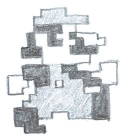
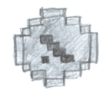


MATH 124: Matrixology (Linear Algebra)
Level Donkey Kong (1981), 6 of 10
University of Vermont, Spring 2015



Dispersed: Thursday, February 26, 2015.
Due: By start of lecture, Tuesday, March 17, 2015.
Sections covered: 3.6, 4.1, 4.2.

Some useful reminders:
Instructor: Prof. Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 12:30 to 3:00 pm Mondays
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2015-01UVM-124
Textbook: "Introduction to Linear Algebra" (3rd or 4th edition) by Gilbert Strang (published by Wellesley-Cambridge Press).

- All questions are worth 3 points unless marked otherwise.
- Please use a cover sheet and write your name on the back and the front of your assignment.
- You must show all your work clearly.
- You may use Matlab to check your answers for non-Matlab questions (usually Qs. 1–8).
- Please list the names of other students with whom you collaborated.

1. Find bases for the four subspaces associated with A, possibly known as Prince Humperdinck:

$$\begin{bmatrix} 1 & 2 & 4 \\ 2 & 4 & 8 \end{bmatrix}$$

   You can do this most easily and most joyfully by finding the reduced row form of both A and A^{T}.

2. True or false (give a reason if true or a counterexample if false):

   (a) If $m = n$ then the row space of A equals the column space.

   (b) The matrices A and $-A$ share the same four subspaces.

   (c) If A and B share the same four subspaces then A is a multiple of B.

3. Suppose the 3 by 3 matrix $\mathbf{A}$ is invertible (hint: what will $\mathbf{R_A}$ be?). Write down bases for the four subspaces of $\mathbf{A}$, and also for the 3 by 6 matrix $\mathbf{B} = [\mathbf{A}\ \mathbf{A}]$ (i.e., two copies of $\mathbf{A}$ placed side by side).

4. Draw the 'big picture' for the following matrix:

$$\begin{bmatrix} 1 & 2 \\ 3 & 6 \end{bmatrix}$$

(Hint: first find the rank r, and the dimensions and bases for all four subspaces.)

On your diagram, please indicate subspace name, dimensions, and indicate how a point in row space maps to column space.

Note: this is not the abstract big picture but rather the particular big picture of this A. So please sketch the actual subspaces of A.

5. If S is a subspace of a vector space V, then we use the notation $S^\perp$ for its orthogonal complement.

   (a) If S is the subspace of R^3 containing only the zero vector, what is $S^\perp$?

   (b) If S is spanned by $\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}$, what is $S^\perp$?

   (c) If S is spanned by $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, what is $S^\perp$?

6. Construct a matrix with the required property or explain why you can't:

   (a) Row space contains $\begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 5 \\ 3 \\ 5 \end{bmatrix}$, and nullspace contains $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$.

   (b) $Ax = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ has a solution and $A^{\mathrm{T}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

7. Find $\vec{p}$, the projection of $\vec{b}$ onto the vector $\vec{a}$ given

$$\vec{b} = \begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix} \text{ and } \vec{a} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}.$$

Also write down the error vector $\vec{e}$ and check for orthogonality: $\vec{p}^{\mathrm{T}}\vec{e} = 0$ (calculus notation: $\vec{p} \cdot \vec{e} = 0$).

8. (Q 3ish, Section 4.2)

For the preceding problem, find the projection 3×3 matrix $P = \vec{a}\vec{a}^{\mathrm{T}}/(\vec{a}^{\mathrm{T}}\vec{a})$. Verify that $P\vec{b} = \vec{p}$ and show that $P^2 = P$.

(The value in having P is that we can reuse it to project any $\vec{b}$. I know this is exciting for you.)

9. Matlab question:

   Taking the same matrix from the previous assignment:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix}$$

   use Matlab's rref command to find the following matrices along with a basis for the row space of each:

   (a) $\mathbf{R_A}$,

   (b) $\mathbf{R_{AA^{\mathrm{T}}}}$,

   (c) $\mathbf{R_{A^{\mathrm{T}}A}}$.

   Optional: Note any connections between these bases. Can you explain them?

10. Matlab question:

    Taking the transpose of the matrix in the preceding question

$$A^{\mathrm{T}} = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 6 \end{bmatrix}$$

    use Matlab's rref command to find a basis for the row space of $\mathbf{A}^{\mathrm{T}}$ by first finding the reduced row echelon form $\mathbf{R_{A^{\mathrm{T}}}}$.

    In terms of $\mathbf{A}$'s four fundamental subspaces, note which one this basis is for, and show its dimensions make sense with your knowledge of m, n, and r.

    Optional: do you see any connection to the reduced row echelon forms in the preceding question?

11. (Bonus, 1 point)

    What's the main ingredient in vegemite?