero frequencies may induce unexpected behavior in the birth-death criterion in [1]. Essentially, whenever "Coryphaeus" does not appear in the corpus for a year, it will be considered dead, and if it does appear, it will have a relative frequency exceeding the dead-undead cutoff.

Thus, while the method in [1] provides a reasonable approach to analyzing dynamics and asymmetries in the evolutionary dynamics of a language data set and is informative about birth rates, the results for death rates depend on when the experiment is performed. We proceed to develop an approach that is independent of time boundaries and agnostic to the 1 -grams themselves.

## 3. Tracking language evolution through the flux of words across relative frequency thresholds

We move away from attempting to identify words as having been born or died to exploring the flux of words across fixed relative frequency thresholds. For example, over some time span, we wish to find and count which words decline in prevalence and drop below, say, a relative frequency of $10^{-5}$, along with which words move up above this threshold. With a decay in the birth rates of words, English may be globally "cooling" [2] but we will show that there is still much bubbling within.


Fig. 2. (A) and (C): Birth and death rates for 1-grams for the 2012 version of English Fiction determined using the method proposed in [1]. Curves correspond to different end-of-history boundaries with history running from $y_{1}=1800$ to $y_{2}=1860-2000$ in 20 year increments. Birth rates show clear departures from an overall form as each end of history year is approached. Including words that appear in only one year in a time range eliminates these discrepancies (plot B). Death rates however are strongly affected by the choice of when history ends and this cannot be remedied by modifying the rule for 1-gram death. As the end of history moves forward in time, words that seemed dead are no longer dead for a number of reasons $B$ and $D$ : Birth and death rates as per plots $A$ and $C$ in all respects except now including words that appear only once in a time range-i.e., have a non-zero relative frequency in only one year. Birth rates are now well determined retrospectively from any vantage point of history and an exponential decay appears confirmed. Death rates remain incongruent as in C .

To work at a meaningful temporal scale, we coarse-grain the relative frequencies in the second English Fiction data set at the level of decades-e.g., between 1870-to-1879 and 1990-to-1999-by averaging the relative frequency of each unique word in a given decade over all years in that decade. We weight each year equally. This allows us to conveniently calculate and sort contributions to the Jensen-Shannon divergence (defined below) of individual 1-grams between, for example, any two time periods. To avoid high levels of optical character recognition (OCR) errors for texts typeset prior to the early 19th century, we will concern ourselves going forward with 1 -grams between the years 1820 and 2000. A prevalent example is the long s-e.g., "said" being read as "faid" [7].

### 3.1. Basic stability of Zipf's law

A famous and fundamental scaling for language is Zipf's law [4] which was long held to be that the relative frequency of a word in a corpus scales approximately as the inverse of its size rank, $f \sim r^{-\alpha}$ with $\alpha \simeq 1$. However, recent empirical work has shown that for large corpora, Zipf's law typically exhibits two scaling regimes:
$f \sim\left\{\begin{array}{l}r^{-\alpha} \text { for } r \ll r_{\mathrm{b}}, \\ r^{-\alpha^{\prime}} \text { for } r \gg r_{\mathrm{b}},\end{array}\right.$
where $\alpha^{\prime}>\alpha$, and the transition between scaling regimes around the break point $r_{\mathrm{b}}$ typically occurs over an order of magnitude. Prior work by our group has elsewhere found the break in scaling for Zipf's law to be a result of text mixing [12] (other theories have been put forward [13,14]). The break point $r_{\mathrm{b}}$ can be estimated by average text length, though we cannot do so for the Google Books corpus as the necessary information on individual books is not available.

For the present work, we only need to characterize Zipf's law with its two scaling regimes. In Fig. 4A, we plot Zipf's law for each decade running from the 1830s through to the 1990s. We observe very strong agreement over nearly 200 years of English Fiction. The variations that we do see are (1) the most common words become slightly less common, and (2) the tail becomes slightly fatter as new 1 -grams enter the lexicon.

For the sake of introducing and broadly characterizing word flux, it is sufficient for us to perform a simple measure of the scaling exponents by averaging the Zipf's laws and then using standard linear regression over the ranges indicated in Fig. 4A. We estimate $\alpha^{\prime} \simeq 1.14$ and $\alpha \simeq 1.95$.

