Social Contagion

Principles of Complex Systems
CSYS/MATH 300, Fall, 2011

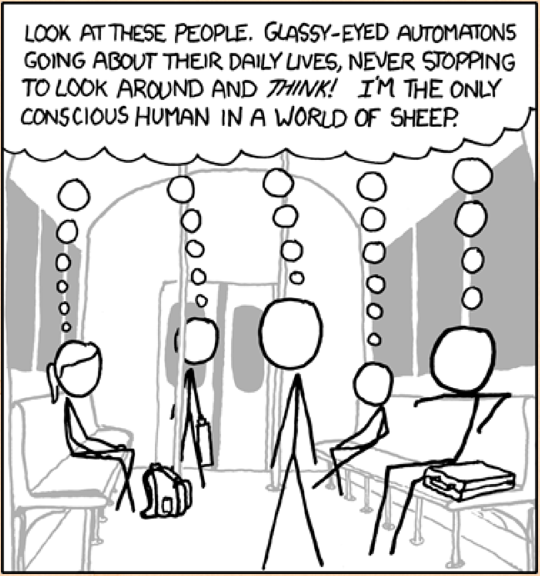
Prof. Peter Dodds

Department of Mathematics & Statistics | Center for Complex Systems | Vermont Advanced Computing Center | University of Vermont

Outline

Outline

Social Contagion



http://xkcd.com/610/ (⊞)

Social Contagion

Social Contagion

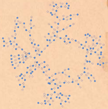
Social Contagion

Examples abound

- fashion
- striking
- smoking (⊞) [6]
- residential segregation [16]
- ipods
- obesity (⊞) [5]
- Harry Potter
- voting
- gossip
- Rubik's cube 
- religious beliefs
- leaving lectures

SIR and SIRS contagion possible

- Classes of behavior versus specific behavior



Social Contagion

Examples abound

- fashion
- striking
- smoking (⊞) [6]
- residential segregation [16]
- ipods
- obesity (⊞) [5]
- Harry Potter
- voting
- gossip
- Rubik's cube 
- religious beliefs
- leaving lectures

SIR and SIRS contagion possible

- Classes of behavior versus specific behavior

Social Contagion

Examples abound

- fashion
- striking
- smoking (⊞) [6]
- residential segregation [16]
- ipods
- obesity (⊞) [5]
- Harry Potter
- voting
- gossip
- Rubik's cube 
- religious beliefs
- leaving lectures

SIR and SIRS contagion possible

- Classes of behavior versus specific behavior

Social Contagion

Examples abound

- fashion
- striking
- smoking (⊞) [6]
- residential segregation [16]
- ipods
- obesity (⊞) [5]
- Harry Potter
- voting
- gossip
- Rubik's cube
- religious beliefs
- leaving lectures

SIR and SIRS contagion possible

- Classes of behavior versus specific behavior: dieting

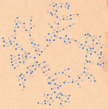
Framingham heart study:

Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking (⊞) [6]
- The spread of spreading (⊞) [5]
- Also: happiness (⊞) [8], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (⊞)

Controversy:

- Are your friends making you fat? (⊞) (Clive Thomspon, NY Times, September 10, 2009).
- Everything is contagious (⊞)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).



Framingham heart study:

Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking (⊞) [6]
- The spread of spreading (⊞) [5]
- Also: happiness (⊞) [8], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (⊞)

Controversy:

- Are your friends making you fat? (⊞) (Clive Thomspon, NY Times, September 10, 2009).
- Everything is contagious (⊞)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

Framingham heart study:

Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking (⊞) [6]
- The spread of spreading (⊞) [5]
- Also: happiness (⊞) [8], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (⊞)

Controversy:

- Are your friends making you fat? (⊞) (Clive Thomspon, NY Times, September 10, 2009).
- Everything is contagious (⊞)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

Framingham heart study:

Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking (⊞) [6]
- The spread of spreading (⊞) [5]
- Also: happiness (⊞) [8], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (⊞)

Controversy:

- Are your friends making you fat? (⊞) (Clive Thomspon, NY Times, September 10, 2009).
- Everything is contagious (⊞)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

Framingham heart study:

Evolving network stories (Christakis and Fowler):

- The spread of quitting smoking (⊞) [6]
- The spread of spreading (⊞) [5]
- Also: happiness (⊞) [8], loneliness, ...
- The book: Connected: The Surprising Power of Our Social Networks and How They Shape Our Lives (⊞)

Controversy:

- Are your friends making you fat? (⊞) (Clive Thomspon, NY Times, September 10, 2009).
- Everything is contagious (⊞)—Doubts about the social plague stir in the human superorganism (Dave Johns, Slate, April 8, 2010).

Social Contagion

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom?
- What kinds of influence response functions are there?
- Are some individuals super influencers?
- The infectious idea of opinion leaders (Katz and Lazarsfeld)[13]

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom?
- What kinds of influence response functions are there?
- Are some individuals super influencers?
- The infectious idea of opinion leaders (Katz and Lazarsfeld)[13]

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom?
- What kinds of influence response functions are there?
- Are some individuals super influencers?
- The infectious idea of opinion leaders (Katz and Lazarsfeld)[13]

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom?
- What kinds of influence response functions are there?
- Are some individuals super influencers?
- The infectious idea of opinion leaders (Katz and Lazarsfeld)[13]

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom?
- What kinds of influence response functions are there?
- Are some individuals super influencers?
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [13]

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers?
- The infectious idea of opinion leaders (Katz and Lazarsfeld)[13]

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers?
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [13]

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers?

- The infectious idea of opinion leaders (Katz and Lazarsfeld)[13]

Social Contagion

Two focuses for us

- Widespread media influence
- Word-of-mouth influence

We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers?
  Highly popularized by Gladwell [9] as ‘connectors’
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [13]

Social Contagion

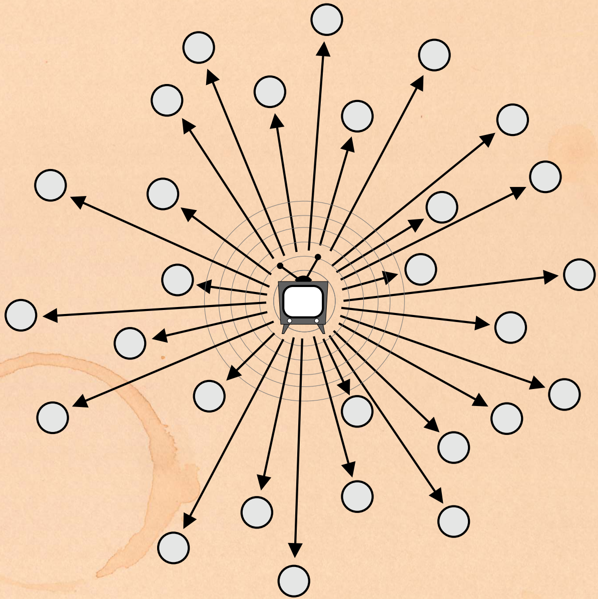
Two focuses for us

- Widespread media influence
- Word-of-mouth influence

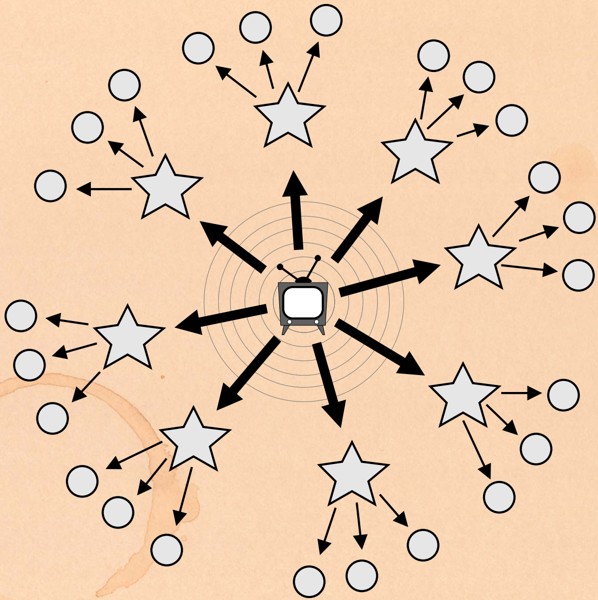
We need to understand influence

- Who influences whom? Very hard to measure...
- What kinds of influence response functions are there?
- Are some individuals super influencers?
  Highly popularized by Gladwell [9] as 'connectors'
- The infectious idea of opinion leaders (Katz and Lazarsfeld) [13]

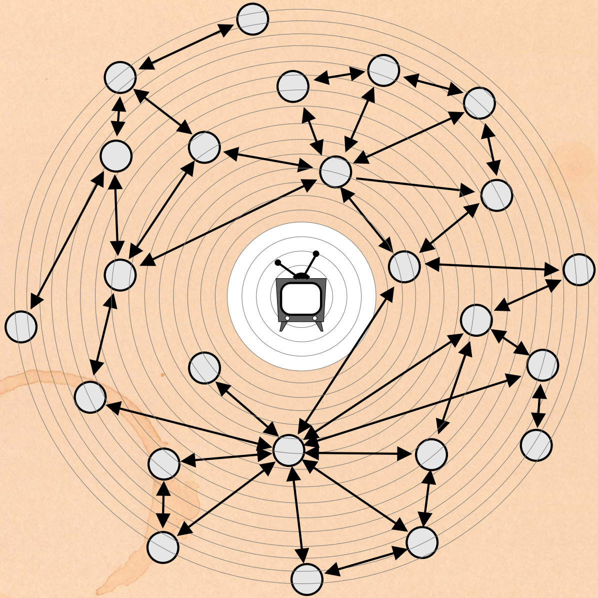
The hypodermic model of influence



The two step model of influence [13]



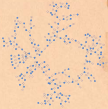
The general model of influence



Social Contagion

Why do things spread?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...



Social Contagion

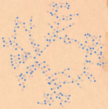
Why do things spread?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...

Social Contagion

Why do things spread?

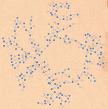
- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...



Social Contagion

Why do things spread?

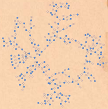
- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...



Social Contagion

Why do things spread?

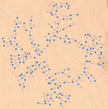
- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...



Social Contagion

Why do things spread?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...



Social Contagion

Why do things spread?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...

Social Contagion

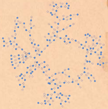
Why do things spread?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...

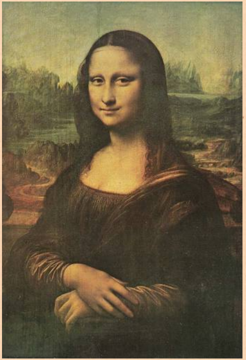
Social Contagion

Why do things spread?

- Because of properties of special individuals?
- Or system level properties?
- Is the match that lights the fire important?
- Yes. But only because we are narrative-making machines...
- We like to think things happened for reasons...
- Reasons for success are usually ascribed to intrinsic properties (e.g., Mona Lisa)
- System/group properties harder to understand
- Always good to examine what is said before and after the fact...

