System Robustness

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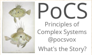
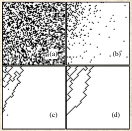
Principles of Complex Systems, Vols. 1, 2, & 3D
CSYS/MATH 6701, 6713, & a pretend number, 2024–2025

Prof. Peter Sheridan Dodds

Computational Story Lab | Vermont Complex Systems Center
Santa Fe Institute | University of Vermont





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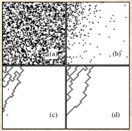

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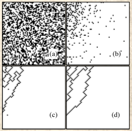
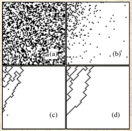


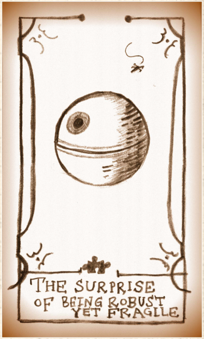
Outline

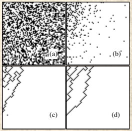
Robustness
HOT theory
Random forests
Self-Organized Criticality
COLD theory
Network robustness

References




THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE MISREADING OF THE BOOK OF SAND
THE MANY TAILS OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE BOOKS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILLSCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS


THE SURPRISE OF BEING ROBUST YET FRAGILE



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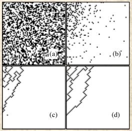
Robustness

- Many complex systems are prone to cascading catastrophic failure:



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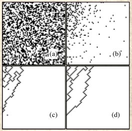

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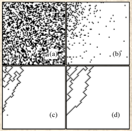

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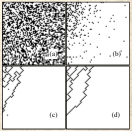

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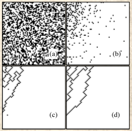

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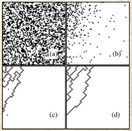

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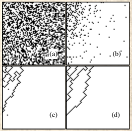

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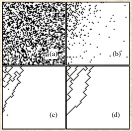

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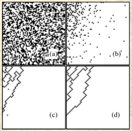

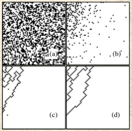

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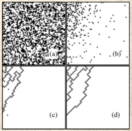

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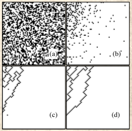

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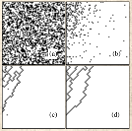
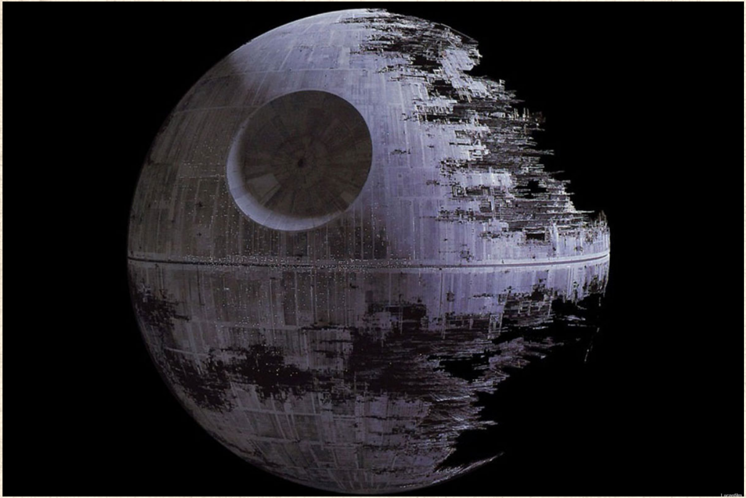
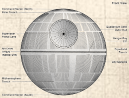
- Robustness and Failure may be a power-law story...

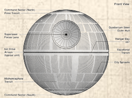


Our emblem of Robust-Yet-Fragile:



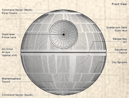


"Trouble ..."



