Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2013
Assignment 8 • code name: Bad Date(s)

Dispersed: Friday, November 1, 2013.
Due: By start of lecture, 1:00 pm, Thursday, November 14, 2013.
Some useful reminders:
Instructor: Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 10:30 am to 11:30 am, Monday, and 1:00 pm to 3:00 pm, Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2013-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. (3 + 3 + 3)

   Solve Krapivsky-Redner's model for the pure linear attachment kernel $A_k = k$.

   Starting point:

   $$n_k = \frac{1}{2}(k-1)n_{k-1} - \frac{1}{2}k n_k + \delta_{k1}$$

   with $n_0 = 0$.

   (a) Determine n_1.

   (b) Find a recursion relation for n_k in terms of n_{k-1}.

   (c) Now find

   $$n_k = \frac{4}{k(k+1)(k+2)}$$

   for all k and hence determine γ.

2. (3 + 3):

   From lectures:

   (a) Starting from the recursion relation

   $$n_k = \frac{A_{k-1}}{\mu + A_k} n_{k-1},$$

and $n_1 = \mu/(\mu + A_1)$, show that the expression for n_k for the Krapivsky-Redner model with an asymptotically linear attachment kernel A_k is:

$$\frac{\mu}{A_k} \prod_{j=1}^{k} \frac{1}{1 + \frac{\mu}{A_j}}.$$

(b) Now show that if $A_k \to k$ for $k \to \infty$ (or for large k), we obtain $n_k \to k^{-\mu-1}$.

3. (3 + 3 + 3)

From lectures, complete the analysis for the Krapivsky-Redner model with attachment kernel:

$$A_1 = \alpha \text{ and } A_k = k \text{ for } k \geq 2.$$

Find the scaling exponent $\gamma = \mu + 1$ by finding μ. From lectures, we assumed a linear growth in the sum of the attachment kernel weights $\mu t = \sum_{k=1}^{\infty} N_k(t) A_k$, with $\mu = 2$ for the standard kernel $A_k = k$.

We arrived at this expression for μ which you can use as your starting point:

$$1 = \sum_{k=1}^{\infty} \prod_{j=1}^{k} \frac{1}{1 + \frac{\mu}{A_j}}$$

(a) Show that the above expression leads to

$$\frac{\mu}{\alpha} = \sum_{k=2}^{\infty} \frac{\Gamma(k+1)\Gamma(2+\mu)}{\Gamma(k+\mu+1)}$$

Hint: you'll want to separate out the $j = 1$ case for which $A_j = \alpha$.

(b) Now use result that [1]

$$\sum_{k=2}^{\infty} \frac{\Gamma(a+k)}{\Gamma(b+k)} = \frac{\Gamma(a+2)}{(b-a-1)\Gamma(b+1)}$$

to find the connection

$$\mu(\mu - 1) = 2\alpha,$$

and show this leads to

$$\mu = \frac{1 + \sqrt{1 + 8\alpha}}{2}.$$

(c) Interpret how varying α affects the exponent γ, explaining why $\alpha < 1$ and $\alpha > 1$ lead to the particular values of γ that they do.

References

[1] P. L. Krapivsky and S. Redner. Organization of growing random networks. Phys. Rev. E, 63:066123, 2001.