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[horton: rivers]

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commonly interbedded with the lavas of steep semiarid Hawaiian volcanoes, especially with those of the dying andesitic third phase of volcanism (Stearns, 1940, p. 1947–1948). They perch water in the overlying lava at the Waihu Springs. At the lower Waihu Springs the same bed of hillwash is underlain by the Waihu fanglomerate. Thus none of the so-called glacial springs on Mauna Kea described by Wentworth and Powers (1943) appear to be perched by drift.

SUMMARY AND CONCLUSIONS

During Wisconsin time Mauna Kea was covered with an icecap several hundred feet thick that extended to an altitude of 10,500 feet and left a distinct terminal moraine and small outwash fans. The Waihu fanglomerate may have been deposited by floods caused by rapid melting of the icecap by lava flows in early Wisconsin time. If not, it is outwash of a pre-Makanaka glacier. All older so-called drift deposits are definitely explosion deposits, possibly hydromagmatic, due to hot lava breaking through the icecap, but they are not indicative of early glaciations. All so-called drift deposits and erratics, supposedly indicating that early icecaps extended as low as 6900 feet on Mauna Kea, are believed to be conglomerates or lag boulders from eroded stream-laid deposits. The absence of deposits of early Pleistocene glaciers is best explained by the hypothesis that the volcano had not at that time reached a height sufficient to nourish glaciers in a tropical climate. An alternate hypothesis is that the deposits lie deeply buried by subsequent lavas.

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EROSIONAL DEVELOPMENT OF STREAMS AND THEIR DRAINAGE BASINS; HYDROPHYSICAL APPROACH TO QUANTITATIVE MORPHOLOGY

BY ROBERT E. HORTON

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ABSTRACT

The composition of the stream system of a drainage basin can be expressed quantitatively in terms of stream order, drainage density, bifurcation ratio, and stream-length ratio.

Stream orders are so chosen that the fingertip or unbranched tributaries are of the 1st order; streams which receive 1st order tributaries, but these only, are of the 2d order; third order streams receive 2d or 1st and 2d order tributaries, and so on, until, finally, the main stream is of the highest order and characterizes the order of the drainage basin.

Two fundamental laws connect the numbers and lengths of streams of different orders in a drainage basin:

(1) The law of stream numbers. This expresses the relation between the number of streams of a given order and the stream order in terms of an inverse geometric series, of which the bifurcation ratio r_b is the base.

(2) The law of stream lengths expresses the average length of streams of a given order in terms of stream order, average length of streams of the 1st order, and the stream-length ratio. This law takes the form of a direct geometric series. These two laws extend Playfair's law and give it a quantitative meaning.

The infiltration theory of surface runoff is based on two fundamental concepts:

(1) There is a maximum or limiting rate at which the soil, when in a given condition, can absorb rain as it falls. This is the infiltration-capacity. It is a volume per unit of time.

(2) When runoff takes place from any soil surface, there is a definite functional relation between the depth of surface detention δ_a, or the quantity of water accumulated on the soil surface, and the rate q_s of surface runoff or channel inflow.

For a given terrain there is a minimum length x_c of overland flow required to produce sufficient runoff volume to initiate erosion. The critical length x_c depends on surface slope, runoff intensity, infiltration-capacity, and resistivity of the soil to erosion. This is the most important single factor involved in erosion phenomena and, in particular, in connection with the development of stream systems and their drainage basins by aqueous erosion.

The erosive force and the rate at which erosion can take place at a distance x from the watershed line is directly proportional to the runoff intensity, in inches per hour, the distance x, a function of the slope angle, and a proportionality factor K_e, which represents the quantity of material which can be torn loose and eroded per unit of time and surface area, with unit runoff intensity, slope, and terrain.

The rate of erosion is the quantity of material actually removed from the soil surface per unit of time and area, and this may be governed by either the transporting power of overland flow or the actual rate of erosion, whichever is smaller. If the quantity of material torn loose and carried in suspension in overland flow exceeds the quantity which can be transported, deposition or sedimentation on the soil surface will take place.

On newly exposed terrain, resulting, for example, from the recession of a coast line, sheet erosion occurs first where the distance from the watershed line to the coast line first exceeds the critical length x_c, and sheet erosion spreads laterally as the width of the exposed terrain increases. Erosion of such a newly exposed plane surface initially develops a series of shallow, close-spaced, shoestring gullies or rill channels. The rills flow parallel with or are consequent on the original slope. As a result of various causes, the divides between adjacent rill channels are broken down locally, and the flow in the shallower rill channels more remote from the initial rill is diverted into deeper rills more closely

adjacent thereto, and a new system of rill channels is developed having a direction of flow at an angle to the initial rill channels and producing a resultant slope toward the initial rill. This is called cross-grading.

With progressive exposure of new terrain, streams develop first at points where the length of overland flow first exceeds the critical length x_c, and streams starting at these points generally become the primary or highest-order streams of the ultimate drainage basins. The development of a rilled surface on each side of the main stream, followed by cross-grading, creates lateral slopes toward the main stream, and on these slopes tributary streams develop, usually one on either side, at points where the length of overland flow in the new resultant slope direction first exceeds the critical length x_c.

Cross-grading and recross-grading of a given portion of the area will continue, accompanied in each case by the development of a new order of tributary streams, until finally the length of overland flow within the remaining areas is everywhere less than the critical length x_c. These processes fully account for the geometric-series laws of stream numbers and stream lengths.

A belt of no erosion exists around the margin of each drainage basin and interior subarea while the development of the stream system is in progress, and this belt of no erosion finally covers the entire area when the stream development becomes complete.

The development of interior divides between subordinate streams takes place as the result of competitive erosion, and such divides, as well as the exterior divide surrounding the drainage basin, are generally sinuous in plan and profile as a result of competitive erosion on the two sides of the divide, with the general result that isolated hills commonly occur along divides, particularly on cross divides, at their junctions with longitudinal divides. These interfluve hills are not uneroded areas, as their summits had been subjected to more or less repeated cross-grading previous to the development of the divide on which they are located.

With increased exposure of terrain weaker streams may be absorbed by the stronger, larger streams by competitive erosion, and the drainage basin grows in width at the same time that it increases in length. There is, however, always a triangular area of direct drainage to the coast line intermediate between any two major streams, with the result that the final form of a drainage basin is usually ovoid or pear-shaped.

The drainage basins of the first-order tributaries are the last developed on a given area, and such streams often have steep-sided, V-shaped, incised channels adjoined by belts of no erosion.

The end point of stream development occurs when the tributary subareas have been so completely subdivided by successive orders of stream development that there nowhere remains a length of overland flow exceeding the critical length x_c. Stream channels may, however, continue to develop to some extent through headward erosion, but stream channels do not, in general, extend to the watershed line.

Valley and stream development occur together and are closely related. At a given cross section the valley cannot grade below the stream, and the valley supplies the runoff and sediment which together determine the valley and stream profiles. As a result of cross-grading antecedent to the development of new tributaries, the tributaries and their valleys are concordant with the parent stream and valley at the time the new streams are formed and remain concordant thereafter.

Valley cross sections, when grading is complete, and except for first-order tributaries, are generally S-shaped on each side of the stream, with a point of contraflexure on the upper portion of the slope, and downslope from this point the final form is determined by a combination of factors, including erosion rate, transporting power, and the relative frequencies of occurrence of storms and runoff of different intensities. The longitudinal profile of a valley along the stream bank and the cross section of the valley are closely related, and both are related to the resultant slope at a given location.

Many areas on which meager stream development has taken place, and which are commonly classified as youthful, are really mature, because the end point of stream development and erosion for existing conditions has already been reached.

When the end point of stream and valley gradation has arrived in a given drainage basin, the remaining surface is usually concave upward, more or less remembling a segment of a parabaloid, ribbed by cross and longitudinal divides and containing interfluve hills and plateaus. This is called a "graded" surface, and it is suggested that the term "peneplain" is not appropriate, since this surface is neither a plane nor nearly a plane, nor does it approach a plane as an ultimate limiting form.

The hydrophysical concepts applied to stream and valley development account for observed phenomena from the time of exposure of the terrain. Details of these phenomena of stream and valley development on a given area may be modified by geologic structures and subsequent geologic changes, as well as local variations of infiltration-capacity and resistance to erosion.

In this paper stream development and drainage-basin topography are considered wholly from the viewpoint of the operation of hydrophysical processes. In connection with the Davis erosion cycle the same subject is treated largely with reference to the effects of antecedent geologic conditions and subsequent geologic changes. The two views bear much the same relation as two pictures of the same object taken in different lights, and one supplements the other. The Davis erosion cycle is, in effect, usually assumed to begin after the development of at least a partial stream system; the hydrophysical concept carries stream development back to the original newly exposed surface.

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The author is indebted to Dr. Howard A. Meyerhoff for many helpful suggestions and criticisms. Grateful acknowledgment is also given Dr. Alfred C. Lane, who, more than 40 years ago, gave the author both the incentive and an opportunity to begin the study of drainage basins with respect to possible interrelations of their hydraulic, hydrologic, hydrophysical, and geologic features.

LIST OF SYMBOLS USED

A = area of drainage basin in square miles.
α = slope angle.
c = distance from stream tip to watershed line.
δ = depth of sheet flow in inches at the stream margin or at the foot of a slope length l_o.
δ_a = average depth of surface detention or overland flow, in inches, on a unit strip of length l_o.
δ_x = depth of sheet flow in inches at a distance x from the crest of the slope or watershed line.
D_d = drainage density or average length of streams per unit of area.
e = energy expended by frictional resistance on soil surface, ft.-lbs. per sq. ft. per sec.
E_a = average erosion over a given strip of unit width and length l_o per unit of time.
e_r = erosion rate or quantity of material, preferably expressed in terms of depth of solid material, removed per hour by sheet erosion.
E_t = total erosion = total solid material removed from a given strip of unit length per unit of time.
f = infiltration-capacity at a given time t from the beginning of rain, inches per hour.
f_c = minimum infiltration-capacity for a given terrain.
f_0 = initial infiltration-capacity at beginning of rain.
F_1 = erosive force of overland flow, lbs. per sq. ft.
F_0 = tractive force of overland flow, lbs. per sq. ft. of surface.
F_s = stream frequency or number of streams per unit area.
i = rain intensity—usually inches per hour.
I = index of turbulence or percentage of the area covered by sheet flow on which the flow is turbulent.
K_a = coefficient in the runoff equation where δ_a is used instead of δ as the depth of sheet flow.
k_e = proportionality factor required to convert the rate of performance of work in sheet erosion into equivalent quantity of material removed per unit of time.
K_f = a proportionality factor which determines the time t_c required for infiltration-capacity to be reduced from its initial value f_0 to its constant value f_c.
K_l = corresponding coefficient (to K_s) in the equation for laminar overland flow.
K_s = constant or proportionality factor in equation expressing runoff intensity in terms of depth δ of overland flow.
l_a = average length of streams of order o.
l_g = maximum length of overland flow on a given area.
l_o = length of overland flow or length of flow over the ground surface before the runoff becomes concentrated in definite stream channels.
l_1, l_2 etc. = average lengths of streams of 1st and 2d orders, etc.
L' = extended stream length measured along stream from outlet and extended to watershed line.
L_0 = total length of tributaries of order o.
M = exponent in the equation: $q_s = K_s\delta^M$, expressing the runoff intensity in terms of depth of sheet flow along the stream margin.
n = surface roughness factor, as in the Manning formula.
N_o = number of streams of a given order in a drainage basin.
N_s = total number of streams in a drainage basin.
N_1, N_2 etc. = total number of streams of 1st, 2d orders, etc.

o = order of a given stream.
q_s = surface-runoff intensity—usually inches per hour.
q_1 = runoff intensity in cubic feet per second from a unit strip 1 foot wide and with a slope length l_o.
ρ = stream length ratio/bifurcation ratio = r_l/r_b.
r_b = bifurcation ratio or ratio of the average number of branchings or bifurcations of streams of a given order to that of streams of the next lower order. It is usually constant for all orders of streams in a given basin.
r_l = stream length ratio or ratio of average length of streams of a given order to that of streams of the next lower order.
r_l' = stream length ratio using extended stream lengths.
r_s = ratio of channel slope to ground slope for a given stream or in a given drainage basin, = s_c/s_g.
R_i = initial surface resistance to sheet erosion, lbs. per sq. ft.
R_1 = subsurface resistance to sheet erosion, lbs. per sq. ft., or resistance of a lower surface or horizon of the soil to erosion after the surface layer of resistance R_i is removed.
σ = supply rate = $i - f$.
s = order of main stream in a given drainage basin.
s_c = channel slope.
s_g = resultant slope of ground surface of area tributary to a given parent stream.
S = surface slope = fall/horizontal length.
t_e = duration of rainfall excess or time during which rain intensity exceeds infiltration-capacity.
v = mean velocity of overland flow, feet per second.
v_x = mean velocity of overland flow at the distance x from the watershed line.
V_d = depth of depression storage on a given area, inches.
w_1 = weight of runoff, including solids in suspension, lbs. per cu. ft.
w_b = width of marginal belt of no erosion = $x_c \sin A$, where A is the angle between the direction of the divide and the direction of overland flow at the divide.
x_c = critical length of overland flow or distance from the watershed line, measured in the direction of overland flow, within which sheet erosion does not occur.

The customary procedure of expressing rainfall, infiltration, and runoff in inches depth per hour, velocities, channel lengths, and discharge rates in foot-second units, and drainage areas in square miles, has been followed, with appropriate interconversion factors in the formulas.

PLAYFAIR'S LAW

More than a century ago Playfair (*in* Tarr and Martin, 1914, p. 177) stated:

> "Every river appears to consist of a main trunk, fed from a variety of branches, each running in a valley proportioned to its size, and all of them together forming a system of vallies, communicating with one another, and having such a nice adjustment of their declivities that none of them join the principal valley either on too high or too low a level."

This has often been interpreted as meaning merely that tributary streams and their valleys enter the main streams and their valleys concordantly. A careful reading of Playfair's law implies that he envisaged a great deal more than merely the concordance of stream and valley junctions. He speaks of the "nice adjustment" of the entire system of valleys and states that each branch runs in a valley "proportioned to its size."

Playfair did pioneer work based on ocular observations. There were available to him neither the results of measurements nor the hydrophysical laws,necessary to their quantitative interpretation. It appears that the time has now come when such a quantitative interpretation can be undertaken.

QUANTITATIVE PHYSIOGRAPHIC FACTORS

GENERAL CONSIDERATIONS

In spite of the general renaissance of science in the present century, physiography as related in particular to the development of land forms by erosional and gradational processes still remains largely qualitative. Stream basins and their drainage basins are described as "youthful," "mature," "old," "poorly drained," or "well drained," without specific information as to how, how much, or why. This is probably the result largely of lack of adequate tools with which to work, and these tools must be of two kinds: measuring tools and operating tools.

One purpose of this paper is to describe two sets of tools which permit an attack on the problems of the development of land forms, particularly drainage basins and their stream nets, along quantitative lines.

An effort will be made to show how the problem of erosional morphology may be approached quantitatively, and even in this respect only the effects of surface runoff will be considered in detail. Drainage-basin development by ground-water erosion, highly important as it is, will not be considered, and the discussion of drainage development by surface runoff will mainly be confined to processes occurring outside of stream channels. The equally important phase of the subject, channel development—including such problems as those of the growth of channel dimensions with increase of size of drainage basin, stream profiles, and stream bends—will not be considered in detail.

STREAM ORDERS

In continental Europe attempts have been made to classify stream systems on the basis of branching or bifurcation. In this system of stream orders, the largest, most branched, main or stem stream is usually designated as of order 1 and smaller tributary streams of increasingly higher orders (Gravelius, 1914). The smallest unbranched fingertip tributaries are given the highest order, and, although these streams are similar in characteristics in different drainage basins, they are designated as of different orders.

Feeling that the main or stem stream should be of the highest order, and that unbranched fingertip tributaries should always be designated by the same ordinal, the author has used a system of stream orders which is the inverse of the European system. In this system, unbranched fingertip tributaries are always designated as of order 1, tributaries or streams of the 2d order receive branches or tributaries of the 1st order, but these only; a 3d order stream must receive one or more tributaries of the 2d order but may also receive 1st order tributaries. A 4th order stream receives branches of the 3d and usually also of lower orders, and so on. Using this system the order of the main stream is the highest.

To determine which is the parent and which the tributary stream upstream from the last bifurcation, the following rules may be used:

(1) Starting below the junction, extend the parent stream upstream from the bifurcation in the same direction. The stream joining the parent stream at

the greatest angle is of the lower order. Exceptions may occur where geologic controls have affected the stream courses.

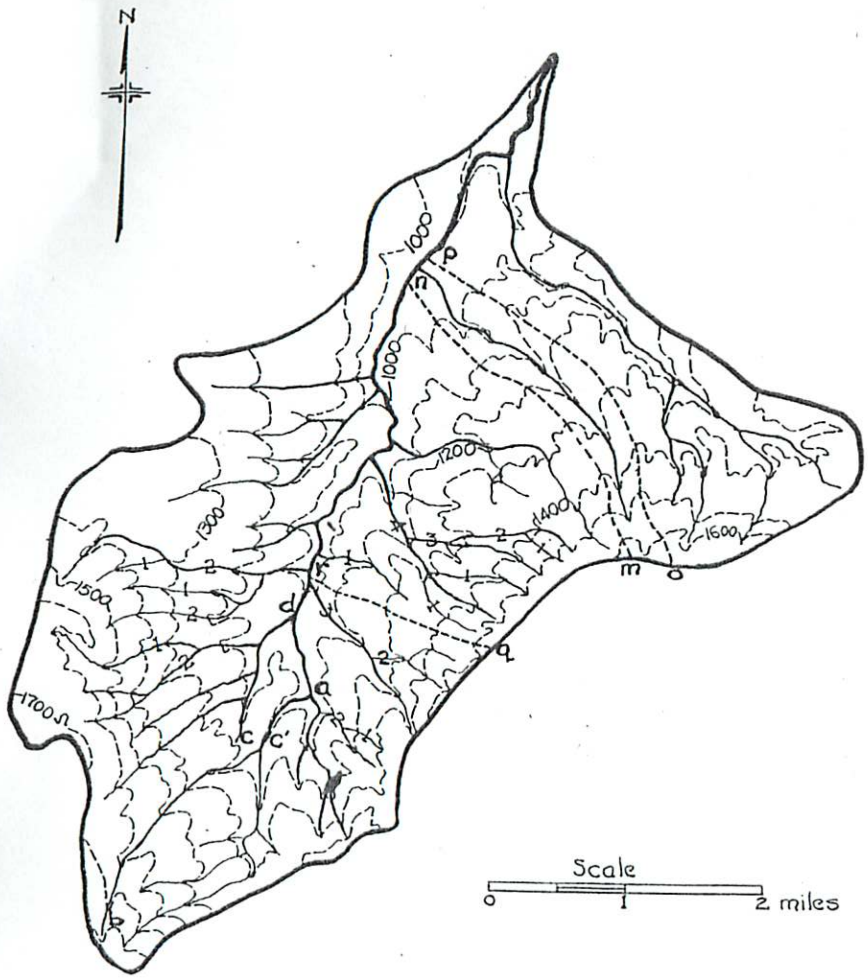


FIGURE 1.—*Well-drained basin*
(Cherry Creek, N. Y., quad., U. S. G. S.)

(2) If both streams are at about the same angle to the parent stream at the junc-
on, the shorter is usually taken as of the lower order.

On Figure 1 several streams are numbered 1, and these are 1st order tributaries.
treams numbered 2 are of the 2nd order throughout their length, both below and
bove the junctions of their 1st order tributaries. The main stream is apparently
'b although it joins *ad* at nearly a right angle. It is probable that the original
urse of the stream was *dcb*, but the portion above *dc* was diverted by headwater
osion into stream *ac'*. The well-drained basin (Fig. 1) is of the 5th order, while the
orly drained basin (Fig. 2) is of the 2d order. Stream order therefore affords a

simple quantitative basis for comparison of the degree of development in the drainage nets of basins of comparable size. Its usefulness as a basis for such comparisons is limited by the fact that, other things equal, the order of a drainage basin or its stream system generally increases with size of the drainage area.

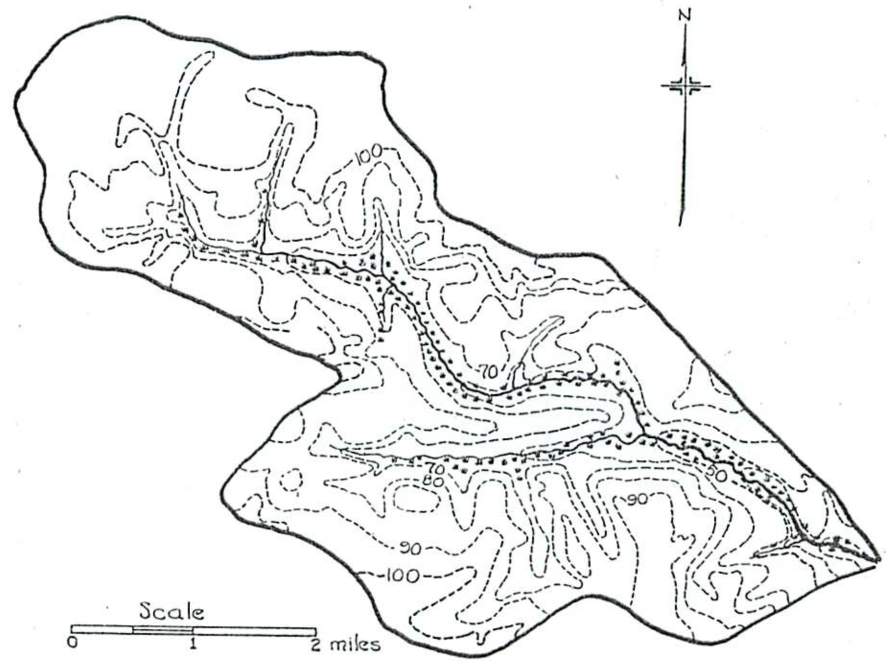


FIGURE 2.—*Flat sandy area, poorly drained*
(Bridgeton, N. J., quad. U. S. G. S.)

DRAINAGE DENSITY

Figures 1 and 2 show two small drainage basins, both on the same scale; one well drained, the other poorly drained. These terms, well drained and poorly drained, while in common use in textbooks on physiography, are purely qualitative, and something better is needed to characterize the degree of drainage development within a basin. The simplest and most convenient tool for this purpose is drainage density or average length of streams within the basin per unit of area (Horton, 1932). Expressed as an equation

$$\text{Drainage density, } D_d = \frac{\Sigma L}{A} \tag{1}$$

where ΣL is the total length of streams and A is the area, both in units of the same system. The poorly drained basin has a drainage density ~~2.74~~ 0.73, the well-drained one, ~~0.73~~ 2.74, or one fourth as great.

For accuracy, drainage density must, if measured directly from maps, be determined from maps on a sufficiently large scale to show all permanent natural stream channels, as do the U. S. Geological Survey topographic maps. On these maps perennial streams are usually shown by solid blue lines, intermittent streams by dotted blue lines. Both should be included. If only perennial streams were included, a drainage basin containing only intermittent streams would, in accordance with equation (1), have zero drainage density, although it may have a considerable degree of basin development. Most of the work of valley and stream development by running water is performed during floods. Intermittent and ephemeral streams carry flood waters, hence should be included in determining drainage density. Most streams which are perennial in their lower reaches or throughout most of their courses have an intermittent or ephemeral reach or both, near their headwaters, where the stream channel has not cut down to the water table. These reaches should also be included in drainage-density determinations.

In textbooks on physiography, differences of drainage density are commonly attributed to differences of rainfall or relief, and these differences in drainage density are largely used to characterize physiographic age in the sense used by Davis (Davis, 1909; Wooldridge and Morgan, 1937). In the poorly drained area (Fig. 2) the mean annual rainfall is about 30 per cent greater than in the well-drained area (Fig. 1). Therefore some other factor or factors are far more important than either rainfall or relief in determining drainage density. These other factors are infiltration-capacity of the soil or terrain and initial resistivity of the terrain to erosion.

LENGTH OF OVERLAND FLOW

The term "length of overland flow," designated l_o, is used to describe the length of flow of water over the ground before it becomes concentrated in definite stream channels. To a large degree length of overland flow is synonomous with length of sheet flow as quite commonly used. The distinction between overland flow and channel flow is not so vague or uncertain as might at first appear. Overland flow is sustained by a relatively thin layer of surface detention. This disappears quickly—often in a few minutes—through absorption by the soil or infiltration after rain ends. Surface detention and surface runoff may, in fact, end before rain ends if, as is often the case, there is at the end of the storm an interval of residual rainfall having an intensity less than the infiltration-capacity. Channel flow is sustained by accumulated channel storage. This drains out slowly and lasts for hours or even days after channel inflow from surface runoff ends.

In addition to its obvious value in various ways in characterizing the degree of development of a drainage net within a basin, drainage density is particularly useful because of the fact that the average length of overland flow l_o is in most cases approximately half the average distance between the stream channels and hence is approximately equal to half the reciprocal of the drainage density, or

$$l_o = \frac{1}{2D_d} \tag{2}$$

Later it will be shown that length of overland flow is one of the most important independent variables affecting both the hydrologic and physiographic development of drainage basins.

In this paper it is frequently assumed for purposes of convenience that the average length of overland flow is sensibly equal to the reciprocal of twice the drainage density. From considerations of the geometry of streams and their drainage areas the author has shown (Horton, 1932) that the average length of overland flow is given by the equation

$$l_o = \frac{1}{2D_d\sqrt{1 - \left(\frac{s_c}{s_g}\right)^2}} \tag{3}$$

where s_c is the channel or stream slope and s_g the average ground slope in the area.

Values of the correction factor or of the ratio $l_o/2D_d$ for different values of the slope ratio s_c/s_g are as follows:

s_c/s_g =	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
$l_o/2D_d$ =	1.86	1.67	1.40	1.25	1.15	1.09	1.05	1.02	1.005

The ground slope or resultant slope of the area tributary to a stream on either side is necessarily always greater than the channel slope since the ground surface has a component of slope parallel with and of the same order of magnitude as that of the stream, and in addition it has a component of slope at right angles to the stream.

Table 4 shows the average channel and ground slopes of streams in the Delaware River and some other drainage basins, derived from topographic maps. The channel slope in these instances is commonly from half to one fourth the ground slope. Often on an area which as a whole is nearly horizontal, but the surface of which is interspersed with hills, the ground slope may be and frequently is two or three times the channel slope. If the channel slope is less than one third the ground slope, the error resulting from the assumption that average length of overland flow is equal to the reciprocal of twice the drainage density may in general be neglected.

STREAM FREQUENCY

This is the number of streams, F_s, per unit of area, or

$$F_s = \frac{N}{A} \tag{4}$$

where N = total number of streams in a drainage basin of A areal units.

Values of drainage density and stream frequency for small and large drainage basins are not directly comparable because they usually vary with the size of the drainage area. A large basin may contain as many small or fingertip tributaries per unit of area as a small drainage basin, and in addition it usually contains a larger stream or streams. This effect may be masked by the increase of drainage density and stream frequency on the steeper slopes generally appurtenant to smaller drainage basins.

COMPOSITION OF DRAINAGE NET

The term "drainage pattern" is used in rather a restricted sense in many books on physiography, implying little more than the manner of distribution of a given set of tributary streams within the drainage basin. Thus, for example, with identically the same lengths and numbers of streams, the drainage pattern may be dendritic, rectangular, or radial. Neither the drainage pattern nor the drainage density, nor both, provide an adequate characterization of the stream system or drainage net in a given basin. There may be various combinations of stream numbers, lengths, and orders which will give the same drainage density, or there may be similar forms of drainage pattern with widely different drainage and stream densities. Something more is needed as a basis for quantitative morphology of drainage basins. The author has therefore coined the expression "composition of a drainage net," as distinguished from "drainage pattern." Composition implies the numbers and lengths of streams and tributaries of different sizes or orders, regardless of their pattern. Composition has a high degree of hydrologic significance, whereas pattern alone has but little hydrologic significance, although it is highly significant in relation to geologic control of drainage systems.

Cotton (1935) and others have used the term "texture" to express composition of a drainage net as related both to drainage density and stream frequency. For quantitative purposes two terms are needed, since two drainage nets with the same drainage densities may have quite different numbers and lengths of streams. Numerical values of drainage density independent of other units are needed for various purposes.

LAWS OF DRAINAGE COMPOSITION

The numbers and lengths of tributaries of different orders were determined for the streams listed in Table 1. The numbers and the lengths of streams varied with the stream order in a manner which suggested a geometrical progression. Plotting the data on semilogarithmic paper it was found (Fig. 3) that the stream numbers fall close to straight lines, and (Fig. 4) the same is true of the stream lengths. From the manner of plotting, these lines are necessarily graphs of geometrical series, inverse for stream numbers of different orders and direct for stream lengths.[1]

From the properties of geometric series, the equation of the lines giving the number N_o of streams of a given order in a drainage basin can be written

$$N_o = r_b^{(s-o)}. \tag{5}$$

From the laws governing geometric series it is easily shown that the number N of streams of all orders is

$$N = \frac{r_b^s - 1}{r_b - 1}. \tag{6}$$

By definition, o is the order of a given class of tributaries, s is the order of the main stream, and r_b is the bifurcation ratio.

The equation correlating the lengths of streams of different orders is, similarly,

$$l_o = l_1 r_l^{o-1}. \tag{7}$$

[1] In the figure the lines of best fit were drawn by inspection. Somewhat more accurate lines could of course be obtained by the method of least squares or the correlation method. However, the agreement of the observed points with the lines located by inspection is so close in most cases that little would be gained in accuracy by the use of these methods.

TABLE 1.—*Characteristics of the drainage nets of certain stream basins*

Stream	Location	Type	Order of Main Stream o	Area Sq. Mi. A	No. of Streams ΣN_o	No. of 1st order streams N_1	Stream Frequency F_s	Drainage Density D_d	Aver. lgth 1st order streams L_1	Bifurcation ratio r_b	Length ratio r_l	ΣL
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Esopus Creek	Olive Bridge, N.Y.	Mountains	5	234	126	90	0.527	0.849	0.99	3.12	2.31	203.2
" "	Lower Area *	Rolling and Plains	7	426.3	361	256	.847	.818	.81	2.27	1.84	348.6
Rondout	Honk Falls, N.Y.	Mountains	4	105.	58	44	.552	1.07	1.08	3.30	2.64	112.6
Putnam Brook	Weedsport, N.Y.	Glacial, Drumlin	4	27	26	18	.963	1.95	.77	2.46	2.74	52.7
Cold Spring Brook	"	"	4	15.8	25	15	1.58	2.025	.58	2.62	2.66	32.
Crane Creek	"	"	5	45.7	48	31	1.05	2.03	.81	2.22	2.30	92.6
Ganargua Creek	Lyons, N.Y.	"	6	299.	269	166	.899	1.628	.87	2.89	2.30	487.1
Keuka Lake	Foot of Lake, N.Y.	Hilly, Dissected	5	161.†	170	124	1.055	1.665	1.16	3.25	1.96	268.
Seneca	" "	"	6	479.†	472	334	.984	1.59	.95	3.15	2.20	762.5
Owasco	Weedsport, "	"	5	200.	265	191	1.325	1.79	.83	3.91	2.22	358.
Thunder Bay River	Alpena, Mich. **	Glacial ~ Flat	4	—	44	33	—	—	—	3.00	—	—

* Ashokan Dam to Saugerties, N.Y. ** Data furnished by Prof. C.O. Wisler. † Land area, excluding lake.