Robustness

- System robustness may result from



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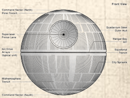

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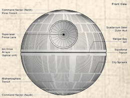

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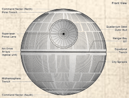

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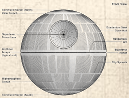

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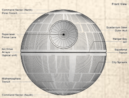

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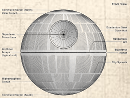

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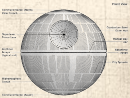
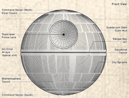
- Great abstracts of the world #73: "There aren't any." [7]



Robustness

Features of HOT systems: [5, 6]



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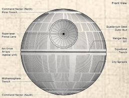
Features of HOT systems: [5, 6]

- High performance and robustness



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Features of HOT systems: [5, 6]

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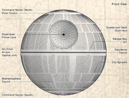

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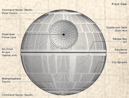

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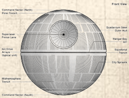

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- Designed/evolved to handle known stochastic environmental variability
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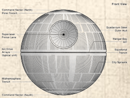
- Power-law distributions appear (of course...)



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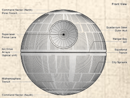
HOT combines things we've seen:

- Variable transformation



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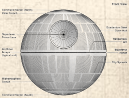

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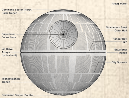

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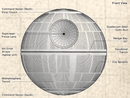

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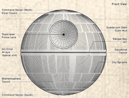

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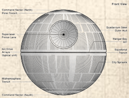

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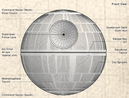
- X has a characteristic size but Y does not



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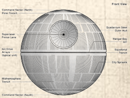
Forest fire example: [5]



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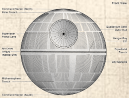

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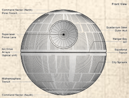

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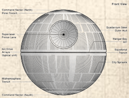
- Sites contain a tree with probability ρ = density
- Sites are empty with probability $1 - \rho$

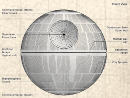


[1] This is bad notation. Would be better to have $N = L \times L$

Robustness

Forest fire example: [5]

- Square $N \times N$ grid[1]
- Sites contain a tree with probability ρ = density
- Sites are empty with probability $1 - \rho$
- Fires start at location (i, j) according to some distribution P_{ij}



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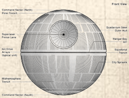

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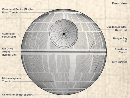

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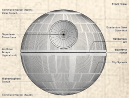

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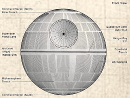
- Connected clusters of trees burn completely
- Empty sites block fire
- Best case scenario:
  Build firebreaks to maximize average # trees left intact given one spark



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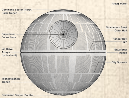

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Forest fire example: [5]

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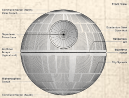
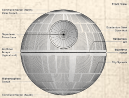


Robustness

Forest fire example: [5]

- Build a forest by adding one tree at a time
- Test D ways of adding one tree



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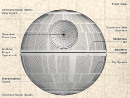
- D = design parameter



Robustness

Forest fire example: [5]

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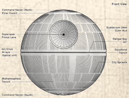
- Average over P_{ij} = spark probability



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- Build a forest by adding one tree at a time
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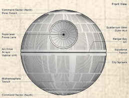
- $D = 1$: random addition



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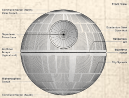
- $D = N^2$: test all possibilities



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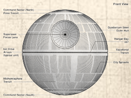
Measure average area of forest left untouched



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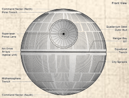

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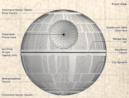
- Yield = $Y = \rho - \langle c \rangle$



Robustness

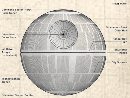
Specifics:

- $$P_{ij} = P_{i;a_x,b_x} P_{j;a_y,b_y}$$

  where

  $$P_{i;a,b} \propto e^{-[(i+a)/b]^2}$$

- In the original work, $b_y > b_x$
- Distribution has more width in y direction.