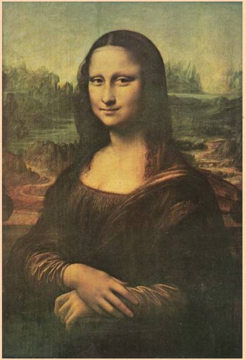


The Mona Lisa



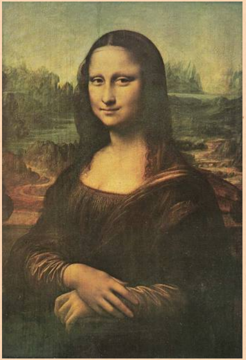
- “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism,

The Mona Lisa



- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism,

The Mona Lisa



- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism,

The Mona Lisa

- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism, parody, ...

The Mona Lisa

- “Becoming Mona Lisa: The Making of a Global Icon”—David Sassoon
- Not the world’s greatest painting from the start...
- Escalation through theft, vandalism, parody, ...

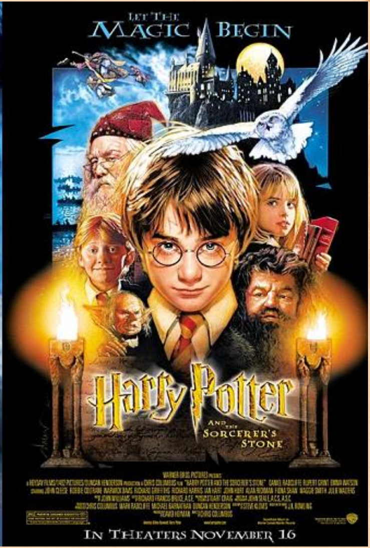
The Mona Lisa

- "Becoming Mona Lisa: The Making of a Global Icon"—David Sassoon
- Not the world's greatest painting from the start...
- Escalation through theft, vandalism, parody, ...

The completely unpredicted fall of Eastern Europe

Timur Kuran: [14, 15] “Now Out of Never: The Element of Surprise in the East European Revolution of 1989”

The dismal predictive powers of editors...



Social Contagion

Messing with social connections

- Ads based on message content
- BzzAgent (⊞)
- Facebook's advertising: Beacon (⊞)

Social Contagion

Messing with social connections

- Ads based on message content
- BzzAgent
- Facebook's advertising: Beacon

Social Contagion

Messing with social connections

- Ads based on message content (e.g., Google and email)
- BzzAgent (⊞)
- Facebook's advertising: Beacon (⊞)

Social Contagion

Messing with social connections

- Ads based on message content (e.g., Google and email)
- BzzAgent (⊞)
- Facebook's advertising: Beacon (⊞)

Social Contagion

Messing with social connections

- Ads based on message content (e.g., Google and email)
- BzzAgent (⊞)
- Facebook's advertising: Beacon (⊞)

Getting others to do things for you

A very good book: 'Influence'[7] by Robert Cialdini (⊞)

Six modes of influence

1. Reciprocation: *The Old Give and Take... and Take*
   e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*
   e.g., Hazing.
3. Social Proof: *Truths Are Us*
   e.g., Catherine Genovese, Jonestown
4. Liking: *The Friendly Thief*
   Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*
   Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*
   Prohibition.

Getting others to do things for you

A very good book: 'Influence' [7] by Robert Cialdini (⊞)

Six modes of influence

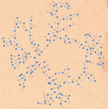
1. Reciprocation: *The Old Give and Take... and Take*
   e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*
   e.g., Hazing.
3. Social Proof: *Truths Are Us*
   e.g., Catherine Genovese, Jonestown
4. Liking: *The Friendly Thief*
   Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*
   Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*
   Prohibition.

Getting others to do things for you

A very good book: 'Influence'[7] by Robert Cialdini (⊞)

Six modes of influence

1. Reciprocation: *The Old Give and Take... and Take*
   e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*
   e.g., Hazing.
3. Social Proof: *Truths Are Us*
   e.g., Catherine Genovese, Jonestown
4. Liking: *The Friendly Thief*
   Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*
   Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*
   Prohibition.

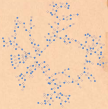


Getting others to do things for you

A very good book: 'Influence' [7] by Robert Cialdini (⊞)

Six modes of influence

1. Reciprocation: *The Old Give and Take... and Take*
   e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*
   e.g., Hazing.
3. Social Proof: *Truths Are Us*
   e.g., Catherine Genovese, Jonestown
4. Liking: *The Friendly Thief*
   Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*
   Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*
   Prohibition.

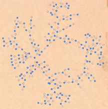


Getting others to do things for you

A very good book: 'Influence' [7] by Robert Cialdini (⊞)

Six modes of influence

1. Reciprocation: *The Old Give and Take... and Take*
   e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*
   e.g., Hazing.
3. Social Proof: *Truths Are Us*
   e.g., Catherine Genovese, Jonestown
4. Liking: *The Friendly Thief*
   Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*
   Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*
   Prohibition.

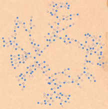


Getting others to do things for you

A very good book: 'Influence' [7] by Robert Cialdini (⊞)

Six modes of influence

1. Reciprocation: *The Old Give and Take... and Take*
   e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*
   e.g., Hazing.
3. Social Proof: *Truths Are Us*
   e.g., Catherine Genovese, Jonestown
4. Liking: *The Friendly Thief*
   Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*
   Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*
   Prohibition.



Getting others to do things for you

A very good book: 'Influence' [7] by Robert Cialdini (⊞)

Six modes of influence

1. Reciprocation: *The Old Give and Take... and Take*
   e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*
   e.g., Hazing.
3. Social Proof: *Truths Are Us*
   e.g., Catherine Genovese, Jonestown
4. Liking: *The Friendly Thief*
   Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*
   Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*
   Prohibition.

Getting others to do things for you

A very good book: 'Influence'[7] by Robert Cialdini (⊞)

Six modes of influence

1. Reciprocation: *The Old Give and Take... and Take*
   e.g., Free samples, Hare Krishnas.
2. Commitment and Consistency: *Hobgoblins of the Mind*
   e.g., Hazing.
3. Social Proof: *Truths Are Us*
   e.g., Catherine Genovese, Jonestown
4. Liking: *The Friendly Thief*
   Separation into groups is enough to cause problems.
5. Authority: *Directed Deference*
   Milgram's obedience to authority experiment.
6. Scarcity: *The Rule of the Few*
   Prohibition.

Social contagion

- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

Social contagion

- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

Social contagion

- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

Social contagion

- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

Social contagion

- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

Social contagion

- Cialdini's modes are heuristics that help up us get through life.
- Useful but can be leveraged...

Other acts of influence:

- Conspicuous Consumption (Veblen, 1912)
- Conspicuous Destruction (Potlatch)

Social Contagion

Some important models

- T ipping models—Schelling (1971) [16, 17, 18]
  - Simulation on checker boards
  - Idea of thresholds
  - Explore the Netlogo (⊞) implementation [21]
- Threshold models—Granovetter (1978) [10]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [1, 2]
  - Social learning theory, Informational cascades,...

Social Contagion

Some important models

- T ipping models—Schelling (1971) [16, 17, 18]
  - Simulation on checker boards
  - Idea of thresholds
  - Explore the Netlogo (⊞) implementation [21]
- Threshold models—Granovetter (1978) [10]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [1, 2]
  - Social learning theory, Informational cascades,...

Social Contagion

Some important models

- T ipping models—Schelling (1971) [16, 17, 18]
    - Simulation on checker boards
    - Idea of thresholds
    - Explore the Netlogo (⊞) implementation [21]
- Threshold models—Granovetter (1978) [10]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [1, 2]
    - Social learning theory, Informational cascades,...

Social Contagion

Some important models

- Tipping models—Schelling (1971) [16, 17, 18]
  - Simulation on checker boards
  - Idea of thresholds
  - Explore the Netlogo (⊞) implementation [21]
- Threshold models—Granovetter (1978) [10]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [1, 2]
  - Social learning theory, Informational cascades,...

Social Contagion

Some important models

- T ipping models—Schelling (1971) [16, 17, 18]
  - Simulation on checker boards
  - Idea of thresholds
  - Explore the Netlogo (⊞) implementation [21]
- Threshold models—Granovetter (1978) [10]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [1, 2]
  - Social learning theory, Informational cascades,...

Social Contagion

Some important models

- T ipping models—Schelling (1971) [16, 17, 18]
  - Simulation on checker boards
  - Idea of thresholds
  - Explore the Netlogo (⊞) implementation [21]
- Threshold models—Granovetter (1978) [10]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [1, 2]
  - Social learning theory, Informational cascades,...

Social Contagion

Some important models

- T ipping models—Schelling (1971) [16, 17, 18]
  - Simulation on checker boards
  - Idea of thresholds
  - Explore the Netlogo (⊞) implementation [21]
- Threshold models—Granovetter (1978) [10]
- Herding models—Bikhchandani, Hirschleifer, Welch (1992) [1, 2]
  - Social learning theory, Informational cascades,...

Social contagion models

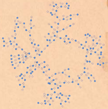
Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter...
- Assumption: level of influence per person is uniform

Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter...
- Assumption: level of influence per person is uniform



Social contagion models

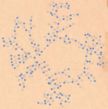
Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter...
- Assumption: level of influence per person is uniform

Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter...
- Assumption: level of influence per person is uniform



Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter...
- Assumption: level of influence per person is uniform

Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter...
- Assumption: level of influence per person is uniform

Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter...
- Assumption: level of influence per person is uniform

Social contagion models

Thresholds

- Basic idea: individuals adopt a behavior when a certain fraction of others have adopted
- 'Others' may be everyone in a population, an individual's close friends, any reference group.
- Response can be probabilistic or deterministic.
- Individual thresholds can vary
- Assumption: order of others' adoption does not matter... (unrealistic).
- Assumption: level of influence per person is uniform (unrealistic).

Social Contagion

Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
- Externalities = Effects on others not directly involved in a transaction
- Examples: telephones, fax machine, Facebook, operating systems
- An individual's utility increases with the adoption level among peers and the population in general

Social Contagion

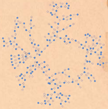
Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
- Externalities = Effects on others not directly involved in a transaction
- Examples: telephones, fax machine, Facebook, operating systems
- An individual's utility increases with the adoption level among peers and the population in general

Social Contagion

Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
- Externalities = Effects on others not directly involved in a transaction
- Examples: telephones, fax machine, Facebook, operating systems
- An individual's utility increases with the adoption level among peers and the population in general



Social Contagion

Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
- Externalities = Effects on others not directly involved in a transaction
- Examples: telephones, fax machine, Facebook, operating systems
- An individual's utility increases with the adoption level among peers and the population in general

Social Contagion

Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
- Externalities = Effects on others not directly involved in a transaction
- Examples: telephones, fax machine, Facebook, operating systems
- An individual's utility increases with the adoption level among peers and the population in general

Social Contagion

Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
- Externalities = Effects on others not directly involved in a transaction
- Examples: telephones, fax machine, Facebook, operating systems
- An individual's utility increases with the adoption level among peers and the population in general

Social Contagion

Some possible origins of thresholds:

- Desire to coordinate, to conform.
- Lack of information: impute the worth of a good or behavior based on degree of adoption (social proof)
- Economics: Network effects or network externalities
- Externalities = Effects on others not directly involved in a transaction
- Examples: telephones, fax machine, Facebook, operating systems
- An individual's utility increases with the adoption level among peers and the population in general

Outline

Social Contagion Models
Background
Granovetter's model
Network version
Groups
Chaos
References

Social Contagion

Granovetter's Threshold model—definitions

- ϕ^* = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$
- ϕ_t = fraction of people 'rioting' at time step t.