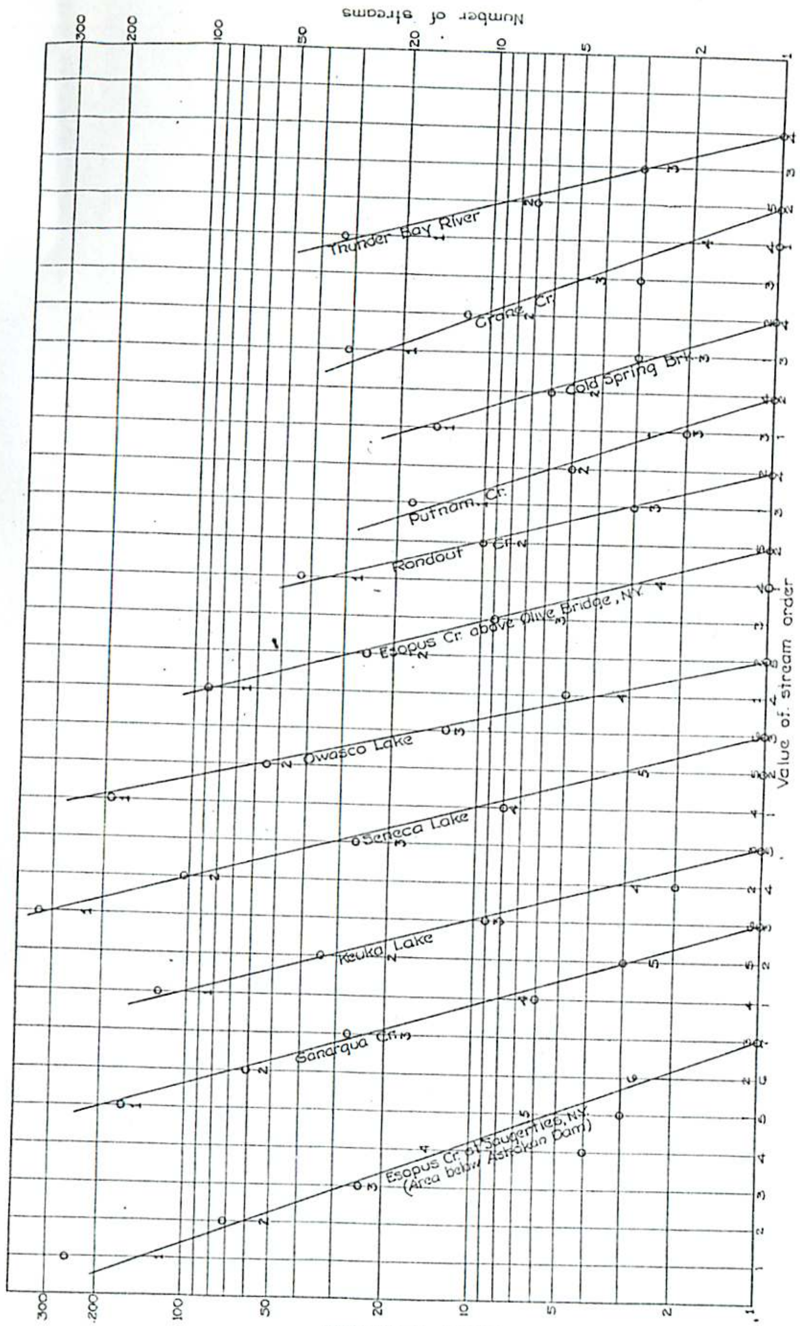


FIGURE 3.—*Bifurcation or relation of stream order to number of streams in different drainage basins*

These equations may appear formidable, but they are merely the statement in symbolic form of the simple algebraic laws of geometric series. Equations (5) and (7) are the most important and are readily solved by means of logarithms.

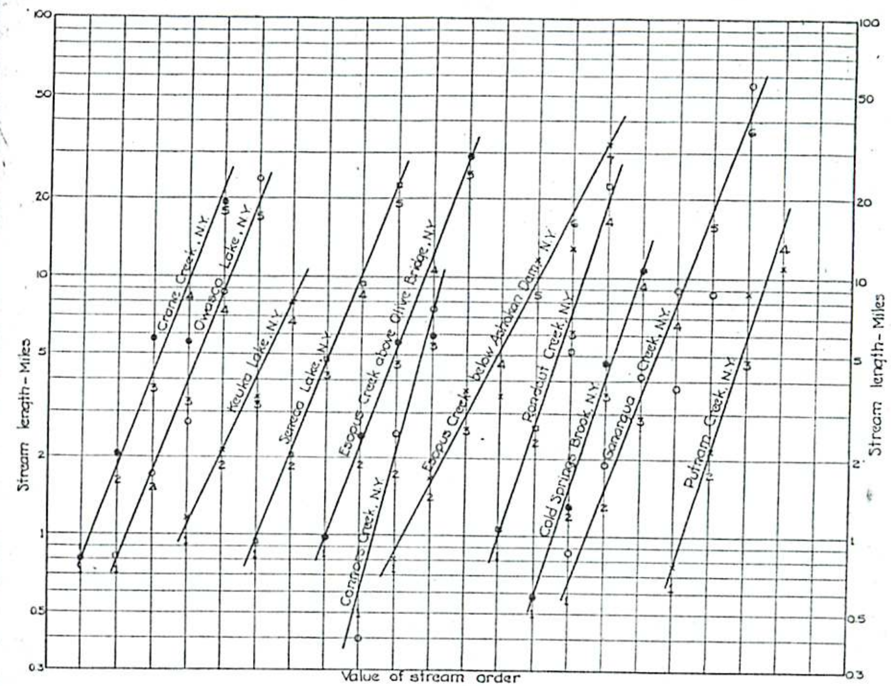


FIGURE 4.—*Relation of stream lengths to stream order in different drainage basins*

As an example, Table 2 shows the observed and computed numbers and lengths of streams of different orders, based on the following values of the variables:

$$r_b = 3.12$$
$$s = 5$$
$$r_l = 2.31$$
$$l_1 = 0.994$$

Actual stream numbers must of course be integers, while the computed numbers may be fractions. Some variation between the computed and observed stream numbers and lengths must be expected, for various reasons. Several drainage basins listed in Table 1 contain large lakes, and the drainage density is less than it would be if the lake did not exist, since there would then necessarily be a stream of the highest order traversing the lake bed. Lower Rondout Creek represents an incomplete drainage basin to which the geometric-series laws do not necessarily apply because it contains a stream or portions of streams of higher order than those originating within this particular area. For some other areas the data were derived from early editions of topographic maps which do not show all the low-order tributaries. In order that the equations shall give correct results the drainage basin must be reasonably homogeneous. This is true of the drainage basin of Esopus Creek above Olive Bridge, which is wholly mountainous. The drainage basin of lower Esopus Creek is, however, rolling and permeable, with great differences in soil, vegetal cover, rainfall, and climate, as compared with the upper basin. Upper Esopus Creek drainage basin

TABLE 2.—*Observed and computed stream lengths and stream numbers*
Drainage basin of Esopus Creek above Olive Bridge, New York.

Stream Order	Number of streams		Average stream length (miles)	
	From topographic maps	By eq. (5)	From topographic maps	By eq. (7)
1	90	94.75	0.994	0.994
2	25	30.37	2.45	2.30
3	9	9.73	5.64	5.31
4	1	3.12	6.00	12.2
5	1	1.00	29.00	28.3

has a much higher bifurcation ratio and stream-length ratio than the lower basin (Table 1), and the composition of the drainage nets is quite different, as shown graphically on Figures 3 and 4.

The importance of these equations lies both in their practical application and in the fact that they represent laws which evolve from physical processes which Nature follows rather closely in the development of stream systems under such diverse conditions as those of upper and lower Esopus Creek. The size of the drainage basin does not enter the equations directly. It is indirectly involved, since the order of the main stream would in general be higher in the larger drainage basin, for two homogeneous drainage basins of different sizes. The order of the main stream is a factor in the equations, and the drainage basin in which the main stream is of the higher order will have, in general, more tributaries of a given order.

The data given in Table 1 cover a wide range of conditions, from precipitous mountain areas, like upper Esopus Creek, and highly dissected areas, like those of Seneca and Owasco lakes, to moderately rolling and flat areas. They cover also drainage basins ranging in size from a few square miles up to several hundred square miles.

The bifurcation ratio (Table 1, column 11) ranges from about 2 for flat or rolling drainage basins up to 3 or 4 for mountainous or highly dissected drainage basins. As would be expected, the bifurcation ratio is generally higher for hilly, well-dissected drainage basins than for rolling basins.

The values of the length ratios (column 12) range from about 2 to about 3; the average is 2.32.

In the examples given in Table 1, the stream lengths were measured to stream tips as shown on U. S. Geological Survey topographic maps. If stream lengths had been measured as extended to watershed lines, the resulting stream-length ratios would have been materially reduced. If c is the average length from the stream tip to the

watershed line, and l_1 and l_2 are actual average stream lengths of two successive orders, l_2 being the higher, then, as computed in Table 1,

$$r_l = \frac{l_2}{l_1}. \tag{8}$$

If measured as extended to the watershed lines,

$$r_l' = \frac{l_2 + c}{l_1 + c}. \tag{9}$$

The quantity r_l' will always be less than r_l. The average value of r_l for the streams listed in Table 1 is 2.32. If the stream lengths were extended to the watershed lines, this value would lie between 2.00 and 2.32. The theoretical value of r_l' for streams flowing into larger streams at right angles is 2.00, but r_l' will be greater for streams entering at acute angles, as do most streams on steeper slopes. The distance along the course of a stream from its mouth extended to the water shed line is called "mesh length." The use of this quantity instead of actual stream length is preferable in physiographic studies, and its use leads to closer agreement with the theoretical values.

In Figures 3 and 4, the agreement between the mean lines for the different streams and the observed data is so close that the two following general laws may be stated regarding the composition of stream-drainage nets:

(1) *Law of Stream Numbers:* The numbers of streams of different orders in a given drainage basin tend closely to approximate an inverse geometric series in which the first term is unity and the ratio is the bifurcation ratio.

(2) *Law of Stream Lengths:* The average lengths of streams of each of the different orders in a drainage basin tend closely to approximate a direct geometric series in which the first term is the average length of streams of the 1st order.

Playfair called attention to the "nice adjustment" between the different streams and valleys of a drainage basin but chiefly with reference to their declivities. These two laws supplement Playfair's law and make it more definite and more quantitative. They also show that the nice adjustment goes far beyond the matter of declivities.

TOTAL LENGTH OF STREAMS OF A GIVEN ORDER

Since the total length of streams of a given order is the product of the average length and number of streams, equations (5) and (7) can be combined into an equation for total stream length of a given order.

The total length L_o of tributaries of order o is:

$$L_0 = l_1 r_b^{s-o} r_l^{o-1} \tag{10}$$

The total lengths of all streams of a given order is the product of the number of streams and length per stream. The number of streams is dependent on the bifurcation ratio r_b and increases with decreasing stream order, while the length per stream is dependent on the stream length ratio r_l and decreases with increasing stream order. Thus the total lengths of streams of a given order should have either a maximum or a minimum value for some particular stream order. A maximum or minimum may not occur because the stream order required to give the maximum or minimum stream length may exceed the order of the main stream, in which case the total lengths of streams of a given order will either increase or decrease progressively with increasing stream order. An exception occurs where r_b and r_l are equal. Then the total lengths of streams of all orders are the same and equal to $l_1 r_b^s$. The ratio of r_l to r_b is designated ρ and is an important factor in relation both to drainage composition and physiographic development of drainage basins. As will be shown later, the value of the ratio $\rho = \frac{r_l}{r_b}$ is determined by precisely those factors—hydrologic, physiographic, cultural, and geologic—which determine the ultimate degree of drainage development in a given drainage basin.

By summation of the total stream lengths for different orders, as given by equation (10), the total stream length within a drainage basin can be expressed in terms of four fundamental quantities: l_1, o_s, r_b, and r_l.

CHANNEL-STORAGE CAPACITY

Natural channel storage is a principal factor in modulating flood-crest intensities as a flood proceeds down a system of stream channels. A knowledge of relative amounts of channel storage at different locations is required for various problems of flood routing and flood control. It can readily be shown that the normal channel storage at a given discharge rate in a given stream channel varies as a simple-power function of the stream length, usually less than the square of the stream length, and this relation can readily be determined from the stage-discharge relation at a gaging station. Equation (10) provides a means of determining total and average stream lengths for each stream order. From this the channel storage provided by each order of streams in the drainage basin can be determined, and by summation the total channel storage in the stream system becomes known. This illustrates the practical application of quantitative physiography to a variety of engineering problems.

Different stream systems may have substantially the same drainage density and yet differ markedly in channel-storage capacity. The higher-order stream channels have larger cross sections and contain more channel storage per unit length than lower-order streams. If r_l/r_b is high, the greater length of larger stream channels may afford greatly increased channel storage per unit of drainage area as compared with a drainage basin with the same drainage density and a lower value of r_l/r_b.

GENERAL EQUATION OF COMPOSITION OF STREAM SYSTEMS

From equation (10) the total length of streams of a given order is:

$$L_o = l_1 r_b^{s-o} r_l^{o-1}$$

The total length of all streams in a drainage basin with the main stream of a given order s is the sum of the total lengths of streams of different orders, or:

$$\Sigma L = l_1 \left[r_b^{s-1} + r_b^{s-2} r_l + r_b^{s-3} r_l^2 + \cdots r_b^0 r_l^{s-1} \right] \tag{11}$$

This equation is cumbersome and can easily be simplified.

Let:

$$\rho = \frac{r_l}{r_b}; \qquad r_l = \rho r_b \tag{12}$$

$$L_o = l_1 \rho^{o-1} r_b^{s-1}. \tag{13}$$

Applying subscripts 1, 2, etc., to designate the total lengths of streams of different orders:

$$\Sigma L = L_1 + L_2 + L_3 + \cdots L_s \tag{14}$$

and from (13):

$$\begin{aligned} \Sigma L &= l_1 r_b^{s-1}(\rho^{1-1} + \rho^{2-1} + \rho^{3-1} + \cdots \rho^{s-1}) \\ &= l_1 r_b^{s-1}(1 + \rho + \rho^2 + \rho^3 + \cdots \rho^{s-1}) \end{aligned} \tag{15}$$

The term in parentheses is the sum of a geometric series with its first term unity and a ratio ρ and is equal to:

$$\frac{\rho^s - 1}{\rho - 1}.$$

Substituting this value in equation (15):

$$\Sigma L = l_1 r_b^{s-1} \cdot \frac{\rho^s - 1}{\rho - 1}. \tag{16}$$

The drainage density is, from (1):

$$D_d = \frac{\Sigma L}{A}.$$

const!

Substituting the value of ΣL from (16):

$$D_d = \frac{l_1 r_b^{s-1}}{A} \cdot \frac{\rho^s - 1}{\rho - 1}. \tag{17}$$

This equation combines all the physiographic factors which determine the composition of the drainage net of a stream system in one expression. Aside from its scientific interest in this respect, it can also be used to determine drainage density. Values of the factor $(\rho^s - 1)/(\rho - 1)$ can be obtained from Figure 5.

RELATION OF SIZE OF DRAINAGE AREA TO STREAM ORDER

Since equation (17) incorporates all the principal characteristics of the stream system of a drainage basin, it may be considered a quantitative generalization of Playfair's law. It can be written in such a form as to give any one of the quantities l_1, D_d, A, r_b, r_l, and s when the other five quantities are known. If the ratio $\rho < 1$, then, for larger values of s, ρ^s is small, and $\rho^s - 1$ may be taken as -1.0. The equation (17) may then be written:

$$s = 1 + \frac{\log [(1 - \rho) D_d A / l_1]}{\log r_b}. \tag{18}$$

If $\rho > 1$, then, for large values of s, ρ^s is sensibly the same as $\rho^s - 1$, and $\rho^s - 1$ is positive. This leads similarly to the equation:

$$(s - 1) \log r_b + s \log \rho = \log \frac{(\rho - 1) D_d A}{l_1}$$

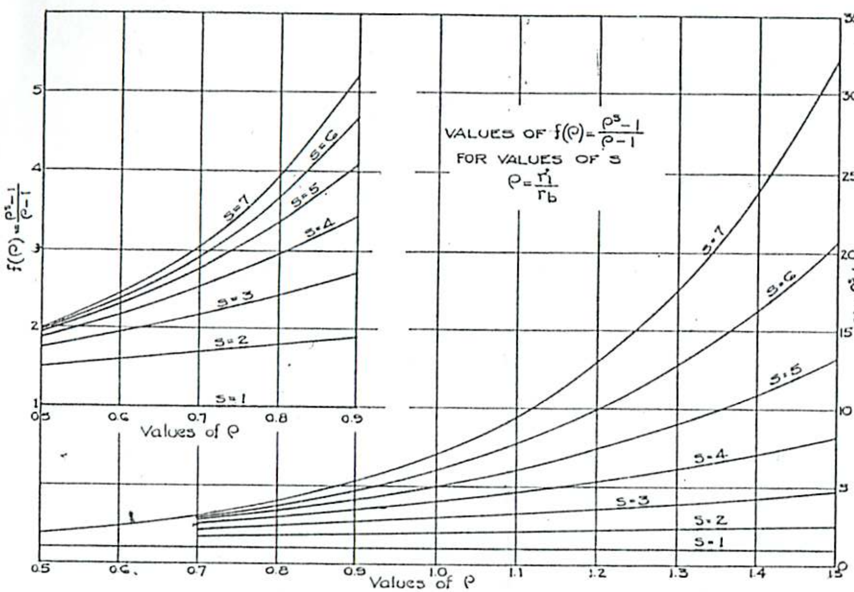


FIGURE 5.—*Diagram of factor $\rho^s - 1/\rho - 1$*

and

$$s = \frac{\log [\overset{1-\rho}{(\rho - 1)} D_d A/l_1] + \log r_b}{\log r_b \cancel{+ \log \rho}}. \tag{19}$$

The quantity $\log r_b$ is small relative to $\log (\rho - 1) D_d A$ and may be neglected, so that in either case the order of the main stream developed in a drainage basin of a given area A increases for larger values of s in proportion to the logarithm of the area A. If, for example, with given values of ρ, D_d, r_b, r_l, and l_1, an area of 10,000 square miles is required to develop a stream order s, then, under the same conditions, in a drainage basin of 100,000 square miles, the main stream would be of order one unit higher, and in a drainage basin of 1,000,000 square miles the main stream would be two units higher in order than in an area of 10,000 square miles. This shows at once why stream systems with extremely high orders do not occur—there is not room to accommodate the requisite drainage basins on the solid surface of the earth. The orders of the Mississippi, Amazon, and other large rivers have not been determined accurately, but the Mississippi River quite certainly does not exceed the 20th order.

From equation (17) drainage density should vary inversely as the drainage area A, other things equal. Actually other things are not equal in drainage areas of different sizes, and, although the bifurcation ratio, stream-length ratio, and average length of

1st order streams may be the same in two drainage basins physiographically similar and of different sizes, the order s of the main stream will in general be larger for the larger drainage basin. As a result the drainage density may increase, decrease, or remain substantially unchanged in two similar drainage basins of different sizes.

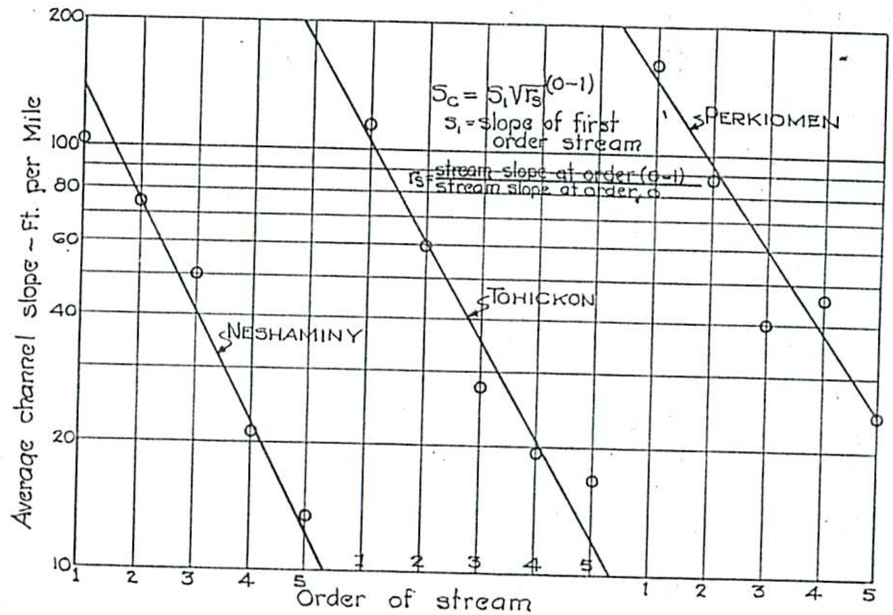


FIGURE 6.—*Law of stream slopes*

Neshaminy, Tohickon, and Perkiomen drainage basins.

LAW OF STREAM SLOPES

In addition to the various quantitative relationships between the different factors involved in drainage composition, expressed by equation (17), there are certain other quantitative relationships. The relation of stream slope to stream order in a given drainage basin is hinted in Playfair's law. As an illustration, the slopes of streams of different orders in the Neshaminy, Tohickon, and Perkiomen drainage basins have been plotted in terms of stream order (Fig. 6), and there is a fairly definite relationship between slope of the streams and stream order, which can be expressed by an inverse geometric-series law.

DETERMINATION OF PHYSIOGRAPHIC FACTORS FOR DRAINAGE BASINS

To determine completely the composition of a stream system it is necessary to know: (1) the drainage area, A, (2) the order s of the main stream, (3) the bifurcation ratio r_b, (4) the stream length ratio r_l, and (5) the length l_s of the main stream or preferably the average length l_1 of 1st order streams. If these data are given, then the drainage density, stream frequency, and other characteristics of the stream system can be determined by calculation, using the equations which have been given.

From equation (5):

$$N_o = r_b^{(s-o)}.$$

If $o = s - 1$:

$$N_{s-1} = r_b.$$

This shows that the bifurcation ratio r_b is equal to the number of streams of the next to the highest order for the given drainage basin.

If the stream numbers for different stream orders are plotted on semilog paper (Fig. 3), the bifurcation ratio r_b can be determined by simply reading from the average line the number of streams of the second highest order.

From equation (7):

$$l_o = l_1 r_l^{o-1}.$$

If $o = 2$,

$$\frac{l_2}{l_1} = r_l.$$

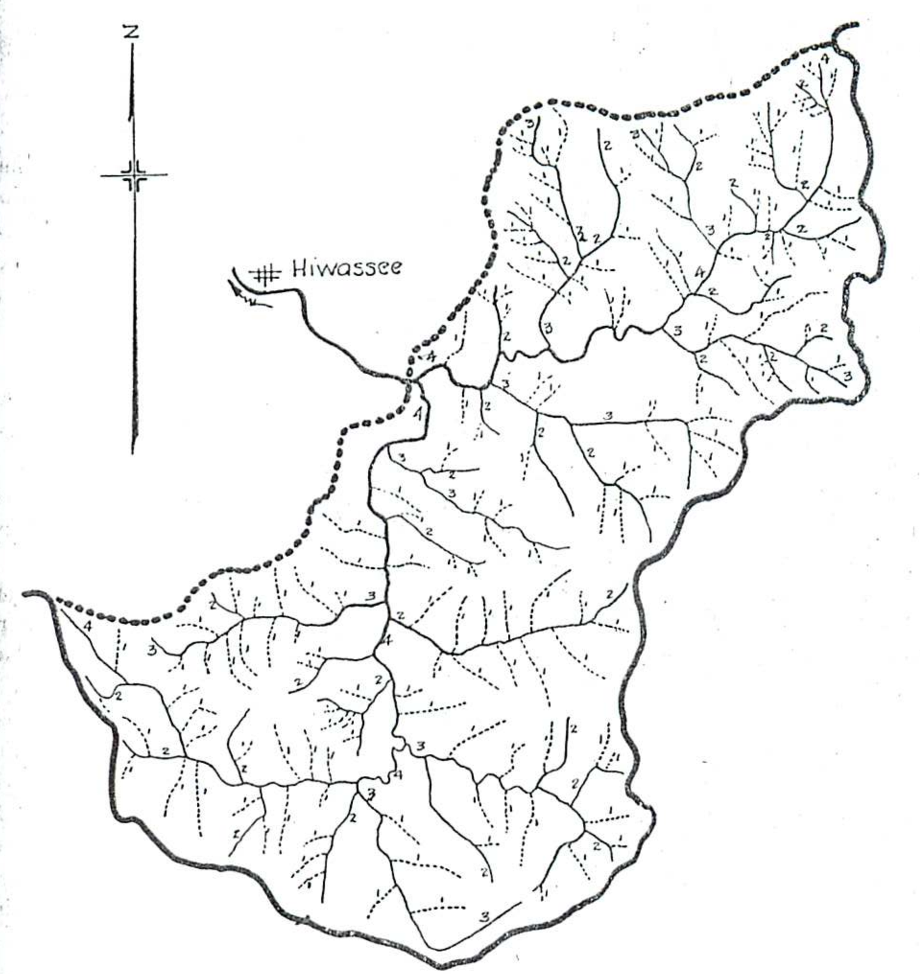
The stream length ratio r_l can therefore be obtained by dividing the average stream length of any order by the average stream length of the next lower order, the values of stream lengths being read from the diagram of stream lengths plotted in terms of stream order. It is preferable to use these data rather than actual measured values, as the number of streams of a given order—particularly the higher order streams—may not be exactly the normal number for the given drainage composition. Stream numbers can, of course, be only integers, and there may be either two, three, or four streams of the second highest order in a given drainage basin where there should be three.

In Table 1 and Figures 3 and 4, the stream lengths and numbers of all orders were determined directly from topographic maps. Where this is done the order s becomes known directly.

In analyzing the drainage net of a stream system it is desirable to trace, with different colors for each order, the stream system from the base map. When the higher-order streams are determined some of the lower-order streams may prove to be the head-water portions of higher-order streams. Figure 7 shows the drainage basin of Hiwassee River above Hiwassee, Georgia, with 1st order streams shown by dotted lines and stream orders indicated by figures.

The determination of stream lengths and orders by direct measurement from maps which are on a sufficiently large scale to show all 1st order streams is so laborious as to be practically prohibitive except for smaller drainage basins.

Fortunately, all the required quantities—l_s, l_1, r_b, r_l, and D_d—can be determined from smaller-scale maps from which the lower-order tributaries are omitted. The maps must show correctly the streams for several of the higher orders. The order of the main stream is of course unknown since it is not in general known which of the lower orders of streams are omitted from the map. The streams shown are assigned orders assuming that the main stream has an unknown order s, the next lower order of stream shown is designated 2, and so on. The number of streams of each assumed



Order	No. of streams	Length miles	Aver. length miles
1	146	72	0.49
2	32	41	1.28
3	9	32.8	3.65
4	2	24.6	12.30
		170.4	

Drainage area = 82.8 sq. mi

D_d = 2.06

FIGURE 7.—*Drainage net, upper Hiwassee River*

order is counted, their stream lengths measured from the map, the results tabulated as follows and plotted as shown by Figure 8A.

Data for Perkiomen Creek

Order	Assumed inverse order	Number of streams	Average stream length (miles)
s	1	1	20.5
$s-1$	2	2	13.75
$s-2$	3	10	3.61
$s-3$	4	32	1.39

If it is assumed that the main stream is of the

4th order, then $l_1 = 1.38$ miles;
5th order, then $l_1 = 0.50$ mile;
6th order, then $l_1 = 0.20$ mile.

Since l_1 is not far from half a mile, the main stream is of the 5th order. From line B (Fig. 8) the number of 2d order streams is 3.15. This is the bifurcation ratio. From line A the lengths of 2d and 1st order streams are, respectively, 1.38 and 0.52 miles. This gives the stream length ratio:

$$r_l = \frac{1.38}{0.52} = 2.70.$$

Data for at least four stream orders are required to determine the order of the main stream from incomplete data by this method. Care must also be used in determining the lines A and B accurately to secure correct results.

The values of the stream lengths as far as known are then plotted on semilog paper (Fig. 8A), in terms of inverse stream orders, a line of best fit drawn to represent the plotted points and this line extended downward to stream length unity or less.

To determine the order of the main stream it is necessary to know the order of magnitude but not the exact value of the average length of streams of the 1st order. The length l_1 of streams of the 1st order is rarely less than a third of a mile, a value which is approached as a minimum limit in mountain regions with heavy rainfall, as in the southern Appalachians. Also it is rarely greater than 2 or 3 miles, values which are approached as maximum limits under some conditions in arid and semiarid regions. Data from which the order of magnitude of l_1 can be determined are always available from some source. In general all that is required is to know whether l_1 is of the order of half a mile, 1 mile, or 2 miles or more. The point at which the stream length shown by the line ab (Fig. 8A) extended downward has a value about the same as the known value of l_1 for the given order indicates the order of the main stream.

This method for determining the order of the main stream is of limited value in some drainage basins, particularly large drainage basins, such as that of the Mississippi River, which are not homogeneous, and where there may be large variations in the length of 1st order streams in different portions of the drainage basins, so that the order of magnitude of l_1 may be difficult to determine. A small portion of a drainage basin, with suitable conditions of high rainfall, steep slopes, etc., may add several units to the value of s for the main stream, although it has little effect on the weighted average value of l_1 for the drainage basin as a whole. For basins which are reasonably homogeneous the method is accurate. Proof of its validity is readily obtained by applying this method to a drainage basin where the values of l_1 and the drainage density D_d have been determined from measurements on a map showing streams of all orders, but using in the determination only the data for streams of higher orders. This was done in preparing Figure 8, which is of the 5th order, although only data for the first four stream orders were used in the computation, it being assumed that l_1 was of an order of magnitude between 1 and 1.5.

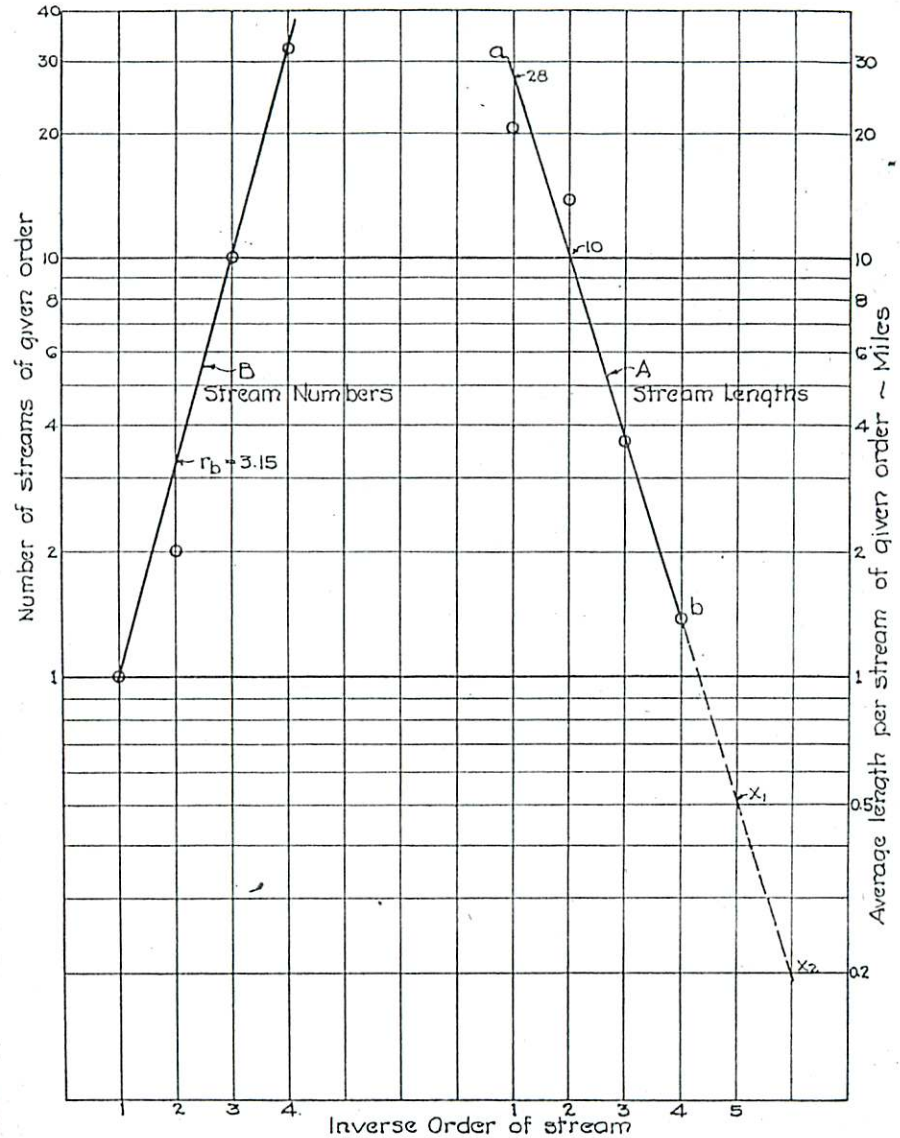


FIGURE 8.—*Graphical determination of stream characteristics*

This determination of s gives also the average length l_1 of 1st order streams. The bifurcation ratio r_b and the stream-length ratio r_l are determined by the slopes of the lines A and B on Figure 8. It is not necessary to know the order of the main stream to determine these quantities. When r_b, r_l, A, s, and l_1 are known, the drainage density can be determined by means of equation (17).

This method of determining s has the advantage that it is at least as accurate when applied to large as when applied to smaller drainage basins. In general, data for more stream orders will be available from a map for a large drainage basin than for a small basin.

Table 3 shows the drainage composition of Neshaminy, Tohickon, and Perkiomen Creek stream systems, derived in the manner described, together with the drainage densities as computed by equation (17) and as derived from direct measurement from topographic maps.

TABLE 3.—*Observed and computed drainage densities, Neshaminy, Tohickon, and Perkiomen creeks*

Item			
1 Stream	*Neshaminy*	*Tohickon*	*Perkiomen*
2 Location	*Below Forks*	*Point Pleasant*	*Near Frederick*
3 Drainage area, square miles	139.3	102.2	152.0
Computed values:			
4 Stream order from map	5	5	5
5 l_s	35.0	33.0	27.0
6 r_b	3.45	3.00	3.15
7 r_l	2.92	2.85	2.70
8 $\rho = r_l/r_b$	0.85	0.95	0.86
9 $f(\rho)$	3.75	4.50	3.80
10 r_b^{s-1}	141.6	81.0	98.4
11 r_b^{s-1}/A	1.02	0.79	0.65
12 (11) × (9)	3.82	3.56	2.47
13 l_1	0.50	0.53	0.52
14 (12) × (13) = D_d	1.91	1.89	1.28
15 Drainage density from map	1.60	1.91	1.24

Drainage densities computed by equation (17) will usually be somewhat higher than those derived directly from maps if stream lengths are measured directly and only to the fingertips of the stream channels, because the stream lengths and mesh lengths are sensibly identical for higher-order streams, whereas there may be 10 to 25 per cent or even 50 per cent difference between stream length and mesh length for low-order streams. In computing drainage density from values of l_1, r_b, and r_l obtained graphically, the computed value corresponds more nearly to drainage density expressed in terms of mesh length than in terms of actual stream length for lower-order streams.