In Fig. 4B, we show in detail how the numbers of 1-grams with relative frequencies exceeding fixed thresholds are stable over time. The only exception is the top 1-gram-always the comma-which gradually deflates in relative frequency (punctured punctuation).

At least in the case of English fiction then, the "bones" of Zipf's law have changed little over the period 1820 to 2000.

But the words underlying Zipf's law have fluctuated in relative frequency, and this is an aspect often overlooked when comparing ranked distributions for any system.

### 3.2. Lexical turbulence: The scaling of word flux across internal frequency thresholds

In Fig. 5, we show word flux as a function of time and frequency threshold. First, In Fig. 5A and B, we display the upward and downward fluxes $\phi_{\text {up }}$ and $\phi_{\text {down }}$ of the number of 1-grams crossing relative frequency thresholds of powers of 10 from $f_{\text {thr }}=10^{-4}$ down to $10^{-7}$. Each point is centered in a decade and represents the total


Fig. 3. Two examples of how a 1-gram may be variously labeled dead or alive depending on the end of history using the criterion in [1]. A. The word 'CHAP' declines in relative frequency over time, from a high of $10^{-3.5}$ to as low as $10^{-7.5}$. Using a twentieth of the median frequency of a 1 -gram as a threshold for birth and death, we see 'CHAP' appears to have "run down the curtain" in 1850 but then re-emerged as alive for 8 subsequent decadal end points. 'CHAP' once again succumbs in 1940 only to stagger on through 2000. This dead-undead cycling can be seen for many words and leads us to exploring how words pass above and drop below fixed relative frequency thresholds. In both plots, the blue region marks the lowest possible relative frequency for each year achieved when a 1 -gram has a count of 1 . B. The word 'Coryphaeus' is a much less frequent word than 'CHAP', and its time series contains a substantial number of zeroes and ones (resting on the top of the blue region). The criterion in [1] leads to a flipping back and forth between being dead and undead at most end-of-history years from 1850 through to 2000.


Fig. 4. A. Overlay of Zipf's laws for all decades running from the 1820 s to the 1990 s for the 2012 English Fiction corpus (color shifts from light gray to black). The observed stability demonstrates that Zipf's law remains largely unchanged for the 2012 English Fiction data set, even though individual words may vary greatly in rank over time. B. In support of A, rank threshold boundaries correspond to nearly constant relative frequency threshold boundaries over many orders of magnitude, with the exception of the top 1-gram (always a comma), which decreases in relative frequency. Points are located at the center of each decade.
number of words moving across a frequency threshold from that decade to the next.

We can see that word flux across frequency thresholds is relatively constant over time. Of the minor modulations we see some consistency across thresholds, notably recent decades for $\phi_{\text {down }}$ and $f_{\text {thr }}=10^{-5}, 10^{-6}$, and $10^{-7}$. Moreover, in comparing Figs. 5A and 5 B, the two fluxes appear to be fairly balanced.

However, word flux does vary strongly with respect to frequency threshold $f_{\text {thr }}$ and we view this as a kind of 'lexical turbulence'. We see in Fig. 5A and B that, as we should expect, the lower the threshold, the higher the flux. The most common words have essentially no turnover (see below) while increasingly rare ones are increasingly volatile.

In Fig. 5C, we attempt to characterize the relationship between word flux and frequency threshold, $f_{\mathrm{thr}}$. We average the fluxes in


Fig. 5. Upward and downward fluxes, $\phi_{\text {up }}$ and $\phi_{\text {down }}$ as a function of relative frequency threshold $f_{\text {thr }}$. These fluxes are the total number of words crossing relative frequency thresholds of $f_{\text {thr }}=10^{-4}, 10^{-5}, 10^{-6}$, and $10^{-7}$ in both upward (Plot A) and downward (Plot B) directions between consecutive decades. Word flux is roughly proportional to the inverse of the relative frequency threshold $f_{\mathrm{thr}}$. For each threshold, the upward and downward flux roughly cancel. For either direction of flux, there appears to be little qualitative difference between the three smallest thresholds. Plot C: Upward and downward fluxes $\phi_{\mathrm{up}}$ and $\phi_{\mathrm{down}}$ from A and B averaged over all decade pairs as a function of relative frequency threshold $f_{\text {thr }}$. The balance between the two fluxes is evident, and the two scaling regimes identified are tied to the break in scaling in Zipf's law (Fig. 4A) [12]. Word flux scales approximately as the inverse of frequency threshold showing that in moving further away from the most common words, the English language becomes more and more alive, churning internally in a regular fashion. Plot D: Using Zipf's law from Fig. 4A, C transformed to show flux as a function of word rank. The scaling regimes connect to those of C through the Zipf exponents $\alpha$ and $\alpha^{\prime}$ (See Eqs. (1, 2), and (3)). The superlinear scaling makes clear that the growth in lexical turbulence with rank $r$ is strong. For example, around $16.8 \%$ of words with $r \leq 10^{5}$ will be replaced every decade.