Social Contagion

Granovetter's Threshold model—definitions

- ϕ^* = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$
- ϕ_t = fraction of people 'rioting' at time step t.

Social Contagion

Granovetter's Threshold model—definitions

- ϕ^* = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$
- ϕ_t = fraction of people 'rioting' at time step t.

Social Contagion

Granovetter's Threshold model—definitions

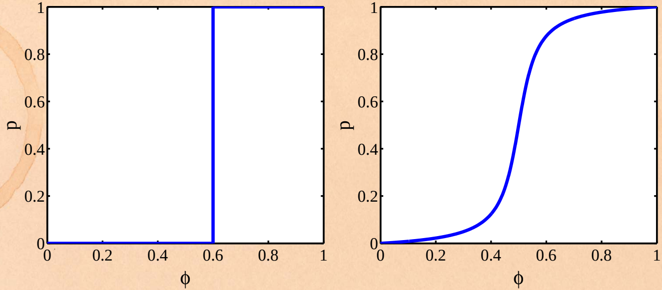
- ϕ^* = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$
- ϕ_t = fraction of people 'rioting' at time step t.

Social Contagion

Granovetter's Threshold model—definitions

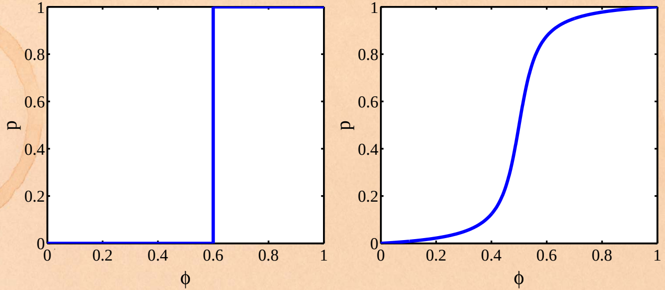
- ϕ^* = threshold of an individual.
- $f(\phi_*)$ = distribution of thresholds in a population.
- $F(\phi_*)$ = cumulative distribution = $\int_{\phi'_*=0}^{\phi_*} f(\phi'_*)\mathrm{d}\phi'_*$
- ϕ_t = fraction of people 'rioting' at time step t.

Threshold models



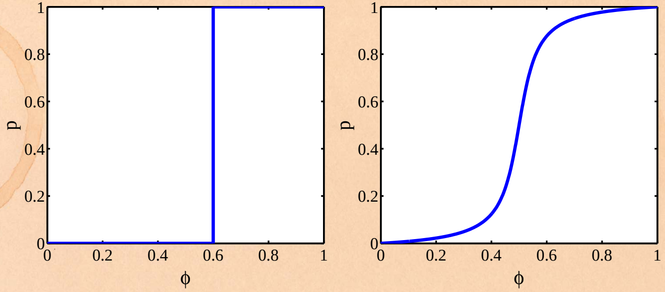
- Example threshold influence response functions: deterministic and stochastic
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Two states: S and I.

Threshold models



- Example threshold influence response functions: deterministic and stochastic
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Two states: S and I.

Threshold models



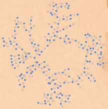
- Example threshold influence response functions: deterministic and stochastic
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Two states: S and I.

Threshold models

- At time $t+1$, fraction rioting = fraction with $\phi_* \le \phi_t$.
- $$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*) \mathrm{d}\phi_* = F(\phi_*)|_0^{\phi_t} = F(\phi_t)$$
- $\Rightarrow$ Iterative maps of the unit interval $[0, 1]$.

Threshold models

- At time $t+1$, fraction rioting = fraction with $\phi_* \le \phi_t$.
- $$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*)\mathrm{d}\phi_* = F(\phi_*)|_0^{\phi_t} = F(\phi_t)$$
- $\Rightarrow$ Iterative maps of the unit interval $[0, 1]$.

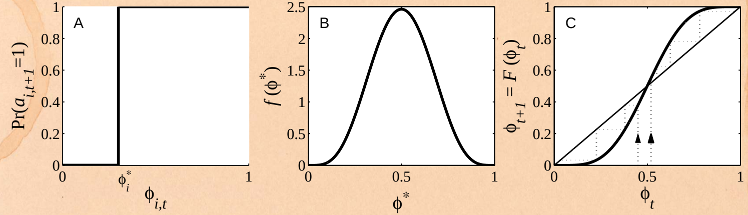


Threshold models

- At time $t+1$, fraction rioting = fraction with $\phi_* \leq \phi_t$.
- $$\phi_{t+1} = \int_0^{\phi_t} f(\phi_*)\mathrm{d}\phi_* = F(\phi_*)|_0^{\phi_t} = F(\phi_t)$$
- $\Rightarrow$ Iterative maps of the unit interval $[0, 1]$.

Threshold models

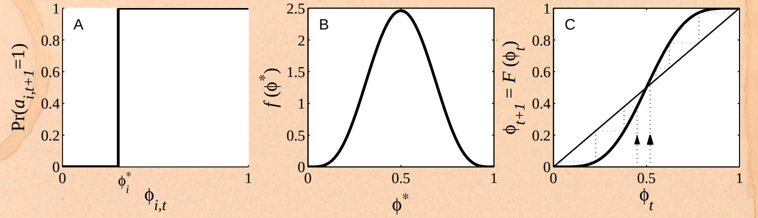
Action based on perceived behavior of others.



- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a Critical mass model

Threshold models

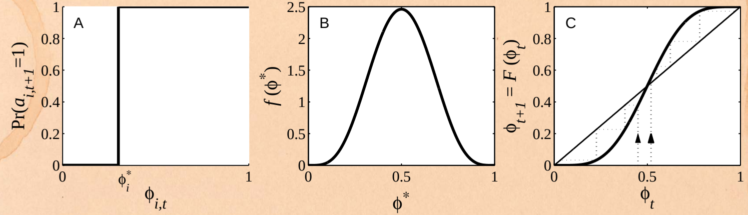
Action based on perceived behavior of others.



- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a Critical mass model

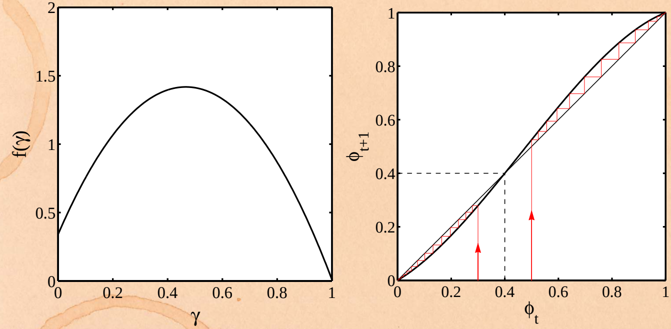
Threshold models

Action based on perceived behavior of others.



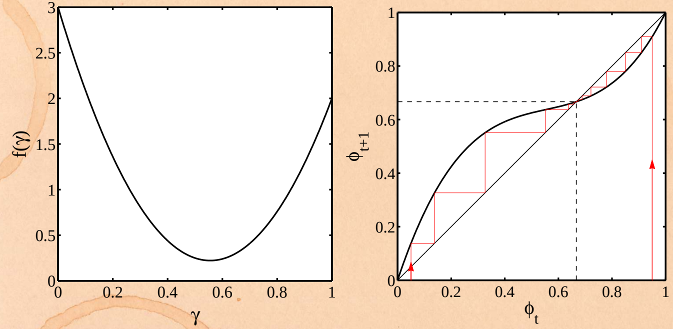
- Two states: S and I.
- ϕ = fraction of contacts 'on' (e.g., rioting)
- Discrete time update (strong assumption!)
- This is a Critical mass model

Threshold models



- Another example of critical mass model...

Threshold models



- Example of single stable state model

Threshold models

Implications for collective action theory:

1. Collective uniformity $\not\Rightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes

Threshold models

Implications for collective action theory:

1. Collective uniformity $\not\Rightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes

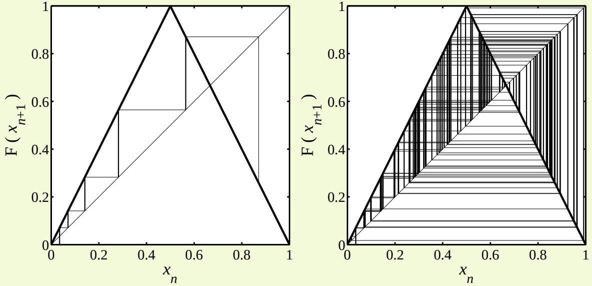
Threshold models

Implications for collective action theory:

1. Collective uniformity $\not\Rightarrow$ individual uniformity
2. Small individual changes $\Rightarrow$ large global changes

Threshold models

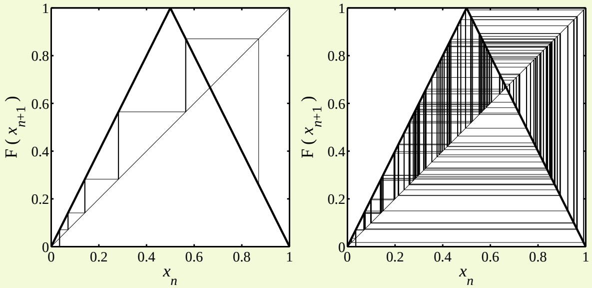
Chaotic behavior possible [12, 11]



- Period doubling arises as map amplitude r is increased.
- Synchronous update assumption is crucial

Threshold models

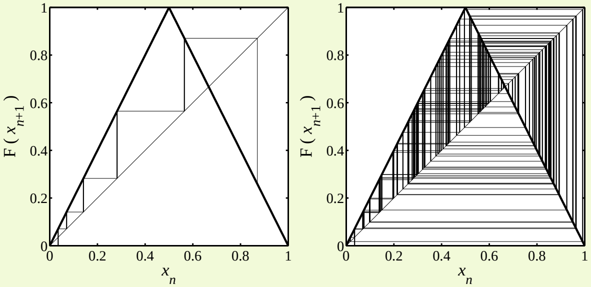
Chaotic behavior possible [12, 11]



- Period doubling arises as map amplitude r is increased.
- Synchronous update assumption is crucial

Threshold models

Chaotic behavior possible [12, 11]



- Period doubling arises as map amplitude r is increased.
- Synchronous update assumption is crucial