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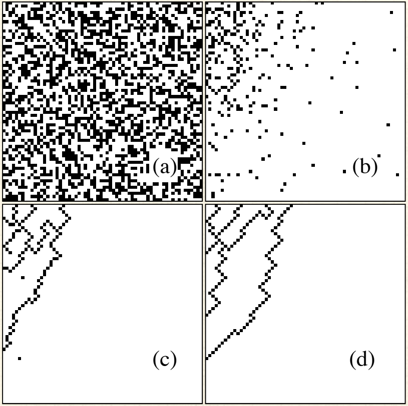
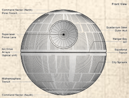


$N = 64$

(a) $D = 1$
(b) $D = 2$
(c) $D = N$
(d) $D = N^2$

- P_{ij} has an asymmetric, offset normal decay
- White square = tree
- Black square = no tree



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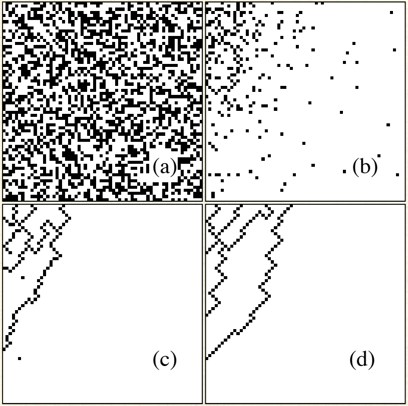

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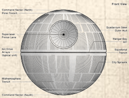

- Optimized forests do well on average

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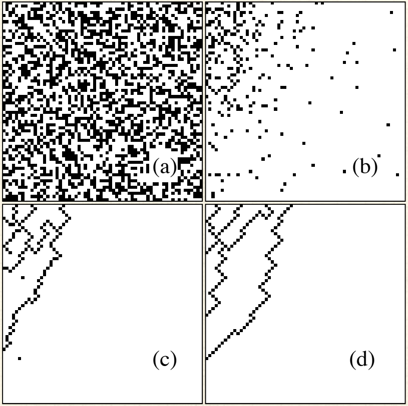

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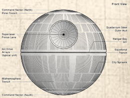
- But rare, extreme events occur



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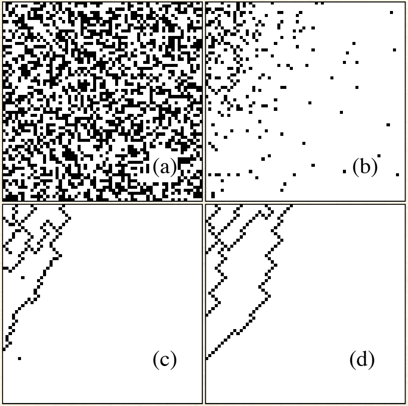

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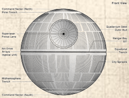

- Optimized forests do well on average (robustness)
- But rare, extreme events occur

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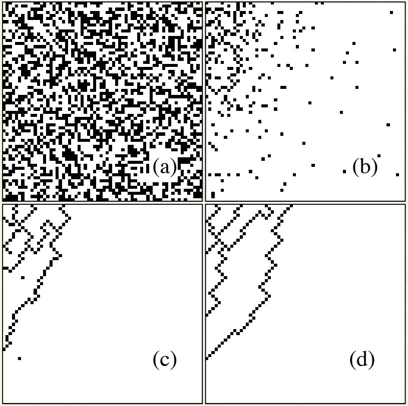

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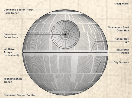

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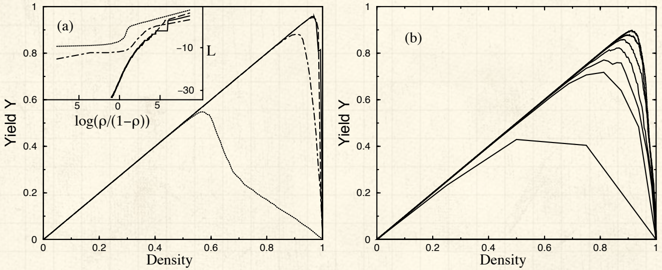
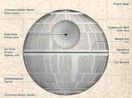


FIG. 2. Yield vs density $Y(\rho)$: (a) for design parameters $D = 1$ (dotted curve), 2 (dot-dashed), N (long dashed), and N^2 (solid) with $N = 64$, and (b) for $D = 2$ and $N = 2, 2^2, \ldots, 2^7$ running from the bottom to top curve. The results have been averaged over 100 runs. The inset to (a) illustrates corresponding loss functions $L = \log[\langle f\rangle/(1 - \langle f\rangle)]$, on a scale which more clearly differentiates between the curves.