Fig. 5A and B, and plot them as a function of $f_{\text {thr }}$. The averages for $\phi_{\text {up }}$ and $\phi_{\text {down }}$ are indistinguishable to the eye, confirming the balance suggested in Figs. 5A and 5 B.

We have at hand evidence of lexical turbulence through an apparent inverse scaling of word flux across frequency thresholds, and we mark two possible scaling regimes:
$\phi \sim\left\{\begin{array}{l}f_{\mathrm{thr}}{ }^{-\mu} \text { for } f_{\mathrm{thr}} \ll f_{\mathrm{b}}, \\ f_{\mathrm{thr}}{ }^{-\mu^{\prime}} \text { for } f_{\mathrm{thr}} \gg f_{\mathrm{b}},\end{array}\right.$
where $\mu \simeq 0.77$ and $\mu^{\prime} \simeq 1.10$, and $f_{\mathrm{b}}$ is the scaling break point.
In Fig. 5D, we also show how flux scales with word rank $r$. To do so, we have used the average form of Zipf's law in Fig. 4A to map frequency to rank. The evident upper limit for flux is $\phi=r$, marked by gray area in Fig. 5D.

We are able to connect the scaling break for flux with respect to both $f_{\text {thr }}$ and $r$ to the scaling break in Zipf's law. Combining Eqs. (1) and (2), we have
$\phi \sim \begin{cases}r^{\nu}=r^{\alpha \mu^{\prime}} & \text { for } r \ll r_{\mathrm{b}}, \\ r^{\nu^{\prime}}=r^{\alpha^{\prime} \mu} & \text { for } r \gg r_{\mathrm{b}} .\end{cases}$
We measure the lower and upper exponents in Fig. 5D as 1.23 and 1.47, and these compare favorably using the equation above and the exponents measured in Figs. 4 and 5 D: $v=\alpha \mu^{\prime} \simeq 1.14 \times 1.10 \simeq 1.25$ and $v^{\prime}=\alpha^{\prime} \mu \simeq 1.95 \times 0.77 \simeq 1.50$.

Now, both lower and upper scalings of flux with rank are superlinear meaning that the lexical turbulence increases strongly with rank-relatively more and more words are turned over the further
we move up in word rank (down in relative frequency thresholds). Clearly this scaling cannot be sustained as eventually we would have $\phi>r$. For the 2012 English Fiction corpus, we see that the lexicon is exhausted before such a possibility comes about.

For the top 100 words, we see the lexicon is strongly conserved-crystallized-with on average $2.4 \%$ of words turning over every decade. But the superlinear scaling means the lexicon becomes increasingly volatile. As we travel out to the top $10^{5}$ words, the flux has grown to a considerable $16.8 \%$ per decade.

There are some important limitations to our findings. The time scale of comparison, which is here decade-to-decade, will affect the scaling as well, i.e., we need to consider $\phi(r, \tau)$ where $\tau$ is the length in years of adjacent periods. Clearly, the smaller the time scale of comparison, the less the degree of lexical turbulence which must tend toward 0 .

We stress that the scalings indicated for flux are intended only to be rough estimates, and we will stop well short of proclaiming a set of universal exponents. Future work will need to be performed across many languages and using better curated and different corpora.

In sum, we find that despite a steady decline in word birth rate for the 2012 English Fiction corpus-two orders of magnitude over two hundred years (Fig. 2)-the flux of words across frequency thresholds in the Zipf distribution has remained essentially constant in magnitude and scaling. Our next and last task will be to explore the individual words most strongly contributing to this lexical turbulence.