Outline

Threshold model on a network

Many years after Granovetter and Soong's work:

"A simple model of global cascades on random networks"
D. J. Watts. Proc. Natl. Acad. Sci., 2002 [20]

- Mean field model → network model
- Individuals now have a limited view of the world

Threshold model on a network

Many years after Granovetter and Soong's work:

"A simple model of global cascades on random networks"
D. J. Watts. Proc. Natl. Acad. Sci., 2002 [20]

- Mean field model → network model
- Individuals now have a limited view of the world

Threshold model on a network

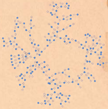
Many years after Granovetter and Soong's work:

"A simple model of global cascades on random networks"
D. J. Watts. Proc. Natl. Acad. Sci., 2002 [20]

- Mean field model → network model
- Individuals now have a limited view of the world

Threshold model on a network

- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)

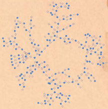


Threshold model on a network

- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)

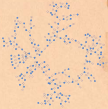
Threshold model on a network

- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)



Threshold model on a network

- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)



Threshold model on a network

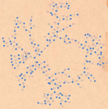
- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)

Threshold model on a network

- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)

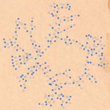
Threshold model on a network

- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)



Threshold model on a network

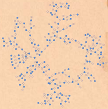
- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)



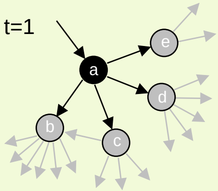
The UNIVERSITY of VERMONT

Threshold model on a network

- Interactions between individuals now represented by a network
- Network is sparse
- Individual i has k_i contacts
- Influence on each link is reciprocal and of unit weight
- Each individual i has a fixed threshold ϕ_i
- Individuals repeatedly poll contacts on network
- Synchronous, discrete time updating
- Individual i becomes active when fraction of active contacts $\frac{a_i}{k_i} \geq \phi_i$
- Individuals remain active when switched (no recovery = SI model)

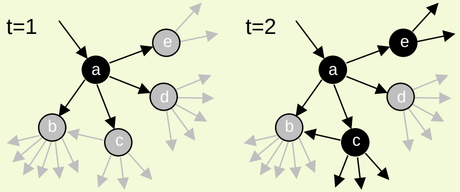


Threshold model on a network



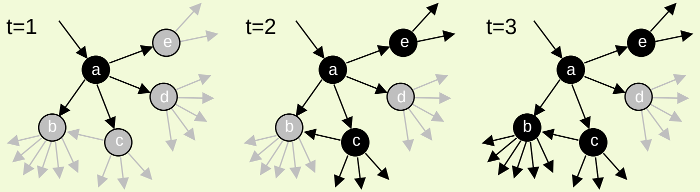
- All nodes have threshold $\phi = 0.2$.

Threshold model on a network



- All nodes have threshold $\phi = 0.2$.

Threshold model on a network



- All nodes have threshold $\phi = 0.2$.

Snowballing

The Cascade Condition:

1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?

Snowballing

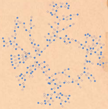
The Cascade Condition:

1. If one individual is initially activated, what is the probability that an activation will spread over a network?
2. What features of a network determine whether a cascade will occur or not?

Snowballing

First study random networks:

- Start with N nodes with a degree distribution p_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition



Snowballing

First study random networks:

- Start with N nodes with a degree distribution p_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition

Snowballing

First study random networks:

- Start with N nodes with a degree distribution p_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition

Snowballing

First study random networks:

- Start with N nodes with a degree distribution p_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition

Snowballing

First study random networks:

- Start with N nodes with a degree distribution p_k
- Nodes are randomly connected (carefully so)
- Aim: Figure out when activation will propagate
- Determine a cascade condition

Snowballing

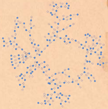
Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to at least 1 more infected link, then activation spreads.
- We need to understand which nodes can be activated when only one of their neigbors becomes active.

Snowballing

Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to at least 1 more infected link, then activation spreads.
- We need to understand which nodes can be activated when only one of their neigbors becomes active.



Snowballing

Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to at least 1 more infected link, then activation spreads.
- We need to understand which nodes can be activated when only one of their neigbors becomes active.

Snowballing

Follow active links

- An active link is a link connected to an activated node.
- If an infected link leads to at least 1 more infected link, then activation spreads.
- We need to understand which nodes can be activated when only one of their neigbors becomes active.

The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables* [20]
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.

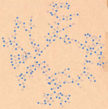
The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables* [20]
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.



The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables*[20]
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.

The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables*[20]
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.

The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables* [20]
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.

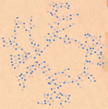
The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables* [20]
- Cluster of vulnerables = critical mass
- Network story: 1 node → critical mass → everyone.



The most gullible

Vulnerables:

- We call individuals who can be activated by just one contact being active vulnerables
- The vulnerability condition for node i:

$$1/k_i \geq \phi_i$$

- Which means # contacts $k_i \leq \lfloor 1/\phi_i \rfloor$
- For global cascades on random networks, must have a *global cluster of vulnerables*[20]
- Cluster of vulnerables = critical mass
- Network story: 1 node $\rightarrow$ critical mass $\rightarrow$ everyone.

Cascade condition

Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k.
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

Cascade condition

Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k.
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$

Cascade condition

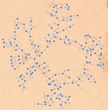
Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k.
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$



Cascade condition

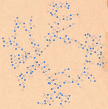
Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k.
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$



Cascade condition

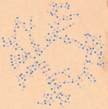
Back to following a link:

- A randomly chosen link, traversed in a random direction, leads to a degree k node with probability $\propto kP_k$.
- Follows from there being k ways to connect to a node with degree k.
- Normalization:

$$\sum_{k=0}^{\infty} kP_k = \langle k \rangle$$

- So

$$P(\text{linked node has degree } k) = \frac{kP_k}{\langle k \rangle}$$



Cascade condition

Next: Vulnerability of linked node

- Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*)\mathrm{d}\phi'_*$$

- If linked node is vulnerable, it produces $k - 1$ new outgoing active links
- If linked node is not vulnerable, it produces no active links.

Cascade condition

Next: Vulnerability of linked node

- Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*)\mathrm{d}\phi'_*$$

- If linked node is vulnerable, it produces $k-1$ new outgoing active links
- If linked node is not vulnerable, it produces no active links.

Cascade condition

Next: Vulnerability of linked node

- Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) \mathrm{d}\phi'_*$$

- If linked node is vulnerable, it produces $k-1$ new outgoing active links
- If linked node is not vulnerable, it produces no active links.

Cascade condition

Next: Vulnerability of linked node

- Linked node is vulnerable with probability

$$\beta_k = \int_{\phi'_*=0}^{1/k} f(\phi'_*) \mathrm{d}\phi'_*$$

- If linked node is vulnerable, it produces $k-1$ new outgoing active links
- If linked node is not vulnerable, it produces no active links.

Cascade condition

Putting things together:

- Expected number of active edges produced by an active edge:

$$R = \underbrace{\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} +$$

$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$

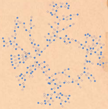
Cascade condition

Putting things together:

- Expected number of active edges produced by an active edge:

$$R = \sum_{k=1}^{\infty} \underbrace{(k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} \quad +$$

$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$



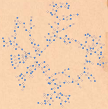
Cascade condition

Putting things together:

- Expected number of active edges produced by an active edge:

$$R = \sum_{k=1}^{\infty} \underbrace{(k-1)\cdot\beta_k\cdot\frac{kP_k}{\langle k\rangle}}_{\text{success}} + \underbrace{0\cdot(1-\beta_k)\cdot\frac{kP_k}{\langle k\rangle}}_{\text{failure}}$$

$$= \sum_{k=1}^{\infty}(k-1)\cdot\beta_k\cdot\frac{kP_k}{\langle k\rangle}$$



Cascade condition

Putting things together:

- Expected number of active edges produced by an active edge:

$$R = \underbrace{\sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}}_{\text{success}} + \underbrace{0 \cdot (1-\beta_k) \cdot \frac{kP_k}{\langle k \rangle}}_{\text{failure}}$$

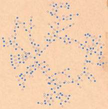
$$= \sum_{k=1}^{\infty} (k-1) \cdot \beta_k \cdot \frac{kP_k}{\langle k \rangle}$$

Cascade condition

So... for random networks with fixed degree distributions, cacades take off when:

$$\sum_{k=1}^{\infty}(k-1)\cdot\beta_k\cdot\frac{kP_k}{\langle k\rangle}\geq 1.$$

- β_k = probability a degree k node is vulnerable.
- P_k = probability a node has degree k.



Cascade condition

Two special cases:

- (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

Cascade condition

Two special cases:

- (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty}(k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty}(k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

Cascade condition

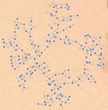
Two special cases:

- (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$



Cascade condition

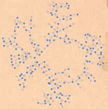
Two special cases:

- (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- (2) Giant component exists: $\beta = 1$

$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$



Cascade condition

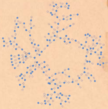
Two special cases:

- (1) Simple disease-like spreading succeeds: $\beta_k = \beta$

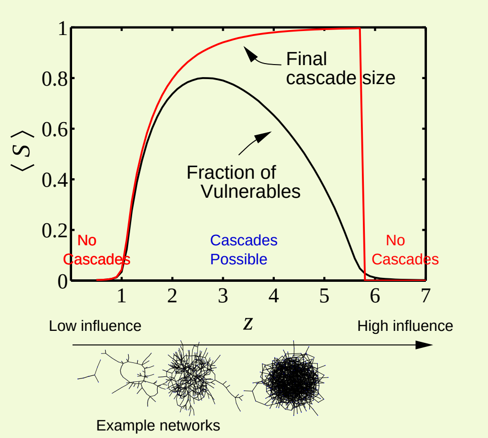
$$\beta \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$

- (2) Giant component exists: $\beta = 1$

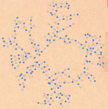
$$1 \cdot \sum_{k=1}^{\infty} (k-1) \cdot \frac{kP_k}{\langle k \rangle} \geq 1.$$



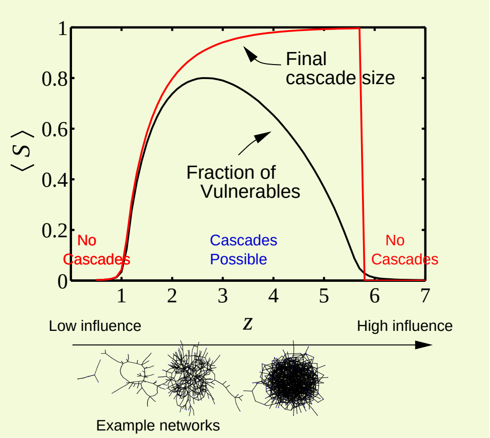
Cascades on random networks



- Cascades occur only if size of max vulnerable cluster > 0.
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