[5]



HOT Forests:

- Y = ‘the average density of trees left unburned in a configuration after a single spark hits.’ [5]

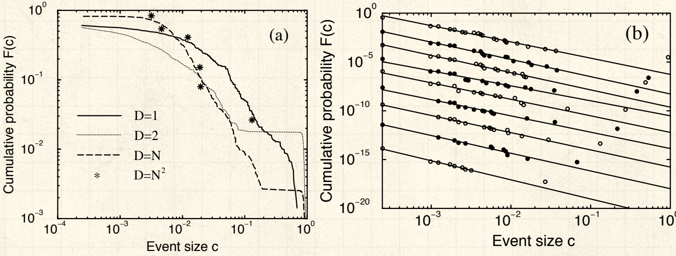
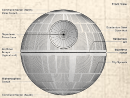
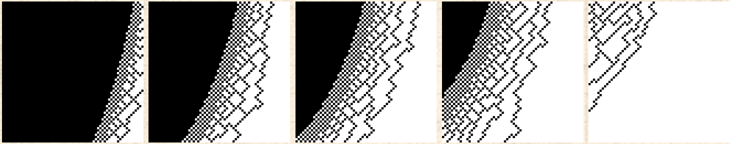


FIG. 3. Cumulative distributions of events $F(c)$: (a) at peak yield for $D = 1, 2, N$, and N^2 with $N = 64$, and (b) for $D = N^2$, and $N = 64$ at equal density increments of 0.1, ranging at $\rho = 0.1$ (bottom curve) to $\rho = 0.9$ (top curve).



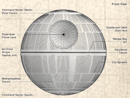
Variable density story does not hold up:



HOT model simulations for:[2]

- $N = 64$, $D = N^2 = 4{,}096$
- $N = 128$, $D = N^2 = 16{,}384$
- $N = 256$, $D = N^2 = 65{,}536$ (symmetric)
- $N = 256$, $D = N^2 = 65{,}536$ (skewed)

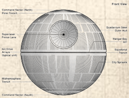
- Density measure should be for forested part only.[3]
- Distribution is missing spike for size zero forests.
- Distribution tail grows with tree addition.



[2] Simulations and videos by David Matthews, PoCS 2020

[3] And it would be high, far above p_c

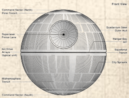
Outline



Random Forests

$D = 1$: Random forests = Percolation [11]

- Randomly add trees.



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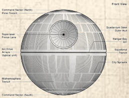

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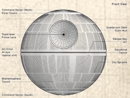

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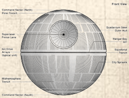
- Only at ρ_c, the critical density, is there a power-law distribution of tree cluster sizes.



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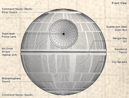
- Forest is random and featureless.



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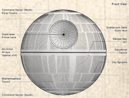
- Highly structured.
- Claim power law distribution of tree cluster sizes for a broad range of ρ, including below ρ_c (but model's dynamic growth path is odd).



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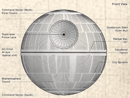

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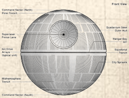

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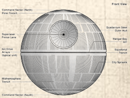

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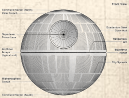

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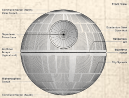

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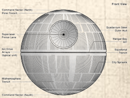
- Growth is key to toy model which is both algorithmic and physical.
- HOT theory is more general than just this toy model.



HOT forests—Real data:

"Complexity and Robustness," Carlson & Dolye [6]

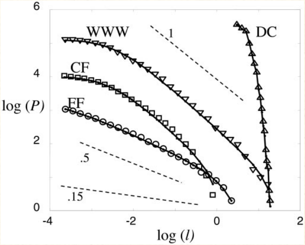
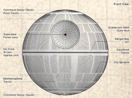


Fig. 1. Log–log (base 10) comparison of DC, WWW, CF, and FF data (symbols) with PLR models (solid lines) (for $\beta = 0, 0.9, 0.9, 1.85$, or $\alpha = 1/\beta = \infty, 1.1, 1.1, 0.054$, respectively) and the SOC FF model ($\alpha = 0.15$, dashed). Reference lines of $\alpha = 0.5$, 1 (dashed) are included. The cumulative distributions of frequencies $\mathcal{P}(l \geq l_i)$ vs. l_i describe the areas burned in the largest 4,284 fires from 1986 to 1995 on all of the U.S. Fish and Wildlife Service Lands (FF) (17), the >10,000 largest California brushfires from 1878 to 1999 (CF) (18), 130,000 web file transfers at Boston University during 1994 and 1995 (WWW) (19), and code words from DC. The size units [1,000 km^2 (FF and CF), megabytes (WWW), and bytes (DC)] and the logarithmic decimation of the data are chosen for visualization.