RELATION OF GEOLOGIC STRUCTURES TO DRAINAGE COMPOSITION

The examples of drainage composition shown in Table 1 and in Figures 3 and 4 in nearly all cases represent special or abnormal conditions, such as the presence of large lakes in several of the drainage basins. While this table agrees well with the laws of stream numbers and stream lengths even with such pronounced geologic control of topography as that afforded by the drumlin areas in the Ganargua Creek drainage basin, there are other conditions where geologic controls apparently exert a definite influence on drainage composition. Figure 9 shows two drainage basins the boundaries of which are definitely fixed by geologic structures.

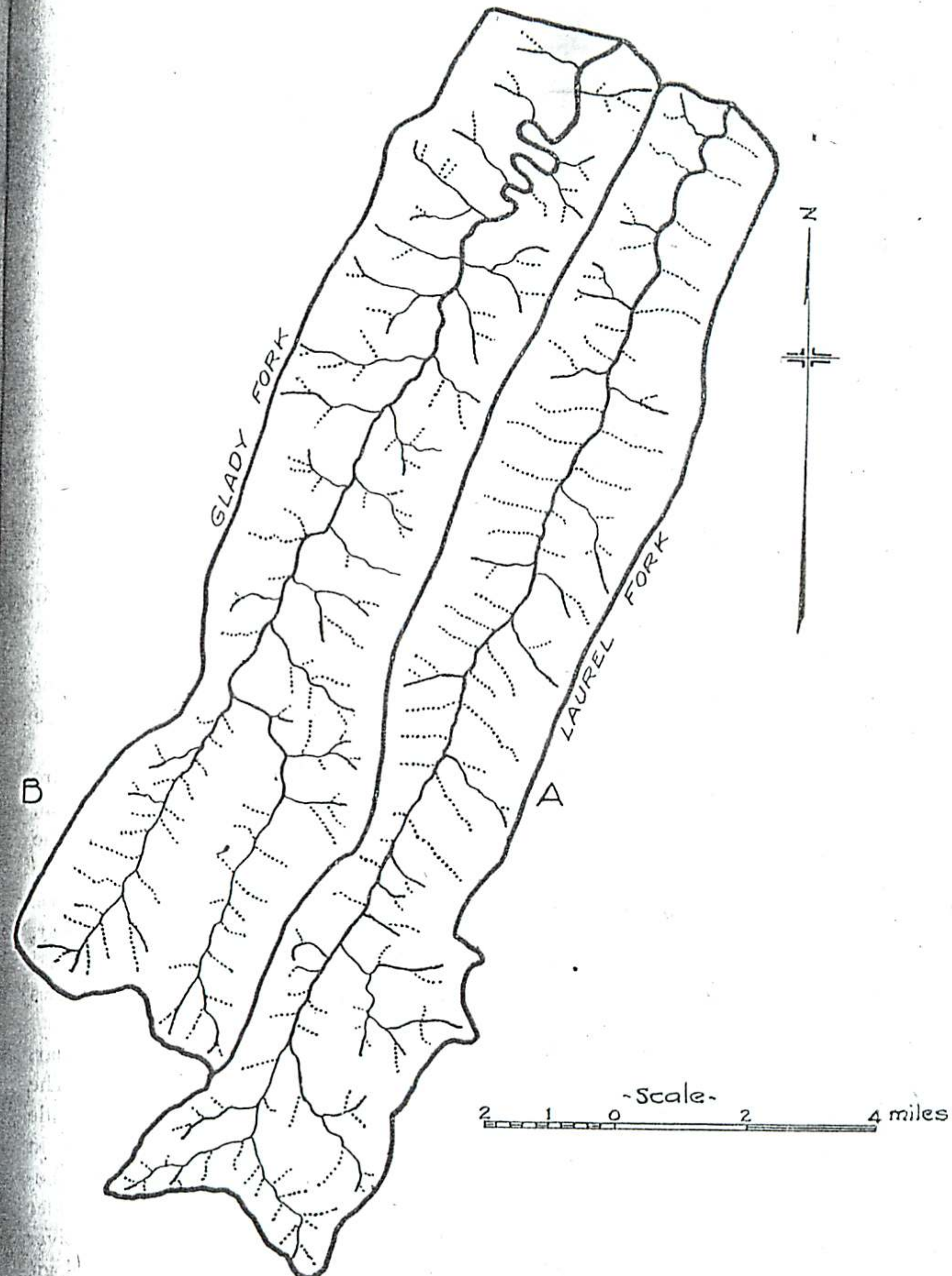


FIGURE 9.—*Drainage patterns of Laurel Fork and Glady Fork, Cheat River drainage basin*
(From Beverly, W. Va., quad., U. S. G. S.)

Data of stream lengths and stream numbers in these basins are as follows:

	Order	Length (miles)	Number of streams	Average length (miles)
Basin "A"—Laurel Fork 50.8 sq. mi.	1	47.80	94	0.51
	2	23.36	26	0.90
	3	4.10	4	1.02
	4	19.62	1	19.62
		94.88		

$$D_d = \frac{94.88}{50.8} = 1.87$$

	Order	Length (miles)	Number of streams	Average length (miles)
Basin "B"—Glady Fork 55.44 sq. mi.	1	38.42	95	0.41
	2	34.76	37	0.94
	3	6.50	6	1.08
	4	20.80	1	20.80
		100.48		

$$D_d = \frac{100.48}{55.44} = 1.82$$

In both streams the law of stream numbers is closely obeyed. The law of stream lengths is approximately obeyed for lower-order streams, but in both basins it is necessary, in order that the area should be drained, that the main stream should have a length sensibly equal to that of the drainage basin; this requires that the length of the main stream should be much greater than it ordinarily would be for a drainage basin of the same order, of normal form.

One may naturally ask whether stream systems in similar terrain and which are genetically similar should not have identical or nearly identical stream composition. Data for the drainage basins of Neshaminy, Tohickon, and Perkiomen creeks in the Delaware River drainage basin near Philadelphia, Pennsylvania, are given in Table 3 and on Figure 10. The drainage patterns of these streams are shown on Figure 11. The physiographic characteristics of these three drainage basins are closely similar.

Tables 4 and 5 show, respectively, drainage composition of streams in the upper Delaware River drainage basin and drainage composition of several small tributaries of Genesee River in western New York.

The streams in the upper Delaware River drainage basin are generally similar, with the exception of Neversink River, in topography, geology, and climate. The various morphologic factors for these basins are of the same order of magnitude although not numerically identical. The tributaries of Genesee River listed in Table 5 represent areas at various locations around the margins of this basin between Lake Ontario and the New York-Pennsylvania State line and comprise a wider range of geologic and topographic conditions than occurs in the Delaware River drainage basins, and there are correspondingly greater variations in the morphologic factors, particularly bifurcation ratio, length of 1st-order streams, and drainage density.

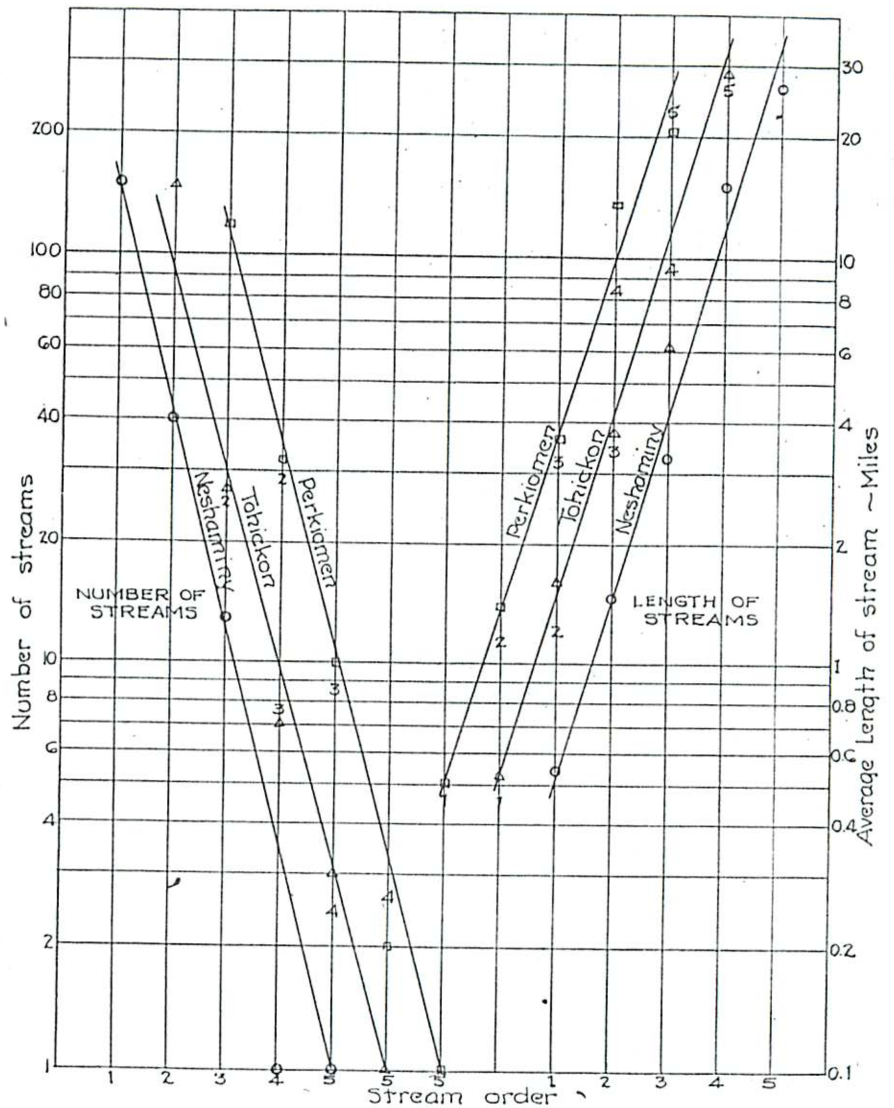


FIGURE 10.—*Stream numbers and stream lengths*
Neshaminy, Tohickon, and Perkiomen drainage basins.

It is found from plotting stream lengths and stream orders, subject to the limiting conditions already described, that both these laws are quite closely followed. Departures from the two laws will, however, be observed, and if other conditions are normal these departures may in general be ascribed to effects of geologic controls. As a rule the law of stream numbers is more closely followed than the law of stream lengths; Nature develops successive orders of streams by bifurcation quite generally

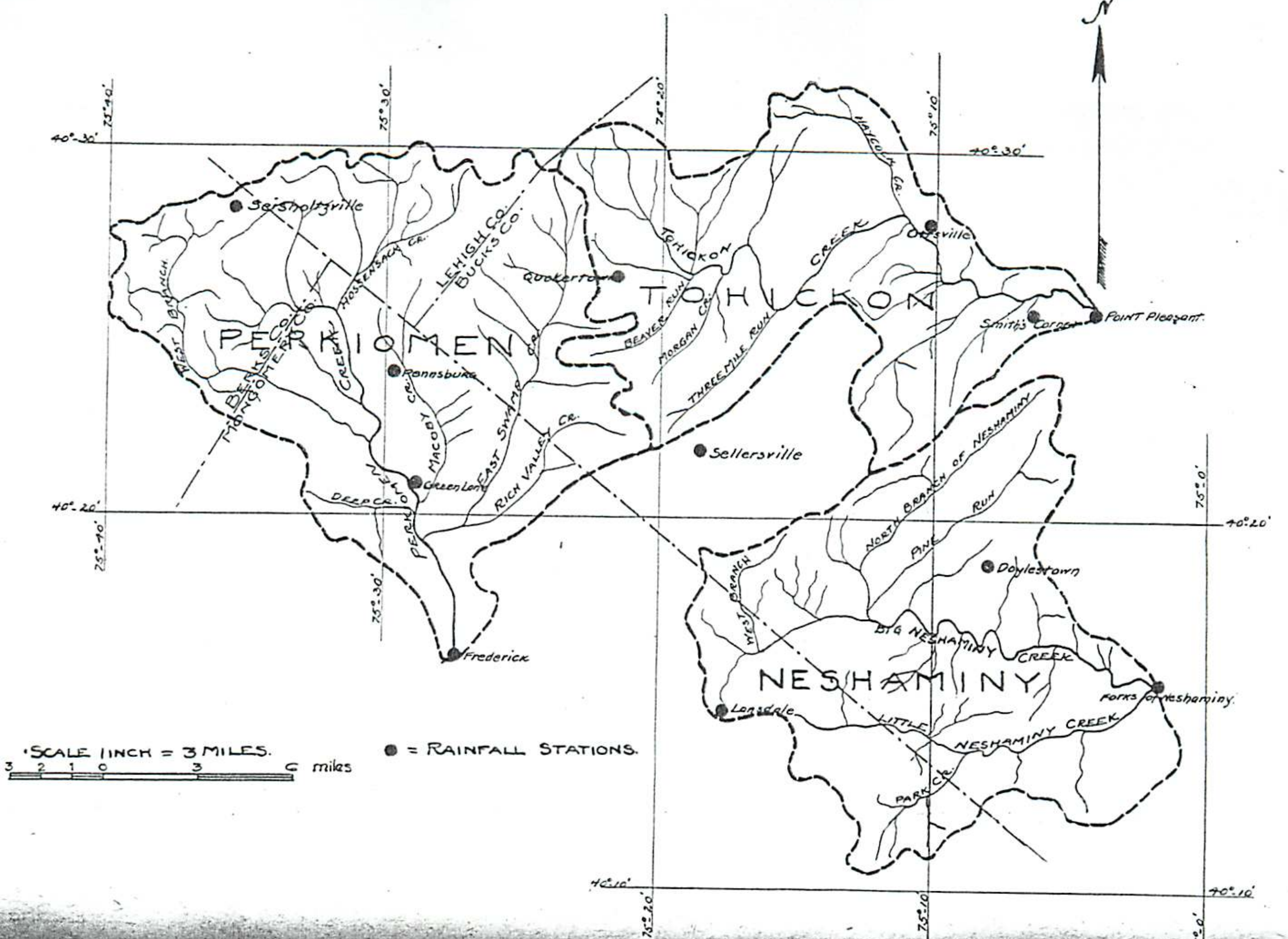


FIGURE 11.—Drainage patterns

TABLE 4.—*Physiographic factors for drainage basins tributary to Delaware River*

Item No.	Item		Formula or Method	East Branch Delaware	Beaver Kill	Little Delaware	East Branch Delaware	Willowemoc	Beaver Kill	Neversink
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
1	Stream			E. Br. Del. R.	Beaver Kill	Little Del R.	E. Br. Del. R.	Willowemoc	Beaver Kill	Neversink
2	Location			Fish Eddy	Cooks Falls	Delhi	Downsville	Livingston Manor	Beaverkill	Claryville
3	Drainage Area	Sq. Mi	A	785	241	50.3	373	63.8	61.0	68.6
4	Length Main stream	Miles (a)	L_s	56.5	28.5	15.0	38.0	14.5	13.0	13.0
5	Width ratio		A/L_s^2	0.246	0.297	0.224	0.258	0.303	0.361	0.406
6	Channel Slope ~ Ft. per mile	S_c	$\frac{\Sigma E_1 - \Sigma E_2}{\Sigma L}$	338	210	296	406	224	307	336
7	Ground " " " "	S_g		1000	779	959	1074	725	1033	1043
8	Slope ratio	r_s	S_c/S_g	0.338	0.270	0.308	0.377	0.309	0.297	0.323
9	Order of main stream	(a)	s	5	4	4	5	4	3	3
10	Average length 1st order streams	(a)	l_1	0.93	0.72	0.82	0.88	0.80	1.05	0.87
11	Bifurcation ratio	(a)	r_b	4.2	5.0	3.0	3.6	2.7	4.5	4.6
12	Stream length ratio	(a)	r_l	2.77	3.12	2.59	2.60	2.54	3.51	3.71
13	Ratio ρ	(a)	r_l/r_b	0.66	0.62	0.86	0.72	0.94	0.78	0.80
14	Average length overland flow	miles	$l_o = \frac{1}{2D_d}\sqrt{1-(S_c/S_g)^2}$	0.346	0.405	0.324	0.317	0.360	0.401	0.438
15	Drainage density		D_d	1.36	1.19	1.47	1.46	1.32	1.19	1.08
16	Latitude			42-00	42-00	42-15	42-10	42-00	42-05	41-55
17	Longitude			74-50	74-40	74-50	74-40	74-40	74-40	74-30

(a) From Bien's Atlas of N.Y. State. Other data are from U.S.G.S. maps in a uniform manner, regardless of geologic controls. Stream lengths, on the other hand, may be definitely limited by geologic controls, such as fixed boundaries of the outline of the drainage basin.

TABLE 5.—*Physiographic factors—tributaries of Genesee River, western New York*

Item No.	Item		Slader Creek	Gates Creek	Rush Creek	Red Creek	Spring Creek	Stony Creek
(1)	(2)		(3)	(4)	(5)	(6)	(7)	(8)
1	Stream		Slader Crk.	Gates Crk.	Rush Crk.	Red Crk.	Spring Crk.	Stony Crk.
2	Location		Canaseraga	at mouth	Fillmore	at mouth	at mouth	at mouth
3	Drainage Area	Sq. Mi.	15.8	20.2	43.3	25.3	20.3	22.2
4	Width ratio		1.12	1.26	0.39	0.65	1.44	0.89
5	Order main stream	s	3	4	3	3	3	4
6	Stream numbers							
	First order		24	32	38	14	15	25
	Second .		4	10	10	4	4	8
	Third .		1	3	1	1	1	3
	Fourth .			1				1
7	Average stream length ~ miles							
	First order		0.61	0.49	0.48	1.29	1.22	0.81
	Second .		2.12	0.95	1.15	1.50	2.00	1.28
	Third .		3.75	2.75	10.50	6.25	3.75	3.25
	Fourth .			4.00				5.00
8	Average stream slope ~ Ft./mi							
	First order		271	89	171	26	37	287
	Second .		121	41	80	18	39	196
	Third .		89	39	49	7	11	169
	Fourth .			25				120
9	Bifurcation ratio	r_b	4.40	3.10	8.10	3.90	3.90	3.10
10	Stream length ratio	r_l	2.21	2.00	4.59	2.57	1.78	1.85
11	Total stream length	mi.	27.00	37.50	60.25	30.25	30.00	38.75
12	Drainage density	D_d	1.71	1.86	1.41	1.20	1.47	1.73
13	Ratio ρ	r_l/r_b	0.50	0.64	0.57	0.66	0.46	0.60
14	Latitude ~ north		42-25	42-50	42-25	43-05	43-05	42-30
15	Longitude ~ west		77-45	77-30	78-05	77-40	78-05	77-40

INFILTRATION THEORY OF SURFACE RUNOFF

GENERAL STATEMENT

The factors and formulas given serve as measuring tools for the quantitative comparison of upland features of drainage basins. Quantitative science develops by the correlation of observed relationships through scientific laws and principles, which may therefore be described as operating tools. Two principal kinds of operating tools are needed in connection with upland erosion: (1) the laws governing the sheet flow of surface runoff, and (2) the laws governing (a) soil resistivity to erosion, (b) erosive force, (c) erosive power of sheet flow, and (d) transporting power of sheet flow. The first of these tools is supplied by the infiltration theory of surface runoff, developed by the author (Horton, 1935; 1937; 1938). Only a few salient features of this theory are pertinent to the present discussion.

The infiltration theory of surface runoff is based on two fundamental concepts:

(1) There is a maximum limiting rate at which the soil when in a given condition can absorb rain as it falls. This is the infiltration-capacity (Horton, 1933).

(2) When runoff takes place from any soil surface, large or small, there is a definite functional relation between the depth of surface detention δ_a or the quantity of water which accumulates on the soil surface, and the rate of surface runoff or channel inflow q_s.

These two concepts, in connection with the equation of continuity or storage equation, form the basis of the infiltration theory. It has hitherto been assumed that surface runoff was some definite fraction of rain. If that were true, then all rains, however low their intensity, should produce runoff. This is not an observed fact.

INFILTRATION-CAPACITY

Infiltration-capacity, f, is governed by physical laws and processes which involve the simultaneous downward flow of water and the upward flow of displaced air through the same system of soil pores (Horton, 1940; Duley and Kelly, 1941) and is used in the sense of a limiting rate of flow, like the capacity of a water pipe.

The infiltration capacity of a given terrain, including soil and cover, is controlled chiefly by (1) soil texture, (2) soil structure, (3) vegetal cover, (4) biologic structures in the soil, especially at and near the surface, including plant roots and root perforations, earthworm, insect, and rodent perforations, humus, and vegetal debris, (5) moisture content of the soil, and (6) condition of the soil surface, whether newly cultivated, baked, or sun-cracked. Temperature is probably also a factor, although its effect is often masked by biologic factors, which also vary with temperature and season.

The infiltration-capacity of a given area is not usually constant during rain but, starting with an initial value f_0, it decreases rapidly at first, then after about half an hour to 2 or 3 hours attains a constant value f_c. The relation of the infiltration-capacity to duration of rain can be expressed accurately by the following equation, with f, f_0, and f_c in inches per hour:

$$f = f_c + (f_0 - f_c)e^{-K_f t} \tag{20}$$

where e is the base of Naperian logarithms, t is time from beginning of rain, in hours, and K_f is a proportionality factor (Horton, 1939; 1940). This equation can easily be derived on the assumption that infiltration-capacity is governed chiefly by the condition of the soil surface and is reduced at the beginning of rain by effects which result from the energy of falling rain and which operate after the manner of exhaustion phenomena. These effects include packing of the soil surface, breaking down of the crumb structure of the soil, swelling of colloids, and the washing of fine material into the larger pores in the soil surface.

As an example typical of many experimental determinations of the change of infiltration-capacity during rain, the values of f have been computed at different times, t, from the beginning of rain for a soil with initial infiltration-capacity f_0 = 2.14 in. per hour and which drops to a constant value f_c = 0.26 in. per hour in 2 hours. The quantity K_f determines the rate of change of infiltration-capacity during rain for a given rain intensity and in this case has the value 3.70.

t:	0.0	0.2	0.4	0.6	0.8	1.0	1.5	2.0
f, in. per hr.:	2.14	1.16	0.69	0.46	0.36	0.31	0.28	0.26

These values of f show that the infiltration-capacity drops off rapidly at first, then more slowly as it approaches f_c. Between rains drying out of the soil and restoration of crumb structure leads to restoration of f toward or to its initial value. The geomorphic significance of this decrease of infiltration-capacity during rain is illustrated by the fact that if infiltration-capacity remained constant at the high value it usually has at the beginning of rain, then there would be little surface runoff or soil erosion.

It is the minimum value f_c of infiltration-capacity which predominates during most of long or heavy rains which are chiefly effective in producing floods and sheet erosion. Infiltration-capacity is highest and least variable for pure coarse sands. For terrain with such soils, the infiltration-capacity may always exceed the rain intensity, so that even in winter, when the ground is frozen, no surface runoff occurs. The infiltration-capacity concept accounts for the absence of sheet erosion and the meager development of drainage in many sandy regions, even with abundant rainfall.

"Transmission-capacity" is the volume of flow per unit of time through a column of soil of unit cross section, with a hydraulic gradient unity or with a hydraulic head equal to the length of the soil column. Infiltration-capacity and transmission-capacity are related, but they are not identical; the infiltration-capacity is usually less than the transmission-capacity. Under conditions where transmission-capacity prevails, the soil column is fully saturated, and the entire cross section of the void space participates in hydraulic flow. When infiltration-capacity prevails, the soil column is not usually fully saturated, and air must escape upward through the soil as fast as water flows downward into the soil. This occupies a fraction, though usually only a small fraction, of the pore space. Also, because of the surface reduction of infiltration-capacity, already described, the soil surface cannot usually absorb water as fast as water can flow downward through the interior of the soil mass. As a result the soil is not saturated appreciably above its capillary-capacity, even during the heaviest rains. This fact can readily be verified during a rain by picking up a handful of garden soil at a depth of a few inches. As a rule, no water can be squeezed out of the moist soil although, if saturated, water could readily be squeezed out. A common fallacy is the statement that during a storm a severe flood was produced by the soil becoming saturated. This happens only in the case of heavy clay soils. Even if the water table rises to the soil surface, as it sometimes does in swampy areas, the soil can still absorb water, which appears later, somewhere, as ground-water or wet-weather seepage. As an example, in the central New York flood of July 1935, analyses showed that, in the Cayuga and Seneca drainage basins, the terrain maintained an infiltration-capacity of 0.2 to 0.3 inch per hour over the areas where the most intense rainfall and runoff occurred.

Infiltration-capacity of a given terrain can be determined in several ways, either with fully controlled conditions or for a drainage basin as a whole, under natural conditions. Thousands of determinations of infiltration-capacity have been made. These and other similar data, when more fully analyzed and classified, will, it is believed, form one of the most important tools for a quantitative study of drainage-basin morphology.

Rain falling at an intensity i which is less than f will be absorbed by the soil surface as fast as it falls and will produce no surface runoff. The rate of infiltration is then less than the infiltration-capacity and should not be designated "infiltration-capacity." If the rain intensity i is greater than the infiltration-capacity f, rain will be

absorbed at the capacity rate f; the remaining rain is called "rainfall excess." This accumulates on the ground surface and for the most part produces runoff, and the difference between rain intensity and infiltration-capacity in such a case is denoted by σ and designated the supply rate ($\sigma = i - f$). For a constant rain intensity i, in inches per hour, the runoff intensity q_s, in inches per hour, approaches the supply rate σ asympototically as a maximum or limiting value as the rain duration increases (Horton, 1939; Beutner, Gaebe, and Horton, 1940). The total surface runoff is approximately equal to the total supply σt_e, where t_e is the duration of rainfall excess.

OVERLAND OR SHEET FLOW

In the minds of most persons the term sheet flow probably implies a greater depth of flow than usually occurs. Sheet erosion is used in contradistinction to channel erosion, and the use of sheet flow to describe overland flow not concentrated in channels larger than rills is appropriate, but it may not imply flow to depths measured in feet or even in inches but rather in fractions of an inch.

Since 1 inch per hour equals approximately 1 second-foot per acre or 640 c.s.m.[2], and an acre is 208 feet square, the surface-runoff intensity q_1 in cubic feet per second from a unit strip 1 foot wide and a slope length l_o will be:

$$q_1 = 0.000023\, l_o q_s \tag{21}$$

where q_s is the runoff intensity in inches per hour. Discharge = Depth × Velocity, or if δ is the depth of sheet flow, in inches, and v the velocity in feet per second, $q_1 = \frac{v\delta}{12}$. It follows that the depth of sheet flow at any point on a slope where the slope length is l_o will be:

$$\delta = \frac{0.000277 l_o q_s}{v}. \tag{22}$$

On a gently sloping lawn, with a length of overland flow of 100 feet and a velocity of a quarter of a foot per second, a depth of surface detention of 0.11 inch will produce 1 inch runoff per hour. Walking over such a lawn while this runoff intensity is occurring, one may not notice that surface runoff is taking place; yet it is this same unobtrusive and almost imperceptible overland flow which, with greater depths and larger volumes and on longer slopes, is largely responsible for carving the landscape of drainage basins into observed forms.

LAW OF OVERLAND FLOW

The velocity of turbulent hydraulic flow is expressed in terms of the Manning formula:

$$v = \frac{1.486}{n} R^{2/3}\sqrt{S} \tag{23}$$

where v is the mean velocity in feet per second, n is the roughness factor, having the same general meaning for sheet flow as for channel flow, R is the hydraulic radius or

[2] c.s.m. = cubic feet per second per square mile.

ratio of area of cross section to wetted perimeter. For sheet or overland flow, R becomes identical with the depth δ. S is the slope.[3] Since discharge or volume of flow per time unit per unit width equals the product: Velocity × Depth × Width, the runoff intensity in inches per hour from a strip of unit width, for turbulent flow, can be expressed by:

$$q_s = K_s \delta^{5/3} \tag{24}$$

where K_s is a constant for a given strip of unit width, having a given slope, roughness, and slope length.

A similar equation:

$$q_s = K_l \delta^2 S \tag{25}$$

can be derived from Poiseuille's law for nonturbulent or laminar flow.

Overland flow may be either wholly turbulent, wholly laminar, or partly turbulent and partly laminar—patches of laminar flow being interspersed with turbulent flow or vice versa. Since the equations for turbulent and laminar flow are of the same form, it follows that for either laminar or turbulent flow, or for mixed flow, the relation between depth of surface detention and runoff intensity, in inches per hour, should be a simple power function of the depth of surface detention or:

$$q_s = K_s \delta^M \tag{26}$$

where q_s is the runoff intensity in inches per hour, δ is the depth of surface detention at the lower end of the slope, in inches, K_s is a coefficient involving slope, length of overland flow, surface roughness, and character of flow, and the exponent M has a value of 5/3 for fully turbulent flow.

Except for very slight depths of surface detention, this simple law of surface runoff is remarkably well verified by plot experiments (Fig. 12). The circles (Fig. 12) indicate points derived directly from the hydrograph, and the solid lines the resulting relation curves plotted logarithmically. The points fall almost precisely on the relation lines, indicating an accurate functional relationship between δ_a and q_s.

Except on steep slopes there are always depressions, often small but numerous, on a natural soil surface. If the derived points were plotted for smaller depths than those shown on Figure 12, the corresponding relation lines would curve off to the left, indicating that the power-function relation of q_s to detention depth changes for very slight depths of surface detention. This represents the effect of depression storage. When runoff is taking place, flow through the depressions also occurs, although usually slowly, and hence the full cross sections of the depressions participate in determining the law of overland flow. The runoff becomes zero, however, when the depth of detention is reduced to the depth of depression storage V_d, although water still remains in the depressions. Consequently for slight depths the relation lines curve to the left.

[3] Expressed hydraulically as the ratio: fall/horizontal length. For steeper slopes the sine of the slope angle should be used in place of S.

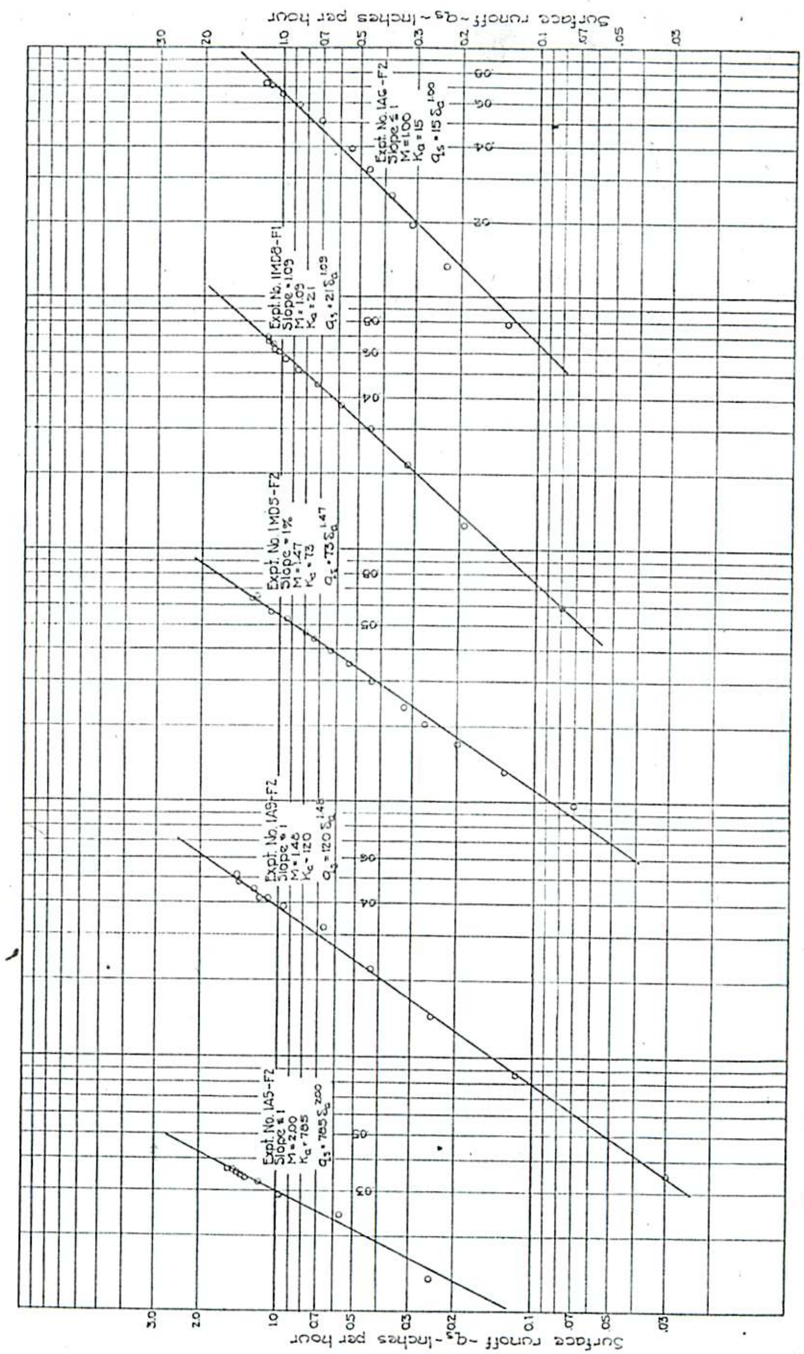


FIGURE 12.—*Relation of surface-runoff intensity (q_s, inches per hour) to average depth of surface detention (in inches)*

Concho River drainage basin sprinkled plot experiments.