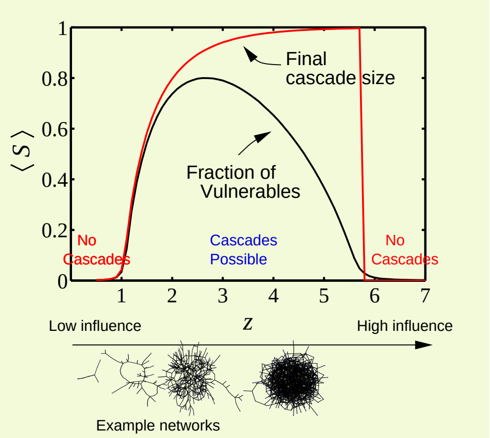


Cascades on random networks



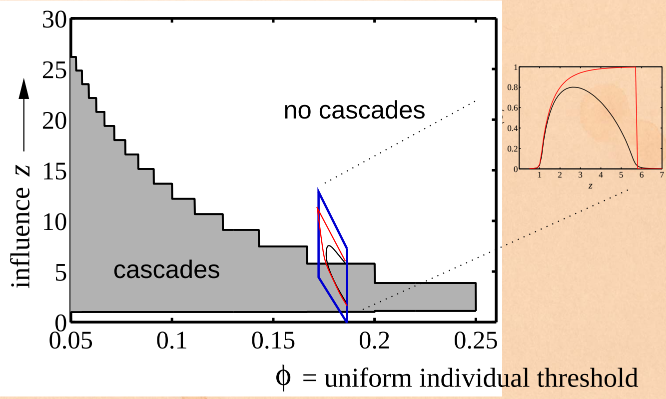
- Cascades occur only if size of max vulnerable cluster > 0.
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

Cascades on random networks



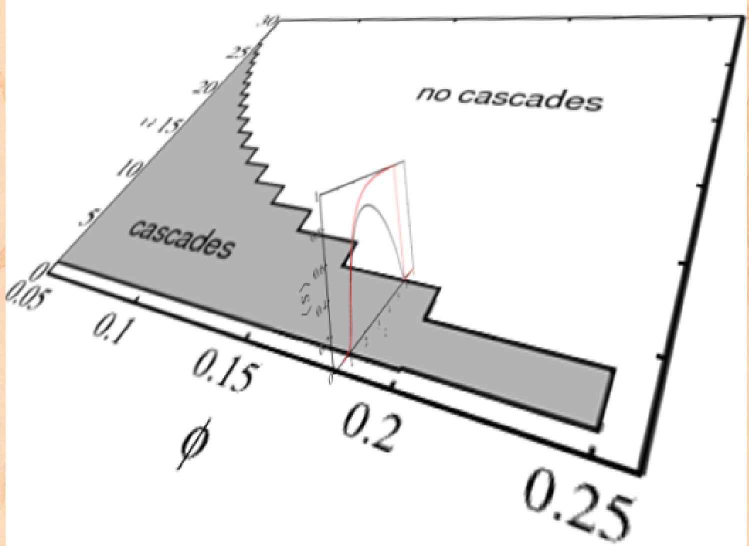
- Cascades occur only if size of max vulnerable cluster > 0.
- System may be 'robust-yet-fragile'.
- 'Ignorance' facilitates spreading.

Cascade window for random networks



- 'Cascade window' widens as threshold ϕ decreases.
- Lower thresholds enable spreading.

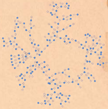
Cascade window for random networks



Cascade window—summary

For our simple model of a uniform threshold:

1. Low $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.
2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in "Cascade window."



Cascade window—summary

For our simple model of a uniform threshold:

1. Low $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.
2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in "Cascade window."

Cascade window—summary

For our simple model of a uniform threshold:

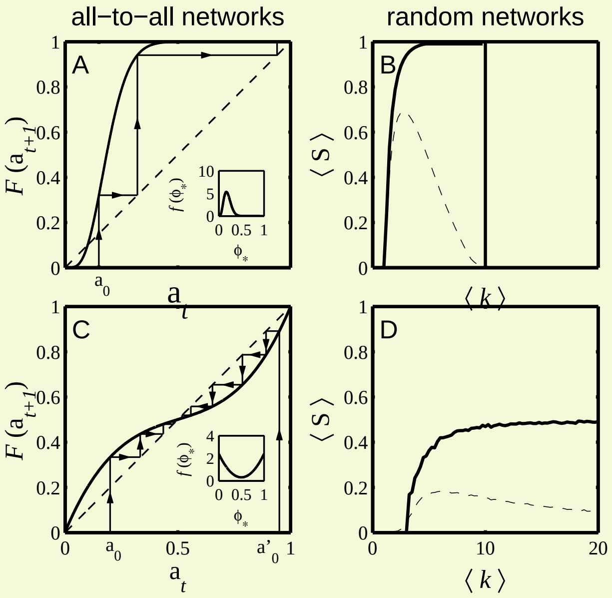
1. Low $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.
2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in "Cascade window."

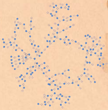
Cascade window—summary

For our simple model of a uniform threshold:

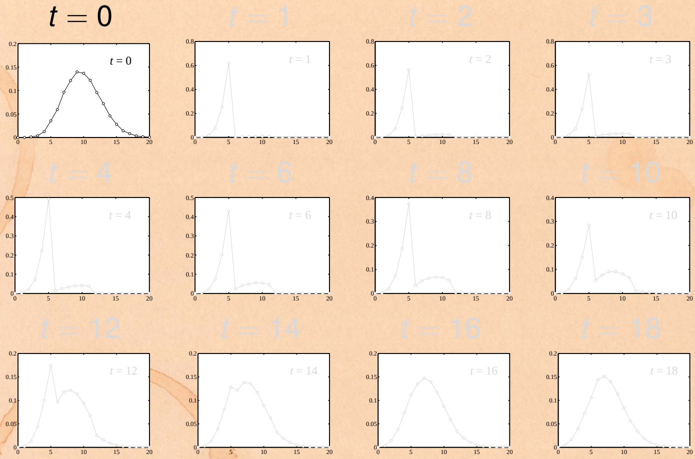
1. Low $\langle k \rangle$: No cascades in poorly connected networks. No global clusters of any kind.
2. High $\langle k \rangle$: Giant component exists but not enough vulnerables.
3. Intermediate $\langle k \rangle$: Global cluster of vulnerables exists. Cascades are possible in "Cascade window."

All-to-all versus random networks



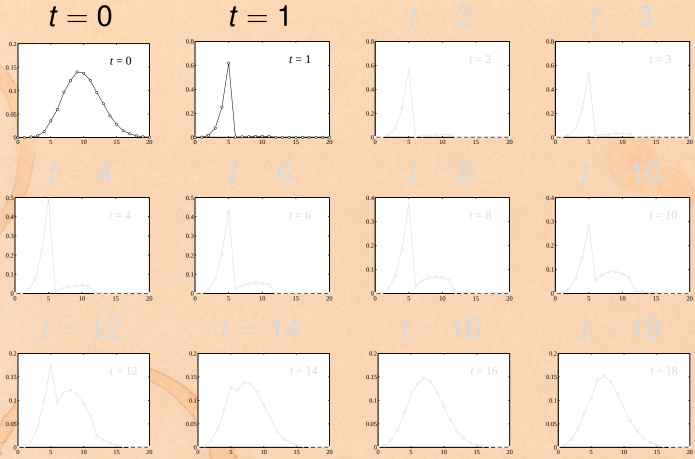


Early adopters—degree distributions

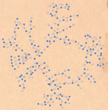


$P_{k,t}$ versus k

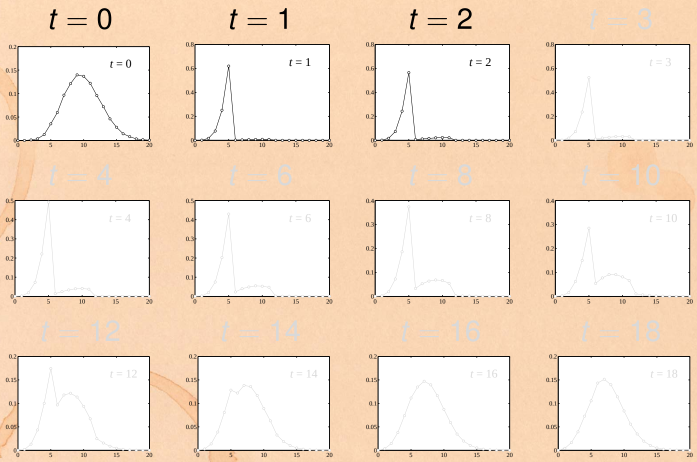
Early adopters—degree distributions



$P_{k,t}$ versus k

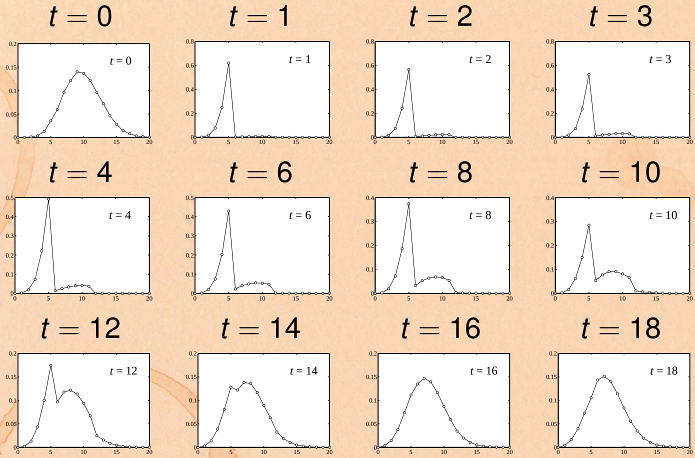


Early adopters—degree distributions

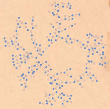


$P_{k,t}$ versus k

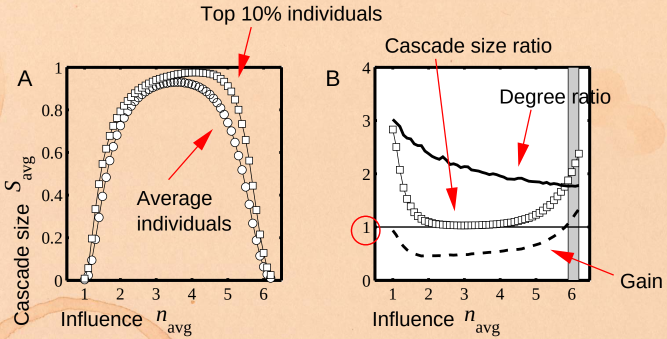
Early adopters—degree distributions



$P_{k,t}$ versus k

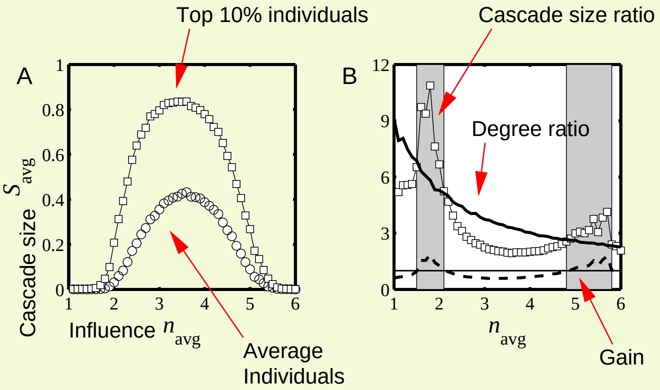


The multiplier effect:



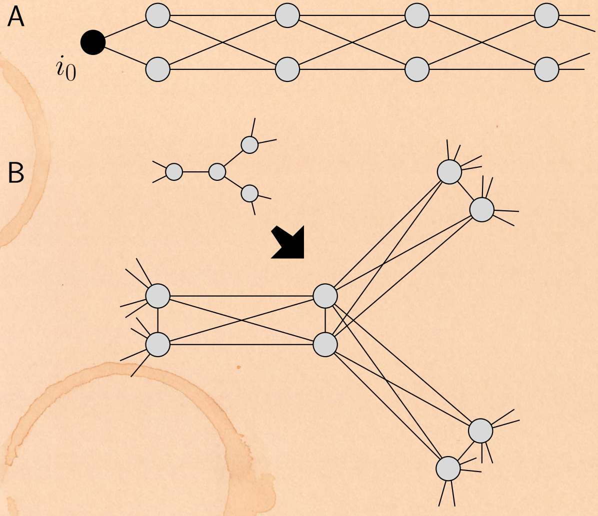
- Fairly uniform levels of individual influence.
- Multiplier effect is mostly below 1.