- PLR = probability-loss-resource.
- Minimize cost subject to resource (barrier) constraints: $C = \sum_i p_i l_i$ given $l_i = f(r_i)$ and $\sum r_i \leq R$.
- DC = Data Compression.

- These are CCDFs (Eek: P, $P(l \geq l_i)$)



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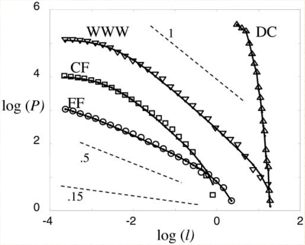

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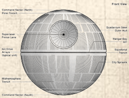
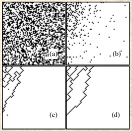
- Horror: log. Screaming: "The base! What is the base!? You monsters!"



HOT theory:

The abstract story, using figurative forest fires:



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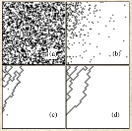
- Given some measure of failure size y_i and correlated resource size x_i with relationship $y_i = x_i^{-\alpha}, i = 1, \ldots, N_{\text{sites}}$.



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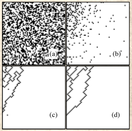

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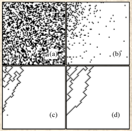
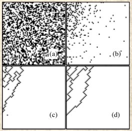


1. Cost: Expected size of fire:

$$C_{\text{fire}} \propto \sum_{i=1}^{N_{\text{sites}}} p_i a_i .$$

a_i = area of ith site's region, and p_i = avg. prob. of fire at ith site over some time frame.



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2. Constraint: building and maintaining firewalls.
Per unit area, and over same time frame:

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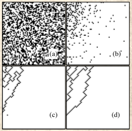

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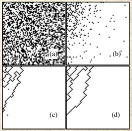

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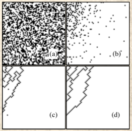

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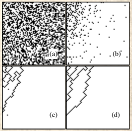

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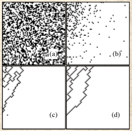
3. Insert assignment question to find:

$$\boxed{\mathbf{Pr}(a_i) \propto a_i^{-\gamma}.}$$

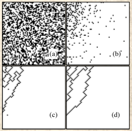


Continuum version:

1. Cost function:

$$\langle C \rangle = \int C(\vec{x}) p(\vec{x}) \mathrm{d}\vec{x}$$

where C is some cost to be evaluated at each point in space $\vec{x}$ (e.g., $V(\vec{x})^{\alpha}$),



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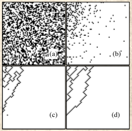

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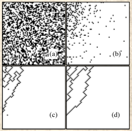

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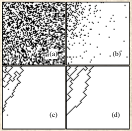

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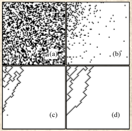
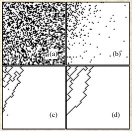
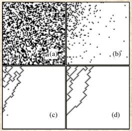
- For spatial systems with barriers: $\beta = d$.