## 4. Fine-grained exploration of flux across frequency threshold boundaries

We now begin to examine the specific 1-grams that cross relative frequency thresholds as we move from decade to decade. We first describe the very limited flux across the $f_{\text {thr }}=10^{-2}$ boundary and then investigate the richer transitions for the lower thresholds $10^{-3}$ down to $10^{-6}$.

Flux across the $10^{-2}$ boundary between consecutive decades is almost nonexistent from the 1820 s to the 1990 s. The 1 -grams that do achieve such a crossing make for a short list of three:

- Between the 1820 s and 1830 s, the semicolon falls below the $10^{-2}$ relative frequency threshold.
- Between the 1840 s and 1850 s, " $I$ " rises above the $10^{-2}$ relative frequency threshold.
- Between the 1910 s and 1920 s, "was" rises above the $10^{-2}$ relative frequency threshold.

This is the entirety of the flux across the $10^{-2}$ threshold from 1820 to 2000 showing once again that the regime of 1 -grams above this frequency (roughly the top 101-grams) is extremely stable. The eleven 1 -grams with relative frequency above a threshold of $10^{-2}$ in the 1990s in decreasing order of frequency are: the comma ",", the period ".", "the", the quotation mark """, "to", "and", "of", "a", ‘ 'I", "in", and "was".

### 4.1. Jensen-Shannon divergence and individual 1-gram contributions

To enable us to make better sense of the detailed flux across lower frequency thresholds, we need some way of assigning some kind of weight of importance to each 1-gram involved in the flux. To do so, we start with a standard measure for comparing two probability distributions, the Jensen-Shannon divergence (JSD) [15]. We will then decompose the JSD into contributions from individual 1grams which in turn will afford a simple ranking of 1 -grams. We note that other approaches to determining the salience of words are possible such as the different lens generated by the use of the partial KL in [16].

Given two corpora with 1 -gram distributions $P$ and $Q$, the JSD between $P$ and $Q$ may be expressed as
$D_{J S}(P \| Q)=H(M)-\frac{1}{2}[H(P)+H(Q)]$,
where $M=\frac{1}{2}(P+Q)$ is the mixed distribution of the two years, and $H(P)=-\sum_{i} p_{i} \log _{2} p_{i}$ is the Shannon entropy [17] of the original distribution. The JSD is symmetric and bounded between 0 bits and 1 bit, and these bounds are only attained when the distributions are identical and free of overlap, respectively.

Helpfully, the JSD is a linear combination of contributions due to individual words and can be expressed as $D_{J, i, i}(P \| Q)=\sum_{i} D_{J S, i}(P \| Q)$. The contribution from the $i$ th word to the divergence between the two distributions, as derived from Eq. (4), is given by
$D_{J S, i}(P \| Q)=m_{i} \cdot \frac{1}{2}\left[r_{i} \log _{2} r_{i}+\left(2-r_{i}\right) \log _{2}\left(2-r_{i}\right)\right]$,
where $r_{i}=\min \left(p_{i}, q_{i}\right) / m_{i}$. The contribution from an individual word is therefore proportional to the average frequency of the word $m_{i}$ and also depends on the ratio between the smaller and average frequencies, $r_{i}=p_{i} / m_{i}$. We write the contribution of the $i$ th word as:
$D_{J S, i}(P \| Q)=m_{i} C\left(r_{i}\right)$,
where $C\left(r_{i}\right)=\frac{1}{2}\left[r_{i} \log _{2} r_{i}+\left(2-r_{i}\right) \log _{2}\left(2-r_{i}\right)\right]$.


Fig. 6. Percent of (Jensen-Shannon divergence) JSD in English Fiction 2012 corpus due to words increasing in relative frequency of use for successive decades (dark gray), and decades three apart (light gray; e.g., 1990s versus 1960s). The contribution for successive decades is nearly always more than half-the exceptions are between the 1820 s, 1840 s, and 1970 s, and their successive decades. For decades three apart, the contribution is always greater than $50 \%$. The JSD between successive decades also shows peaks in the vicinity of major conflicts.

