Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2013
Assignment 7 • code name: "These go to eleven." (⊞)

Dispersed: Thursday, October 24, 2013.
Due: By start of lecture, 1:00 pm, Thursday, October 31, 2013.
Some useful reminders:
Instructor: Peter Dodds
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E-mail: peter.dodds@uvm.edu
Office hours: 10:30 am to 11:30 am, Monday, and 1:00 pm to 3:00 pm, Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2013-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. (3 + 3):

   Consider a modified version of the Barabàsi-Albert (BA) model [1] where two possible mechanisms are now in play. As in the original model, start with m_0 nodes at time $t = 0$. Let's make these initial guys connected such that each has degree 1. The two mechanisms are:

   M1: With probability p, a new node of degree 1 is added to the network. At time $t + 1$, a node connects to an existing node j with probability

   $$P(\text{connect to node } j) = \frac{k_j}{\sum_{i=1}^{N(t)} k_i} \quad (1)$$

   where k_j is the degree of node j and $N(t)$ is the number of nodes in the system at time t.

   M2: With probability $q = 1 - p$, a randomly chosen node adds a new edge, connecting to node j with the same preferential attachment probability as above.

   Note that in the limit $q = 0$, we retrieve the original BA model (with the difference that we are adding one link at a time rather than m here).

   In the long time limit $t \to \infty$, what is the expected form of the degree distribution P_k?

Do we move out of the original model's universality class?

Different analytic approaches are possible including a modification of the BA paper, or a Simon-like one (see also Krapivsky and Redner [2]).

(3 points for set up, 3 for solving.)

References

[1] A.-L. Barabási and R. Albert. Emergence of scaling in random networks. Science, 286:509–511, 1999.

[2] P. L. Krapivsky and S. Redner. Organization of growing random networks. Phys. Rev. E, 63:066123, 2001.