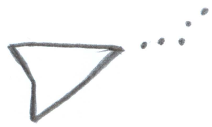
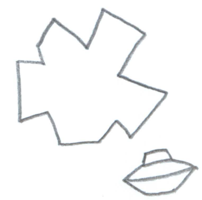


MATH 124: Matrixology (Linear Algebra)
Level Asteroids (1979), 4 of 10
University of Vermont, Spring 2015



Dispersed: Thursday, February 12, 2015.
Due: By start of lecture, Thursday, February 19, 2015.
Sections covered: 3.1, 3.2, 3.3, 3.4.

Some useful reminders:
Instructor: Prof. Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 2 to 2:45 pm, Mondays; 3 to 3:45 pm Tuesdays; and 1 to 2:30 pm Wednesdays
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2015-01UVM-124
Textbook: "Introduction to Linear Algebra" (3rd or 4th edition) by Gilbert Strang (published by Wellesley-Cambridge Press).

- All questions are worth 3 points unless marked otherwise.
- Please use a cover sheet and write your name on the back and the front of your assignment.
- You must show all your work clearly.
- You may use Matlab to check your answers for non-Matlab questions (usually Qs. 1–8).
- Please list the names of other students with whom you collaborated.

1. (Based on Q. 5, Section 3.1)

   Find the condition(s) on b_1, b_2, and b_3 for $\vec{b}$ to belong to $C(\mathbf{A})$, the column space of the matrix $\mathbf{A}$, and write down $C(\mathbf{A})$:

   $$\mathbf{A} = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -6 \end{bmatrix}.$$

   In doing so, find $\mathbf{R_A}$, the reduced row echelon form of $\mathbf{A}$.

2. Given the same $\mathbf{A}$ as in the preceding question:
   (a) What are the pivot variable(s) and the free variable(s) for this particular $\mathbf{A}$?

   (b) Find a description of the nullspace, $N(\mathbf{A})$.

   (c) What are m, n, and the rank r?

3. Consider a system $[\ \mathbf{A} \mid \vec{b}\ ]$ which reduces to

$$[\ \mathbf{R_A} \mid \vec{d}\ ] = \left[\begin{array}{cccc|c} 1 & 0 & 2 & 3 & 4 \\ 0 & 1 & -2 & 5 & -1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right].$$

(a) What are m, n, and r for this system?

(b) What is the general solution $\vec{x} = \vec{x}_p + \vec{x}_h$? Which part of the solution belongs in the nullspace of $\mathbf{A}$?

(c) For this $\mathbf{A}$, how many solutions are possible depending on $\vec{b}$: 0, 1, and/or ∞?

4. Finding a description of $C(\mathbf{A})$, the column space of $\mathbf{A}$, is the same as solving a homogeneous/nullspace problem ($\mathbf{A}\vec{x} = \vec{0}$).

Consider the following pre-reduced system:

$$[\ \mathbf{R_A} \mid \vec{d}\ ] = \left[\begin{array}{cccc|c} 1 & -2 & 4 & -1 & b_1 \\ 0 & 0 & 0 & 0 & -b_1 + 4b_2 \\ 0 & 0 & 0 & 0 & -6b_1 + 25b_2 + 2b_3 \end{array}\right].$$

Both the last two entries in $\vec{d}$ have to be zero for the system to have a solution.

Represent the resulting equations as a 2×3 system $\mathbf{A}\ \vec{b} = 0$. Solve this homogeneous equation to find the column space of $\mathbf{A}$.
Hint—Some help with this Inception-like problem:

Direct link: `http://www.youtube.com/v/_K75f8slhTc?rel=0`

5. (Q 23, 3.2)

(a) Construct a matrix whose column space contains $\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 3 \\ 1 \end{bmatrix}$ and whose nullspace contains $\begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$.

(b) Is your answer to (a) unique? Can more than one matrix have the same column space and nullspace?

(c) Without performing row reduction on your answer above, construct $\mathbf{R_A}$, the reduced row echelon form of all matrices that have the column space and null space given above.

6. Find the nullspace of the matrix $\mathbf{A} = [\ 1 \quad 3 \quad -2 \quad 7\ ]$.

7. (Q 26, 3.3)

   Find the reduced row echelon form $\mathbf{R_A}$ and rank r for the following matrix, showing how these change with c:

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 & 2 \\ 2 & 2 & 4 & 4 \\ 1 & c & 2 & 2 \end{bmatrix}$$

8. (from Q 10, 3.1) Is this subset of R^3 also a subspace? Why or why not?

$$S = \left\{ \vec{x} \in R^3 \text{ such that } \ x_1 x_2 x_3 = 0 \right\}.$$

   Sketch S.

   Hint: For a subset of a vector space to be a subspace, it has to satisfy the three criteria given in class. If you find one criterion is not satisfied, then you can stop—the subset cannot be a subspace.

9.

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 & 2 & 1 & 1 & 1 \\ 2 & 4 & 1 & -1 & 2 & 1 & -2 \\ 1 & 2 & 2 & 1 & 1 & -1 & 2 \\ 1 & 2 & 1 & 0 & 1 & 1 & -1 \\ -1 & -2 & 1 & 2 & 1 & 2 & -2 \end{bmatrix}$$

   a. Use the matlab function rref to obtain the reduced row echelon form $\mathbf{R_A}$ of $\mathbf{A}$.
   b. Use the matlab function null($\mathbf{A}$,'r') to obtain a matrix $\mathbf{S}$ whose columns form a basis for the nullspace of A.
   c. Compute the product of $\mathbf{R_A}$ and $\mathbf{S}$. Explain the answer you find.

10. **a** Using Matlab's command rank find the rank of $\mathbf{A}$ given in the preceding question.

    b Now find the rank of $\mathbf{A}^{\mathrm{T}}$.

    c Find the rank of a matrix $\mathbf{A}$ of size 100 by 100 filled with ones.

Use Matlab's ones command:

```
A = ones(100,100);
rank(A)
```

Can you guess the rank of a matrix $\mathbf{A}$ filled with ones and of arbitrary size (meaning m and n can be any integer ≥ 1)?

11. (The bonus; 1 point)

    Can individuals of the species Sarcophilus harrisii really spin like mini tornadoes?

    Also, members of this species greatly enjoy eating food and have disproportionately strong jaws. According to the Wikipedia, what percentage of their bodyweight can they allegedly eat in half an hour?