INDEX OF TURBULENCE

The exponent in the surface-runoff equation is not the same for all the lines (Fig. 12). This exponent, as indicated by the slopes of the lines, came out 2.0 for the line at the left, 1.48 and 1.47 for the next two lines, and 1.09 and 1.00 for the lines at the right. This suggests that in these cases the flow was not wholly ordinary turbulent flow.

For fully turbulent flow the exponent M should be 5/3, while for fully laminar flow the exponent M should be 3.0.

Where the exponent $M > 5/3$, the larger exponent can readily be explained if part of the overland flow is laminar flow, and this is quite certain to occur where the flow is alternately in thin films on ridges in the form of laminar flow, and through depressions wholly or partly as turbulent flow. An index of turbulence applicable to such a case is expressed by the equation:

$$I = \tfrac{3}{4}(3 - M). \tag{27}$$

If the flow is fully turbulent and $M = 5/3$, this gives $I = 1.0$. If the flow is fully laminar and $M = 3.0$, this gives $I = 0$.

By transposition, the runoff exponent M can be expressed in terms of the index of turbulence:

$$M = 3 - \tfrac{4}{3} I. \tag{28}$$

Turbulence in overland flow increases downslope from the watershed line. In laboratory and field-plot experiments with plot lengths l_o usually less than 25 feet, the flow over surfaces without vegetal cover is usually partially turbulent. On long natural slopes, with l_o much greater—frequently 1000 feet or more—the flow is fully turbulent except for extremely slight depths or close to the head of the slope.

TYPES OF OVERLAND FLOW

The study of overland flow in accordance with the infiltration theory has revealed various phenomena of microhydraulics not commonly present in ordinary channel flow.

Partially turbulent flow described may be considered "mixed flow." In general it consists of turbulent flow interspersed with laminar flow.

If the area on which flow occurs is covered with grass or other close-spaced vegetation, the flow may be "subdivided." Part of the energy available for overcoming resistance is expended on the grass blades and stems, reducing the amount of energy available for expenditure on the soil surface. For the limiting condition of complete subdivision of the flow, all the resistance to flow would be due to the vegetation, and the law of overland flow would be:

$$q_s = K_a \delta_a. \tag{29}$$

The velocity of overland flow would be sensibly constant regardless of the depth of surface detention. Some experiments show substantially this condition. Because of the increased resistance, the depth of surface detention required to carry a given rate of runoff is very greatly increased, and the velocity of overland flow is correspondingly decreased where there is dense cover of grass, grain, or similar vegetation. This is an important fact in relation both to surface runoff and soil erosion.

It is often found in runoff-plot experiments that the hydrograph does not have a smooth surface but is broken into irregular waves or surges. This type of flow may be designated "surge" flow and may be due to several causes:

(1) Under certain hydraulic conditions steady flow cannot occur even on a smooth, unchanging surface (Jeffreys, 1925).

(2) Plant debris, especially of the sand-burr type, may be loosened and carried along with the flow, forming debris dams, behind which the water piles up, and these hold back the water temporarily and then release it in relatively large volumes, producing irregular waves (Beutner, Gaebe, and Horton, 1940).

(3) Active surface erosion may produce a succession of irregular waves due either to mud or mud-and-debris dams of the type last described, to the breaking down of divides between natural depressions, or to the lateral incaving of the walls of gullies (Horton, 1939). Erosion may produce traveling mud dams or mud flows similar to those sometimes produced on a larger scale in mountain canyons by cloudburst storms. Wherever a mud or debris dam is formed, water accumulates behind it until presently the dam moves down the slope with the accumulated water behind it. In case of surge flow or traveling back-water due to debris dams, there is often no consistent relation between depth of overland flow and runoff intensity.

RAIN-WAVE TRAINS

On slopes which are not too flat, shallow flow in the form of a uniform sheet may be hydraulically impossible. The flow then takes the form of wave trains or series of uniformly spaced waves in which nearly all the runoff is concentrated. The author has twice observed such rain-wave trains in intense storms. They occur most commonly in rains of high intensity, particularly those of the cloudburst type, characterized by their ability to tear up sod on slopes and carry fences and other large debris into stream channels. Rain-wave trains occur only under suitable hydraulic conditions and have been described in another paper (Horton, 1939). Observation strongly indicates that rain-wave trains may be important in initiating erosion on sloping lands. If, for example, the waves are 6 feet apart, then each wave contains as much water as would be contained in a length of slope of 6 feet with uniform flow. The successive waves, with their concentration of runoff and energy, can strike sledge-hammer blows on obstructions. They may initiate erosion where it would never occur from the same runoff intensity with steady flow. The difference between the two cases is like that of breaking a rock with a few sledge-hammer blows, when a million taps with a pencil tip would expend the same amount of energy but produce no effect.

The following types of sheet or overland flow take place:

Type of flow	*M*	*I*
Pure laminar	3	0
Mixed laminar and turbulent	3 to 5/3	0 to 1
Turbulent	5/3	1.0
Subdivided or superturbulent	5/3 to 1	1 to 1⅓
Surge flow	Indefinite	—
Rain-wave trains	Indefinite	—

PROFILE OF OVERLAND FLOW

It can readily be shown from the infiltration theory that the profile of sheet or overland flow, or the relation of depth δ of surface detention to the distance x downslope from the watershed line, is expressed by a simple parabolic or power function

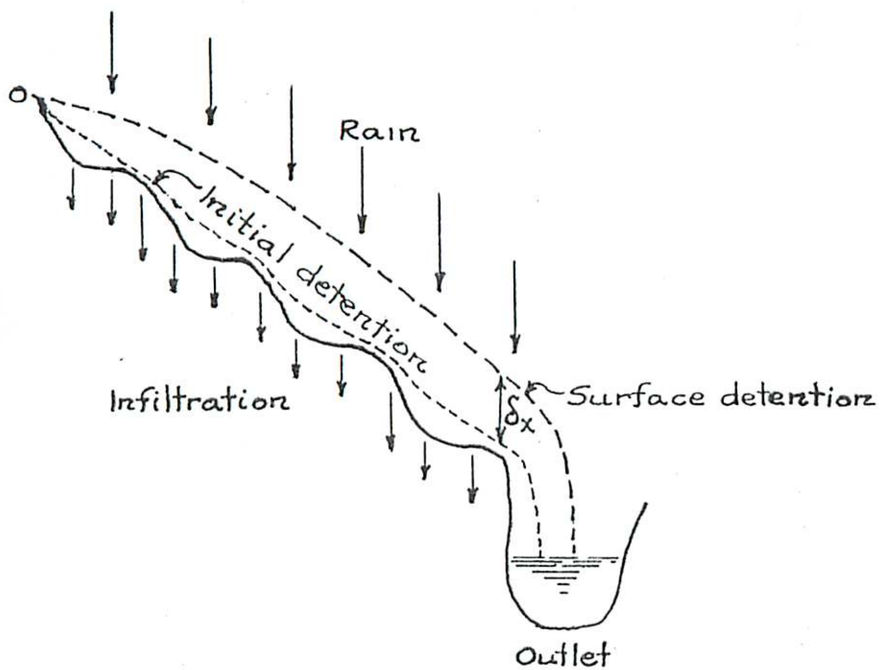


FIGURE 13.—*Half section of a small drainage basin*
Illustrating runoff phenomena. Vertical scale greatly exaggerated.

(Horton, 1938). This relation is illustrated on Figure 13. A similar power function expresses the relation of velocity of overland flow in terms of distance from the watershed line (Horton, 1937). For turbulent flow:

$$\delta_x = \left(\frac{\sigma}{K_s} \frac{x}{l_o}\right)^{3/5} \tag{30}$$

$$v_x = \frac{1.486}{n} \left(\frac{\delta_x}{12}\right)^{2/3} \sqrt{S} \tag{31}$$

and, in general, for δ_x in inches, for any type of flow:

$$\delta_x = \sqrt[M]{\frac{\sigma}{K_s} \frac{x}{l_o}} \tag{32}$$

and for velocity in feet per second:

$$v_x = 0.2836 \frac{\sqrt{S}}{n} \left(\frac{\sigma}{K_s} \frac{x}{l_o}\right)^{\frac{2}{3M}} \tag{33}$$

where σ = supply rate, in inches per hour; l_o = total length of slope on which overland flow occurs, in feet; x = distance, in feet, downslope from the watershed line; K_s is a coefficient derived from the Manning formula for turbulent flow, and approximately applicable to other types of flow. Its value is:

$$K_s = \frac{1020\sqrt{S}}{In\ l_o} \tag{34}$$

and the exponent:

$$\frac{2}{3M} = \frac{2}{9.0 - 4I} \tag{35}$$

in which S is the slope, I the index of turbulence, l_o the length of overland flow, and n is a roughness factor, of the same type as the roughness factor in the Manning formula. The equations for turbulent flow are derived directly from the Manning formula and the law of continuity and are rational. The equations for other types of flow are closely approximate. The depth δ_x as given by these equations is the total depth of surface detention, including depression storage. The equations for δ_x fail at points close to the watershed line if, as is often the case, depression storage persists to the watershed line. In the equations both for turbulent and other types of flow, it is assumed that the velocity varies as $\sqrt{S}$, as for turbulent flow. This may not be entirely correct although numerous experiments indicate that, in mixed flow, most of the resistance is that due to turbulence. Theoretically, for laminar flow the velocity should vary directly as the slope, not as $\sqrt{S}$.

These equations apply primarily to steady flow. Experiments show that they are, however, closely approximate during the early stages of runoff, while surface detention is building up to its maximum value. While detailed discussion of the effect of the various factors on surface-runoff phenomena cannot be undertaken here, comparison of the equations shows that the velocity of overland flow increases, while the depth at a given point x decreases, as the slope increases. Increasing roughness of the surface decreases the velocity but increases the depth of surface detention.

The equations for depth and velocity profiles, in conjunction with that for K_s, are of fundamental importance in relation to erosional conditions, since they express the two factors, δ_x and v_x, which control the eroding and transporting power of sheet flow, in terms of the independent variables which govern surface-runoff phenomena. There are six variables: (1) rain intensity, i; (2) infiltration-capacity, f; (3) length of overland flow, l_o; (4) slope, S; (5) surface-roughness factor, n; (6) index of turbulence or type of overland flow, I. To apply these equations to erosion and gradational problems one must also have laws governing the relation of velocity and depth of overland flow to the eroding and transporting power of overland flow.

SURFACE EROSION BY OVERLAND FLOW

SOIL-EROSION PROCESSES

There are always two and sometimes three distinct but closely related processes involved in surface erosion of the soil: (1) tearing loose of soil material; (2) transport or removal of the eroded material by sheet flow; (3) deposition of the material in transport or sedimentation. If (3) does not occur, the eroded material will be carried into a stream.

Every farmer has noticed that the spots most vulnerable to erosion are the steeper portions of the hill or valley slopes, neither at the crest nor at the bottom of the hill

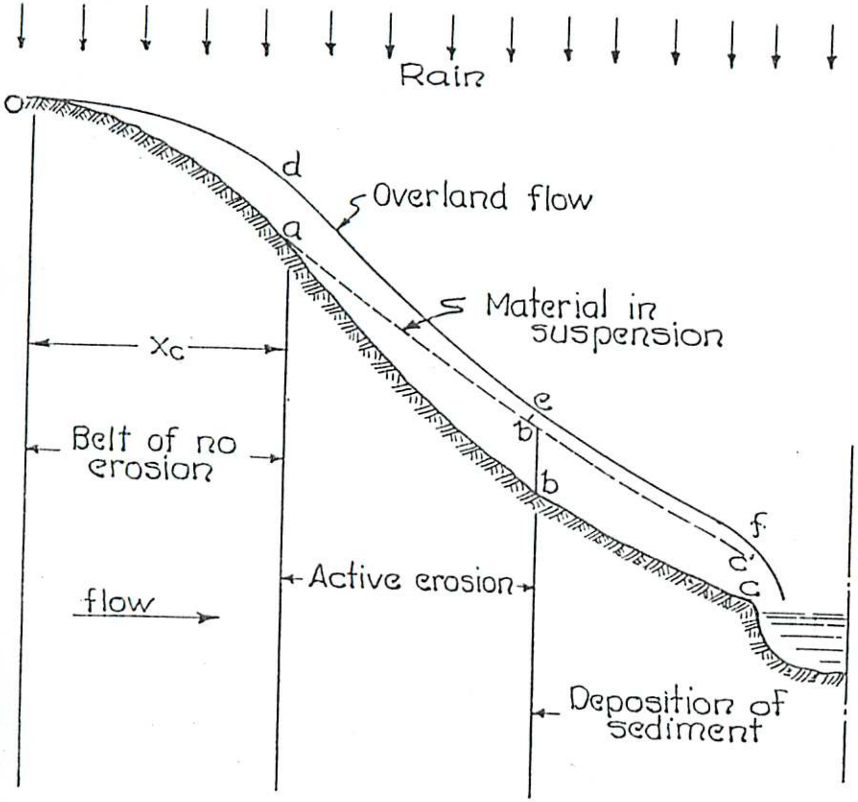


FIGURE 14.—*Half profile of a valley slope*
Illustrating soil-erosion processes.

but intermediate. All soils possess a certain resistivity to erosion, and this resistivity may be increased greatly by a vegetal cover, especially a good grass sod. The underlying soil may have a much smaller resistivity to erosion, and, if the surface conditions are changed by cultivation or otherwise so as to destroy the surface resistance, erosion will begin on land which has not hitherto been subject to erosion.

Figure 14 shows a half profile of a typical stream valley slope, with the vertical scale greatly exaggerated. The line *oabc* represents the soil-surface profile—flat in the region *o*, near the crest, steepest in the region *ab*, about mid-length of the slope, and relatively flat at the foot of the slope, in the region *bc*. The line *odef* represents the surface of sheet or overland flow in an intense rain, the depth of overland flow increasing downslope from *o* toward *f*. In the region *oa* no erosion occurs throughout

a distance x_c from the crest of the slope, and this is called the belt of no erosion. Here the energy of the sheet or overland flow is not sufficient to overcome the initial resistance of the soil surface to erosion, even in the most intense storm. In the belt *ab*, mid-length of the slope and where the slope is steepest, active erosion occurs.

Beginning at *a* the amount of material carried in suspension by the overland flow is proportional to the ordinate between the dotted line *ab′c′* and *abc*. At *a* it is zero; at *b* it is represented by the vertical intercept *bb′*. Beginning at *a* a given volume of water, for example, the water flowing over 1 square foot of soil surface, picks up a certain amount of eroded soil matter and carries it in suspension. Passing over the next adjacent square foot of area the same water picks up another increment of soil matter and holds it in suspension, and so on, the amount of material in suspension increasing until at some point *b* the overland flow is fully charged with material in suspension and can carry no more. Between the point *b* and the stream channel, no material is carried away because any material picked up must be replaced by an equal quantity of material deposited from that already in suspension. If the slope decreases as shown on the diagram, then the ability of the overland flow to carry away material may decrease, in which case deposition of material or sedimentation on the surface will occur instead of erosion.

RESISTANCE TO EROSION

The physical factors governing soil erosion are: (1) initial resistivity, R_i; rain intensity, i; infiltration-capacity, f; velocity and energy of overland flow or eroding force, F. The breaking down of the soil structure, tearing the soil apart and lifting or rolling soil particles or aggregates, requires the expenditure of energy. Erosion can occur at a given location only where the amount of energy expended as frictional resistance on the soil surface exceeds the amount of energy required to overcome the initial resistance of the soil to erosion. An exception occurs in some cases where the soil is churned up into a semifluid mass by intense rain before surface runoff begins, producing high initial erosion rate. Sustained erosion can occur only where the condition above described is fulfilled.

The term "soil" as related to surface erosion includes not only the soil substance but also the vegetal cover and the structures—physical and biologic—in the surface layers of soil. Soils are of two general classes: (1) indigenous, or those formed by weathering of underlying parent rock, either igneous or sedimentary. Such soils generally prevail outside of glaciated and loess-covered regions. For some types of rock the formation of soil *in situ* is extremely slow. After a shallow surface layer of soil is formed, the formation of additional soil is restricted by the previously formed soil cover, but even in full exposure the rate of soil formation from many types of consolidated and igneous rocks is so slow that when the soil cover has been removed the land becomes worthless. (2) Preformed and transported soils. These consist of rock material comminuted by glacial or aeolian action and transported and deposited. Such soil is often a mixture of transported and indigenous soil material and includes sedimentary soils deposited on lake or ocean floors and afterward exposed. Transported soils, particularly those of glacial origin, are often highly fertile at the time they are laid down, as is evidenced by the growth of thrifty forest vegetation within a few years on soils recently exposed by glacial retreat. Erosion of transported soils, while a serious menace, is not in general so completely destructive as in case of indigenous soils unless the land is gullied and scarified to such an extent as to make cultivation impracticable. With equal runoff intensity the resistance of soil material to erosion generally increases with the fineness of the soil particles or soil texture, the resistance being small for fine uncemented sands but so high for cemented hardpan and tough clay that erosion rarely if ever occurs even on bare soil.

Resistance to erosion is, however, governed more largely by vegetal cover, biologic structures, and physical structure of the soil in the surface layers than by soil structure. A soil which forms a hard crust on drying may be highly resistant to erosion although the same soil when newly cultivated erodes easily. The coherence of soil particles and consequently the resistance to erosion is generally increased by the presence of colloidal matter, particularly that of vegetal origin. Vegetal cover is the most important factor in relation to initial resistance to soil erosion. Its effects on the resistivity of the soil to erosion are complex but include:

(1) Vegetal cover breaks the force of raindrops, thereby reducing the effect of the energy of falling rain in breaking down the crumb structure of the soil and packing the soil surface. For some soils with little coherence, breaking down of the crumb structure by rain impact reduces the soil to a fluid condition, readily susceptible to erosion, while for other soils packing of the soil surface tends to increase the resistance to erosion.

(2) A grass sod operates somewhat like a carpet covering the underlying soil and tends strongly to inhibit erosion.

(3) Fine soil particles adhere to root hairs and plant roots near the soil surface and act strongly as a soil binder. In a forest similar effects are produced largely by the grass cover but are accentuated by differences in soil structure as between natural or undisturbed and cultivated soils and by the presence of an undisturbed humus layer near the soil surface. In addition there is often a dense matting of roots of trees, herbaceous vegetation, and litter within a forest. Some of the runoff may be subsurface runoff and pass through this mat of litter and roots but at so greatly reduced velocity as to inhibit erosion. Factors have been devised which stress the resistivity of soil material to erosion in terms of the chemical and physical composition of the soil. Such factors are, however, inadequate to express the resistivity of a given terrain to erosion because of the predominant effect of vegetation and soil structure and condition, which are not reflected in indexes of the erodibility of the soil material itself.

The resistivity of a given terrain to surface erosion can be expressed quantitatively in terms of the force in pounds per square foot required to institute erosion. This quantity can readily be determined on a given soil surface from measurements of the distance from the watershed line downslope to a point where erosion begins.

Nearly all the factors which control resistance of a soil to erosion also control infiltration-capacity of the soil. At a given point on a given slope and with a given rain intensity the erosion rate is governed by various factors, one of the most important of which is the infiltration-capacity of the soil. In many instances factors which tend to promote a high resistance to erosion also tend to restrict or reduce the in-

filtration-capacity and vice versa. Consequently open-textured, coarse, sandy soils, such as soils of sand dunes, with little vegetal cover, may never be subject to erosion even in the most intense rains although the soil has little resistance to erosion, because the infiltration-capacity is so high that little or no surface runoff ever occurs.

ERODING FORCE

Erosion by aqueous agencies involves three processes: (1) dislodgment or tearing loose of soil material and setting it in motion. This is called "entrainment." (2) transport of material by fluid motion. (3) sedimentation or deposition of the transported material.

Let x = distance from divide or watershed line, measured on and along the slope (not horizontally); δ_x = depth of overland flow at x, in inches; w_1 = weight per cubic foot of water in runoff, including solids in suspension; α = slope angle; v = velocity of overland flow at x, in feet per second. The energy expended in frictional resistance per foot of slope length on a strip 1 foot wide running down the slope, per unit of time, for steady flow, will be equal to the energy of the volume of water passing over a unit of area per unit of time. This is the product of the weight, fall, and velocity, or:

$$e = w_1 \frac{\delta_x}{12} v \sin \alpha. \tag{36}$$

The energy equals the force times the distance moved. Hence the force exerted parallel with the soil surface per unit of slope length and width is:

$$F_1 = \frac{e}{v} = w_1 \frac{\delta_x}{12} \sin \alpha. \tag{37}$$

Equation (37) is known as the DuBoys formula and is a rational expression which may be used as a basis for determining the eroding force per square foot of soil surface. F_1 is the force available to dislodge or tear loose soil material. Sometimes not all this force is expended in tearing loose soil material; on a grass-covered surface a large portion of it may be expended in frictional resistance on grass blades and stems and have little effect in tearing loose soil material unless or until the grass is broken over by the impact of the overland flow. However, these factors are best included in the resistivity R_i of the soil to erosion.

As shown in connection with surface runoff, for turbulent flow the depth of overland flow at the distance x from the watershed line can be expressed in terms of slope, runoff intensity, and surface roughness:

$$\delta_x = \left(\frac{\sigma\, nx}{1020}\right)^{3/5} \cdot \frac{1}{S^{0.3}}. \tag{38}$$

The slope S is ordinarily expressed as the tangent of the slope angle α or as tan α. Also, for steady overland flow, $\sigma = q_s$, in inches per hour. Substituting these values in (38):

$$\delta_x = \left(\frac{q_s nx}{1020}\right)^{3/5} \cdot \frac{1}{\tan^{0.3} \alpha}.$$

Substituting this value of δ_x in (37) gives as the total eroding force at x:

$$F_1 = \frac{w_1}{12}\left(\frac{q_s n x}{1020}\right)^{3/5} \cdot \frac{\sin\alpha}{\tan^{0.3}\alpha}. \tag{39}$$

CRITICAL LENGTH x_c—BELT OF NO EROSION

As indicated, erosion will not occur on a slope unless the available eroding force exceeds the resistance R_i of the soil to erosion. The eroding force increases downslope from the watershed line (Equation 39). The distance from the watershed line to the point at which the eroding force becomes equal to the resistance R_i is called the "critical distance" and is designated x_c. Between this point and the watershed line no erosion occurs. This strip adjacent to the watershed line, and immune to erosion, is designated the "belt of no erosion." An expression for the width of the belt of no erosion can readily be obtained from equation (39) by substituting R_i for F_1, making $x = x_c$, and solving the equation for x_c. The runoff is free from sediment where erosion begins, and $w_1 = 62.4$ lbs. per cu. ft.

$$1020\left(\frac{12}{62.4}\right)^{5/3} = 65.0.$$

Substituting this constant in (39) gives:

$$x_c = \frac{65}{q_s n}\left(\frac{R_i}{f(S)}\right)^{5/3} \tag{40}$$

where $f(S)$ is a function expressing the effect of slope on the critical length x_c and is given by the equation:

$$f(S) = \frac{\sin\alpha}{\tan^{0.3}\alpha}. \tag{41}$$

Numerical values of the slope function $f(S)$ are given on Figure 15. For slopes less than 20°, $f(S)$ increases nearly in proportion to the slope. The critical length x_c varies inversely as the runoff intensity q_s in inches per hour, inversely as the roughness factor n, and directly as the 5/3 power of the resistance R_i (equation 40).

Table 6 gives numerical values of x_c for R_i = 0.01, 0.05, 0.10, 0.20, and 0.50 lb. per square foot, for slope angles of 5°, 10°, and 20°, and for four different runoff intensities. These are computed for the roughness factor $n = 0.10$ but can easily be applied to other roughness factors, since the value of x_c is the same if the product $q_s n$ is the same.

Renner (1936) observed the percentages of areas having different slope angles which were subject to erosion in the Boise River drainage basin, Idaho; his results are shown on Figure 16. The extent to which erosion occurred on a given slope increased to a maximum on a 40-degree slope and thereafter decreased to zero approaching a 90-degree slope angle.

A comparison of the results of Renner's observations with the value of the slope function $f(S)$ is also given on Figure 15, and the two curves are in close agreement. Equation (41) for the slope function is rational in that it is based directly on fundamental physical laws and principles with respect to small and moderate slopes. For such slopes both Renner's observations and observations by Fletcher and Beutner (1941) show that for slopes of less than about 20 degrees the amount of erosion generally increases about in proportion to the slope. For steeper slopes the slope

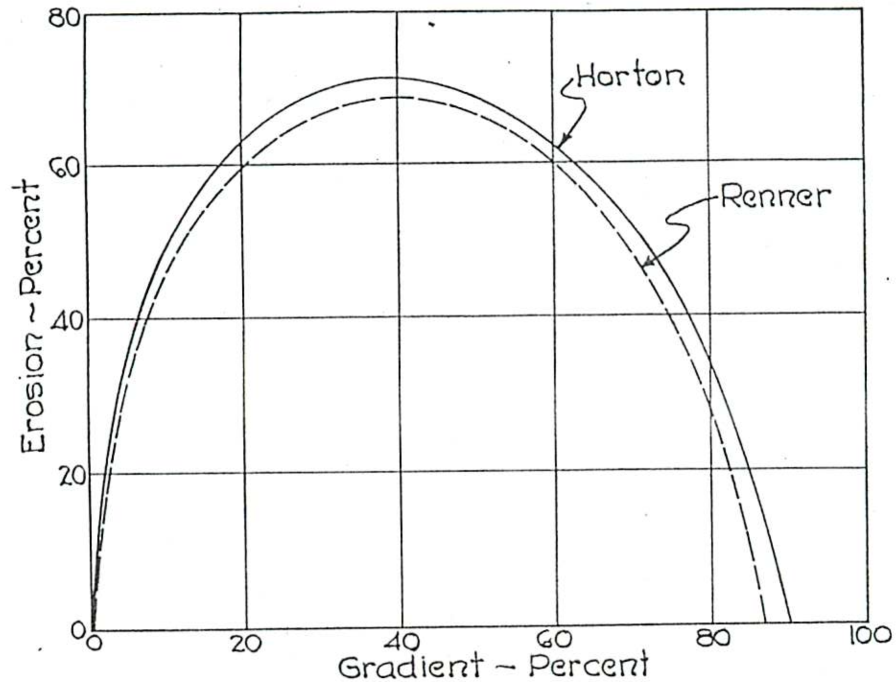


FIGURE 15.—*Horton slope function for surface erosion*

$f(s) = \dfrac{\sin\alpha,}{\tan^{.3}\alpha}$, where α = slope angle.

function $f(S)$ must be considered as empirical, but its validity is confirmed by comparison with Renner's observations. This function is predicated on uniform turbulent flow. For very steep slopes such flow cannot occur.

The critical length x_c is the most important factor in relation to the physiographic development of drainage basins by erosion processes and also in relation to erosion control. The value of x_c (Table 6) is highly sensitive to changes in the variables by which it is controlled, in particular the resistance R_i and the runoff intensity q_s. With a newly cultivated bare soil, with R_i small, 0.05 lb. per square foot, for example, with a 10-degree slope and a runoff intensity of 1 in. per hour, the width of the belt of no erosion would be 35.1 feet, whereas on the same terrain, but with a good, well-developed grass sod to protect the soil, and R_i increased to 0.5 lb. per square foot, the belt of no erosion would be 1573 feet wide. The width of the belt of no erosion varies with the rain intensity, and, consequently, regions with frequent storms of high rain intensity are much more subject to erosion, other things equal, than regions with lower rain intensities and less frequent heavy storms. Furthermore, in a given

TABLE 6.—*Critical length x_c for various values of R_i*

Slope angle α, and runoff intensities q_s, with roughness factor $n = 0.10$: $x_c = \frac{65}{q_s n}\left(\frac{R_i}{(fS)}\right)^{5/3}$.

R_i	α (degrees)	q_s, inches per hour: 0.5	1.0	1.5	2.0
		$1/q_s n$: 20	10	6.67	5.00
.01	5	10.66	5.33	3.55	2.67
.01	10	4.80	2.40	1.60	1.20
.01	20	2.28	1.14	0.76	0.57
.05	5	153.4	76.7	51.16	38.3
.05	10	70.2	35.1	23.41	17.6
.05	20	32.6	16.3	10.87	8.2
.10	5	487.6	243.8	162.61	121.9
.10	10	218.4	109.2	72.84	54.6
.10	20	102.8	51.4	34.28	25.7
.20	5	1535.2	767.6	512.0	383.8
.20	10	689.0	344.5	229.8	172.2
.20	20	325.0	162.5	108.4	81.3
.50	5	7046.0	3523.0	2349.8	1762.0
.50	10	3146.0	1573.0	1049.2	787.0
.50	20	1478.0	739.0	4929.0	369.0

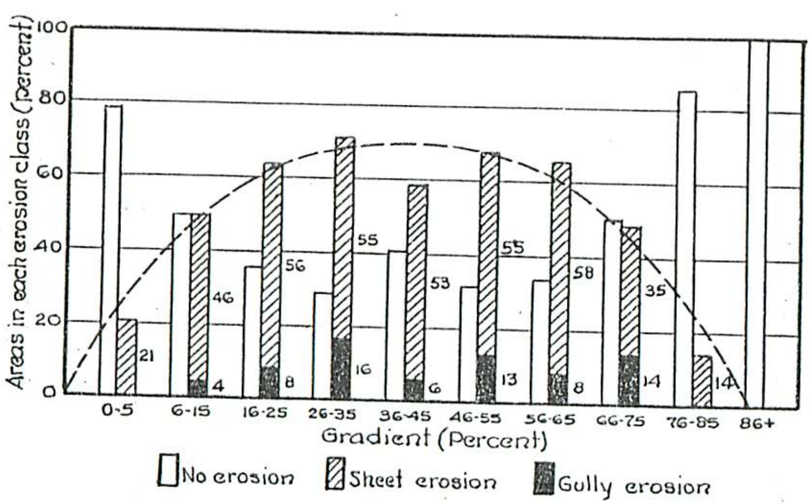


FIGURE 16.—*Gradient and degree of erosion*

(F. G. Renner)

locality, long intervals may elapse during which no erosion occurs on a given slope, than a rain of sufficiently high intensity may cause erosion on a part of the slope.

For example, with a moderately good grass-covered slope, with $R_i = 0.20$ and a slope angle of 5°, a slope 700 feet in length would not be subject to erosion with runoff intensities less than 1 in. per hour, whereas with 2 in. per hour runoff intensity nearly the entire lower half of the slope would be subject to erosion.

Most slopes do not have a uniform gradient from the watershed line to a stream but are flattest near the summit, steepest in the middle portion, and again flat adjoining the stream. For such a slope the belt of no erosion will usually comprise all the upper, flatter portion. If, for example, the slope length is 2000 feet, $q_s = 1.5$ in. per hour, and the mid-portion of the slope has a gradient of 10° and a resistivity of 0.5 lb. per square foot, erosion will begin 1049 feet from the watershed line. If the lower 250 feet of the slope is flatter (its gradient being 5°), then the length of overland flow required to produce erosion with this slope would be 2350 feet. Consequently, no erosion would occur on the lower or flatter portion of the slope. This example shows why erosion is generally confined to the steeper, middle portion of a given slope (Fig. 14).

As another example, on a slope which has been moderately well protected by grass cover, with $R_i = 0.20$, slope angle 5°, and with the limiting maximum value of runoff intensity for the given terrain 2 in. per hour, the width of the belt of no erosion in the maximum storm would be 384 feet. If the slope length was 400 feet, erosion would occur only at the foot of the slope and at rare intervals. If the resistivity of the soil was reduced, for example, by overgrazing, to 0.1 lb. per square foot or half its former value, then the width of the belt of no erosion in a maximum storm would be reduced to 122 feet, and some erosion would occur on the lower part of the slope, while for a runoff intensity of 1 in. per hour the width of the belt of no erosion would be 243 feet. Under the changed conditions some erosion would occur in all storms with runoff intensities exceeding 0.5 in. per hour. An area having a low resistance to erosion and on which erosion occurs over nearly the entire area in the more intense storms becomes partially immune to erosion in lighter storms. Between storms of maximum intensity, resistivity to erosion may be built up by the growth of grass or trees so that when the next succeeding maximum storm occurs the surface resistivity is increased, and the areal extent of erosion greatly diminished. In this manner Nature tends to correct the deleterious effects of surface erosion. Another result of importance is the fact that, on an area subject to erosion only in maximum storms, the total amount of erosion over a given time interval—a century, for example—may be relatively small, while, if the area is subject to erosion in storms with moderate as well as maximum runoff intensities, then because of the much greater frequency of storms of lower runoff intensities, the total erosion will be enormously increased. Not uncommonly the entire surface of the soil is removed in a century or less.