The multiplier effect:



- Skewed influence distribution example.

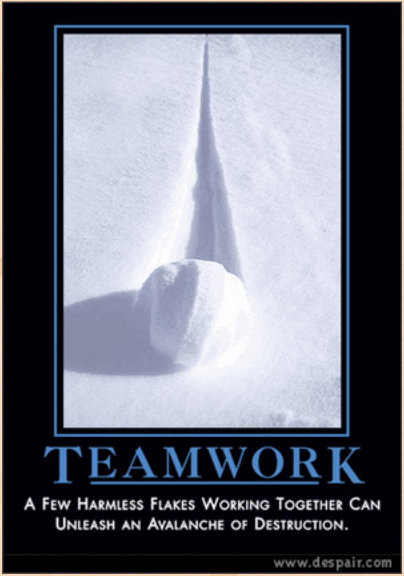
Special subnetworks can act as triggers



- $\phi = 1/3$ for all nodes

Outline

The power of groups...

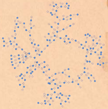


despair.com

"A few harmless flakes working together can unleash an avalanche of destruction."

Extensions

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: group structure



Extensions

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: group structure

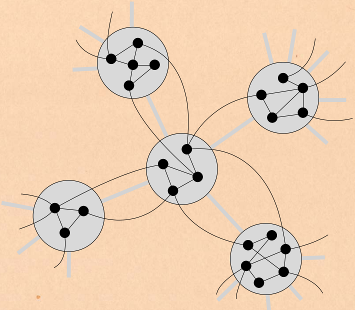
Extensions

- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: group structure

Extensions

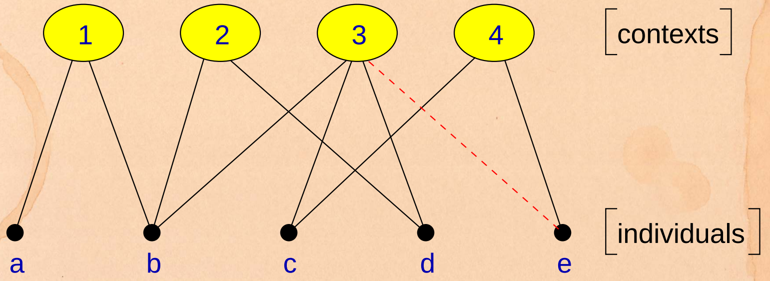
- Assumption of sparse interactions is good
- Degree distribution is (generally) key to a network's function
- Still, random networks don't represent all networks
- Major element missing: group structure

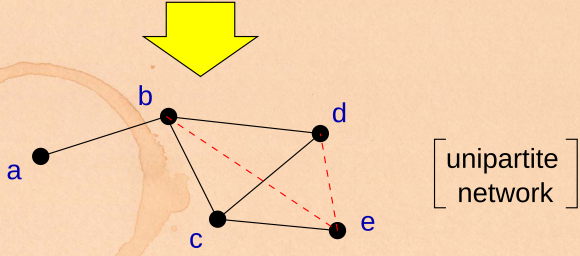
Group structure—Ramified random networks



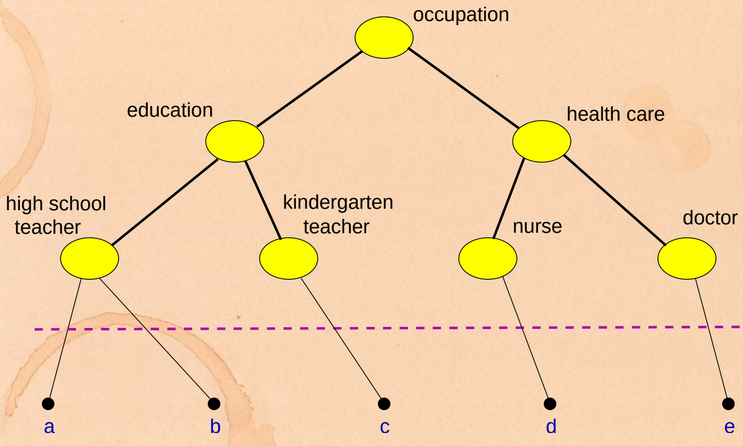
p = intergroup connection probability
q = intragroup connection probability.

Bipartite networks

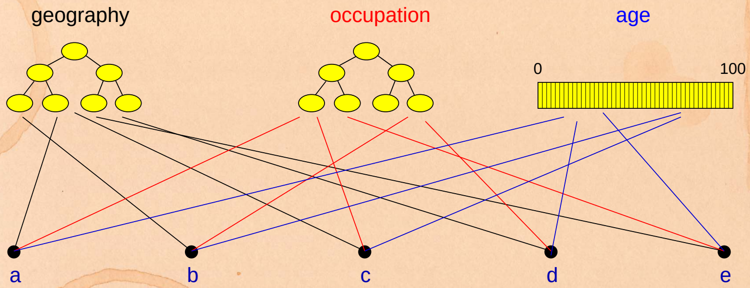




Context distance



Generalized affiliation model



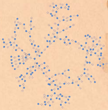
(Blau & Schwartz, Simmel, Breiger)

Generalized affiliation model networks with triadic closure

- Connect nodes with probability $\propto \exp^{-\alpha d}$ where
  α = homophily parameter
  and
  d = distance between nodes (height of lowest common ancestor)
- τ_1 = intergroup probability of friend-of-friend connection
- τ_2 = intragroup probability of friend-of-friend connection

Generalized affiliation model networks with triadic closure

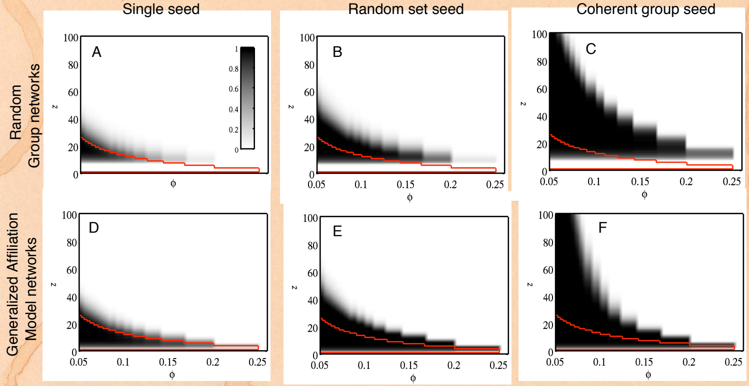
- Connect nodes with probability $\propto \exp^{-\alpha d}$
  where
  α = homophily parameter
  and
  d = distance between nodes (height of lowest common ancestor)
- τ_1 = intergroup probability of friend-of-friend connection
- τ_2 = intragroup probability of friend-of-friend connection



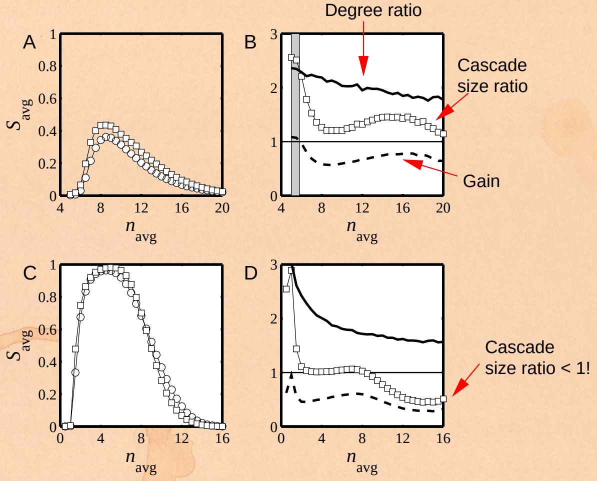
Generalized affiliation model networks with triadic closure

- Connect nodes with probability $\propto \exp^{-\alpha d}$ where
  α = homophily parameter
  and
  d = distance between nodes (height of lowest common ancestor)
- τ_1 = intergroup probability of friend-of-friend connection
- τ_2 = intragroup probability of friend-of-friend connection

Cascade windows for group-based networks

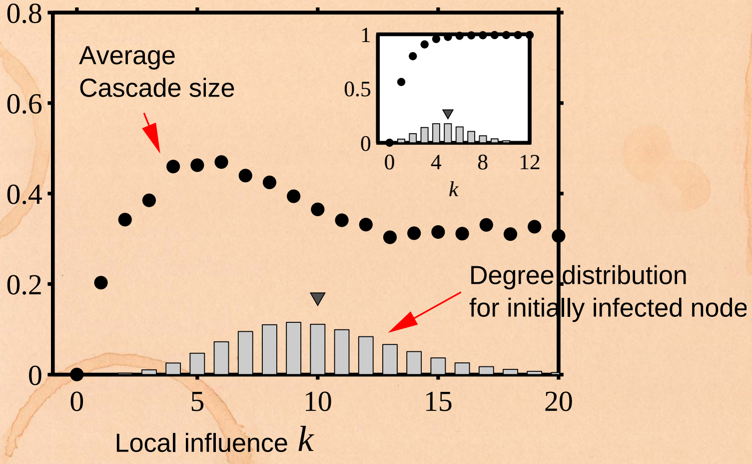


Multiplier effect for group-based networks:



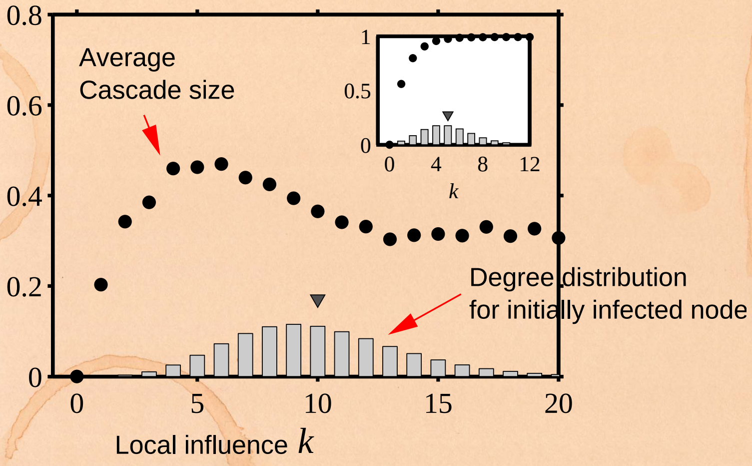
- Multiplier almost always below 1.