Words with larger average frequencies $\left(m_{i}\right)$ yield larger contribution signals as do those with smaller ratios $\left(r_{i}\right)$. A commonly occurring 1 -gram changing subtly can produce a large signal. So can an uncommon or new word given a sufficient shift in probability. The quantity $C\left(r_{i}\right)$ is concave (up) and symmetric about $r_{i}=1$, where the frequency remains unchanged ( $p_{i}=q_{i}=m_{i}$ ) yielding no contribution. If a word appears or disappears between two decades (e.g., $p_{i}=0$ and $q_{i}>0$ ), then the contribution is maximized at precisely the average frequency of the word in question.

### 4.2. Asymmetry in Jensen-Shannon divergence measures between decades

As we show in Fig. 6, more than half of the JSD between a given decade and the next is typically due to contributions from words increasing in relative frequency. The JSDs between 1820s, 1840s, and 1970s and their successive decades are the only exceptions. Moreover, when the time differential is increased to three decades, no exceptions remain. This asymmetry is sympathetic to the lexicon enjoying new words but relatively few true deaths (Section 2).

We note relative extrema of the inter-decade JSD in the vicinity of major conflicts. Between the 1860s and successive decade, words on the rise contribute substantially to the JSD. This is consistent with words not relatively popular during wartime (specifically the American Civil War) being used more frequently in peacetime. A similar tendency holds for the JSD between the 1910s (World War I) and the 1920s. This is not as apparent in the JSD between the 1910s and the 1940s, possibly because the 1940s coincide with World War II. The absolute maximum for the single-decade curve corresponds to the divergence between the 1950s and 1960s. This suggests a strong effect from social movements. For the 3-decade split, the absolute peak comes from the JSD between the 1940s and 1970s, which are certainly decades of starkly different character.

### 4.3. Fine-grained exploration of flux across frequency threshold

 boundaries: $\mathrm{f}_{\text {thr }}=10^{-3}$We conclude our analysis with a series of observations on which words contribute to flux between a number of example decade pairs and across the frequency thresholds $10^{-3}, 10^{-4}, 10^{-5}$, and $10^{-6}$. For thresholds of $10^{-5}$ and below, we omit signals corre-


Fig. 7. Words crossing relative frequency threshold of $10^{-3}$ between consecutive decades. Signals for each pair of decades are sorted and weighted by contribution to the Jensen-Shannon divergence (JSD) between those decades. Bars pointing to the right represent words that rose above the threshold between decades. Bars pointing left represent words that fell. In parentheses in each title is the total percent of the JSD between the given pair of decades that is accounted for by flux over the $10^{-3}$ threshold.
sponding to references to specific years, as such references would otherwise overwhelm the charts for these thresholds. We prepare the reader by noting that these final sections are somewhat detailed in nature.

But we also add that any study of texts reduced to 1 -grams should in some fashion "look at the words" themselves, for the very
least as a sanity check on code and more deeply to find the story behind observed summarized dynamics and patterns [18,19].

The set of 1 -grams with relative frequencies above $10^{-3}$ is also fairly stable. From Zipf's law for the 2012 English Fiction corpus (Fig. 4), we know that this threshold is typically exceeded by the 100 most common 1-grams. Viewing language as code, these top 1001 -
grams are fundamental elements in the construction of meaningful statements and comprise around $55 \%$ of all 1 -grams.

We should expect limited turnover for these core 1 -grams, and indeed the flux of 1 -grams across the $10^{-3}$ boundary between consecutive decades is entirely captured by Fig. 7.

We generate these and all subsequent "JSD word shift" figures by:

- Finding all 1-grams that either move above or below a given threshold between two decades;
- Ranking these 1 -grams by their contributions to the JSD measured between the same decades; and
- Plotting downward flux 1 -grams with their contributions as bars to the left, and upward flux 1-grams with their contributions as bars to the right. We leave aside all 1-grams representing years.

In taking a close look at Fig. 7, we see that parentheses drop in relative frequency of use between the 1840s and 1850 s and cross back over the threshold after the American Civil War (between the 1860 s and 1870s). The same is true for before and after World War II (between the 1930s and 1940s and between the 1940s and 1950s, respectively). Beyond these, the flux is entirely due to proper words (not punctuation). For example, "made" fluctuates up and down over this threshold repeatedly over the course of a century. Between the 1870s and the 1880s, "made", which sees slightly increased use, is the only word to cross the threshold. The most crossings is 12 , which occurs between the first two decades. Also, "great" struggled over the first 5 decades and eventually failed to remain great by this measure. "Mr." fluctuated across the threshold between the 1830s and 1910s. More recently, from the 1930s on, "They" has been making its paces up and down across the threshold.