Another factor of importance in relation to erosion is that the soil surface, if protected by vegetation, has commonly a resistance to erosion many times greater than the underlying, unprotected soil. If the surface protection is removed and a maximum storm occurs, erosion will then take place at a rate governed by the lower resistance of the underlying soil. The soil once exposed to direct erosion may then be rapidly removed. Soil removed in the belt of erosion may either be carried away or deposited farther downslope. The manner in which these combined effects develop and control the forms of valley cross sections is considered later.

EROSION RATE

If a factor k_e be introduced in equation (37) to reduce the erosive force to terms of quantity of soil removed, as, for example, the depth in inches of soil material removed per hour, then the erosion rate at the point x would be, making $F_1 = e_r \cdot \frac{1}{k_e}$:

$$e_r = k_e w_1 \frac{\delta_x}{12} \sin \alpha. \tag{42}$$

This equation is rational in form and in fact if the rate at which soil material is torn loose is proportional to the force available from frictional resistance on the soil surface. It relates, however, only to the rate at which soil material can be torn loose and does not take into account the ability of overland flow to transport material in suspension. Equation (42) is limited in its applicability to cases where the erosion rate is less than the transporting power.

TOTAL EROSION AND EROSION DEPTH

Beyond the critical distance x_c and where the overland flow is not loaded to capacity with solid matter in transport, the erosion rate at a given point x may be assumed to be proportional to the net eroding force. Introducing a proportionality factor k_e in equation (39) to reduce erosion force to equivalent depth of solid soil material removed from the surface per unit of time, and making F_1 equal the erosion rate e_r and subtracting the value of F_1 at x_c gives:

$$e_r = \frac{k_e w_1}{12} \left(\frac{q_s n}{1020}\right)^{3/5} f(S) \; (x^{3/5} - x_c^{3/5}). \tag{43}$$

The total erosion between x_c and x is:

$$E_t = \int_{x_c}^{x} e_r \, dx$$

or, letting:

$$B = \frac{5}{8} \frac{k_e w_1}{12} \left(\frac{q_s n}{1020}\right)^{3/5} f(S) \tag{44}$$

and integrating equation (43):

$$E_t = B(x^{8/5} - x_c^{3/5} x). \tag{45}$$

Substituting the slope length l_o for x,

$$E_t = B(l_o^{8/5} - l_o x_c^{3/5}) \tag{46}$$

$$= B l_o (l_o^{3/5} - x_c^{3/5}). \tag{47}$$

The quantity ordinarily measured or measurable in the field is the total erosion per storm. Equation (47) can be used to work back from measured total erosion to the physical characteristics of the terrain which govern erosion rate.

The average depth of erosion between x_c and x is:

$$\frac{E_t}{(l_o - x_c)}.$$

The average depth of erosion is commonly expressed in terms of depth on the entire area, not merely the depth on the part of the area where erosion occurs. When so expressed the average depth of erosion is:

$$E_a = \frac{E_t}{x} = B(l_o^{3/5} - x_c^{3/5}). \tag{48}$$

The coefficient B in this equation is 5/8 of the coefficient of the term containing x in equation (43). Consequently, the average erosion depth over a given area is, for turbulent flow, 5/8 of the erosion depth for the same time interval at the point x.

If the value of x_c is determined from field observation, together with the average erosion depth, the slope length and runoff intensity being known, it becomes possible to determine for a given field or area the erosion force F_1 and the constant k_e. The latter is:

$$k_e = \frac{\text{Erosion depth}}{\text{Eroding force}}.$$

These equations form a practical working basis for determining the erosion constants R_i and k_e and for comparing the erosion conditions on a quantitative basis for different areas.

RELATION OF EROSION TO SLOPE LENGTH

The average depth of erosion on a given slope increases as the 3/5 power of the slope length minus the 3/5 power of the quantity x_c, which is constant for a given slope and storm (equation 48). For example, the relative erosion rates for different slope lengths, with x_c = 100 ft. and B taken at unity, are as follows:

l_o	=	100	200	500	1000	2000	5000
$l_o^{3/5}$	=	15.85	24.00	40.3	63.10	95.0	166.2
E_a	=	0.00	8.15	24.45	47.25	79.15	140.35

For a soil with R_i and x_c each zero, the relative erosion rates would be as shown in the second line of the table—*i.e.*, proportional to $l_o^{3/5}$. The actual relation of erosion to slope length is, however, not quite so simple. In equation (48) and in computing the figures given above, surface-runoff intensity q_s has been assumed constant for all slope lengths. Other things equal, runoff intensity in a given storm decreases somewhat as the slope length l_o increases. Also, if erosion rate is determined as an average for a year or for several storms, the width of the belt of no erosion will vary in different storms. There will in general be more storms producing erosion on the middle and lower than on the upper portions of the slope.

Comparable determinations of soil loss by erosion over a period of 4 to 7 years have been made by the U. S. Soil Conservation Service at several stations, using slope lengths of 145.2, 72.6, and 36.3 feet. Some of the reported results are shown on Figure 17 (Bennett, 1939, p. 152) Differences of soil type, slope and resistivity, rainfall, and infiltration-capacity account for differences in soil loss for a given slope length at the different stations. Variations of these conditions in different portions of the same slope length account for small differences in the relation of slope length to erosion at a given station. In all these experiments the soils were under cultivation, producing corn or cotton, and the values of x_c were small, particularly in summer. The results of these experiments cannot be directly compared with equation (48) because x_c is unknown, and x_c was probably much greater in winter than in summer.

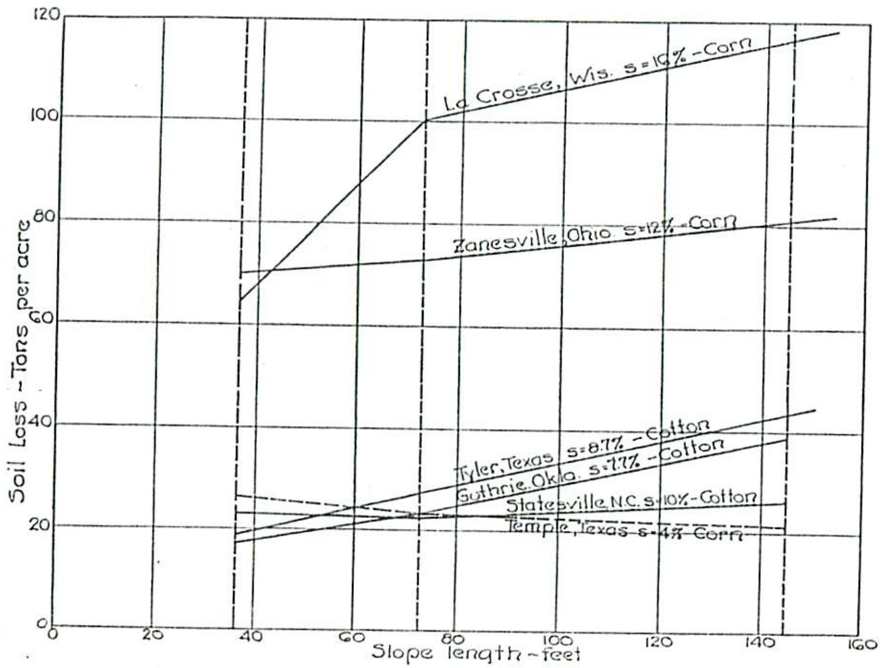


FIGURE 17.—*Relation of erosion to slope length*

From field experiments of U. S. Soil Conservation Service.

Allowing for differences, the results are entirely consistent with equation (48). As shown by equation (48) and also by these experiments, the greater the rain intensity and erosion, the greater, in general, will be the variation of erosion rate with slope length. Rains of low intensity may produce erosion on scattered patches of soil as a result of local variations of infiltration-capacity and resistivity and with little relation to slope length.

RAIN INTENSITY AND EROSION

Maximum rain intensities in a given locality generally occur in storms of the summer thunderstorm type. For such storms the highest rain intensity usually occurs before the middle of the storm and frequently within a few minutes after the beginning of rain. Some soils are easily pulverized when excessively dry but possess coherence through the operation of capillary force when partially dried after a gradual wetting. If an abrupt intense rain occurs on such a noncoherent soil the soil may be beaten into a pasty semifluid mass by rain of high intensity before runoff begins and before the soil surface becomes protected by surface detention. Such a semifluid

mass of soil may accumulate on the surface until it becomes sufficiently fluid, or the runoff intensity becomes sufficiently great, to overcome its plastic resistance to flow. It is then carried into the stream by surface runoff *en masse*. The combination of sudden high rain intensity on previously dried soil of low coherence is not uncommon

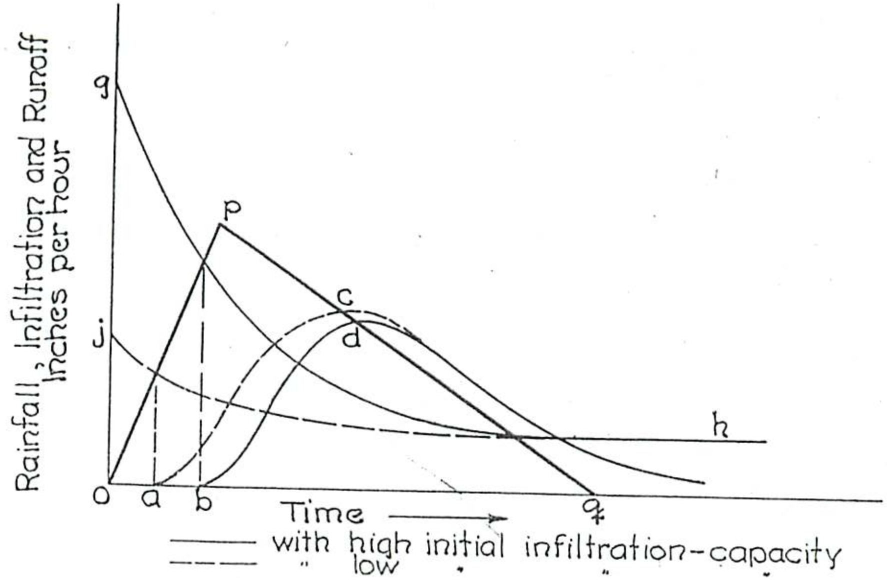


FIGURE 18.—*Relation of initial infiltration-capacity to erosion*

in summer storms in semiarid regions. In addition the initial infiltration-capacity of the dry soil is likely to be abnormally high, and this intensifies the effect of rain impact by increasing the time during which the soil is directly exposed without a protective cover of surface detention.

The combination of the conditions described frequently produces what is referred to as a "cloudburst flood." The term "cloudburst flood" is used because of the characteristics of the flood rather than those of the rain which produces it. Meteorological conditions are, however, involved. Measurements show that, while such floods may carry large volumes of solids, they often carry surprisingly little water as runoff.

In Figure 18 with a rain pattern *opq*, conditions for initially high and initially low infiltration-capacity are shown by solid and dotted lines, respectively. With a high initial infiltration-capacity *og* there is no surface detention or runoff during the interval *ob* during which rain intensity has risen nearly to the maximum. With a previously wet and packed soil and low initial infiltration-capacity *oj*, surface detention and runoff begin earlier at *a*, while the rain intensity is still low. Conditions such as those first described occur both on the upland and in stream channels previously dry. These conditions in both cases usually produce a cloudburst flood, characterized by a wall of turbid water or fluid mixed with debris traveling down the stream channel.

The solid material carried along in cloudburst floods is popularly believed to be derived from stream banks and channels. This may be true, but most of it may be derived from the upland, as is evidenced by the presence of fences, trees, logs, sheep, and other objects derived from the upland enmeshed in the semifluid mass. When

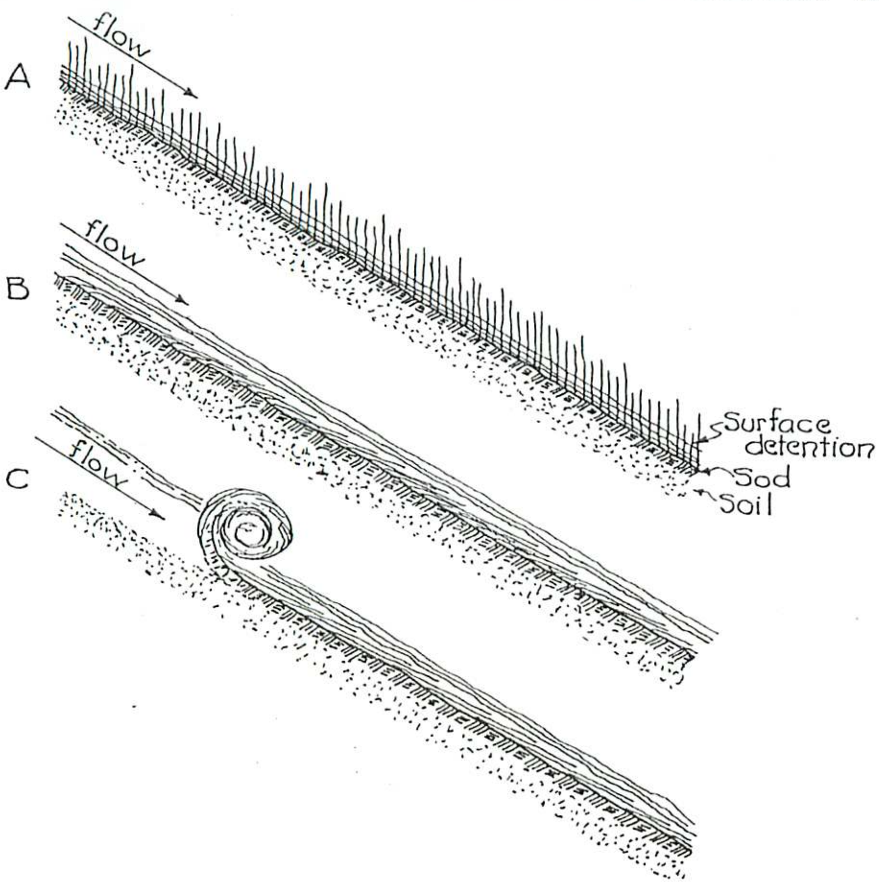


FIGURE 19.—*Erosion of sodded area initiated by the breaking down of grass cover in intense rains*

the flood wave debouches from a mountain canyon, the water in and behind it may escape laterally or by infiltration. The mud flow then slows down, sometimes traveling so slowly that a person could keep pace with it. Finally the mud stops flowing, dries out, and becomes a "fossil cloudburst." Many such "dry floods" may be observed below the mouths of certain canyons in the Wasatch Mountains in Utah. Sometimes one or more fossil floods are superposed pick-a-back fashion. An accumulation of such mud flows may form a debris cone (Horton, 1938; Bailey, 1935; Bailey *et al.*, 1934).

Hydraulic conditions do not permit the occurrence of shallow steady flow on steep slopes (Horton, 1938; Jeffreys, 1925); the runoff water is concentrated in a succession or train of more or less uniformly spaced waves. These waves concentrate the

impact of overland flow on irregularities or obstructions on the surface and greatly accentuate surface erosion. While grass or close-growing crops are in general an excellent preventive of erosion, they may contribute to active upland erosion in cloudburst floods. As long as the grass remains standing it decreases the velocity but increases the depth of surface detention, and the resistance to runoff is exerted at right angles to the grass stems and has little tendency to pull the sod loose (Fig. 19). When a certain depth and velocity of overland flow is attained the grass begins to flatten down, like the tipping down of a row of dominoes standing on end by pushing the first domino against its neighbor. Then (Fig. 19B) the pull of frictional resistance is exerted parallel with the surface. A bit of sod projecting more, or which is relatively less firmly held, than its neighbors is torn loose by the impact of the wave train, and erosion begins. A dense grass sod may be torn up and rolled down the slope like a snowball (Fig. 19C). Often several parallel strips of sod are torn off from the same slope, each strip a few feet in width. The less resistant underlying soil is thus exposed to erosion, and a cycle of erosion may begin on a slope which has been immune to erosion for centuries.

Thus the conditions which initiate soil erosion and govern its rate of occurrence are simply and readily expressible in quantitative form in terms of known independent variables under some conditions, while under other conditions the factors are so complicated that neither the necessary and requisite conditions to cause erosion nor the rate of its occurrence can be predicted or expressed accurately in quantitive terms.

TRANSPORTATION AND SEDIMENTATION

The transportation or carrying forward of eroded material by overland flow or in stream channels takes place in various ways: (1) as bed load or material moved chiefly by being rolled or pushed along the ground surface or along the bottom of the stream channel; (2) material called "suspended load," which is held more or less continuously in suspension by upward currents due to turbulence; (3) material held permanently in suspension by molecular agitation, called the "Brownian movement"; (4) solids in chemical solution in the water. The last mode of transport, while it is the most important process in connection with ground-water erosion, is relatively low in order of importance in connection with surface runoff.

Bed load comprises materials of sizes ranging from large boulders down through cobblestones, shingle, pebbles, gravel, and coarse sand. Mud flows are an important process in the transport of material in overland flow. This is, properly speaking, bed load. As the terms are ordinarily applied in connection with the dynamics of streams, there is no sharp line of demarcation between bed load and suspended load, the former term applying to material carried along, on, or near, the solid boundary. Flat fragments, such as shingle, are transported by sliding or as bed load. Round particles, such as fine gravel, may be transported by a combination of rolling, sliding, and jumping. This is also counted as bed load. The process of transport of particles by hopping from point to point in semielliptic arcs has been described (Gilbert, 1914) as "saltation."

In turbulent flow, eddies thrown off at the solid boundary surfaces have an upward component of velocity. At the same time there is a gradual settling of the water between eddies, creating a downward current equal in volume to that of the upward current in the eddies. The upward velocity and the magnitude and frequency of eddies increase with the velocity of flow and with the roughness of the boundary surface. The eddies are slowly dissipated by viscous resistance as they proceed upward. Both experiment and theory indicate that an eddy originates near the boundary of the fluid as a vortex ring system consisting of the vortex ring and its surrounding field. The fluid comprised in the ring retains its identity throughout the life of the ring, as in the case of the familiar smoke ring. The fluid comprising the field is continually changing, like the water in a wave.

Saltation in its simplest form involves the picking up of solid particles by ascending eddies. Those lifted and transported by the field are usually carried only a short distance; those entrapped near the center of the section of a vortex-ring may be carried much farther, until they are thrown out of the ring by centrifugal force. These two processes are more or less distinct although both depend on the laws of vortex motion. For this reason mathematical analyses of bed load and suspension transport without taking vortex motion into account are likely to prove inadequate and unsatisfactory. A given particle may be thrown out of one eddy at a certain level above the channel bottom, picked up by another, and carried forward, and so buffeted about, like a player in a football scrimmage. It may remain a long time in suspension, finally reaching the bottom, only to rest for a moment and then embark on another wild escapade.

Various attempts have been made to derive empirical expressions for rate of erosion from runoff-plot experiments. In general the conditions of the experiments have been such that it cannot be determined certainly whether the results represent the rate at which the given surface runoff could erode soil material or the ability of the sheet flow to transport such eroded material.

Most of the work done on sediment transport has been in connection with stream channels. Turbulent flow consists of laminar flow on which is superposed the effect of the transverse motion of eddies. If the flow is turbulent, then only a minute fraction of the energy consumed would be required to provide an equal mean velocity of laminar flow. The remaining energy becomes, in effect, latent at the boundary by conversion into rotational energy of vortex motion.

Two principal results follow: (1) The mean velocity is reduced from that for laminar to that for turbulent flow; (2) the velocity distribution is changed from that for laminar to that for turbulent flow.

For the usual slight depths of sheet flow the energy actually used in translational motion of the fluid is a much larger fraction of the total energy available than for types of flow commonly occurring in stream channels. The relative roughness is usually much greater for overland flow than for channel flow. Sand particles 0.001 foot in diameter with overland flow 0.01 foot in depth correspond in relative roughness to boulders 1 foot in diameter in a stream channel 10 feet in depth. Because of these and other differences the extent to which experiments and analyses for channels are applicable to sediment transport in sheet or overland flow is an open question.

Channel flow is ordinarily turbulent except in lakes, while overland flow, except on steeper slopes, usually comprises a mixture of areas on which the flow is turbulent and depressions through which the flow may be laminar.

Much more experimental and analytical work is needed on this problem. However, the following facts appear to be well established and suffice for present purposes:

(1) The transporting power of sheet flow increases with the amount of eddy energy due to surface resistance.

(2) Kinetic energy varies as the square of velocity, and transporting power of sheet flow must vary at least as the square—perhaps as some higher power—of the velocity.

(3) There is a maximum or limiting volume of eroded material which can be transported in suspension by a unit volume of overland flow at a given velocity.

ORIGIN AND DEVELOPMENT OF STREAM SYSTEMS AND THEIR VALLEYS BY AQUEOUS EROSION

RILL CHANNELS AND RILLED SURFACE

The first step toward the gradation of newly exposed sloping terrain is the development of shallow parallel gullies wherever the length of overland flow is greater than the limiting critical distance x_c. These are "rill channels," and a surface covered with such channels is a "rilled surface." Rill channels are usually relatively uniform, closely spaced, and nearly parallel channels of small dimensions which are initially developed by sheet erosion on a uniform, sloping, homogeneous surface. They are sometimes described as "shoestring gullies," but the term "gully" as ordinarily understood connotes larger and less regular channels developed by sheet erosion at a later stage. A rilled surface presents a striated appearance in plan and a finely serrated appearance in cross section.

Excellent examples of rilled surfaces may be found in newly made road cuts, on the slopes of highway and reservoir embankments, spoil banks, and mine dumps. In road cuts, water often drains onto the newly made slope from above the edge of the cut, in which case the rill channels extend the full length of the slope. If such drainage does not occur then the rill channels invariably begin at a little distance below the top edge of the cut. Actually the value of the critical length x_c may be very small, a few inches to a few feet, on newly exposed steep slopes, and the fact that the rills do not extend to the top of the slope would not ordinarily be noticed. Where a rilled surface develops on a newly exposed slope the usual result is the development of a deep central master rill or gully, with more or less parallel, shallower, shoestring rills, decreasing in depth and frequency, on both sides of the master gully. These shoestring rills do not generally survive. The deeper ones close to the master gully are absorbed by the master rill by bank caving or are destroyed by the breaking down of the narrow ridges between them. Those more remote are later obliterated when lateral slope has developed sufficiently to permit cross flow.

On some newly exposed lands, with high infiltration-capacity and high resistivity to erosion, the length of slope from the major divide to the downslope edge of the area may never exceed x_c. Under these conditions a rilled surface may not develop. This condition often occurs on sand-dune areas and in some glaciated areas with deep permeable soils, especially where grass or other vegetal cover develops soon after the disappearance of glacial ice. In the latter case x_c on the newly formed surface may exceed the values of l_o pertaining to the drainage of melt-water from the ice sheet, a rilled surface may not develop, and the topography will remain much the same as when the ice disappeared, except that gradation by solution may take place. In desert regions, with suitable relations between the rain intensity, infiltration-capacity, surface resistivity, and the slope, a rilled surface may develop with little or no cross-grading, so that surface gradation may never extend beyond the rill stage.

ORIGIN OF RILL CHANNELS

"Sheet erosion" implies the formation of either a rilled or gullied surface. From the discussion thus far it would appear that overland flow downstream from the critical point x_c on a smooth uniform surface should remove a uniform layer of soil instead of producing a rilled surface.

The question may fairly be asked: Why does a drainage basin contain a stream system? Surface runoff starts at the watershed line as true sheet flow, without channels. Even below the critical distance x_c it should apparently continue as such sheet flow combined with sheet erosion. Why, then, do rill channels develop? The answer is that channels start to develop where there is an accidental concentration of sheet flow. Accidental variations of configuration may provide the requisite initial conditions where a local area has a lateral slope joining a longitudinal slope or where two lateral slopes join and form a trough.

Most cases of active erosion observed at present represent conditions where there is or has been a protective vegetal cover and the initial resistivity of the soil surface is greater than that of the immediately underlying subsoil.

Consider a point upslope from x_c. If, as a result of change in cover conditions, either the resistance R_i or the infiltration-capacity is reduced, the point x_c may move upslope from the given point, which will then be susceptible to erosion. When the remaining protective cover is broken through at a given point, a channel or gully will form which will proceed rapidly upslope, chiefly by headward erosion, because of the lower resistivity of the underlying soil.

This, however, is not the mode of origin of rill channels, which, it must be presumed, often form on new terrain without vegetal cover and with a value of R_i sensibly the same at and to some depth below the soil surface. Slight accidental variations of topography may produce a sag in which the depth of sheet flow is a maximum at the point a (Fig. 20), the line bb' representing the water surface at maximum runoff intensity. As a result of the greater depth at a, erosion will be most rapid at that point, and increased channel capacity will be provided at a, and part of the water which originally flowed in shallower depths on the adjacent area will be diverted into this enlarged channel. This may accelerate the process until the entire flow is concentrated in the rill channel (Fig. 20). This does not involve headward erosion in the ordinary sense. However, when a rill channel has once formed, sheet flow coming down the slope upstream from the head of the rill will be deflected toward and diverted into the rill channel, thus providing a means of rapid headward extension of the rill.

This process of rill formation can often be observed on a cultivated slope during

heavy runoff. The size and spacing of the rill channels vary with the slope, runoff intensity, and length of overland flow, ranging from a few inches apart on a cultivated slope to many feet or yards apart on long slopes with low runoff intensity and higher erosive resistance. In some areas in abandoned lake beds or exposed coastal

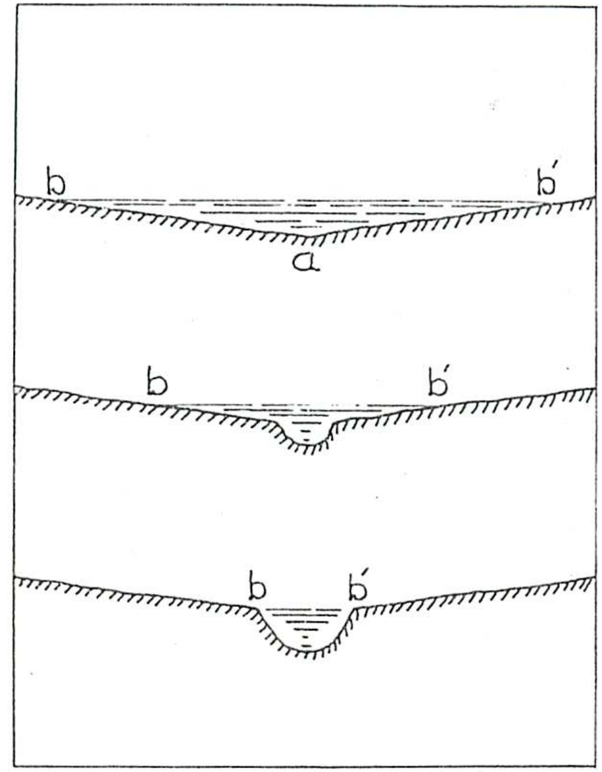


FIGURE 20.—*Successive stages of rill-channel development*

belts in arid regions stream development has never progressed beyond the rill stage. Such an example is given on the Moon Mountain, Arizona-California, quadrangle, U. S. G. S. topographic map (Fig. 21). Later stages of stream-channel development belong to the domain of channel dynamics and involve velocity distribution, silt equilibrium, and other factors which cannot be considered fully here.

The ultimate dimensions of a stream channel are, as indicated by Playfair's law, such that it is adapted to the area which it drains. Stream channels tend to acquire ultimate dimensions such as to carry all or most of the flood waters of the stream. This is largely because most surface erosion and channel erosion occur during floods.

CROSS-GRADING AND MICROPIRACY

A system of parallel shoestring gullies is transformed to a dendritic drainage net as the result of the tendency of the water to flow along the resultant slope lines and

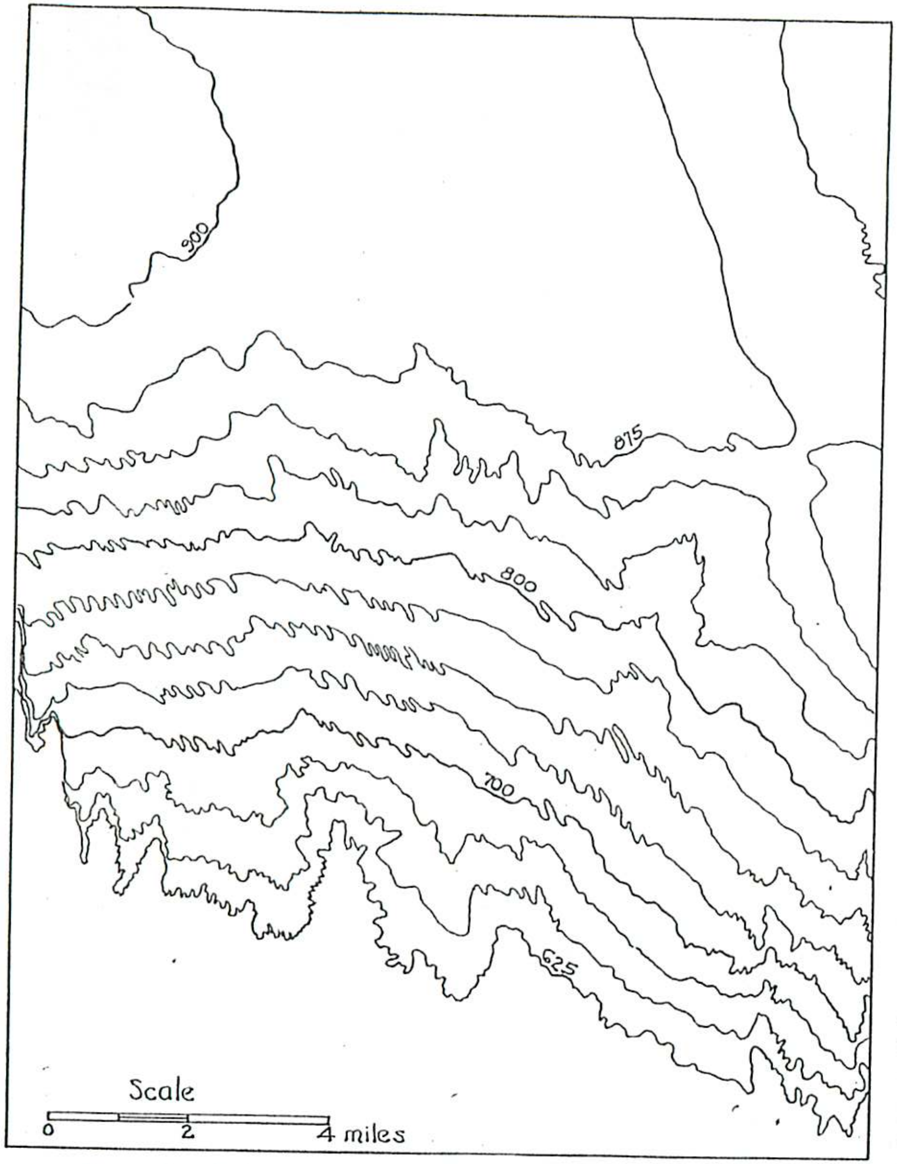


FIGURE 21.—*Portion of Moon Mountain, Ariz.-Calif., quadrangle, U. S. G. S.*

is a direct consequence of the overtopping and breaking down of intermediate ridges between gullies by overland flow during heavier storms.

The deepest and widest rill develops where the net length $l_o - x_c$ in which erosion can occur is greatest. If x_c varies, this may not occur where the total length l_o of overland flow is greatest. The longest, deepest, and strongest rill channel will be called the "master" rill. Owing to smaller values of $l_o - x_c$, proceeding away from the master rill on each side, the rills will be shallower, or, considering two adjacent rills, the bottom of the one farther from the master rill will be higher.

When a storm occurs exceeding in intensity preceding storms on the newly exposed areas, the divide between two rills may be broken down at its weakest point by (1) caving in of the divide between two rills, diverting the higher into the lower rill; (2) erosion of the divide by the deeper or lower rill, thus diverting the higher rill; (3) overtopping of the divide at the low point by the higher rill, again diverting it into the lower rill. This breaking down of divides between adjacent rill channels and diverting the higher into the lower rills is described as micropiracy. Micropiracy much resembles stream capture by lateral corrasion, but micropiracy results chiefly from water overtopping a low spot in the narrow ridge between two rills. Micropiracy obliterates the original system of rills and their intermediate ridges on a uniform newly exposed surface. The process of erosion, in the course of development of a stream system and its accompanying valleys destroys most of the record of their origin. Ultimately the original slope parallel with the stream is replaced on each side of the stream by a new slope deflected toward the stream. This process is described as "cross-grading."

