PoCS What's The Story?

Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2014
Assignment 5 • code name: Timmy the Dog

Dispersed: Tuesday, September 30, 2014.
Due: By start of lecture, 1:00 pm, Thursday, October 16, 2014.
Some useful reminders:
Instructor: Peter Dodds
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E-mail: peter.dodds@uvm.edu
Office hours: 2:30 pm to 3:45 pm on Tuesday, 12:30 pm to 2:00 pm on Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2014-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. (3 + 3) The 1-d percolation problem:

   Consider an infinite 1-d lattice forest with a tree present at any site with probability p.

   - Find the distribution of forest sizes as a function of p. Do this by moving along the 1-d world and figuring out the probability that any forest you enter will extend for a total length ℓ.
   - Find p_c, the critical probability for which a giant component exists.

     Hint: One way to find critical points is to determine when certain average quantities explode. Compute $\langle l \rangle$ and find p such that this expression goes boom (if it does).

2. Show analytically that the critical probability for site percolation on a triangular lattice is $p_c = 1/2$.

Hint—Real-space renormalization gets it done.:

Direct link: `http://www.youtube.com/v/JlkbU5U7QqU?rel=0`

3. (3 + 3)

   Coding, it's what's for breakfast:

   (a) Percolation in two dimensions (2-d) provides a classic, nutritious example of a phase transition.

   Your mission, whether or not you choose to accept it, is to code up and analyse the L by L square lattice percolation model for varying L.

   Take $L = 20$, 50, 100, 200, 500, and 1000.

   (Go higher if you feel $L = 1000$ is for mere mortals.)

   (Go lower if your code explodes.)

   Let's continue with the tree obsession. A site has a tree with probability p, and a sheep grazing on what's left of a tree with probability $1 - p$.

   Forests are defined as any connected component of trees bordered by sheep, where connections are possible with a site's four nearest neighbors on a lattice.

   Do not bagelize (or doughnutize) the landscape (no periodic boundary conditions—boundaries are boundaries).

   (Note: this set up is called site percolation. Bond percolation is the alternate case when all links between neighboring sites exist with probability p.)

   Steps:

i. For each L, run N_{tests}=100 tests for occupation probability p moving from 0 to 1 in increments of 10^{-2}. (As for L, use a smaller increment if that's just how you do things.)

ii. Determine the fractional size of the largest connected forest for each of the N_{tests}, and find the average of these, S_{avg}.

iii. On a single figure, for each L, plot the average S_{avg} as a function of p.

(b) Comment on how $S_{\text{avg}}(p; N)$ changes as a function of L and estimate the critical probability p_c (the percolation threshold).

Helpful reuse of code (intended for black and white image analysis): You can use Matlab's bwconncomp to find the sizes of components. Very nice.

4. (3 + 3)

(a) Using your model from the previous question and your estimate of p_c, plot the distribution of forest sizes for $p \simeq p_c$ for the largest L your code and psychological makeup can withstand. (You can average the distribution over separate simulations.)

Comment on what kind of distribution you find.

(b) Repeat the above for $p = p_c/2$ and $p = p_c + (1 - p_c)/2$, i.e., well below and well above p_c.

Produce plots for both cases, and again, comment on what you find.