### 4.4. Fine-grained exploration of flux across frequency threshold boundaries: 1970s-1980s and 1980s-1990s

We now choose a few interesting decade-to-decade transitions to delve into for the flux at the lower frequency thresholds of $f_{\text {thr }}=10^{-4}, 10^{-5}$, and $10^{-6}$. Returning to Fig. 5 , we know that for each threshold between $10^{-4}$ and $10^{-7}$, the upward and downward flux roughly cancel. For both upward and downward flux, there appears to be little qualitative difference between the three smallest thresholds of $10^{-4}, 10^{-5}$, and $10^{-6}$. For these thresholds, the downward flux between the 1950s and the 1960s is a minimum then increases over the next two pairs of consecutive decades, and then dips again between the 1980 s and 1990 s. For $f_{\text {thr }}=10^{-4}$, the increase between the 1960s and 1970s and the next pair of decades is more noticeable for the downward flux, as is the decrease between the last two pairs of decades.

Based on these observations, we will examine the flux for two decade pairs in this section: 1970s-1980s and 1980s-1990s. In the following two sections, we will consider the 1930s-1940s transition because of the historical importance of these decades, and then finally the 1960s-1970s transition to show some peculiarities of word flux.

We begin by displaying in Fig. 8 the top 60 flux 1-grams across $f_{\text {thr }}=10^{-4}$ between the 1970 s and the 1980 s, and in Fig. 9, we show all 55 flux words between the 1980s and the 1990s for the same $f_{\text {thr }}$. In these and all subsequent figures, we use the same format as Fig. 7.

Between each pair of decades, we see reduced relative use of particularly British words, including "England" between the first two decades and "King", "George", and "Sir" between the latter two. We also see reduced use of more formal-sounding words, such as "character", "manner", and "general" between the first two decades and "suppose", "indeed", and "hardly" between the latter two. Increasing are physical and emotional words. Those between the first

JSD flux contributions: 1970s to 1980s Relative frequency threshold: $f_{\mathrm{thr}}=10^{-4}$


Fig. 8. The top 1 -grams crossing relative frequency threshold of $10^{-4}$ between the 1970s and 1980s. Words are rank-ordered by their contribution to the JensenShannon divergence (JSD) between those decades. Bars pointing to the right represent 1 -grams that rose above the threshold between decades, and bars pointing left represent 1 -grams that fell. (The first signal is the asterisk "*".) We omit references to years in this and all subsequent figures.
two decades include "stared", "breath", "realized", "shoulder" and "shoulders", "coffee", "guess", "pain", and "sorry." Between the latter two, we see "chest", "skin", "whispered", "hit", "throat", "hurt", "control", and "lives." Also included are "phone" and "parents."

In Figs. 10 and 11, we display the top 60 flux words, not counting references to years, across the $10^{-5}$ threshold between the same pairs of decades. Many of the words declining below the threshold between the 1970s and 1980s are unusual spellings such as "tho", proper names like "Balzac", or words from non-English languages like "une." Increasing across this threshold between the first two decades are a plethora of mostly female proper names, with "Jessica" and "Megan" leading. Also seen are "KGB" and "jeans." ("KGB" decreases in the 1990s, as does "Russians.") Increasing between the 1980s and 1990s are a few proper names; however, most of the signals here are social and sexual in nature, and in part point to the inclusion of academic, literary criticism. These include "lesbian" and "lesbians", "AIDS", and "gender" in the top positions. Also

JSD flux contributions: 1980s to 1990s Relative frequency threshold: $f_{\text {thr }}=10^{-4}$


Fig. 9. Words (not counting references to years) crossing relative frequency threshold of $10^{-4}$ between the 1980s and 1990s. See the caption for Fig. 8 for details. Note that only 55 words make this transition.
included are both "homosexuality" and the more general "sexuality." We also see "girlfriend", "boyfriend", "feminist", and "sexy."