The initiation of cross-grading is illustrated on Figure 22, which shows a plan of a small area of newly exposed land, *aa'bb'*. The line *cc'* marks the downslope limit of the belt of no erosion, *aa'cc'*. The critical distance x_c is assumed to vary with slope, infiltration-capacity, and initial resistance to erosion. Rills develop downslope from *cc'*, and their development is followed by cross-grading, as shown on the cross sections taken on the line *dd'* and numbered 1 to 4, inclusive. The line of resultant slope in each case is in the direction shown by the arrows, and the rills increase in depth and degree of gradation at a given time proceeding away from the lateral boundaries toward the initial or master rill. In section 2 (Fig. 22) the rilled surface has developed, but flat "lands" still persist between rills, and the resultant slope is still parallel with the original slope. In section 3 some rills have combined by cross-grading, creating slight cross slopes, but the overland flow is still carried chiefly by rills parallel with the original slope. In section 4, with increased cross slopes and perhaps a heavier storm, active cross-grading has taken place, especially mid-length of and in the lower portion of the original slope; the direction of overland flow is no longer parallel with the original slope, but overland flow takes place partly in the rills and partly across the intervening ridges, somewhat as shown on a larger scale on Figure 23, and in detail for a single pair of rills on Figure 24. This process can sometimes be observed in heavy storms on lands cultivated nearly but not quite parallel with the contours, the tillage marks corresponding to the original rills above described.

In Figure 22, section 5, the original serrated rilled surface has been obliterated by cross-grading and is replaced by an irregularly roughened surface on which a new rilled surface tends to develop, with flow lines parallel with the resultant slope. This represents the end point of the first stage of valley gradation and stream development. At this stage there exists only one stream in the area, and this follows the course of the initial or master rill. This idealized picture of the cross-grading of a rilled surface is based in part on field observations of eroding slopes and in part on the observed manner of erosion of experimental plots, using artificial rainfall. Under natural conditions the results are rarely so uniform as those shown on Figure 22.

A break across a rill divide may result from numerous causes, such as a rock or

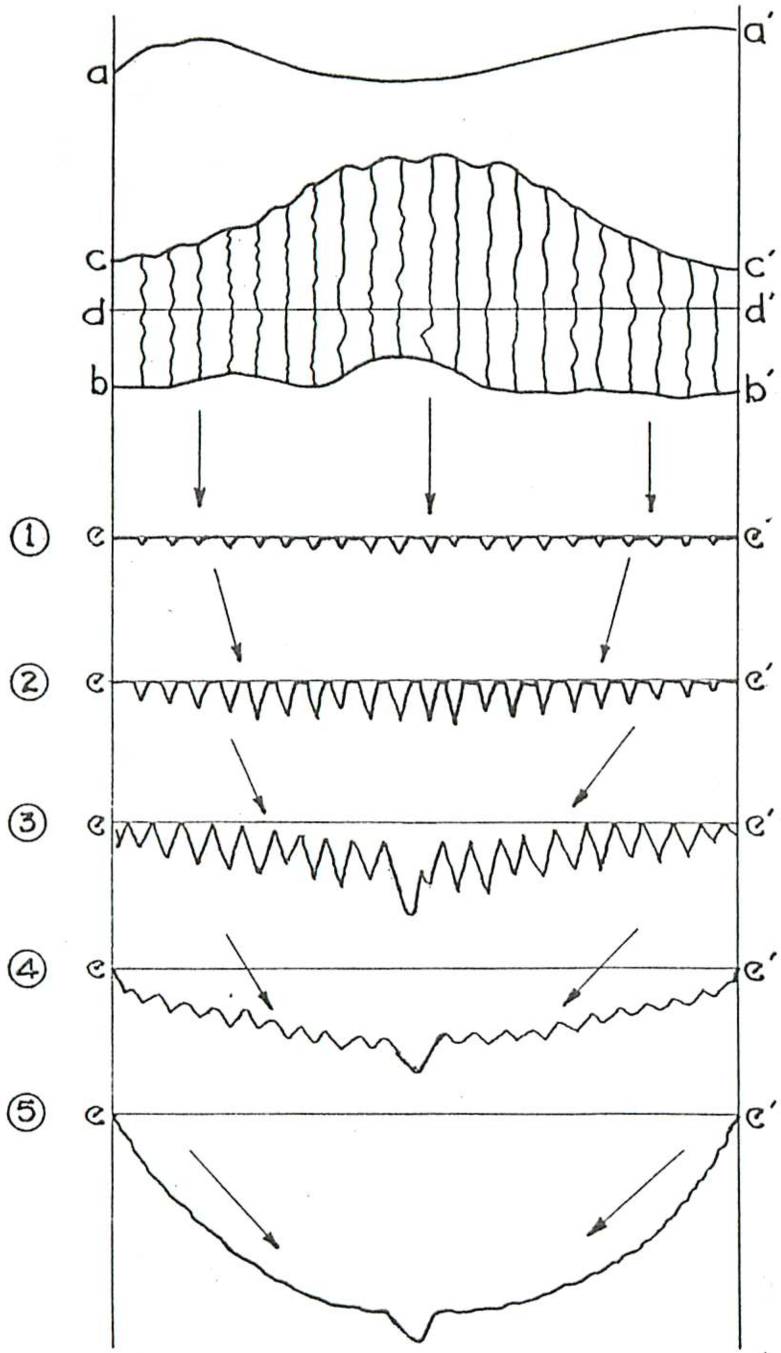


FIGURE 22.—*Development of a valley by cross-grading*

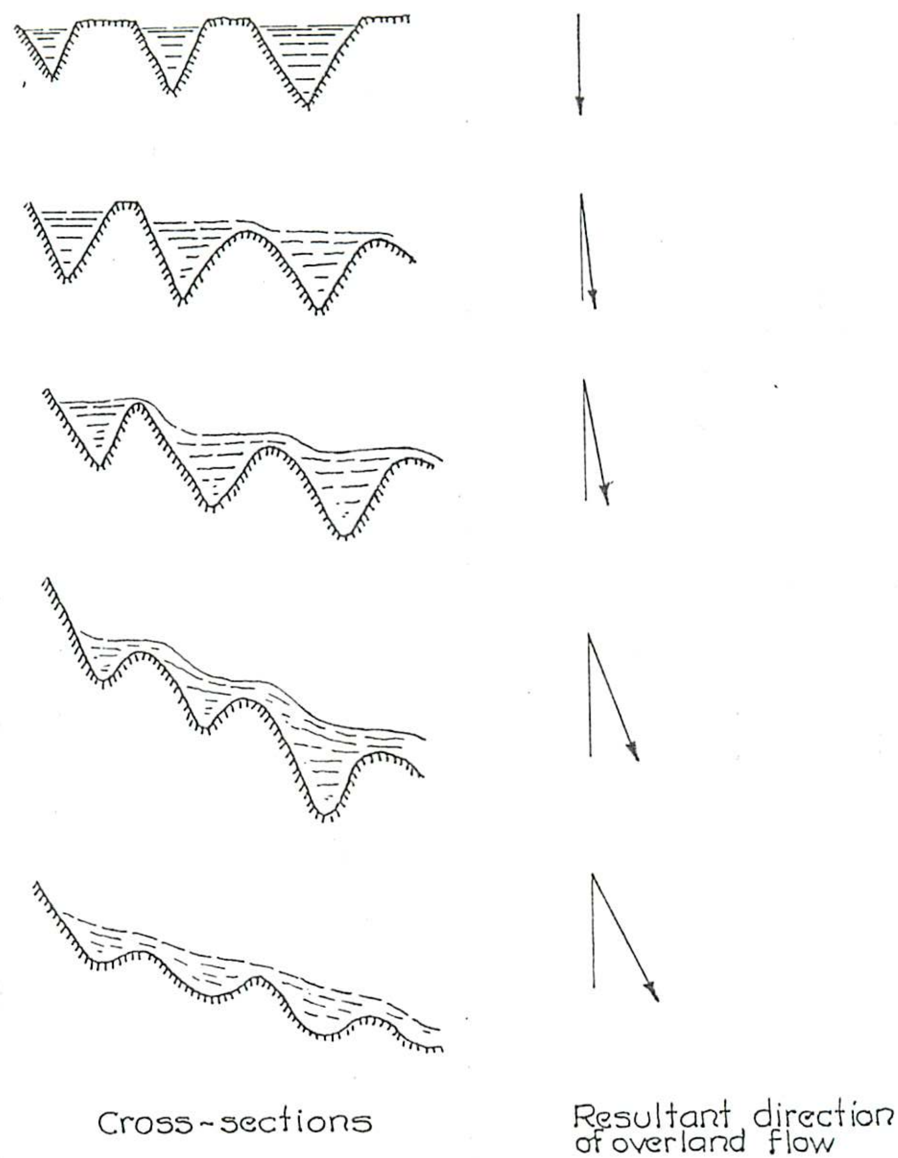


FIGURE 23.—*Successive stages of rill obliteration*

obstruction in the path of the rill, causing back-water upstream therefrom, or the caving in of the bank of the rill, thus obstructing its flow and producing a side outlet. A very common cause is an accidentally low divide between two adjacent rills. In most cases the channel of the diverted rill will be obstructed just downstream from the point of diversion. If the break supplies a free outlet from the diverted rill, then immediately downstream from the break, as at *x* (Fig. 24, B), there will be little or no flow, and erosion will cease, while erosion just above the break will continue. The channel above x will quickly become deeper than the abandoned rill channel below x. There will be thrust, as indicated by the downslope arrows on Figure 24, at the point of diversion, resulting in a tendency toward the formation of a rounded

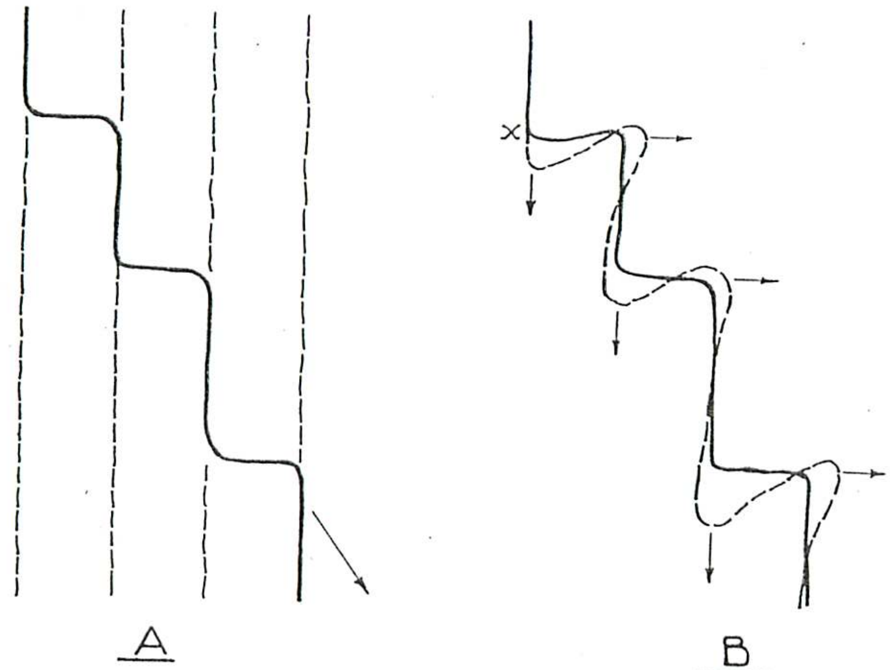


FIGURE 24.—*Development of a stream from a rill system by cross-grading*
(A) Showing angular plan of initial stream; (B) Development of bends by thrust at angles of an initial stream.

bend by impact erosion. Similar thrust across the slope will occur at the second angle of each cross rill diversion, with similar results.

Micropiracy tends to give the resulting stream at first a more or less angular course (Fig. 24) which, as the angular bends are rounded by erosion, will finally develop a more or less tortuous stream course following generally the line of resultant slope. On steeper slopes the resulting stream tends strongly to maintain a straight course and has eroding power sufficient to do so. On flatter slopes, with greatly reduced eroding power, centrifugal force around the initial angular stream bends tends to enlarge their radii. Weak bends merge with stronger ones until ultimately a system of stream meanders is developed. Since the master rill either initially or by cross-grading, on a given slope, ultimately becomes a permanent stream, it appears that conditions favoring the formation of stream bends on flatter slopes are inherent with the origin of the streams.

The elimination of rill systems on gentle slopes is very different from the development of mature meander belts such as those observed in natural streams. Even where stream bends originate in this way the evidence of the intermediate steps is eliminated in the course of the process.

This is not the sole explanation of the origin of stream bends, for individual bends arise from other and accidental causes. The fact that bends occur generally in rather definite systems on flatter stream slopes and in material that is almost perfectly homogeneous seems to require something more than purely fortuitous causes to explain their origin. It is at least significant to find that a stream in its initial stage, as it merges from a rill system, provides the necessary conditions for the development of stream bends on flat slopes.

HYDROPHYSICAL BASIS OF GEOMETRIC-SERIES LAWS OF STREAM NUMBERS AND STREAM LENGTHS

General statement.—A conventionalized illustration is given of the main steps involved in the development of a drainage net, showing the hydrophysical basis of the geometric-series laws of stream numbers and stream lengths. A square area with its diagonal parallel with the direction of slope will be assumed. This roughly approximates in form a typical ovoid drainage basin.

First stage.—On Figure 25, *oabc* is a uniform surface sloping toward o. The maximum length of overland flow is the diagonal length $l_o = ob$. A storm produces surface runoff of intensity sufficient to reduce the critical distance x_c to some value $bg < l_o$. Sheet erosion can then occur over the area *ofgh*. The runoff intensity q_1, in cubic feet per second per unit width, will increase proceeding from a and c toward the center line nearly in proportion to the length of overland flow. There will be no erosion at f or h. The erosion intensity will increase from these points along the line fh toward the center line. The maximum intensity of surface runoff cannot occur until surface detention is built up to a point where the inflow to and outflow from surface storage are equal. Since the unit runoff intensity q_1, other things equal, increases nearly as the length of overland flow, the critical intensity necessary to induce erosion will be exceeded first along the center line of the area and then progressively later proceeding toward f and h (Fig. 25, a). Sheet erosion will begin first along the diagonal line og, spreading more or less rapidly toward the points f and h. In a typical storm the duration of surface-runoff intensity adequate to produce erosion will also be greatest near the center line of the area, decreasing toward the sides. Consequently, erosion will begin first, be most intense, and last longer near the center line of the area. A series of more or less parallel gullies will develop (Fig. 25, a), decreasing in depth and frequency, proceeding away from og, forming a rilled surface.

In Figure 25a the lines of overland flow are parallel with the central or master rill. Actually as the valley develops, the lines of overland flow will follow the resultant slope lines converging toward the central rill first at acute angles and, as the slope increases, at increasingly larger angles, approaching right angles as a limiting case.

This is the first stage of stream and valley development. It may later produce further upslope erosion in more intense storms. At the end of the first stage (Fig. 25a), there is a single central stream, with a V-shaped valley.

Second stage.—The development of the first stream (Fig. 25a) has divided the area and reduced the maximum length of overland flow on the remaining areas on each side to $\frac{1}{2}\, l_o$ or to half the initial maximum length of overland flow. Lateral tributaries have not yet developed because (1) Lateral flow cannot occur unless there is a lateral component of slope, and (2) until the lateral slope extends far enough from the main stream so that in a given storm the value of the critical length x_c is reduced

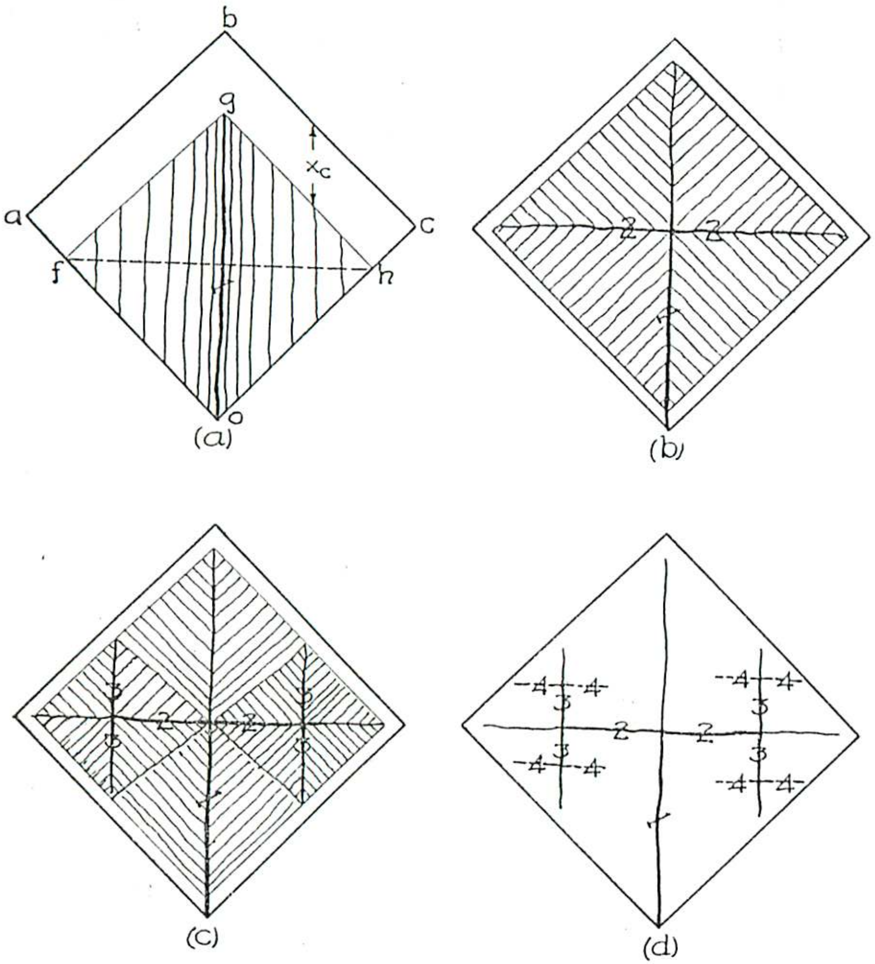


FIGURE 25.—*Development of a drainage net in a stream basin* (Schematic).

to less than the width of the lateral slope, or in case of complete development of the initial valley, the value x_c must be reduced to less than $\frac{1}{2}\,l_o$.

The second stage involves the development of lateral tributaries and their valleys. During a storm of sufficient intensity to meet the prerequisites described, a pair of master rills or lateral tributaries with accompanying shoestring gullies will develop, one from each of the equal areas on opposite sides of stream 1 (Fig. 25b). These lateral streams are designated "2," and their development and that of their lateral slopes will follow the same course as in the case of stream 1. At the end of the second stage the area will appear as shown by Figure 25b. In the meantime, further headward development or downward gradation, or both, of the valley of stream 1 may have occurred.

Subsequent stages.—At the end of the second stage the maximum remaining length of overland flow in the area has been again halved or reduced to $\frac{1}{4}\,l_o$, and a still more intense flood is required to reduce the critical length x_c to a value less than this and permit the development of a third group of lateral tributaries. When such a flood occurs, each of the No. 2 tributaries will develop a pair of lateral tributaries, designated "3" on Figure 25c.

The development of lateral slopes adjacent to a given stream brings in overland flow from additional areas, increasing downstream, and accelerates grading of the main stream and its immediate valley. The stream system at the end of the fourth stage is shown by Figure 25d. The number of the stage of stream development corresponds to the order of the main stream. The main stream is of the 1st order at the end of the 1st stage, and of the 4th order at the end of the 4th stage.

Development of lateral tributaries and the manner in which they develop is the direct consequence of (1) the existence of a critical length x_c of overland flow required to institute erosion; (2) the operation of cross-grading. Drainage patterns, while invariably following the two fundamental geometric-series laws as to stream length and stream numbers, can still develop in an infinite variety of ways.

Two questions naturally arise: (1) What would happen if the newly exposed area was a continuous belt along the coast line, with no lateral boundaries? (2) Why and how are the boundaries of drainage basins developed? This case will be considered later.

ADVENTITIOUS STREAMS

Differences occur between the development of streams under natural conditions and those assumed in the example because:

(1) The drainage area is usually not rectangular but ovoid.

(2) Newly formed tributaries follow in general the resultant slope of the cross-graded areas on which they develop and hence enter the parent stream at more or less acute angles; the steeper the slope, the more acute is the angle of stream entrance. Tributaries thus tend to be longer than if they entered at right angles, and the stream-length ratio is consequently usually greater than 2.0.

(3) There are nearly always variations—sometimes large variations—of infiltration-capacity in different parts of the area.

(4) There are also variations—sometimes extreme—in the initial resistance of the terrain to erosion, as, for example, where part of the area is in consolidated and part in unconsolidated material, or part covered with vegetation and part bare. As a result of these departures from hypothetical conditions, the following results often occur:

In certain parts of the area the length of overland flow to the parent stream or its larger tributaries may be less relative to x_c than on the remaining areas tributary to the last group of streams developed. Then some 1st, and perhaps 2nd, order tributaries develop, entering the main stream or larger streams directly and not through higher-order tributaries. These streams, which result from accidental variations of conditions within the area, may appropriately be designated "adventitious" streams. The development of adventitious streams increases the number of streams of lower orders and tends to make the bifurcation ratio greater than 2.0, as it usually is for natural streams. Lateral slope also increases the length of tributaries and makes the stream-length ratio also greater than 2.0. An increase of r_l is also

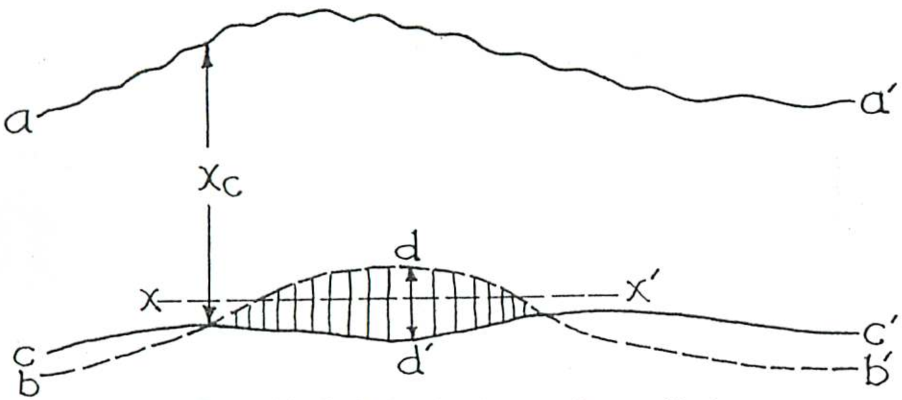


FIGURE 26.—*Beginning of erosion on newly exposed land*

produced because streams do not extend to the watershed line although they may extend by headward erosion to a distance less than the critical minimum value of x_c from the watershed line. Adventitious streams usually increase the bifurcation ratio more than the stream-length ratio is increased by the conditions described, with the result that the ratio $\frac{r_l}{r_b}$ is in general a fraction, and the total length of streams of a given order is not constant but decreases proceeding from the lowest to the highest stream orders. Adventitious streams do not in general develop simultaneously with larger streams in the basin but are developed later as the development of the stream system approaches maturity.

STREAM DEVELOPMENT WITH PROGRESSIVELY INCREASING LAND-EXPOSURE COMPETITION

For illustration the exposure of coast marginal lands will be assumed to be nearly a uniform homogeneous sloping plane, extending from a divide line aa' (Fig. 26) to the new coast line cc'. It is assumed that the soil surface in the newly exposed belt $aa'cc'$ is initially bare and has a certain infiltration-capacity f and a surface resistance to erosion R, such that the critical length x_c required to permit surface erosion to occur in the most intense rain is as shown on the diagram. The dashed line bb' is at a distance x_c from the watershed line aa'. As long as the coast line is within the belt $aa'bb'$, no streams will develop, runoff will be in the form of direct sheet flow to the new coast line, and no erosion takes place. There will be irregularities in the watershed line aa' and in the coast line cc', and, when the length of overland flow exceeds x_c at some point d, erosion will begin at that point. When the coast line has reached the position cc' there will be a small area, as outlined by a dashed line, within which $l_o > x_c$, and within this area sheet flow will produce erosion and a series of rill

channels parallel with the direction of the initial slope surface. The first rill channel will be at dd', where the length of overland flow first exceeds the critical length x_c. As the coast line recedes the belt in which erosion can occur will increase laterally and longitudinally, and the system of rill channels will be extended correspondingly

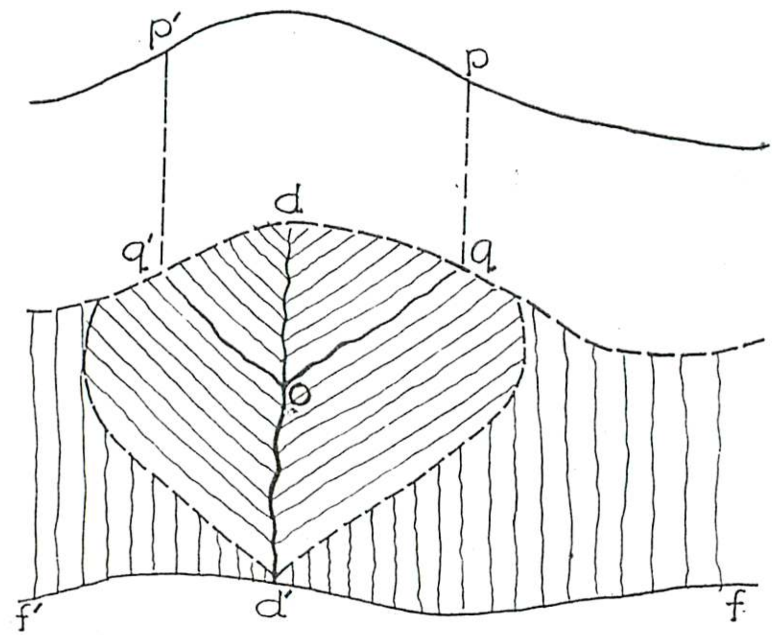


FIGURE 27.—*Development of first pair of tributaries on new stream system*

in both directions. The rill at dd' was first formed and has been longest subject to erosion and will become the master rill. Cross-grading will take place, producing new components of slope toward the rill dd' and obliterating the original rill channels. New rill channels will develop following the new direction of slope, on each side of the original stream dd' (Fig. 27). In general the lengths of these new rill channels will increase proceeding down the slope from the line cc'. At some point o (Fig. 27) a new rill channel will have a greater length oq and greater runoff than rills between o and d. It will have developed earlier than rill channels entering the parent stream dd' between o and d'. It therefore has greater runoff and a longer duration of runoff in which to cut its channel than rills formed farther down the slope. Such rill channels will survive as a tributary stream. Such a rill channel occurs on each side of the parent stream in the vicinity of o, and cross-grading toward these tributary streams will also occur. Cross-grading of the areas adjacent to these two tributaries will produce cross-graded slopes on either side of each tributary (Fig. 28), until there is again a location on each of these areas favorable for the development of tributaries, and new tributaries will develop, usually one on each of the two preceding tributaries, as at m, n, and p (Fig. 28). This process will continue until finally there is no land surface above the mouths of the original tributaries where the length of overland flow exceeds the critical length x_c.

If the coast line recedes farther (Fig. 29), the area upslope from oo' on the right hand side of the stream dd' is tributary to the stream oq. The original rill channels parallel with dd' upslope from oo' have been obliterated, and the runoff from the area

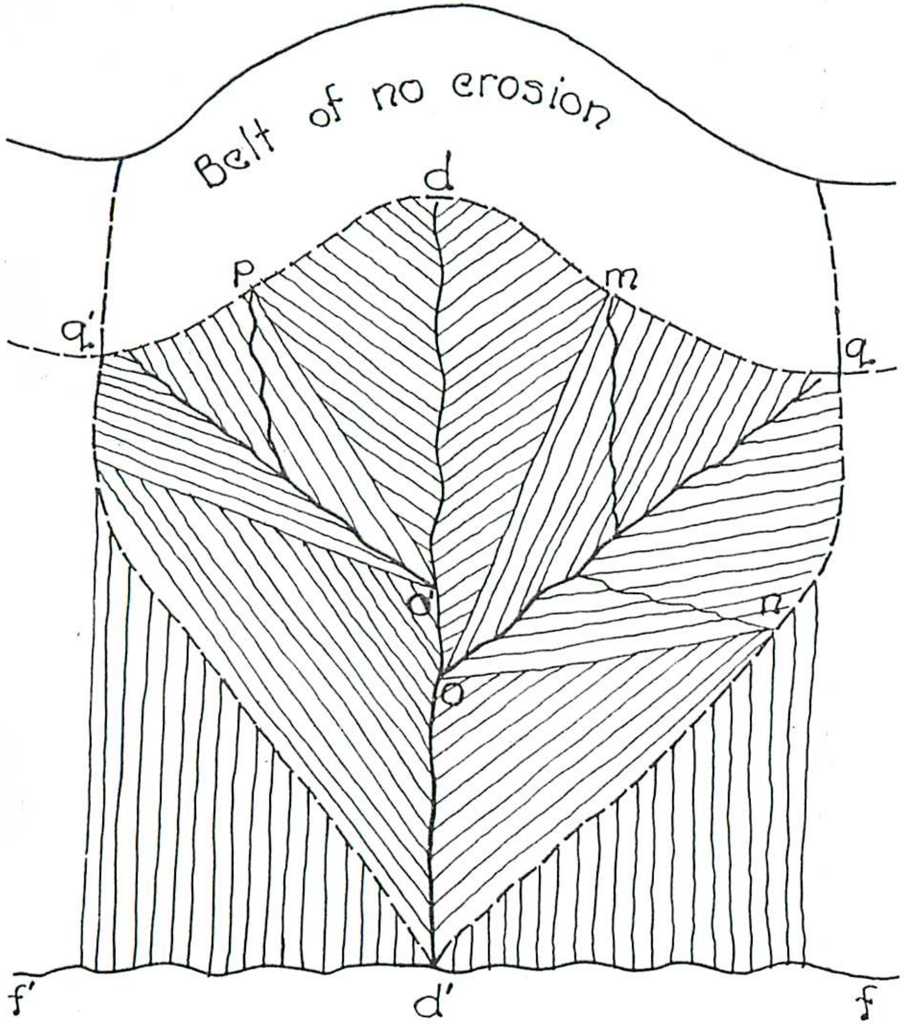


FIGURE 28.—*Lines of flow after cross-grading of first pair of tributary areas*

$oo'o''$ now enters stream oq. As the coast line recedes a new system of rill channels parallel with dd' develops downslope from oo', and the area $oo'd'd''$ will become cross-graded toward dd'. When the length of overland flow within the area $oo'd'd''$ becomes sufficiently great at some point q', a new tributary $q'r$ will develop along the line of the resultant slope, and its basin will in turn be developed by cross-grading. There must be a certain minimum space or intercept between tributaries of the main stream to provide adequate length of overland flow to permit a lower tributary to

the main stream to develop. Furthermore, the tributary $q'r$, having developed much later than the tributary oq, will extend its drainage area laterally more slowly than the latter, with the result that the drainage basin will tend to have an ovoid outline (Fig. 29).

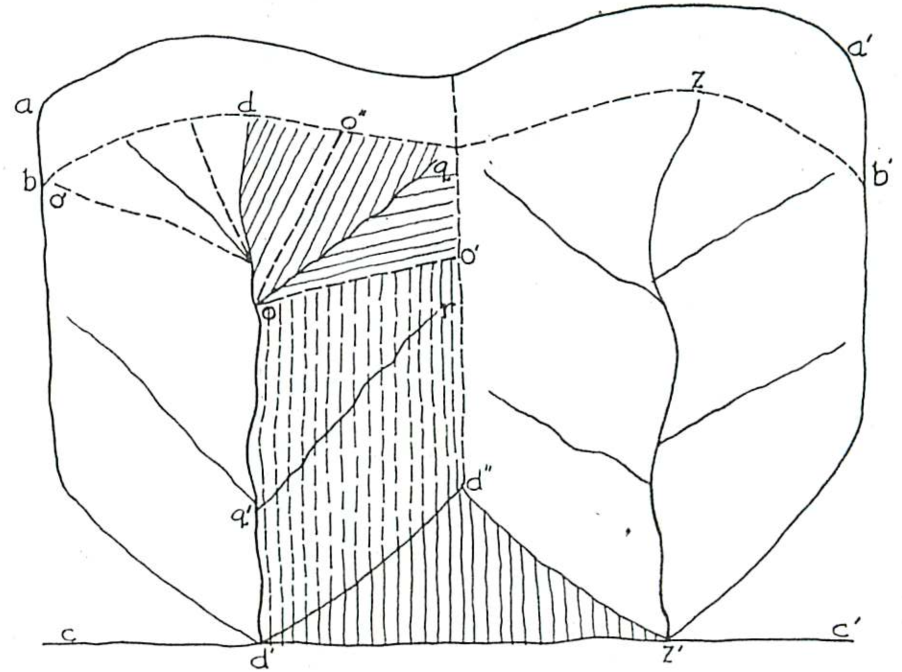


FIGURE 29.—*Development of lower pairs of main tributaries*

Another stream may also develop at zz' in the same manner as the stream at dd'. The development of this stream may have begun either a little earlier or a little later than the stream dd', and the final location of the lateral divide between the two drainage basins will be determined by the conditions of competition. The older stream will absorb the greater part of the area between the two streams. A marginal area of direct drainage $d'd''z'$ is left between the two major drainage basins. If the length of overland flow here becomes sufficient, an intermediate subordinate stream will develop.