Assortativity in group-based networks



- The most connected nodes aren't always the most 'influential.'
- Degree assortativity is the reason.

Assortativity in group-based networks

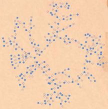


- The most connected nodes aren't always the most 'influential.'
- Degree assortativity is the reason.

Social contagion

Summary

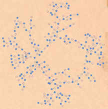
- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.



Social contagion

Summary

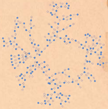
- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.



Social contagion

Summary

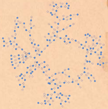
- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.



Social contagion

Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.



Social contagion

Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.

Social contagion

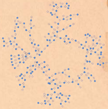
Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.

Social contagion

Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.



Social contagion

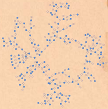
Summary

- 'Influential vulnerables' are key to spread.
- Early adopters are mostly vulnerables.
- Vulnerable nodes important but not necessary.
- Groups may greatly facilitate spread.
- Seems that cascade condition is a global one.
- Most extreme/unexpected cascades occur in highly connected networks
- 'Influentials' are posterior constructs.
- Many potential influentials exist.

Social contagion

Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth.
  (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.



Social contagion

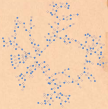
Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth.
  (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.

Social contagion

Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth.
  (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.



Social contagion

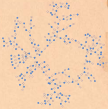
Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth.
  (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.

Social contagion

Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth.
  (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.



Social contagion

Implications

- Focus on the influential vulnerables.
- Create entities that can be transmitted successfully through many individuals rather than broadcast from one 'influential.'
- Only simple ideas can spread by word-of-mouth.
  (Idea of opinion leaders spreads well...)
- Want enough individuals who will adopt and display.
- Displaying can be passive = free (yo-yo's, fashion), or active = harder to achieve (political messages).
- Entities can be novel or designed to combine with others, e.g. block another one.

Outline

Chaotic contagion:

- What if individual response functions are not monotonic?
- Consider a simple deterministic version:
  - Node i has an 'activation threshold' $\phi_{i,1}$

    ... and a 'de-activation threshold' $\phi_{i,2}$
  - Nodes like to imitate but only up to a limit—they don't want to be like everyone else.

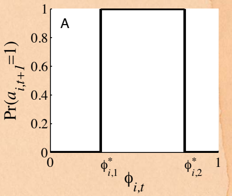
Chaotic contagion:

- What if individual response functions are not monotonic?
- Consider a simple deterministic version:

- Node i has an 'activation threshold' $\phi_{i,1}$

  ... and a 'de-activation threshold' $\phi_{i,2}$

- Nodes like to imitate but only up to a limit—they don't want to be like everyone else.

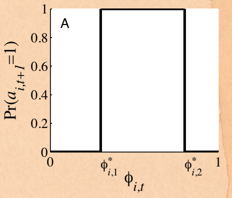


Chaotic contagion:

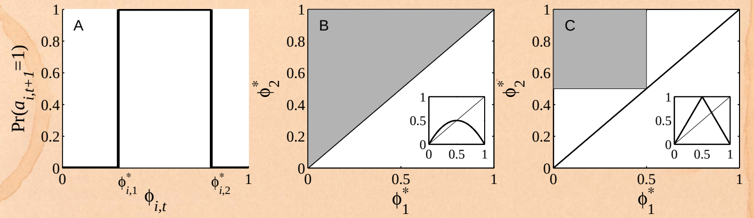
- What if individual response functions are not monotonic?
- Consider a simple deterministic version:

- Node i has an 'activation threshold' $\phi_{i,1}$

  ...and a 'de-activation threshold' $\phi_{i,2}$
- Nodes like to imitate but only up to a limit—they don't want to be like everyone else.

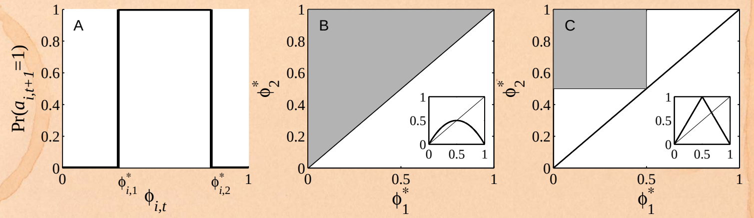


Two population examples:



- Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- Insets show composite response function averaged over population.
- We'll consider plot C's example: the tent map.

Two population examples:



- Randomly select $(\phi_{i,1}, \phi_{i,2})$ from gray regions shown in plots B and C.
- Insets show composite response function averaged over population.
- We'll consider plot C's example: the tent map.

Chaotic contagion

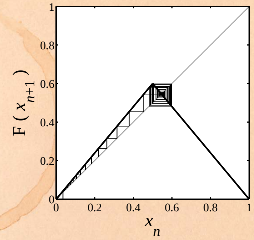
Definition of the tent map:

$$F(x) = \begin{cases} rx \text{ for } 0 \le x \le \frac{1}{2}, \\ r(1-x) \text{ for } \frac{1}{2} \le x \le 1. \end{cases}$$

- The usual business: look at how F iteratively maps the unit interval $[0, 1]$.

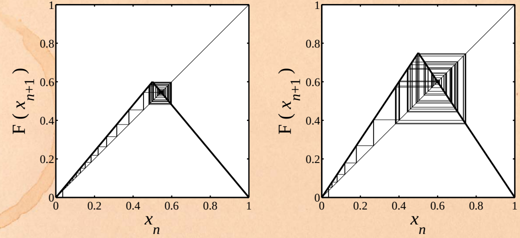
The tent map

Effect of increasing r from 1 to 2.



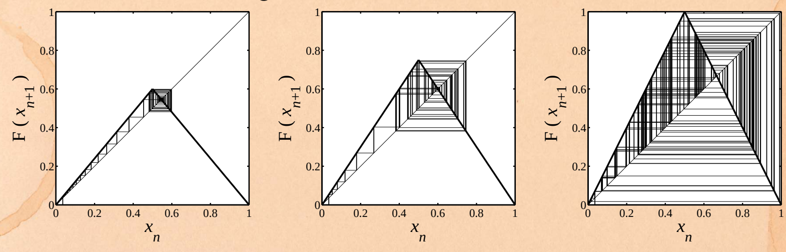
The tent map

Effect of increasing r from 1 to 2.



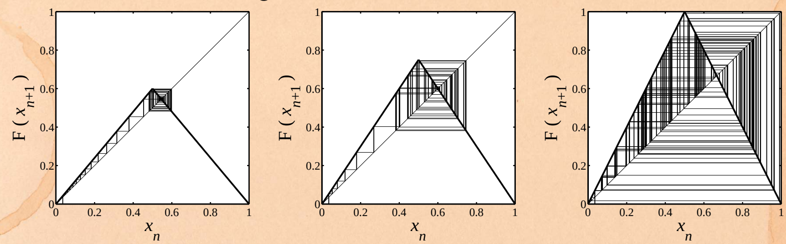
The tent map

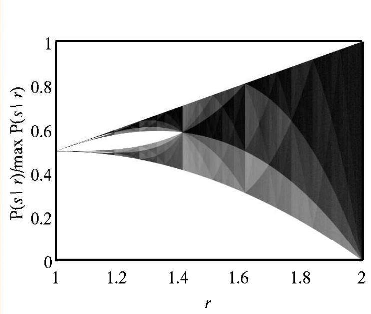
Effect of increasing r from 1 to 2.



The tent map

Effect of increasing *r* from 1 to 2.



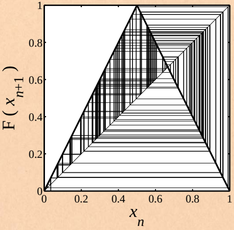


Orbit diagram:

Chaotic behavior increases as map slope *r* is increased.

Chaotic behavior

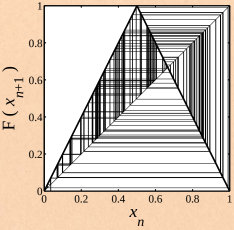
Take $r = 2$ case:



- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

Chaotic behavior

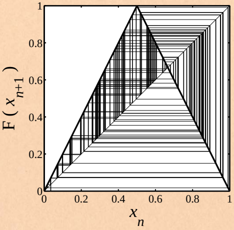
Take $r = 2$ case:



- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

Chaotic behavior

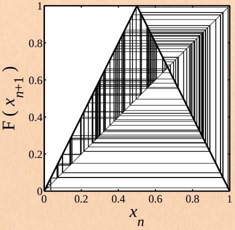
Take $r = 2$ case:



- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

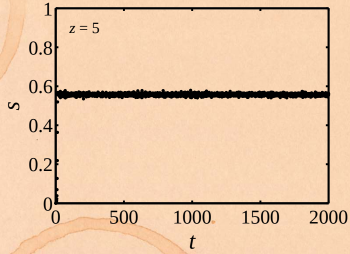
Chaotic behavior

Take $r = 2$ case:

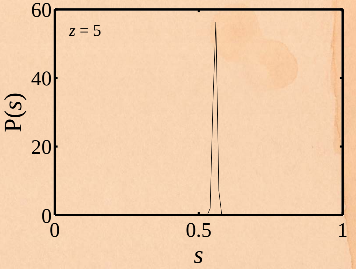


- What happens if nodes have limited information?
- As before, allow interactions to take place on a sparse random network.
- Vary average degree $z = \langle k \rangle$, a measure of information