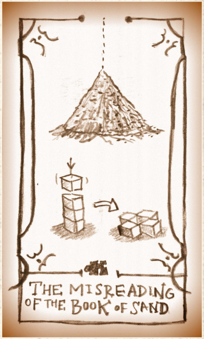
The HOT model in the wild



Outline



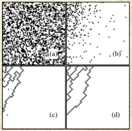

THE SUN
THE GOLDEN AGE OF REDUCTIONISM
THE MANIFESTO
WELL OF SCALES
THE LAWS OF DECAYING POWER
THE UNEXPECTED TRANSFIGURATION
MINISTRY OF RANDOM WALKS
THE RICH GET RICHER
THE DREAM OF EFFICIENT LANGUAGE
THE LAW OF DIGITS FIRST
THE SURPRISE OF BEING ROBUST YET FRAGILE
THE MANY TAILS OF MANY TAILS
THE MEASURING
THE DATA POOR
THE POURING DATA
THE CHARMS OF THE MANY WRONG PATHS
THE BOOKS THAT SHOULD NOT BE OPENED
THE KNOWLEDGE OF DEAD MONKS
THE EMERGENCE OF STRUCTURE
THE EMERGENCE OF DESTRUCTION
THE EMERGENCE OF THINKING
THE EMERGENCE OF STORIES
THE MECHANICS STATISTICAL
THE THRESHOLDS OF PERCOLATION
THE NETWORKS COMPLEX
THE ELEMENTS OF WEBS
THE BIRDS OF A FEATHER
THE BIRDS OF NOT A FEATHER
THE ARK OF THE RANDOM NETWORK
THE STRANGENESS OF FRIENDS
THE SMALLNESS OF WORLDS
THE THEORY OF SIX DEGREES
THE NETWORKS FREE OF SCALE
THE CONTAGION
THE CONFUSION OF CONTAGIONS
THE DISEASE OF RANDOM MIXING
THE UNENDING PANDEMICS
THE THRESHOLDS OF THE MIND
THE SOCIAL WILD
THE SOCIAL CONTAGION
THE RATIO OF GAIN
THE MECHANICS OF FAME
THE TALK OF FAME
THE STORIES CONTAGIOUS
THE JOHN DORY
THE ADJACENT NARRATIVES
THE STORYTELLERS
THE FATED ASTRAL PATH
THE PATHS OF UNIVERSALITY
THE DEPENDENT PATHS
THE TROUSERS OF BIFURCATING REALITIES
THE UNKNOWN MECHANISM
THE LANDSCAPES OF FORKING PATHS
THE NETWORKS OF BLOOD
THE TREES OF REALITY
THE ORDERS OF STREAMS
THE LAWS OF BRANCHING
THE SCALING OF BROKEN HIERARCHIES
THE BRAID OR THE BRANCH
THE LAW OF OPTIMAL FORKS
THE CHURCH OF QUARTEROLOGY
THE TRUTHICIDE
THE FAMILY OF PANCAKE COWS
THE FAMILY OF SPHERICAL COWS
THE IMPROBABLE PACHYDERM
THE ILL-SCALING OF ALLOMETRIC BODIES
THE SUPPLY MOST FAIR
THE SUPPLY TO THE MOST
THE DARK ART OF GENERATING FUNCTIONS
THE RANDOM SUM OF RANDOMNESS
THE TWO-SIDED WORLDS
THE STORIES AND THE TROPES
THE TRUTH OF DIFFUSION
THE UNVEILING OF HIDDEN POWERS
THE SCRYING FOR THE SHAPES OF THINGS
THE LIMITS OF ORGANIZATIONS


THE MISREADING OF THE BOOK OF SAND

SOC theory

SOC = Self-Organized Criticality

- Idea: natural dissipative systems exist at ‘critical states’;



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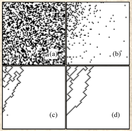

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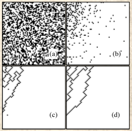

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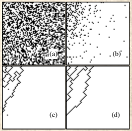

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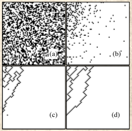

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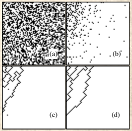

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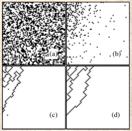
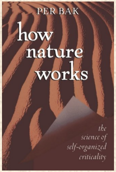
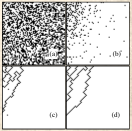
- Much criticism and arguing...

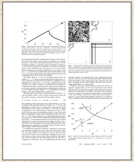




"How Nature Works: the Science of Self-Organized Criticality" by Per Bak (1997). [2]

Avalanches of Sand and Rice ...





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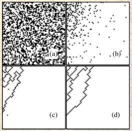
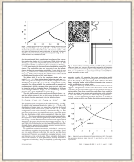
Proc. Natl. Acad. Sci., **99**, 2538–2545, 2002. [6]

HOT versus SOC

- Both produce power laws





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HOT versus SOC

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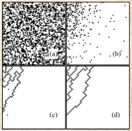
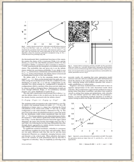
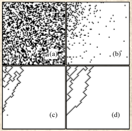


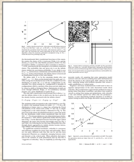


"Complexity and robustness"
Carlson and Doyle,
Proc. Natl. Acad. Sci., **99**, 2538–2545, 2002. [6]