The appearance of the final stream systems in the two drainage basins will be somewhat as shown by Figure 30.

Two major factors control the development not only of the drainage basin of a given stream but the systems of drainage basins tributary to a new coast line:

(1) Streams develop successively at points where the length of overland flow becomes greater than the critical length x_c.

(2) Competition results in the survival of those streams which have the earliest start or had the greatest length of overland flow, or both, and which are therefore able to absorb their competitors by cross-grading.

END POINT OF STREAM DEVELOPMENT

Stream development on a newly exposed slope continues until the greatest remaining length of overland flow is less than the critical distance x_c required to institute erosion.

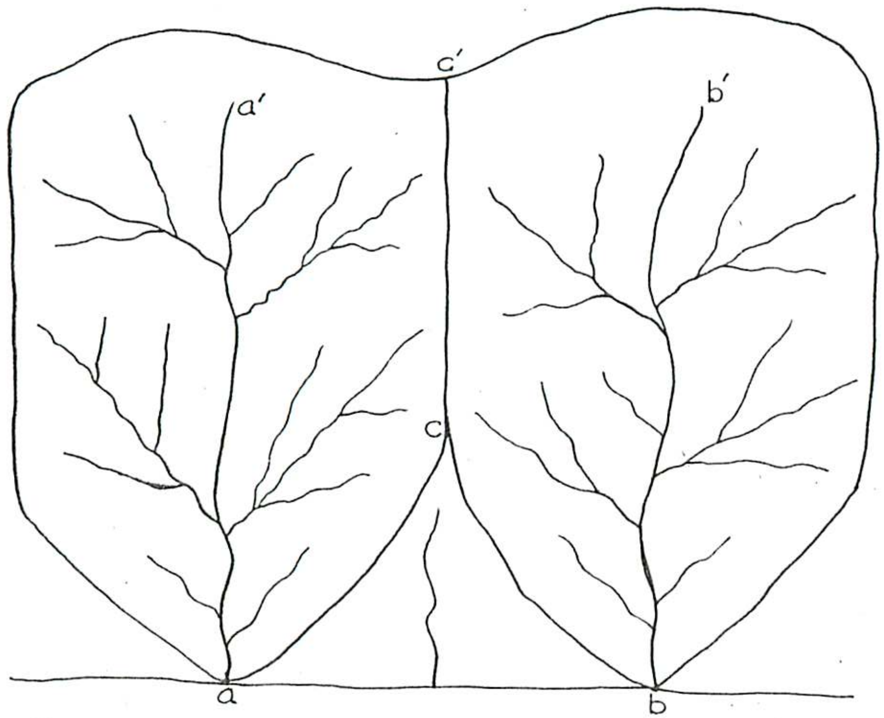


FIGURE 30.—*Final development of two adjacent drainage basins on newly exposed land*

At a certain stage of gradation (Fig. 31) the stream *oa* has developed with a drainage basin *ocd*. Before cross-grading of this area the critical length x_c is, for example, equal to that shown by the line *mm′* on the insert, and this is less than *oa*. After cross-grading of the area *ocd* this critical length is somewhat reduced by increased resultant slope and is now *mn*. The greatest lengths of overland flow on the areas *oca* and *oad* are now along the slope lines *de* and *ce*, but these are both less than *mn*. Hence no additional streams will develop in the area *ocd*.

The upper ends of the streams in a drainage basin will extend at least to the distance x_c from their watershed line, measured in the direction of slope. They may be extended closer to the watershed line by headward erosion, under suitable conditions.

For streams to be perennial at their sources there must be ground-water flow at the head of the stream channel. In regions where there is a permanent ground-water horizon under the drainage basin the most common condition is that the stream is intermittent for a distance downstream from the point where its channel begins. Figure 32 shows the profile at the head of a stream. The watershed line is at *a*, and a definite channel begins at *b*. There is a water table underneath the headwater

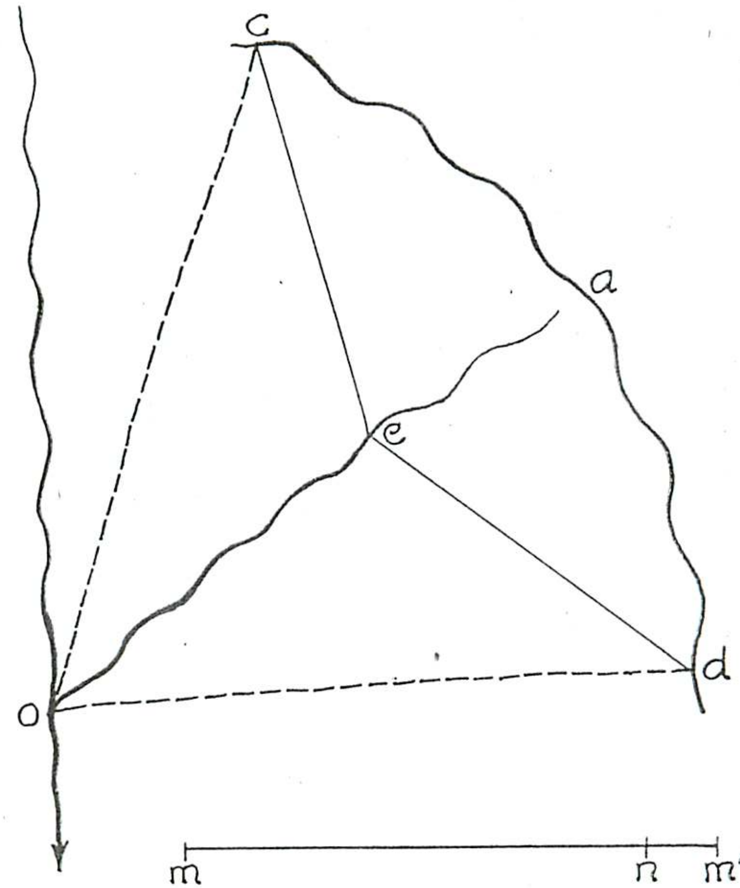


FIGURE 31.—*End point of stream development*

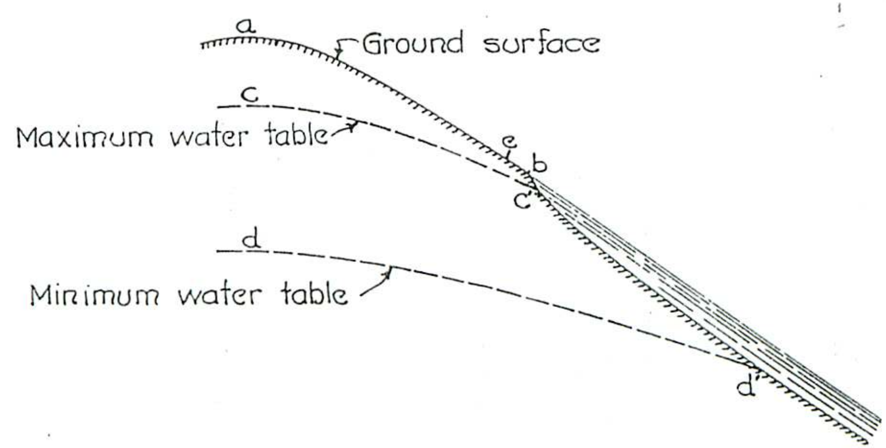


FIGURE 32.—*End point of a definite stream channel*

belt of no erosion *ab*, the maximum ground-water table is at *cc'*, and the minimum at *dd'*. Between *cc'* and *dd'* the stream is intermittent. At *c'* part of the infiltration on the upper drainage area enters the stream. At times of maximum surface runoff the ground-water flow may represent a considerable fraction of the total flow. If,

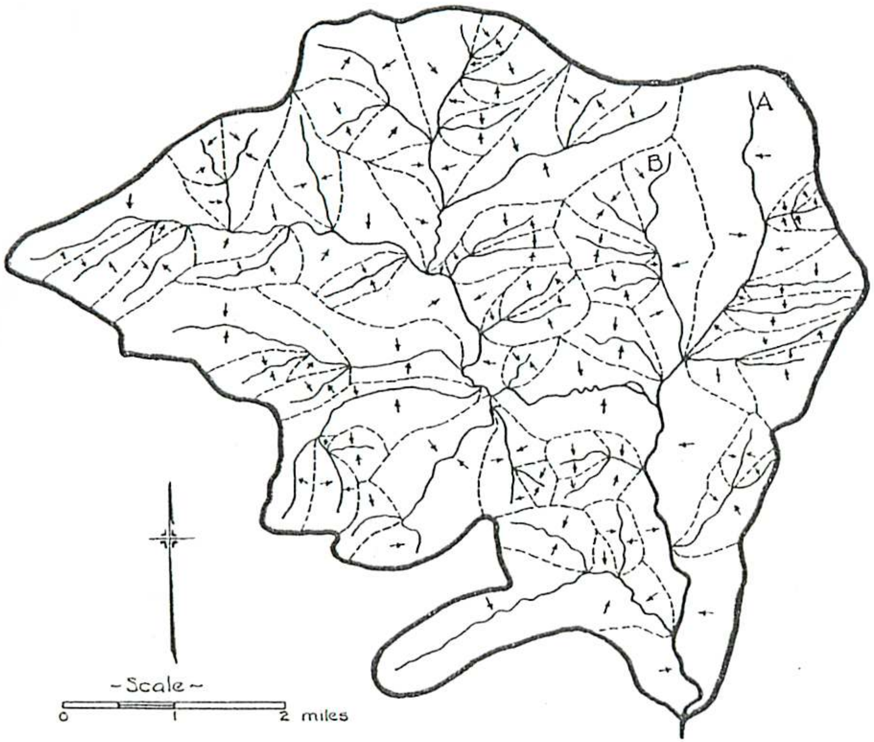


FIGURE 33.—*Drainage basin of Pennypack Creek*
Above Valley Falls, Pa., showing subareas from which surface runoff is derived.

for example, the ground-water flow is one fourth of the total flow at *c'*, then, if the channel extended a little farther upslope to *e*, the maximum runoff would be reduced one fourth by elimination of ground water. There is therefore an abrupt and sometimes considerable change in the total runoff at about the point where the maximum level of the water table intersects the stream channel. Surface runoff plus ground-water flow can generally extend the channel upstream farther by headward erosion than could surface runoff alone. Hence the channel usually ends near the point where ground-water flow is no longer effective. Ground-water flow at *c'* is intermittent, but it usually continues much longer than surface runoff and by maintaining the soil at the head of the stream channel moist and soft it promotes extension of the channel by headward erosion and bank caving.

The final results of stream development under natural conditions are illustrated by Figure 33. Some of the streams in the lower part of the basin are clearly adventitious. There are several drainage basins, such as *A* and *B*, where tributaries have

developed only on one side of the parent stream, leaving, in this case, an isolated plateau in the interfluve area, although the drainage development of the basin is evidently mature.

STREAM-ENTRANCE ANGLES

From geometrical considerations the following equation has been obtained for the entrance angle between a tributary and the higher-order stream which it enters (Horton, 1932)[4]:

$$\cos z_c = \frac{\tan s_c}{\tan s_g}$$

where z_c is the entrance angle between the two streams; s_c is the channel slope of the parent or receiving stream; s_g is the ground slope or resultant slope, which is here assumed to be the same as the slope of the tributary stream.

Values of the entrance angle computed by this equation for different values of the ratio s_c/s_g are as follows:

s_c/s_g =	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
z_c =	25.5°	36.8°	45.5°	37.0°	60.0°	66.2°	72.3°	78.3°	84.2°

As shown by Table 4, stream slopes are always less than the adjacent ground slope, and tributaries should enter the confluent stream at acute angles when the slopes of the channels of the tributary and confluent streams are nearly the same. The equation takes on the indeterminate form 0/0 if the two slopes s_c and s_g are equal. This means that the two streams will be parallel and will not join. Three cases will be considered for purposes of illustration.

CASE 1—FLAT STREAMS DEVELOPED ON A FLAT AREA: When the parent stream has developed and cross-grading has proceeded to a point where a pair of tributaries develop, the parent stream will in general have cut into the initial surface to some depth, and its stream slope in the vicinity of the debouchure of the tributaries will be materially less steep than the original slope, while the slopes of the tributaries as they approach the parent stream will be materially steeper than the original slope. As a consequence, instead of the ratio s_c/s_g being close to unity, this ratio will seldom have a value greater than 1/2 or 1/3, and the tributaries will not enter the main stream at acute angles, as would be the case if s_c and s_g were nearly equal, but will more generally enter the parent stream at angles of 60° to 80°. On extremely flat surfaces in humid regions a swampy condition often prevails, and stream-entrance angles are but little subject to control by erosion conditions. On semiarid plains where little erosion occurs, acute entrance angles of tributaries to the parent stream may sometimes be observed.

CASE 2—FLAT VALLEY SLOPE WITH MODERATE TO STEEP ADJACENT GROUND SLOPE: Under these conditions the ratio s_c/s_g is nearly always low, and the stream-entrance angles to the main or parent stream are commonly 60° or greater. As the

[4] Derivation of this equation is given correctly in the reference cited. Interpretation of the equation as there given is incomplete and not wholly correct.

stream system develops, the slope of the main stream steepens proceeding upstream, and the lateral ground slopes also steepen proceeding upstream. The ratio s_c/s_g may remain sensibly constant, or it may either increase or decrease. Most commonly it decreases to some extent. Quite generally the entrance angles of tributaries to the main or initial stream are quite uniform and range from 60° upward, decreasing somewhat upstream.

CASE 3—TRIBUTARIES ON A STEEP SLOPE: Tributaries developed on the same slope generally run nearly parallel, and if the main valley is relatively flat they will enter the parent stream at an angle of 90°, representing a limiting condition which is approached but not often attained. Tributaries developed on the same lateral slope may of course join and are especially likely to join where drainage development is incipient, as on steep, rocky slopes and in semiarid regions where tributary development has been arrested at the end of the rill stage. Parallel tributaries which join on a steep slope under these conditions commonly have an acute angle of juncture. In this case the ratio s_c/s_g is close to unity.

DRAINAGE PATTERNS

Much has been written regarding the forms of drainage patterns. They are usually classified as dendritic (treelike), rectangular or trellised, radial, and centripetal. The terms radial and centripetal commonly refer to the arrangement of a group of drainage patterns originating at or converging to a common point and do not refer in general to the pattern in an individual drainage basin. All drainage patterns of individual drainage basins are treelike, but different patterns resemble the branchings of different kinds of trees and range from those with branches entering the parent stream nearly at right angles, to those with tributaries nearly parallel and entering their parent streams at small angles. The form of the drainage pattern depends to a large extent on the relation of the slope of the parent stream to the resultant ground slope after cross-grading. If this ratio increases with successive cross-gradings, stream-entrance angles of successive tributaries are somewhat more acute for successively lower-order streams, affording the most usual type of dendritic drainage pattern.

On a relatively flat surface the directions of resultant overland flow after the first cross-grading are nearly at right angles to the initial stream, and the second series of streams developed enter the parent stream nearly at right angles. Cross-grading of the areas tributary to these streams produces but a slight change in the slope ratio s_c/s_g, so that the next order of streams also enters the parent streams more or less nearly at right angles. In this way a rectangular drainage pattern is developed.

If, on a steep, sloping, original surface, the headwater divide forms roughly an arc of a circle, then the first two tributaries developed will enter the parent stream from opposite sides at nearly the same point (Fig. 34). These streams will develop long tributaries nearly parallel with the initial stream, giving rise to a centripetal drainage pattern (Fig. 34).

On flat slopes each successive cross-grading of a given subarea changes the direction of the next stream to develop on the area through an angle approaching 90° as a

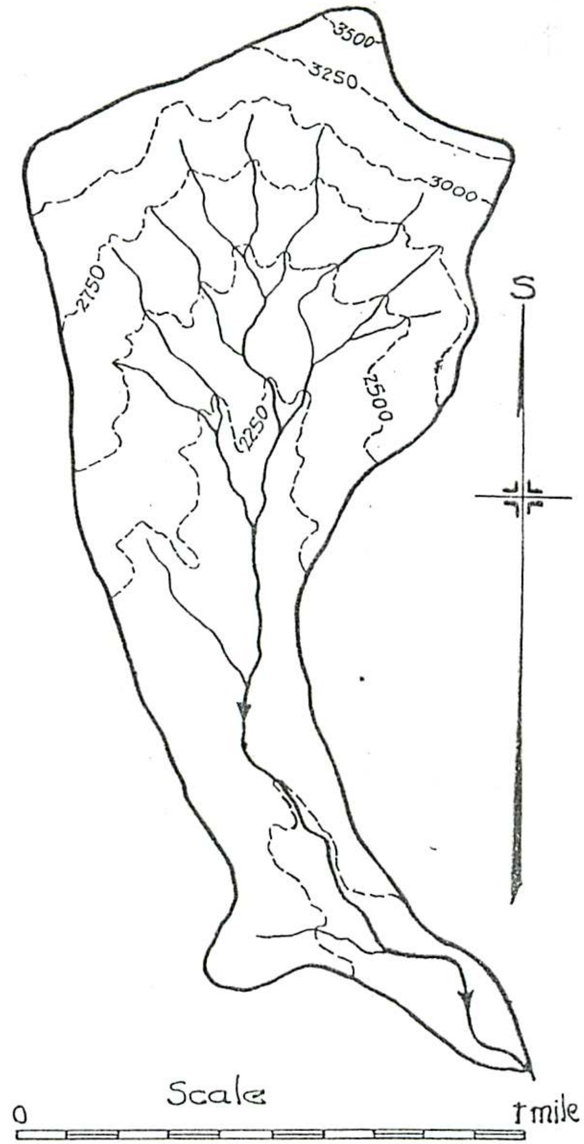


FIGURE 34.—*Centripetal drainage pattern*
Payne Creek, Ga., Mulky Gap quad., U. S. G. S.—T. V. A.

limit and changes the direction of overland flow through a corresponding angle. The direction of resultant cross-graded slope at the end of a given stage becomes the direction of the stream of the next succeeding stage. The directions of streams and of resultant slopes will change through nearly a right angle with each successive stage of stream development and cross-grading, and the directions of streams and of resultant slopes tend generally to be the same in any two stages of stream development which are either both even numbered or both odd numbered.

ASYMMETRICAL DRAINAGE PATTERNS

Because newly developed tributaries enter their parent streams at acute angles, they divide their tributary areas into two parts such that the remaining upslope tributary areas are larger than those on the downslope side, using the terms "upslope" and "downslope" with reference to the two sides of the tributary. Because of inequality of area, width, and slope on the two sides of a tributary, the next lower order of tributaries may develop with two or three tributaries on the upslope side and fewer or none on the downslope side, a common phenomenon, particularly in mountain areas. Since the average elevation of the upslope area is greater than that of the downslope area, this phenomenon is sometimes attributed to increase of rainfall with elevation. It may occur, however, as the result of differences of tributary area and length of overland flow on the upslope and downslope sides of the parent streams, independently of variation of rainfall or runoff on the drainage basin. Burch Creek and Reels Creek drainage basins (Utica, New York, quad., U. S. Geological Survey) afford examples of asymmetrical drainage-basins.

PERCHED OR SIDEHILL STREAMS

In general, streams follow the bottoms of the valleys in which they are located. Small—usually 1st order—streams are occasionally perched precariously on the side slopes of graded valleys of higher-order streams. The course of such a stream is often more nearly parallel with the antecedent slope than with the cross-graded slope. At the foot of the slope the stream often turns abruptly and debouches into the parent stream at nearly a right angle (Fig. 35). Evidently gradation of the valley of the parent stream *cd* reached the stage shown in the figure before the slope became steep enough to reduce the critical length x_c below the maximum length l_o of overland flow on the right-hand side, and l_o became greater than x_c only when gradation of the valley slope had reached the end point. A weak stream, *ab*, then developed by micropiracy and cross-grading, but owing to some local cause, such as increased resistivity of the soil to erosion at increased depth below the original surface, this stream was unable to develop a valley of its own by further cross-grading and so remained high above the parent stream on the antecedent rilled surface, until, with increasing volume and slope, it turned nearly a right angle as it entered the parent stream.

REJUVENATED STREAMS; EPICYCLES OF EROSION

In the preceding sections it has been assumed that: (1) Uplift or exposure of new terrain took place continuously though not necessarily at a uniform rate, the region finally becoming stable; (2) the initial resistance R_i of the soil surface to erosion remained constant. The effect of subsequent further elevation or subsequent subsidence of an area on which a stream system has already developed has been extensively discussed in connection with the Davis erosion cycle (Wooldridge and Morgan, 1937) and will not be considered further here. Before leaving the general subject of stream development and valley gradation consideration will be given to the effect of (1) differences between surface and subsurface resistivity to erosion, (2) changes in the surface resistivity to erosion.

The term "rejuvenated stream" is applied to a stream system in which a renewed cycle of erosion begins and which may extend the drainage net after it has reached maturity. Rejuvenation may result from several causes, although in the Davis

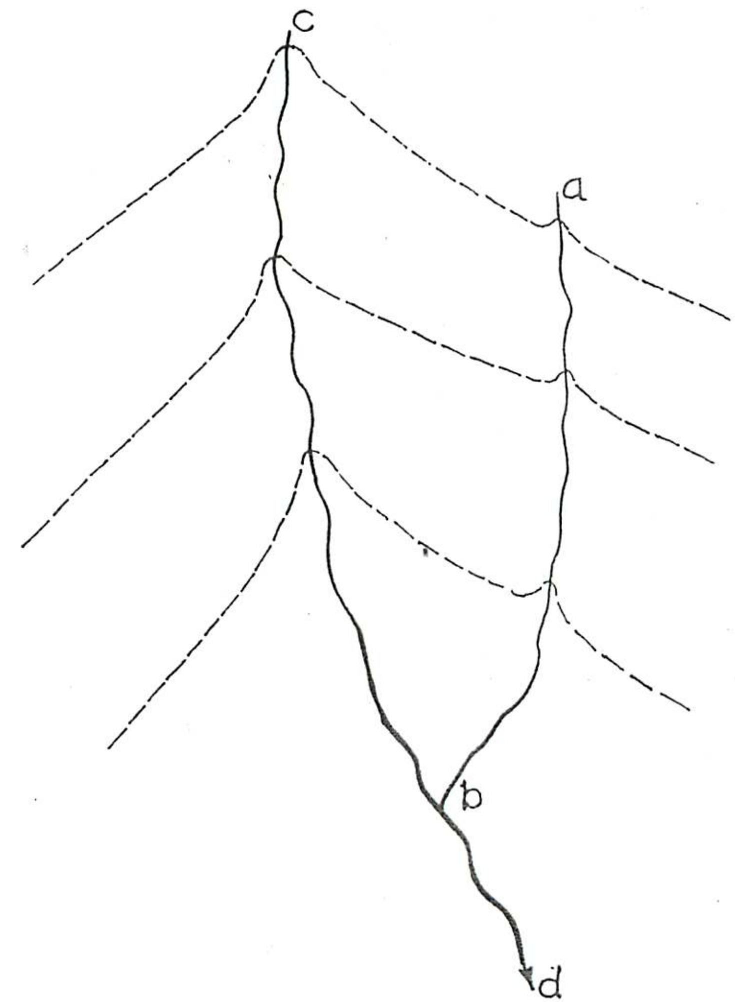


FIGURE 35.—*Perched or hillside stream*

sense the term is applied chiefly where it results from widespread geologic changes such as renewed uplift, folding, and tilting.

Accelerated or decreased erosion may result without any such geologic changes if the original terrain varies in erosional resistivity or infiltration-capacity proceeding downward from the surface. Then, as erosional gradation takes place, changes in the critical length of overland flow x_c will occur, and if these changes are abrupt they may result in important effects, either (1) marked increase in drainage density and extension and number of minor tributaries, if R_i and x_c decrease downward from the surface, or (2) abandonment and fossilization of pre-existing streams and tributaries, if R_i and x_c increase with increased gradation.

A third condition may also bring about changes in erosion rate and stream development which is more common than rejuvenation due to strictly geologic causes. This occurs where, as the result chiefly of climatic or cultural changes, there is a change in the surface-erosional resistivity or infiltration-capacity of the terrain which brings about changes in the critical length x_c and in the consequent development of drainage.

Accelerated erosion due to the removal or replacement of an initially resistant surface by a less resistant surface has been appropriately described by Bailey (1935) as an epicycle of erosion. This term is appropriate since it implies a marked changed in erosional and gradational activity, superposed on the normal erosional conditions. Changes in erosional conditions brought about by dust storms and the formation of loess veneer on soil surfaces, and changes in erosional activity resulting from improper cultivation of the soil, deforestation, fires, or overgrazing of range lands, afford excellent examples of epicycles of erosion.

Where a less permeable and more resistant surface layer of soil or sod overlies weaker or more permeable subsoil, there will be in effect two different values of x_c, one pertaining to the surface layer, the other to the underlying material. This occurs where well-established grass or other vegetal cover overlies a noncohesive sandy soil or where there is a layer of loess or similar fine-textured material, with moderate or high cohesiveness, overlying more permeable and less cohesive material, such as sand.

If the overlying resistive material is broken through, the value of x_c pertaining to the underlying material governs subsequent stream development. In such cases the development of a drainage net is likely to be erratic and sporadic. On much of the area there may be but few streams. This will be true where the larger or surficial value of erosive resistance R_i and critical distance x_c are effective. At other locations where the smaller subsurface values of R_i and x_c have become effective, active and extensive stream development may take place. Extensive plains, for the most part undisturbed by erosion, may be dissected by rapidly growing and irregularly branching systems of gullylike channels. This condition exists in the Pontotoc Ridge region of the Little Tallahatchie, Mississippi, drainage basin, where deep incoherent sand is overlain with a thin veneer of fine uniform loessal silt. In this region x_c for the underlying sand is practically zero, and stream development may extend far above the x_c limit for the surface material as a result of headward erosion. The author has observed gullies in the Pontotoc Ridge region which in some cases have extended not only to but somewhat beyond the topographic boundaries of their drainage basins (Happ *et al.*, 1940). This has resulted from the slumping of masses of earth from the nearly vertical and sometimes undermined scarp formed by the erosion of the deep, incoherent sand.

The destruction of vegetation by smelter fumes early in the present century in the vicinity of Ducktown and Copper Hill, Tennessee, brought about a new erosion cycle. Glenn's early report (1911) and the author's later observations show that forest and hills sometimes protected the sod locally even where the trees were killed,

and where the sod was protected no erosion occurred. As described by Glenn (1911, p. 78):

> "The erosion starts near the bottom of a slope, and where the soil is porous rapidly cuts a steep-sided gully to a depth of 5 to 12 feet below the surface, where the underlying schist is as a rule still measurably firm. After a gully has reached its limit in depth it widens until its walls coalesce with the walls of adjacent gullies, by which time most of the soil has been removed."

Over much of the denuded area erosion has not been as complete as that above described. Narrow flat lands still persist between the parallel gullies, and uneroded, nearly flat summits of the hills are conspicuous. In some cases the gullies afford excellent examples of cross-grading in progress, with remnants of the antecedent rill surface still visible.

The erosional topography of this region was essentially mature before denudation took place wherever there was a well-established sod cover. The resistivity of the underlying soil to erosion is, however, so small that, lacking protection, the critical distance x_c is reduced nearly but not quite to zero. Consequently the walls between initial parallel ridges on steep slopes have sometimes coalesced, as described by Glenn. Within a few years after destruction of the vegetation the drainage density was increased locally from ten to one hundred fold, and where this occurred the end point of the new erosion cycle was quickly attained.

In the gully formation in the Pontotoc Ridge region in Mississippi and in the vicinity of Ducktown, Tennessee, surface and subsurface resistance R_i differed, the surface resistance being initially greater and the terrain initially stable against erosion. Reduction of surface resistance resulted from improper cultivation in the Pontotoc Ridge region and from partial destruction of vegetal cover by smelter fumes in the Ducktown region, and an active epicycle followed in each case. The formation of arroyos on overgrazed land affords another example of an epicycle of erosion where the value of x_c is less for underlying soil than for undisturbed surface cover.

DRAINAGE-BASIN TOPOGRAPHY

MARGINAL BELT OF NO EROSION; GRADATION OF DIVIDES

In addition to controlling the drainage density and the composition of the drainage pattern and fixing the end point of development of a stream system on a given area, the critical distance x_c and the belt of no erosion which it produces govern the degree of gradation which can occur on a given area and the extent of gradation along and adjacent to both exterior and interior watershed lines or divides.

If the angle between the watershed line aa' (Fig. 36A) and the direction of overland flow is A, then for a given critical length x_c there will be a belt of no erosion on the given side of the watershed line having a width

$$w_v = x_c \sin A.$$

This marginal belt of no erosion $aa'cc'$ is relatively permanent. It is widest, other things equal, where the direction of overland flow is most nearly normal to the watershed line; this is usually around the headwaters of an exterior divide. The width of the marginal belt of no erosion decreases for a given x_c as the direction of overland

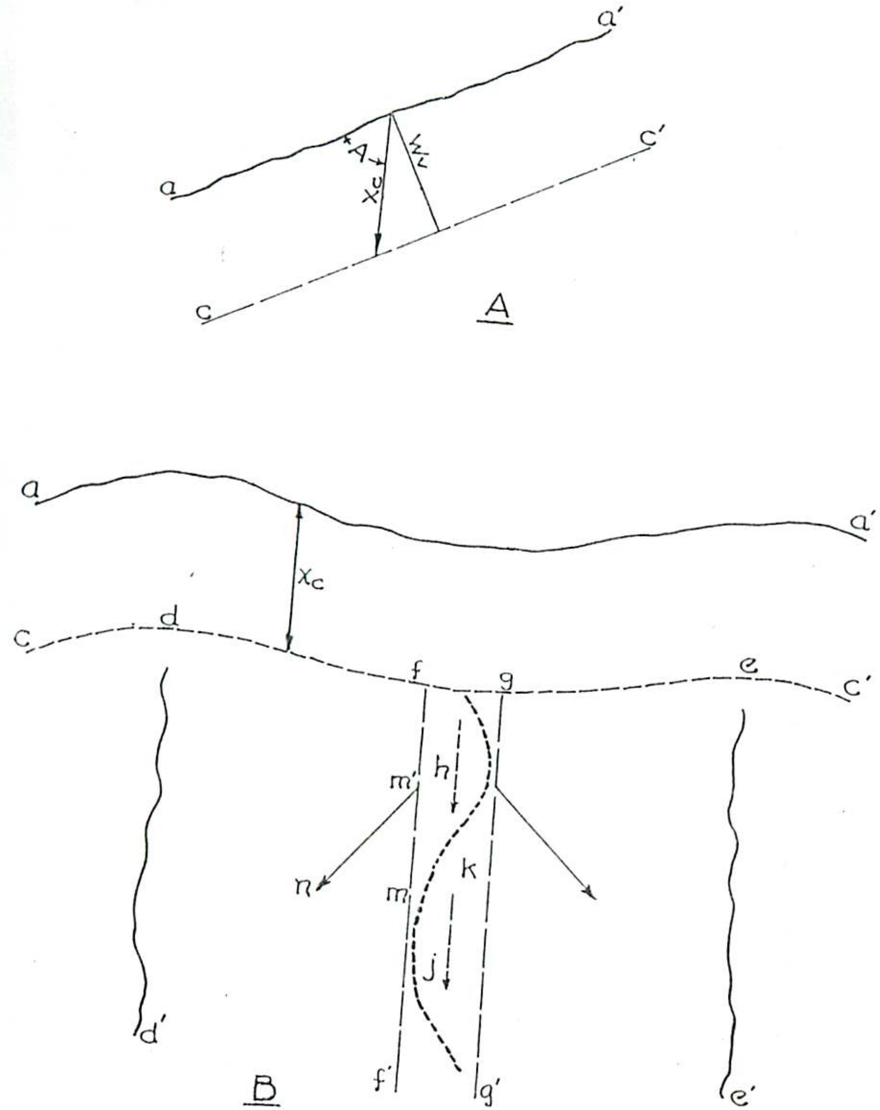


FIGURE 36.—*Belts of no erosion*
(A) Width of belt of no erosion. (B) Longitudinal belt of no erosion.

flow becomes more nearly parallel with the direction of the divide, a condition which commonly occurs along lateral segments of the main divide surrounding a drainage basin.

If aa' (Fig. 36B) represents the exterior divide at the head of a newly exposed area, then, with sufficient newly exposed surface, streams will develop, starting at d and e. The entire slope from cc' to the outlet is subject to sheet erosion. Cross-grading begins adjacent to the streams and spreads laterally until there remains a narrow belt $ff'gg'$ not yet cross-graded. Dashed arrows indicate directions of overland flow

antecedent to, and solid arrows the corresponding directions with, cross-grading. This belt has, however, been previously subject to sheet erosion since it lies downslope from the headward belt of no erosion, and the direction of overland flow is parallel with the slope. The profile of the belt $ff'gg'$ is concave, and it lies, except at its ends, considerably below the original slope. The narrow belt $ff'gg'$ is still subject to cross-grading. Slight variations in surface conditions will divert most of the surface runoff at a given location, as at h, into one stream or the other. The divide between the streams will move away from the stream into which the diversion occurs. The direction of overland flow on the diverted area will swing around until it is more or less parallel with that on the adjacent cross-graded slope, and a belt of no erosion will develop on the side of the divide on which diversion occurs. This belt will have a width $x_c \cos A$, where A is the runoff angle between the diverted surface runoff and the antecedent slope. This angle will vary from zero to $mm'n$, and the width of the belt of no erosion on the given side will vary accordingly.