We show in Figs. 12 and 13 the flux across a threshold of $10^{-6}$ between the 1970s and 1980s, and the 1980s and 1990s (again, not counting years). The first of these is not particularly topical, though we do see "AIDS" increase above this threshold a decade prior to its increase over $10^{-5}$ as seen in Fig. 11. For the second pair of decades, we find some surprising signals. In particular, while increases in "HIV" and "bisexual" make the list (similarly to many signals in Fig. 11), as do "fax", "laptop", and "Internet", a great swath of the signals are accounted for by one franchise. We note increases in "Picard", "TNG", "Sisko", and "DS9." These latter signals should serve as a reminder that the word distributions in library-like Google Books corpus [7], even for fiction, do not remotely resemble the contents of normal conversations (at least not for the general population). However, we do observe signals arising at this threshold

JSD flux contributions: 1970s to 1980s Relative frequency threshold: $f_{\mathrm{thr}}=10^{-5}$


Fig. 10. Words (not counting references to years) crossing relative frequency threshold of $10^{-5}$ between the 1970s and 1980s. See the caption for Fig. 8 for details.
from factors external to the imaginings of specific authors. It would therefore be premature to dismiss the contributions at this threshold because of an apparent overabundance of "Star Trek." In fact, because "The Next Generation" and "Deep Space 9" aired precisely during these two decades, an abundance of "Star Trek" novels in the English Fiction data set is actually quite encouraging, because these novels do exist, are available in English, and are (clearly) fiction.

The cultural signals change as we dial down the frequency threshold. We typically find that thresholds of $10^{-4}$ and above produce signals with little to no noise. This is not surprising because this relative frequency roughly corresponds to rank threshold for the 1000 most common words (see Fig. 4) in the data set. Using a threshold of $10^{-5}$ (fewer than 10,000 words fall above this frequency in any given decade), we see some noise (mostly in the form of familiar names), but still observe many valuable signals. Only when the threshold is reduced to $10^{-6}$ does the overall texture of the signals become questionable as a result of a variety of

JSD flux contributions: 1980s to 1990s Relative frequency threshold: $f_{\text {thr }}=10^{-5}$


Fig. 11. Words (not counting references to years) crossing relative frequency threshold of $10^{-5}$ between the 1980 s and 1990s. See the caption for Fig. 8 for details.
proper nouns far less familiar than those observed with the previous threshold. However, at this threshold, we nevertheless observe several early signals of real social importance.

### 4.5. Fine-grained exploration of flux across frequency threshold boundaries: 1930s-1940s

Curiously, between the 1930s and 1940s the volume of flux across each threshold is not atypical (see Fig. 5). Moreover, the asymmetry between the JSD contributions between those decades is very low. Yet it is obvious that we should expect signals of historical significance between these two decades, and indeed we do once we examine the dynamics of individual 1-grams. In Figs. 14 and 15 , we see words crossing the $10^{-4}$ and $10^{-5}$ thresholds, respectively (with references to years omitted in Fig. 15). For the higher threshold, only 56 words cross. The most noticeable such words that are more commonly used in the 1940s are "General" and "German." Also, "killed" appears in this list. Words used less frequently include

JSD flux contributions: 1970s to 1980s Relative frequency threshold: $f_{\text {thr }}=10^{-6}$


Fig. 12. Words (not counting references to years) crossing relative frequency threshold of $10^{-6}$ between the 1970s and 1980s. See the caption for Fig. 8 for details.
"pleasure", "garden", and "spirit." For the lower threshold, we see the signals from prolific authors as in our previous paper [7], particularly Upton Sinclair's character, Lanny Budd. We also see more Nazis ("Nazi" and "Nazis").

### 4.6. Fine-grained exploration of flux across frequency threshold boundaries: 1960s-1970s

Last, we include one of the more colorful examples. In Fig. 16, we show signals (not including years) for words crossing the $10^{-5}$ threshold between the 1960s and 1970s. Profanity dominates. We see references to The World According to Garp ("Garp") and, again, to "Star Trek" ("Kirk" this time). We also see more "computer", "TV", and, per The Graduate, "plastic." Signals also appear for "blacks" and "homosexual", for narcotics ("drug" and "drugs"), and a changing role for police ("enforcement" and "cop").