Invariant densities—stochastic response functions

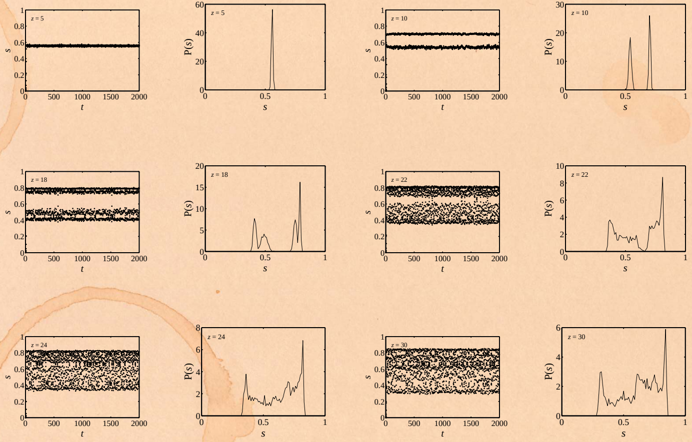


activation time series

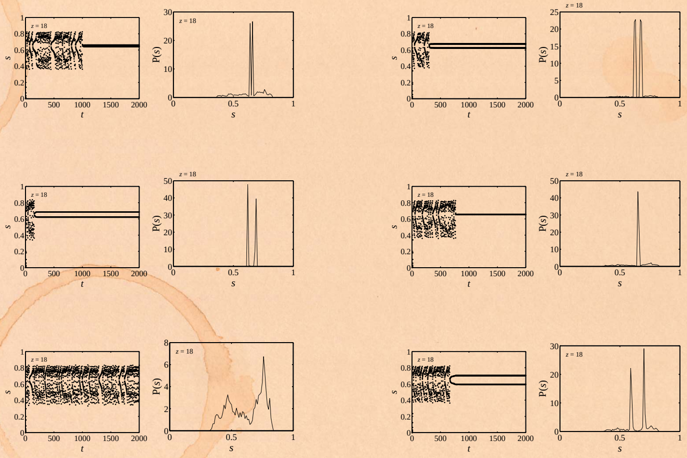


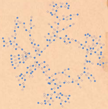
activation density

Invariant densities—stochastic response functions

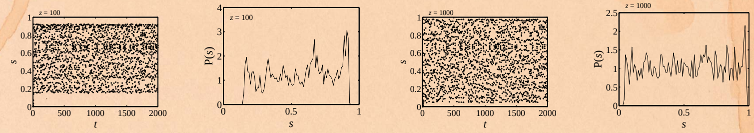


Invariant densities—deterministic response functions for one specific network with $\langle k \rangle = 18$



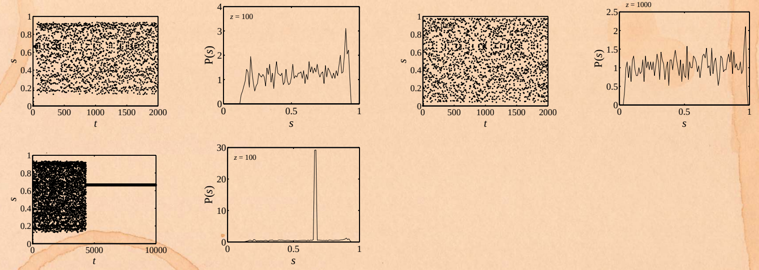


Invariant densities—stochastic response functions



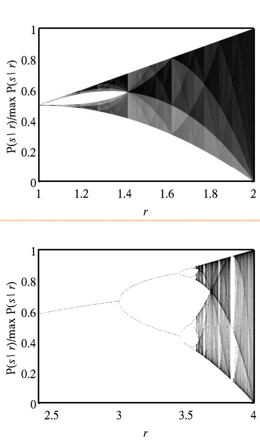
Trying out higher values of $\langle k \rangle \ldots$

Invariant densities—deterministic response functions

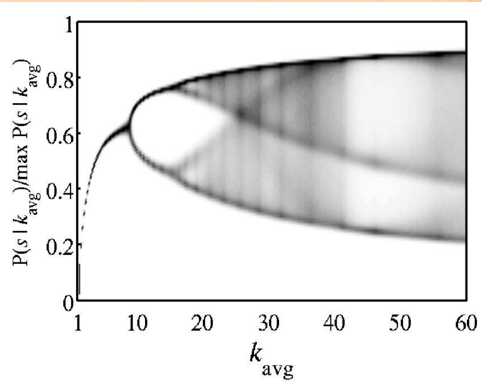


Trying out higher values of $\langle k \rangle$...

Connectivity leads to chaos:



Stochastic response functions:



Chaotic behavior in coupled systems

Coupled maps are well explored (Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

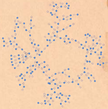
- $\mathcal{N}_i$ = neighborhood of node i

1. Node states are continuous
2. Increase δ and neighborhood size $|\mathcal{N}|$

$\Rightarrow$ synchronization

But for contagion model:

1. Node states are binary
2. Asynchrony remains as connectivity increases



Chaotic behavior in coupled systems

Coupled maps are well explored (Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

- $\mathcal{N}_i$ = neighborhood of node i

1. Node states are continuous
2. Increase δ and neighborhood size $|\mathcal{N}|$ $\Rightarrow$ synchronization

But for contagion model:

1. Node states are binary
2. Asynchrony remains as connectivity increases

Chaotic behavior in coupled systems

Coupled maps are well explored (Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

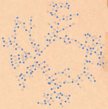
- $\mathcal{N}_i$ = neighborhood of node i

1. Node states are continuous
2. Increase δ and neighborhood size $|\mathcal{N}|$

$\Rightarrow$ synchronization

But for contagion model:

1. Node states are binary
2. Asynchrony remains as connectivity increases



Chaotic behavior in coupled systems

Coupled maps are well explored (Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

- $\mathcal{N}_i$ = neighborhood of node i

1. Node states are continuous
2. Increase δ and neighborhood size $|\mathcal{N}|$

$\Rightarrow$ synchronization

But for contagion model:

1. Node states are binary
2. Asynchrony remains as connectivity increases

Chaotic behavior in coupled systems

Coupled maps are well explored (Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

- $\mathcal{N}_i$ = neighborhood of node i

1. Node states are continuous
2. Increase δ and neighborhood size $|\mathcal{N}|$

$\Rightarrow$ synchronization

But for contagion model:

1. Node states are binary
2. Asynchrony remains as connectivity increases

Chaotic behavior in coupled systems

Coupled maps are well explored (Kaneko/Kuramoto):

$$x_{i,n+1} = f(x_{i,n}) + \sum_{j \in \mathcal{N}_i} \delta_{i,j} f(x_{j,n})$$

- $\mathcal{N}_i$ = neighborhood of node i

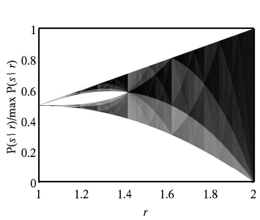
1. Node states are continuous
2. Increase δ and neighborhood size $|\mathcal{N}|$

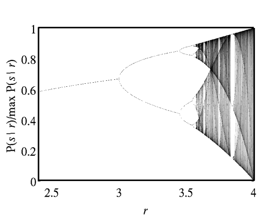
$\Rightarrow$ synchronization

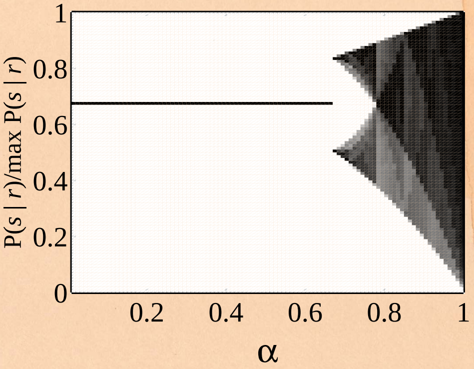
But for contagion model:

1. Node states are binary
2. Asynchrony remains as connectivity increases

Bifurcation diagram: Asynchronous updating







References I

[1] S. Bikhchandani, D. Hirshleifer, and I. Welch.
A theory of fads, fashion, custom, and cultural change as informational cascades.
J. Polit. Econ., 100:992–1026, 1992.

[2] S. Bikhchandani, D. Hirshleifer, and I. Welch.
Learning from the behavior of others: Conformity, fads, and informational cascades.
J. Econ. Perspect., 12(3):151–170, 1998. pdf (⊞)

[3] J. M. Carlson and J. Doyle.
Highly optimized tolerance: A mechanism for power laws in designed systems.
Phys. Rev. E, 60(2):1412–1427, 1999. pdf (⊞)

References II

[4] J. M. Carlson and J. Doyle.
Highly optimized tolerance: Robustness and design in complex systems.
Phys. Rev. Lett., 84(11):2529–2532, 2000. pdf (⊞)

[5] N. A. Christakis and J. H. Fowler.
The spread of obesity in a large social network over 32 years.
New England Journal of Medicine, 357:370–379, 2007. pdf (⊞)

[6] N. A. Christakis and J. H. Fowler.
The collective dynamics of smoking in a large social network.
New England Journal of Medicine, 358:2249–2258, 2008. pdf (⊞)

References III

[7] R. B. Cialdini.
Influence: Science and Practice.
Allyn and Bacon, Boston, MA, 4th edition, 2000.

[8] J. H. Fowler and N. A. Christakis.
Dynamic spread of happiness in a large social network: longitudinal analysis over 20 years in the Framingham Heart Study.
BMJ, 337:article #2338, 2008. pdf (⊞)

[9] M. Gladwell.
The Tipping Point.
Little, Brown and Company, New York, 2000.

[10] M. Granovetter.
Threshold models of collective behavior.
Am. J. Sociol., 83(6):1420–1443, 1978. pdf (⊞)

References IV

[11] M. Granovetter and R. Soong.
Threshold models of diversity: Chinese restaurants, residential segregation, and the spiral of silence.
Sociological Methodology, 18:69–104, 1988. pdf (⊞)

[12] M. S. Granovetter and R. Soong.
Threshold models of interpersonal effects in consumer demand.
Journal of Economic Behavior & Organization, 7:83–99, 1986.
Formulates threshold as function of price, and introduces exogenous supply curve. pdf (⊞)

[13] E. Katz and P. F. Lazarsfeld.
Personal Influence.
The Free Press, New York, 1955.

References V

[14] T. Kuran.
Now out of never: The element of surprise in the east european revolution of 1989.
World Politics, 44:7–48, 1991. pdf (⊞)

[15] T. Kuran.
Private Truths, Public Lies: The Social Consequences of Preference Falsification.
Harvard University Press, Cambridge, MA, Reprint edition, 1997.

[16] T. C. Schelling.
Dynamic models of segregation.
J. Math. Sociol., 1:143–186, 1971.

References VI

[17] T. C. Schelling.
Hockey helmets, concealed weapons, and daylight saving: A study of binary choices with externalities.
J. Conflict Resolut., 17:381–428, 1973. pdf (⊞)

[18] T. C. Schelling.
Micromotives and Macrobehavior.
Norton, New York, 1978.

[19] D. Sornette.
Critical Phenomena in Natural Sciences.
Springer-Verlag, Berlin, 2nd edition, 2003.

[20] D. J. Watts.
A simple model of global cascades on random networks.
Proc. Natl. Acad. Sci., 99(9):5766–5771, 2002.
pdf (⊞)

References VII

[21] U. Wilensky.
Netlogo segregation model.
http://ccl.northwestern.edu/netlogo/models/Segregation. Center for Connected Learning and Computer-Based Modeling, Northwestern University, Evanston, IL., 1998.