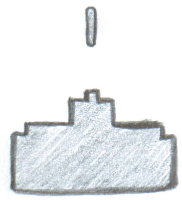
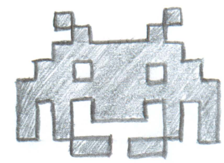


MATH 124: Matrixology (Linear Algebra)
Level Space Invaders (1978), 3 of 10
University of Vermont, Spring 2015



Dispersed: Thursday, January 29, 2015.
Due: By start of lecture, Thursday, February 5, 2015.
Sections covered: 2.5, 2.6, 2.7.

Some useful reminders:
Instructor: Prof. Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 2 to 2:45 pm, Mondays; 3 to 3:45 pm Tuesdays; and 1 to 2:30 pm Wednesdays
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2015-01UVM-124
Textbook: "Introduction to Linear Algebra" (3rd or 4th edition) by Gilbert Strang (published by Wellesley-Cambridge Press).

- All questions are worth 3 points unless marked otherwise.
- Please use a cover sheet and write your name on the back and the front of your assignment.
- You must show all your work clearly.
- You may use Matlab to check your answers for non-Matlab questions (usually Qs. 1–8).
- Please list the names of other students with whom you collaborated.

1. Given a 3x3 matrix A has multipliers $l_{21} = -7/2$, $l_{31} = -3$, and $l_{32} = 4$, write down E_{21}, E_{31}, E_{32}, E_{21}^{-1}, E_{31}^{-1}, E_{32}^{-1}, and the lower triangular matrix L.

2. Using the Gauss-Jordan method, show that the inverse of the general 2x2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad \text{is} \quad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Assume $a \neq 0$ and $ad - bc \neq 0$.

Some plans: (a) Find the elimination matrices E_{21} and E_{12} and the pivot matrix D required to turn A into the identity matrix I (as we did in class; you remember; it was fun...).

(b) you can set up the augmented matrix as follows and reduce it until the left hand side is the identity matrix:

$$A = \left[\begin{array}{cc|cc} a & b & 1 & 0 \\ c & d & 0 & 1 \end{array}\right]$$

3. Find the inverse of the following matrix using the Gauss-Jordan method:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

4. Factorize the following matrix into the product LU:

$$A = \begin{bmatrix} 2 & 3 \\ 6 & 8 \end{bmatrix}.$$

Write down E_{21} and its inverse.

5. Find the LDU factorization of

$$A = \begin{bmatrix} 4 & 3 & 7 \\ 0 & 2 & -3 \\ 0 & 0 & 7 \end{bmatrix}.$$

6. Solve $L\vec{c} = \vec{b}$ to find $\vec{c}$. Then solve $U\vec{x} = \vec{c}$ to find $\vec{x}$. What is A?

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \vec{b} = \begin{bmatrix} 3 \\ 3 \\ 5 \end{bmatrix}.$$

7. For which three values of c is this matrix not invertible and why?

$$A = \begin{bmatrix} 2 & c & c \\ c & c & c \\ 8 & 7 & c \end{bmatrix}.$$

(Hint: for A to be invertible, all its pivots must be $\neq 0$.)

8. **(a)** Find an example pair of 2x2 invertible matrices A and B such that $A + B$ is not invertible.
(b) Find an example pair of 2x2 singular (i.e., non-invertible) matrices A and B such that $A + B$ is invertible.

9. Find A^{T}, A^{-1}, $(A^{-1})^{\mathrm{T}}$, and $(A^{\mathrm{T}})^{-1}$ for

$$\textbf{(a)} \begin{bmatrix} 1 & 0 \\ 9 & 3 \end{bmatrix}$$

Please use the formula for the inverse of a 2×2 matrix.

10. If $A = A^{\mathrm{T}}$ and $B = B^{\mathrm{T}}$ (i.e., A and B are symmetric) which of these matrices are symmetric?:
(a) $ABABA$.
(b) $A^3 - B^3$,
(c) $(A + B)(A - B)$ (hint: expand this one first),

11. Open up Matlab, and compute the inverses for the following three matrices.

    Use Matlab's inv function:

```
>> inv(A)
```

    Note: No need to show this, but you can check by multiplication that you have indeed found the inverse. Also check that $A = LU$ for the matrices shown.

    Adjacent question (unscored): anything interesting about the kinds of matrices you find for L^{-1} and U^{-1}?

    One last check (unscored): Multiply L^{-1} and U^{-1} **in the right order** to obtain A^{-1}.

    (a) $L = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 1/2 & 3 & 1 \end{bmatrix}$, **(b)** $U = \begin{bmatrix} 6 & 4 & 2 \\ 0 & -3 & 3 \\ 0 & 0 & 7 \end{bmatrix}$,

    (c) $A = LU = \begin{bmatrix} 6 & 4 & 2 \\ -12 & -11 & -1 \\ 3 & -7 & 17 \end{bmatrix}$.

12. Find the LU factorization of the following matrices using your BFF Matlab. Use Matlab's lu command:

```
>> [L,U,P] = lu(A)
```

    (a) $\begin{bmatrix} 3 & 1 & 2 \\ 6 & 3 & 4 \\ 3 & 1 & 5 \end{bmatrix}$ **(b)** $\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ **(c)** $\begin{bmatrix} 1 & -1 & 1 & 2 \\ 0 & 2 & 1 & 0 \\ 1 & 3 & 4 & 4 \\ 0 & 2 & 1 & -1 \end{bmatrix}$

13. The bonus one pointer:

    Apart from the platypus, one other kind of mammal lays eggs. What's the name of this crazy beast and what are its young (possibly) called?