HOT versus SOC

- Both produce power laws
- Optimization versus self-tuning
- Claim: HOT systems viable over a wide range of high densities (false)





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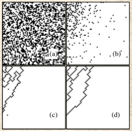

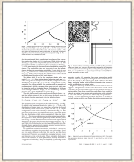

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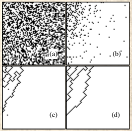
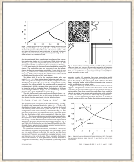
- HOT systems produce specialized structures





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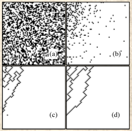
HOT versus SOC

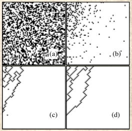
- Both produce power laws
- Optimization versus self-tuning
- Claim: HOT systems viable over a wide range of high densities (false)
- True: SOC systems have one special density
- HOT systems produce specialized structures
- SOC systems produce generic structures



HOT theory—Summary of designed tolerance [6]

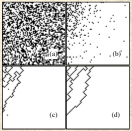
Table 1. Characteristics of SOC, HOT, and data

	Property	SOC	HOT and Data
1	Internal configuration	Generic, homogeneous, self-similar	Structured, heterogeneous, self-dissimilar
2	Robustness	Generic	Robust, yet fragile
3	Density and yield	Low	High
4	Max event size	Infinitesimal	Large
5	Large event shape	Fractal	Compact
6	Mechanism for power laws	Critical internal fluctuations	Robust performance
7	Exponent α	Small	Large
8	α vs. dimension d	$\alpha \approx (d-1)/10$	$\alpha \approx 1/d$
9	DDOFs	Small (1)	Large (∞)
10	Increase model resolution	No change	New structures, new sensitivities
11	Response to forcing	Homogeneous	Variable



Robustness and narrative causality:

Robust-yet-fragile, enstoried.[4]



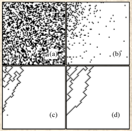
[4] See also: Achilles

Outline

Robustness

- HOT theory
- Random forests
- Self-Organized Criticality
- COLD theory
- Network robustness

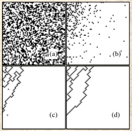
References



COLD forests

Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations [9]



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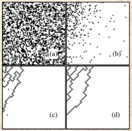
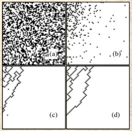
- Weight cost of larges losses more strongly



COLD forests

Avoidance of large-scale failures

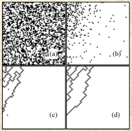
- Constrained Optimization with Limited Deviations [9]
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...



COLD forests

Avoidance of large-scale failures

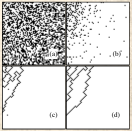
- Constrained Optimization with Limited Deviations [9]
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe



COLD forests

Avoidance of large-scale failures

- Constrained Optimization with Limited Deviations [9]
- Weight cost of larges losses more strongly
- Increases average cluster size of burned trees...
- ... but reduces chances of catastrophe
- Power law distribution of fire sizes is truncated



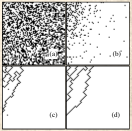
Cutoffs

Observed:

- Power law distributions often have an exponential cutoff

$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

where x_c is the approximate cutoff scale.



Cutoffs

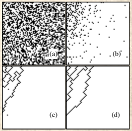
Observed:

- Power law distributions often have an exponential cutoff

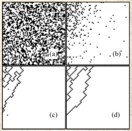
$$P(x) \sim x^{-\gamma} e^{-x/x_c}$$

  where x_c is the approximate cutoff scale.
- May be Weibull distributions:

$$P(x) \sim x^{-\gamma} e^{-ax^{-\gamma+1}}$$



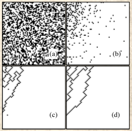
Outline



Robustness

We'll return to this later on (maybe):

- Network robustness.
- Albert et al., Nature, 2000:
  "Error and attack tolerance of complex networks" [1]
- General contagion processes acting on complex networks. [13, 12]
- Similar robust-yet-fragile stories ...



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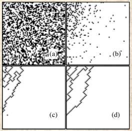

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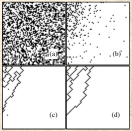

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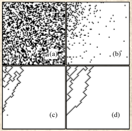

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