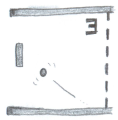
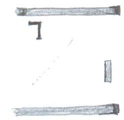


MATH 124: Matrixology (Linear Algebra)
Level Pong (1972), 1 of 10
University of Vermont, Spring 2015



Dispersed: Thursday, January 15, 2015.
Due: By start of lecture, Thursday, January 22, 2015.
Sections covered: 1.1, 1.2, 2.1, 2.2.

Some useful reminders:
Instructor: Prof. Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 12:30 to 3:00 pm Mondays
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2015-01UVM-124
Textbook: "Introduction to Linear Algebra" (3rd or 4th edition) by Gilbert Strang (published by Wellesley-Cambridge Press).

- All questions are worth 3 points unless marked otherwise.
- Please use a cover sheet and write your name on the back and the front of your assignment.
- You must show all your work clearly.
- You may use Matlab to check your answers for non-Matlab questions (usually Qs. 1–8).
- Please list the names of other students with whom you collaborated.

1. (Q 10, Section 2.1) Compute each $A\vec{x}$ by dot products of the rows with the column vector:
(a) $\begin{bmatrix} -4 & 1 & 2 \\ 1 & 2 & 4 \\ -2 & 3 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 3 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 2 & 1 \\ 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix}$

2. Write the following system of linear equations in (a) column form and (b) matrix form:
$$\begin{array}{rcrcrcr} 7x_1 & + & x_2 & + & 2x_3 & = & 4 \\ x_1 & + & 2x_2 & - & 9x_3 & = & 3 \\ 3x_1 & + & 3x_2 & - & 3x_3 & = & -1 \end{array}$$

3. (Q 27, Section 2.1) Solve the following system using elimination and back substitution, and draw the row and column pictures:
$$\begin{array}{rcrcl} x_1 & - & 2x_2 & = & 0 \\ x_1 & + & x_2 & = & 6. \end{array}$$

4. Reduce this system to upper triangular form ($U\vec{x} = \vec{b}$) by row operations:

$$\begin{array}{rcrcrcr} x_1 & + & 2x_2 & + & x_3 & = & 3 \\ 3x_1 & + & 2x_2 & - & x_3 & = & 9 \\ 2x_1 & + & 8x_2 & + & x_3 & = & 11. \end{array}$$

   Write down the three pivots and the three multipliers used in the row operations. Solve by back subsitution for x_1, x_2, and x_3.

5. (a) Which value of k forces a row exchange when you carry out elimination on the following set of linear equations?:

$$\begin{array}{rcrcrcr} x_1 & + & kx_2 & & & = & 0 \\ 3x_1 & - & 3x_2 & + & 3x_3 & = & 0 \\ & & x_2 & + & x_3 & = & 0. \end{array}$$

   (b) Which value of k leads to a "missing" third pivot? (By missing, we mean we find $D_3 = 0$.)
   (c) How many solutions exist when the third pivot is zero: 0, 1, or ∞?

   Hint—Some visual help on setting this problem up:
   `http://www.youtube.com/v/OcgnwAEfmxs?rel=0`

6. How must b_1 and b_2 be related if a solution is possible for the following system?:

$$\begin{array}{rcrcl} 2x_1 & - & x_2 & = & b_1 \\ 6x_1 & - & 3x_2 & = & b_2. \end{array}$$

   Draw the column picture showing example values of b_1 and b_2 that (a) allow a solution and (b) make the system unsolvable.

7. (Q 28, Section 2.1) Write out the following sentence while making the correct choice for each of the four parentheses.

   For four linear equations in two unknowns x_1 and x_2, the row picture will show **(2 or 4) (lines or planes)** in **(2 or 4)**-dimensional space. The column picture is in **(2 or 4)**-dimensional space.

8. (Q 25, Section 2.2) For which two values of a will elimination fail on

$$A = \begin{bmatrix} a & 2 \\ a & a \end{bmatrix}?$$

   By failure, we mean that at least one pivot is zero. Explain what the problem is for each value.

9. Note: For Matlab questions, please just write down the answers you obtain—no need to print anything out.

Solve the following system directly using Matlab:

$$\begin{array}{rcrcr} x_1 & + & 3x_2 & = & -9 \\ 3x_1 & + & x_2 & = & 5. \end{array}$$

Do this by first rewriting the system as $A\vec{x} = \vec{b}$ and then solve using Matlab's backslash operator:

```
>> x = A \ b
```

Note: A is a very special kind of matrix. More later.

10. Let H denote the $n \times n$ Hilbert matrix, whose (i, j) entry is $1/(i + j - 1)$. Write a matlab program to solve $H\vec{x} = \vec{b}$ where $\vec{b}$ is the length n vector of all ones, for $n = 2, 5, 10$.

    Help 1—generate H of size $n \times n$ with the built-in Matlab command **hilb**:

    ```
    >> n = 10;
    >> H = hilb(n);
    ```

    See `http://www.mathworks.com/help/matlab/ref/hilb.html` for more.

    Help 2—generate $\vec{b}$ using **ones**:

    ```
    >> n = 10;
    >> b = ones(n,1);
    ```

    Help 3—You will need to use use the backslash command $x = H\backslash b$.

    Observation for later: If we solved this problem by hand, $\vec{x}$ would contain only integers.

11. (bonus question, 1 point)

    Australia's land area as a percentage of the land area of the US's 48 contiguous states is closest to (a) 5%, (b) 10%, (c) 20%, (d) 50%, or (e) 100%?