JSD flux contributions: 1980s to 1990s Relative frequency threshold: $f_{\text {thr }}=10^{-6}$


Fig. 13. Words (not counting references to years) crossing relative frequency threshold of $10^{-6}$ between the 1980s and 1990s. See the caption for Fig. 8 for details.

We refer the reader to our paper's Online Appendices [9] for figures representing flux across relative frequency thresholds of $10^{-4}, 10^{-5}$, and $10^{-6}$ between consecutive decades over the entire period analyzed (the 1820s to the 1990s).

## 5. Concluding remarks

In seeking to characterize word birth and death for the 2012 Google Books English Fiction corpus, we have identified and characterized what we believe is a fundamental feature of language evolution: lexical turbulence. In general, for any time-coded corpus, we quantify lexical turbulence as the flux $\phi$ of words across a relative frequency of usage threshold between two time periods. We speak of undirected flux $\phi$ because we found that upward and downward flux $\phi_{\text {up }}$ and $\phi_{\text {down }}$ across a threshold were on average well balance, though this may not always be the case.

JSD flux contributions: 1930s to 1940s Relative frequency threshold: $f_{\text {thr }}=10^{-4}$


Fig. 14. Words (not counting references to years) crossing relative frequency threshold of $10^{-4}$ between the 1930 s and 1940s. See the caption for Fig. 8 for details. Note that only 56 words make this transition.

Like the Jensen-Shannon divergence and related measures, lexical turbulence is one way of characterizing the degree of word rank (or relative frequency) variability underlying Zipf's law. The overall form of Zipf's law may be strongly preserved across corpora suggesting stability (Fig. 4) but completely occlude how individual word usage rates are changing (Fig. 5).

Word flux may also be naturally measured across a fixed word rank, with the connection to relative frequency being made through Zipf's law (Fig. 5D). The scaling of word flux with rank is superlinear with a break in scaling tied to that of Zipf's law [12].

We conjecture that word rank may be viewed as roughly analogous to a kind of temperature where the most common words are nearly frozen in usage rates while rarer and rarer words increasingly boil and bubble in their relative frequencies. One metaphor for words sometimes invoked is that of tools [4]. Words form a hierarchy of tools with a crystallized set of the most frequently used

JSD flux contributions: 1930s to 1940s Relative frequency threshold: $f_{\text {thr }}=10^{-5}$


Fig. 15. Words (not counting references to years) crossing relative frequency threshold of $10^{-5}$ between the 1930s and 1950s. See the caption for Fig. 8 for details.

JSD flux contributions: 1960 s to 1970 s
Relative frequency threshold: $f_{\mathrm{thr}}=10^{-5}$


Fig. 16. Words (not counting references to years) crossing relative frequency threshold of $10^{-5}$ between the 1960 s and 1970s. See the caption for Fig. 8 for details.
instruments (comma, period, "the") resting above a vast tool set of increasingly specific uses.

We arrived at the notion of lexical turbulence via our efforts to reproduce the results of [1]. We found general agreement regarding a decay in word birth from 1800 to 2000 but not so for word death. True word death appears to be extremely and durably rare. Overall, the lowering birth rate signals a cooling of language [2] but the time-independent scaling of lexical turbulence shows that the lexicon is constantly turning over.

Using JSD word shifts, we also explored in detail the words dominating the flux across some example frequency thresholds for a number of interesting decade-decades transitions. While extremely specific fiction can be of great interest-whether it be in the form of war novels or volumes from the "Star Trek" franchise-vocabulary from these works is more easily studied when placed in proper context. Dialing down the relative frequency threshold across several orders of magnitude helps to capture this
distinction. However, further experimentation is called for, because an automatic means of separating specific signals from the more general signals (e.g., "Star Trek" from social movements) could afford both a more intuitive grasp of the lexical dynamics and might, ideally, allow investigators to hypothesize causal relationships between exogenous and endogenous drivers of language.

Of many potential directions for future work, several that stand out would be (1) Reproducing the present analysis of lexical turbulence for 2 -grams and 3 -grams which, $n$-grams that are particularly rich in meaning; (2) Quantifying the behavior of lexical turbulence with time (e.g., beyond adjacent decade comparisons as we have done here); (3) Creating toy models of language evolution to attempt to capture lexical turbulence; and (4) Building interactive JSD-based word shifts where corpora, frequency thresholds and year range may be selected to facilitate rapid explorations.

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