At some other location, j, the stream ee' will gain the advantage in competition with gg', and the watershed line will be deflected toward ff'. As a result the watershed line will become sinuous, as shown by the dashed line on Figure 36B. Intermediate between h and j the streams will divide the runoff more or less equally. The watershed line will cross the center of the belt $ff'gg'$, but at this location most of the runoff will have been diverted at h, and there will be less erosion than at either h or j. As a consequence of the competitive development of divides the width of the belt of no erosion will vary from point to point, governed locally by the slope, the direction of overland flow, and the amount of previously undiverted surface runoff originating within the belt of no erosion. The watershed line will be sinuous in plan and profile, and the watershed ridge will be broken up into a series of irregularly spaced hills, often with flat crestal plateaus, and adjacent hill crests will be at about the same elevations. The hills will be separated by saddles, and both will be rounded not only as a result of the manner of their development by aqueous erosion but also by secondary processes, such as earth slips and rain-impact erosion.

A favorable location for flat-top, interfluve hills is at the junction of a longitudinal and a cross divide. Such junctions commonly occur where there is an angle or bend in the parent divide. Under these conditions the flat-top hill usually has an arm extending out onto the interior divide. Flat-top hills and plateaus may also occur at intermediate locations where there is a relatively wide belt of no erosion.

On Figure 37, aa' and bb' are adjacent tributary streams which developed more or less simultaneously on the same side of the parent stream and which flow nearly parallel, and crosswise of the original slope. When these streams have developed on an antecedent slope, cross-grading will occur, spreading laterally on both sides of each stream. Dashed arrows show directions of overland flow on the antecedent surface, and solid arrows show the corresponding directions after cross-grading by the streams aa' and bb'. Most of the surface runoff on the area $aa'm$ will be diverted from the parent stream into stream aa' by cross-grading.

Downslope from aa' this stream can divert only the runoff from the area $aa'c$. Overland flow on the area upslope from the watershed line ac will be parallel to this line, while on the area $acde$ the antecedent direction of overland flow will still persist, and a belt of no erosion will develop. The stream bb' will receive the runoff from the area $bb'f$ on the upslope side and from the area $bb'g$ on the downslope side. The areas aca' and bgb' will have been subject to at least two cross-gradings, and as a result the direction of overland flow on these areas will have been turned nearly

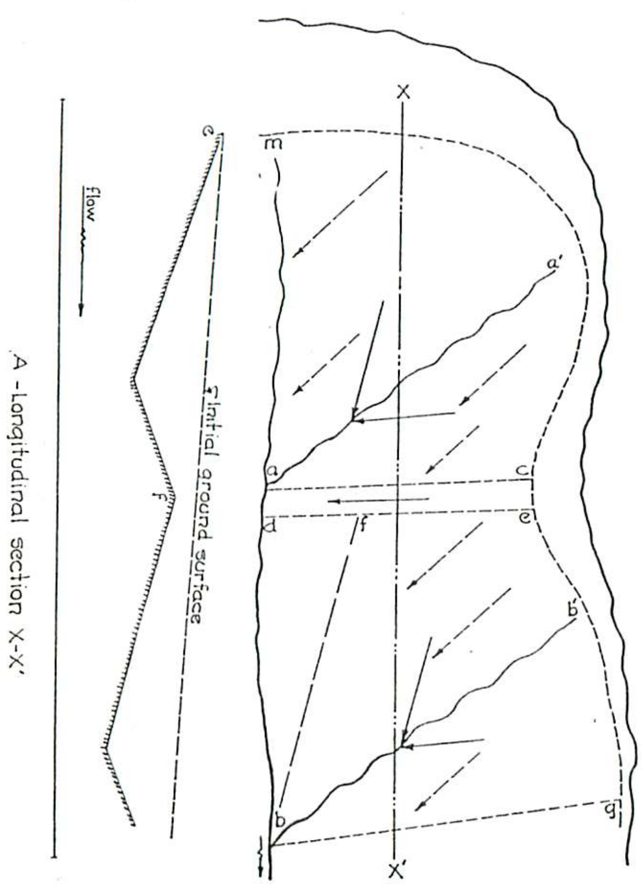


FIGURE 37.—*Belt of no erosion at a cross divide*

through a right angle. The direction of overland flow on such areas downslope from streams running crosswise of the original slope may, of course, have either a downslope component (Fig. 37), or it may have its direction of flow reversed with respect to the original slope. This happens if the direction of overland flow is deflected through more than 90°.

A longitudinal section along the line xx' is shown on Figure 37A. The initial surface is shown by a dashed line. In spite of the fact that e is higher than f, the resultant slope is not materially different on the wide and narrow sides of the valley, a fact often noticed on topographic maps.

It has been shown that the occurrence of a belt of no erosion along an interior divide between streams parallel with the original slope is contingent on the development of components of flow across the divide by micropiracy. A belt of no erosion usually occurs on each side of the divide but is relatively narrow in relation to x_c, and in the vicinity of saddles between crestal hills it may have been subject to cross-grading during its development. If a divide runs crosswise of the drainage basin (*acde*, Fig. 37), the belt of no erosion will temporarily be subject to longitudinal erosion, but presently, as a result of erosional competition, hills and saddles will develop, breaking up the longitudinal components of overland flow into elements each less than x_c, as in case of a longitudinal divide. A crosswise belt of no erosion will usually be wider, and the interfluve hills and plateaus developed thereon will usually be larger and with flatter tops, than in case of a divide running parallel with the original slope.

As a drainage system develops, additional belts of no erosion are introduced along the new interior divides, thereby reducing the portion of the total area over which sheet erosion can occur, other things equal. These later divides have been longer subject to gradation than those developed earlier, and they are generally at lower elevations relative to the original surface. Streams that ultimately become the higher-order streams of the drainage basin usually develop early in the erosion cycle, and their divides are usually higher relative to the original surface than those of lower-order tributaries.

A belt of no erosion once developed persists throughout subsequent stages of gradation although subject to variations in width with subsequent cross-grading of the adjacent terrain.

If the drainage basin of a tributary is narrow and steep on one side, with overland flow at right angles to the divide, the belt of no erosion may extend from the watershed line to the stream on that side, while a flatter slope or overland flow at an acute angle on the opposite side may permit erosion over all or a part of the area on that side.

Discussion thus far has related chiefly to the earlier stages of gradation of a drainage basin where the length of overland flow is generally much greater than the critical length x_c. At later stages of stream development the critical length x_c is decreased by successive subdivisions, with the birth of new tributaries, until finally there remains little or no intermediate length of overland flow between the belts of no erosion and the streams.

A practical illustration of a belt of no erosion is afforded by slope terracing. This introduces a system of artificial watershed lines or cross divides on the terraced slope such that the remaining lengths of overland flow are everywhere less than the critical distance x_c, and the area between a terrace divide and the next one downslope constitutes a belt of no erosion.

Nature accomplishes a similar result in the development of a drainage-basin system by the successive development of tributaries of lower orders, thereby cutting down the length of overland flow until it does not exceed the critical length x_c anywhere within the drainage basin. The drainage net is then complete. Since the development of tributaries of successively lower orders in a stream system does not go on indefinitely, the drainage density approaches a finite limit. Drainage density seldom exceeds 3.0 and commonly is 1.0 to 2.0 in humid regions where soil erosion is active, as shown in column 9 of Table 1.

INTERFLUVE HILLS AND PLATEAUS

Within almost any drainage basin approaching maturity, especially with steeper slopes, relatively flat-topped interfluve hills and plateaus are scattered over the

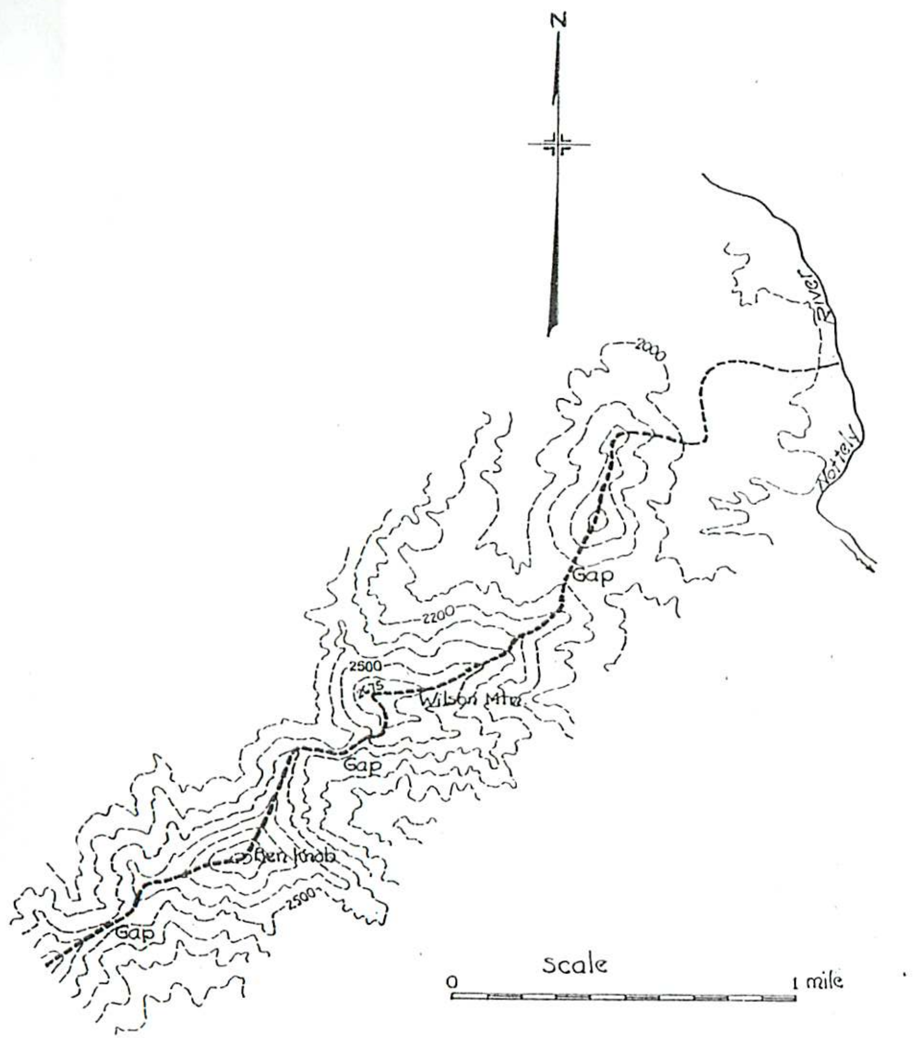


FIGURE 38.—*Topography of an interior cross divide*
(Coosa Bald, Ga., quad., U. S. G. S.—T. V. A.)

area. These hills and plateaus are not, as sometimes described, ungraded areas. They represent remnants of antecedent slopes and are therefore areas on which gradation was arrested when the adjacent streams and valleys developed.

Interior divides running crosswise of the drainage basin have broader belts of no erosion and are more permanent than either interior divides or lateral segments of the main divide running parallel with the original slope. Interior, flat-top, residual hills

are most common along the lines of transverse divides and at their junctions with interior longitudinal divides. Figure 38 shows the topography of a transverse divide in the drainage basin of Nottely River between Wold and Crumby creeks (Coosa Bald, Georgia, quad., U. S. G. S.—T. V. A. map), with interfluve hills (knobs and balds) and saddles (gaps).

TABLE 7.—*Flat tops in upper Esopus Creek drainage basin*

Distance above Olive Bridge (miles)	Elevation of flat tops (feet)	Approximate area (Sq. miles)	Average elevation, major divide (feet)	Elevation of stream (feet)
(1)	(2)	(3)	(4)	(5)
3.5	1500	0.09	1600	600
3.7	2000	0.61	1650	610
5.7	2500	0.11	2200	640
6.4	2500	0.03	2350	660
7.35	3500	0.06	3000	700
7.35	2500	0.51	3000	700
9.2	3500	0.05	3150	740
9.2	2500	0.23	3150	740
9.2	2500	0.16	3150	740
9.7	2500	0.24	3400	760
10.6	3000	0.15	3300	800
10.6	2500	1.00	3300	800
11.9	3000	1.85	3100	900
13.4	2500	0.14	3200	950
14.3	3500	0.05	3450	1000
18.3	3000	0.04	3200	1500

The topographic map of Esopus Creek drainage basin above Olive Bridge in southeastern New York shows several residual flat-top hills and plateaus, all located along the lines of interior cross divides (Phoenicia, Kaaterskill, Margaretville, and Slide Mountain quads., U. S. G. S. topographic maps). Column 2 of Table 7 shows the elevations of the highest closed 500-foot contours. They are given in order of occurrence proceeding upslope from Olive Bridge. Column 3 shows the area within the contour. Above these contours the summits are relatively flat. Column 4 gives the average elevation of the main divide at the same cross section. Flat tops on the same cross divide are usually at nearly the same elevations, but those on adjacent divides, even where at nearly the same distance upslope, may be of quite different elevations; the one located on the divide which developed later is usually lower.

At the head of the drainage basin the marginal belt of no erosion along the main divide represents a portion of the original surface. While the elevations of the summits of flat tops usually increase upslope, those on lateral divides near the head of the drainage basin have been graded somewhat and are consequently usually at a lower elevation than the divide at the head of the basin.

Interior interfluve hills and plateaus are, however, not always lower than the adjacent peripheral divide because of conditions of exposure of the original surface. If the original surface was warped upward or is domelike, interfluve hills may rise above the adjacent main divide. The occurrence of flat-top, interfluve hills and plateaus results from the development of a divide between two streams under competitive conditions and requires only aqueous erosion. The locations and sizes of such hills may, however, be governed by secondary causes.

There may be, on a newly exposed region, local areas where high infiltration-

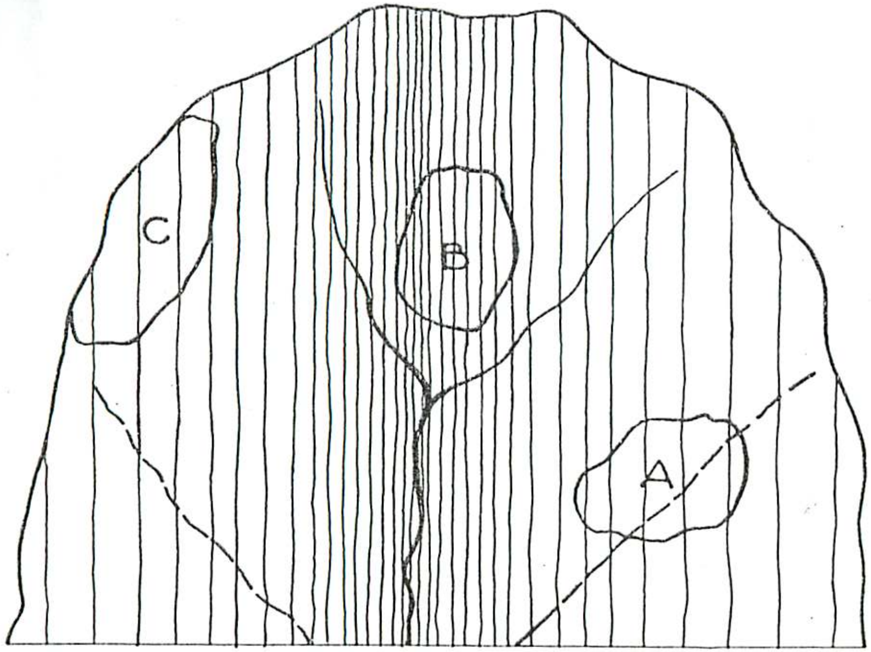


FIGURE 39.—*Origin of ungraded or partially graded interfluve hills and plateaus*

capacity f, large initial resistance R_i, or local flatness of slope, or these combined, make the critical distance x_c abnormally large—for illustration, 100 times as great as for the region in general. On Figure 39, A, B, and C represent such local areas, the lines of overland flow in the first stage of gradation being as indicated. The area A receives such intense runoff at its upslope edge that it may be eroded actively in the first stage of gradation. Area B may be little eroded or eroded only on its downslope portion, while area C will be immune to erosion. In the second and subsequent stages of stream development, none of these areas may be subject to erosional gradation, but they will remain as flat-topped interfluve hills, some of them at the elevations of the original surface, others at somewhat lower elevations.

CONCORDANT STREAM AND VALLEY JUNCTIONS

A new stream develops on a pregraded slope extending away from the parent stream. Thus the new stream enters the parent stream concordantly. A tributary valley has, in general, steeper side slopes and a smaller value of x_c than its parent stream valley. Hence a tributary valley usually grades faster than the coincident gradation of its parent valley, and, although younger, if its stream does not initially

enter the parent stream concordantly, it ultimately reaches the grade of the parent stream and debouches into it concordantly. If unrestricted, the tributary stream would cut below the level of its junction with the parent stream, but, since it cannot discharge below the grade of the parent stream at the junction and can easily maintain its grade at the level of the latter, it continues to discharge into the parent stream concordantly, as stated by Playfair's law.

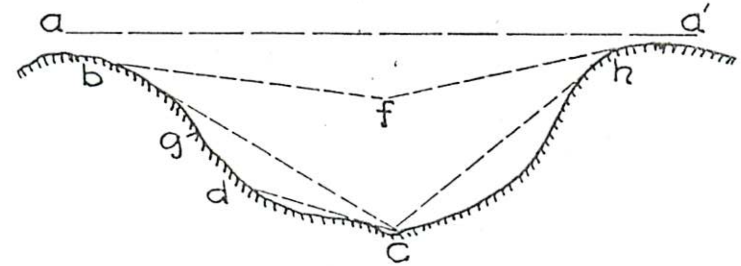


FIGURE 40.—*Gradation of stream valley*

STREAM-VALLEY GRADATION

Stream and valley gradation are closely related. The stream supplies a means of disposal of eroded material from the valley and fixes the minimum level of valley gradation. The valley tributary to the stream supplies the runoff that grades the stream. Stream and valley gradation proceed together, but valley gradation tends to lag behind stream gradation.

In the development of the stream and valley of a first-order tributary, runoff volume adequate to produce erosion close to the stream will be most frequent, and this part of the valley will be cut down rapidly to stream level, progressing backward toward the divide. At any point in the drainage basin downstream from the critical point x_c, the overland flow is increasingly charged with material in transport. This cuts down the transporting power of the overland flow, and near the foot of the slope the eroding and transporting power may be in equilibrium, and further erosion may thereby be inhibited.

Figure 40 represents the cross section typical of a mature tributary stream valley produced by aqueous erosion. As a result of cross-grading and re-cross-grading, the initial surface aa' was cut down to bfh when the stream developed. At each side is a belt of no erosion. For homogeneous material, valley side-slope erosion will not stop at the line bc. If erosion continues until the profile on the left-hand side is bd, the sheet flow, charged with eroded material, arriving at d must be disposed of. Slope is required to carry the water from d to c, and the segment dc will not be graded below this minimum slope. A steeper slope may be maintained from d to c because of sedimentation if the sheet flow from above d is overcharged with sediment with respect to its transporting power at the reduced slope dc.

For turbulent flow the critical length x_c varies inversely as the surface-runoff intensity q_s. Rainstorms range from those with intensities less than infiltration-capacity, and which produce no surface runoff, up to the maximum intensity possible in the given locality. In lighter storms x_c will extend to the stream at c, and no erosion will occur, although there may be runoff. In moderate storms x_c will extend to some point between b and c, and erosion will occur only on the lower portion of the slope. Only in maximum storms will x_c be limited to ab. with consequent erosion or sedimentation throughout the length bc.

As a result of the combination of (1) decreasing frequency of higher rain intensities, (2) the existence of a marginal belt of no erosion, and (3) inhibition of valley-bottom erosion by limited transporting power of overland flow, the valley cross section takes on an ogive or S-shaped form, with a point of contraflexure at some point g and a valley cross section below this point commonly nearly parabolic in form.

A maximum storm produces erosion increasing downslope from x_c and simultaneously provides material in suspension proceeding downslope and thereby increases the width of the belt of sedimentation adjoining the stream. The combined effect is in general to concentrate the steepest-slope angle in the middle portion of the slope extending from the stream to the watershed line.

The slope angle in the belt of steepest slope does not continue to increase indefinitely as erosion continues but approaches a value corresponding to the slope angle for maximum erosion. Obviously if the slope became steeper, the erosion would be decreased instead of increased. This limiting angle of erosion may be either greater than or less than the angle of repose of the soil material when wet. If it is less than the angle of repose, then the slope will ultimately become stable at about the angle of maximum erosion. If the angle of slope for maximum erosion is greater than the angle of repose, the slope will become subject to earth slips as well as to erosion, and each earth slip exposes new material, thus favoring continued erosion. Earth slips do not occur on all mature slopes—in fact, their occurrence is relatively uncommon, and this fact is explained in the manner above described but cannot readily be explained if it is assumed, as has frequently been done heretofore, that erosion continues to increase indefinitely with increase of slope. Land slips, if they occur, generally tend to prolong the process of valley-slope gradation and often decrease the width of the belt of no erosion.

When a first-order tributary develops, the length of overland flow along the course of the rill channels on the antecedent surface is generally less than x_c over most of the area tributary to the stream. Hence only limited subsequent cross-grading can occur. Such cross-grading as does occur is often local and feeble and is usually confined mostly to belts closely adjoining the stream on either side. This often results in a relatively deeply incised V-shaped stream channel adjoined on either side by uneroded upland. In this case the stream and its valley are not truly concordant in the upper reaches of the stream although they become concordant at the mouth of the stream, in accordance with Playfair's law.

Valley gradation, while it may lag behind stream development, is usually close to completion when the latter is completed. As a result the drainage basins of 1st order streams are likely to be born mature or nearly so and yet have the appearance of youth in the physiographic sense, in that their slopes are steep and their immediate stream valleys narrow and V-shaped in cross section. They remain perpetually youthful in appearance not because there is no remaining gradation which could be accomplished but because there are no adequate tools available by which it can be accomplished.

Transverse and longitudinal valley profiles both result from the same hydrophysical processes operating on the same terrain, and they are closely related. The resultant slope is the result of cross-grading at a given point. The transverse slope is the component of resultant slope at a right angle to the stream. Longitudinal slope is the component of the resultant slope parallel with the stream.

Many attempts have been made, beginning with Unwin (1898), to derive an equation for a stream profile. The stream profile at bankful stage is essentially the same as the longitudinal profile of the valley bottom. None of the existing equations is wholly satisfactory. A rational equation for stream profiles must take into account not only the laws of hydraulics and channel dynamics but those also of sheet erosion and storm frequency.

TYPICAL OVOID FORMS OF DRAINAGE-BASINS

Whether a drainage basin develops from the head downstream or from the mouth upstream depends on the manner in which the land becomes exposed. On lands exposed by vertical uplift of oceanic marginal lands, the streams would obviously develop from the headwaters downstream, while on lands exposed by retreat of glacial ice, the drainage basin and stream system may develop from the mouth upstream. In either case the points of first emergence ultimately become either the heads or mouths of principal streams because they provide the greatest lengths of overland flow and hence the points at which l_o first becomes greater than x_c. There is an angle or bend at the point of first emergence which remains as a landward-directed bend at the head of the main divide. Hence the head of a main drainage basin is usually somewhat rounded.

As a result of change of direction of overland flow by cross-grading (Fig. 30), the area acb between two parallel streams never becomes tributary to these streams as long as the coast line remains at ab. Areas tributary to rilled surfaces which initially drained part of the area $aa'bb'$ directly to ab will be captured by competitive cross-grading, and in this way the drainage basin of the streams aa' and bb' will expand laterally until a permanent competitive divide cc' is established.

Similar competitive divides will be established to the right of stream bb' and to the left of stream aa'. An area of direct drainage similar to abc always remains between each two streams, with the result that the final forms of drainage basins are usually pear-shaped or ovoid.

Elsewhere (Horton, 1941) the author has shown that the average of the forms of drainage basins of several of the larger rivers of the world is a slightly asymmetrical, pear-shaped figure with the apex at the outlet end. Also, assuming valley cross sections to be approximately parabolic, it is shown (Horton, 1941) that the trace of the intersection of a parabaloid with its apex upstream and a plane inclined to the axis of the parabaloid is a similar ovoid figure. Hence the hydrophysical laws and processes, as a consequence of which valley sections are generally similar to, though not actually parabolas, lead directly to the development of drainage basins of typical form.

In spite of modifications of individual drainage basins by geologic structures, the semblance to the typical ovoid form is still preserved in most drainage basins.

DEVELOPMENT OF LARGE DRAINAGE BASINS

In this paper, lands exposed by changes of a coast line have been used for purposes of illustration. If an entire continent was exposed progressively the principles outlined would still apply and would lead to the formation of stream systems dependent on the manner of exposure and the position of the continental divide. In North America, for example, upwarping of the northern portion of the continent and a gentle tilt of the remainder to the southeast would provide requisite conditions for the existing stream systems.

Waters of the earth are probably chiefly of volcanic origin. If at an early stage in geologic history a continent was exposed at a time when rainfall was light and oceans shallow or wanting, and there followed a period of gradually increasing rainfall, then a stream system would develop with a tendency to greater concentration in large drainage basins than occurs where exposure of the area is gradual.

If an entire continent was exposed before erosion begins, the maximum length of overland flow for the whole continent would occur at the start. Smaller coastal basins would be more limited in their extension inland than with a gradual exposure of a coastal margin. For such a continental exposure a few major drainage basins would generally develop, draining the greater part of the area. The development of each drainage basin would take place in much the same manner as one of the larger basins on a progressively exposed coastal margin. Smaller coastal streams would be restricted in tributary area by competition with the stronger major streams in much the same way in the two cases.

There would be certain differences in the operation of the processes of erosional gradation for the continental area:

(1) Even an approximation to homogeneity of the area with respect to the controlling factors: rain intensity, infiltration-capacity and surface resistivity to erosion, would not be likely to occur.

(2) A single maximum storm covering the whole of the continent is improbable. A major continental drainage basin would apparently develop in sections, and the sections would combine one after another when erosion attained requisite stages. There would be opportunities for large-scale stream piracy, such as appears to have occurred in the upper part of the area originally tributary to Colorado River.

The development of great drainage basins, such as those of the Mississippi, Amazon, and Nile rivers, must be treated on an individual basis. The recognized persistence of major stream systems, barring such catastrophic events as lava overflows or glacial submergence, suggests that a re-examination of the early erosional history of the earth in the light of hydrophysical processes is well worth while.

DAVIS STREAM-EROSION CYCLE

In discussions of the Davis erosion cycle the cycle is in effect assumed to begin after the development of at least a partial stream system. The initial development of streams is considered to be either fortuitous or governed by antecedent geologic structures and is largely taken for granted (Davis, 1909). The author has considered stream development and drainage-basin topography wholly from the viewpoint

of the operation of hydrophysical processes. In the Davis theory the same subject is treated largely with reference to the effects of antecedent geologic conditions and subsequent geologic changes. The two views bear much the same relation as two pictures of the same object taken in different lights—the results are not necessarily in conflict; each supplements the other. The hydrophysical concept appears to be more fundamental because it carries back to the original, newly exposed surface. In comparing the two viewpoints much depends on the meanings given to the words "youth," "youthful," "maturity," and "peneplain."

As commonly applied in connection with the Davis erosion cycle, "young" and "youthful" relate to an area where there has been but little erosional gradation. "Mature" means that all or nearly all the gradation which can result from the operation of existing agencies has been accomplished. On an area where infiltration-capacity and surface resistance are sufficiently high, little erosional gradation may be possible under existing conditions. Such an area is actually mature although from its surface appearance it would be classified as youthful.

Incipient drainage and low drainage density are accepted as *prima facie* evidence of youth. Extensive dissection and high drainage density on a given area are accepted as necessary and sufficient proof of maturity. These are usually sufficient, but they are not necessary as conditions precedent to maturity. An area of low drainage density and with little dissection may have been born mature if the original infiltration-capacity, initial resistance, and rainfall appurtenant to the drainage area were such that the length of overland flow was not sufficient to induce erosion on any part of the area. Such an area represents erosionally mature topography. It is mature not because much has been accomplished but because nothing more can be accomplished without rejuvenation by geologic agencies or as a result of climatic and cultural changes.

Conversely, but less frequently, an area occurs with moderate dissection and a fairly high drainage density which is in a youthful stage because of a small value of x_c. Such areas are most likely to be found where an epicycle of erosion is in progress. That they are not common elsewhere indicates that gradational development of drainage basins is far more generally complete with respect to pre-existing conditions than is usually supposed or assumed. The error has resulted largely from a careless and, as now appears, unjustifiable application of the Davis concepts rather than from error in the concepts themselves if correctly interpreted. For example, areas on the Highwood, Illinois; Rochester, New York; Dunlop, Illinois; Oak Orchard, New York; Parmelee, North Carolina; Anson, Texas; Fargo, North Dakota-Minnesota; and Oberlin, Ohio, topographic sheets of the U. S. Geological Survey have all been referred to as representing "stream erosion—youth" (Salisbury and Atwood, 1908). In spite of the low drainage density and lack of dissection of most of the areas referred to, the stream systems are all or nearly all relatively mature because of high infiltration-capacity, flat slopes, or high surface resistance to erosion or a combination of these factors. Erosion on many of these areas, if it occurs, must be ascribed to conditions which have induced either a local or general epicycle of erosion over part or all of the area.

There is often need to refer to actual age. Tributary streams and their basins are usually younger, in this sense, than higher-order streams or basins. The terms "little dissected" and "well dissected" seem preferable to "youthful" and "mature" in describing erosional status. There may then be areas which are (1) young, little dissected; (2) young, well dissected; (3) old, little dissected; (4) old, fully dissected.

The term "peneplain" seems inappropriate. The ultimate surface of erosion within a main basin boundary is neither "almost a plane," as the prefix "pene" implies, nor is it usually as close to being a plane as was the original surface area from which it has been derived. It seems better to call it a "base surface" in comparison with the original surface below which the stream basin is developed. The base surface at its downslope end is at "base level" in the usual sense. The base surface is, however, generally concave upward except along divides, and its margins intersect the initial surface around the upstream portion of the drainage area. The ultimate base surface is, under ideal conditions, closely similar to a segment of a parabaloid cut by a plane which is not parallel with the axis of the parabaloid. The parabaloidal surface is ribbed with ridges which represent the divides between streams.

Wooldridge and Morgan (1937) use "the invaluable concept of the cycle of erosion as initiated by W. M. Davis." They state (p. 184):

> "Some writers have argued that the cycle of erosion can never have run its full course and that the peneplain is an unrealized and unrealizable abstraction."

They invoke geologic factors, as others have done, to complete the erosion cycle—in other words, the Davis erosion cycle is not completed by erosion *per se*. The hydrophysical concept neither denies the effects nor invokes the operation of uplift. It carries the matter of basin development only so far as it can be carried by purely erosional processes, and it shows that there is a definite end point to the development of streams and valleys by aqueous erosion and leaves room for the survival of hills and plateaus between valleys and which are not subject to further erosion or peneplanation. It provides a better foundation than has heretofore existed for the interpretation of the effects of changes of geologic conditions in relation both to the subsequent march of the erosion cycle and in relation to changes of drainage patterns and drainage composition occasioned thereby.

In accordance with the hydrophysical concept most of the observed gradation of divides takes place before the streams which are separated by the given divide are developed—in other words, the terrain where the divide is located is graded in advance at a time when sheet erosion is taking place along or across the line which subsequently becomes the divide. Interfluve hills and plateaus are remnants of this pregraded surface, and when once formed they are permanent features of the topography. The whole concept of ultimate development of a peneplain appears to be founded on the idea that grading of interior divides continues indefinitely and is accomplished by the streams they separate, whereas in the case of the hydrophysical concept the gradation is already, for the most part, accomplished when the adjacent streams originate.

On drainage basins near maturity, erosion occurs only sporadically and in local patches where local conditions provide requisite length of overland flow, runoff

volume, slope, and low local resistivity to erosion, or where an epicycle of erosion occurs.

As regards advanced stages of erosional development of a drainage basin the principal differences between the Davis concept and the hydrophysical concept seem to be that the former does not provide any definite end point of erosional development, whereas the hydrophysical concept does provide a definite end point for both stream and valley development.

The physical and mathematical treatment of the subject establishes rational quantitative relationships between the interpretation of observed phenomena accurately and with confidence in the correctness of the results. For example, it has long been known that there is a relation between surface erosion and slope, but without knowing the physical basis of this relationship gross errors could easily be made in interpreting valley-slope erosion. The equations may be applied to the study of individual cases. Such possible applications are of infinite variety and must be left to others. It has been shown that some of the equations are of practical use in engineering problems, as in determining flood-crest modulation by channel storage and the requisite spacing of soil conservation terraces.

In conclusion, what has been given is a framework or outline of drainage-basin development along hydrophysical lines rather than the completed picture. It is hoped that the reader will find that a new, more definite, and more quantitative meaning has been imparted to Playfair's law and the Davis concept of the stream erosion cycle. It is also hoped that the reader will find stimulation to further study and research.

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