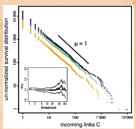
More Mechanisms for Generating Power-Law Size Distributions II

Principles of Complex Systems
CSYS/MATH 300, Fall, 2011

Prof. Peter Dodds

Department of Mathematics & Statistics | Center for Complex Systems | Vermont Advanced Computing Center | University of Vermont



Outline

Growth Mechanisms
- Random Copying
- Words, Cities, and the Web

Optimization
- Minimal Cost
- Mandelbrot vs. Simon
- Assumptions
- Model
- Analysis
- Extra
- And the winner is...?

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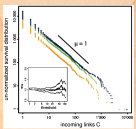

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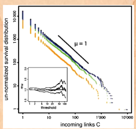
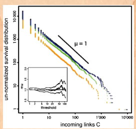


Aggregation

- Random walks represent additive aggregation
- Mechanism: Random addition and subtraction
- Compare across realizations, no competition.
- Next: Random Additive/Copying Processes involving Competition.
- Widespread: Words, Cities, the Web, Wealth, Productivity (Lotka), Popularity (Books, People, ...)
- Competing mechanisms (trickiness)



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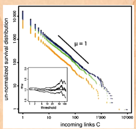

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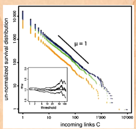

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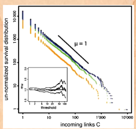

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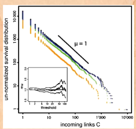

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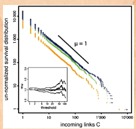

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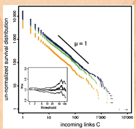

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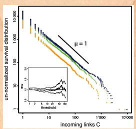

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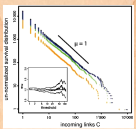

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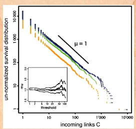

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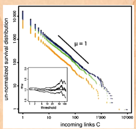

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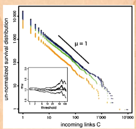


Examples

Recent evidence for Zipf's law...

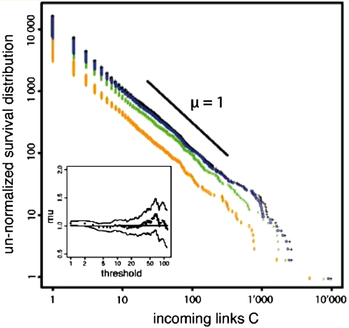
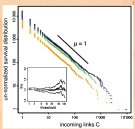


FIG. 1 (color online). (Color Online) Log-log plot of the number of packages in four Debian Linux Distributions with more than C in-directed links. The four Debian Linux Distributions are Woody (19.07.2002) (orange diamonds), Sarge (06.06.2005) (green crosses), Etch (15.08.2007) (blue circles), Lenny (15.12.2007) (black+'s). The inset shows the maximum likelihood estimate (MLE) of the exponent μ together with two boundaries defining its 95% confidence interval (approximately given by $1 \pm 2/\sqrt{n}$, where n is the number of data points using in the MLE), as a function of the lower threshold. The MLE has been modified from the standard Hill estimator to take into account the discreteness of C.

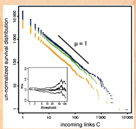
Maillart et al., PRL, 2008:
"Empirical Tests of Zipf's Law Mechanism in Open Source Linux Distribution" [11]



Essential Extract of a Growth Model

Random Competitive Replication (RCR):

1. Start with 1 element of a particular flavor at $t = 1$
2. At time $t = 2, 3, 4, \ldots$, add a new element in one of two ways:
   - With probability ρ, create a new element with a new flavor
   - With probability $1 - \rho$, randomly choose from all existing elements, and make a copy.
   - Elements of the same flavor form a group



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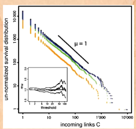

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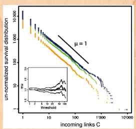

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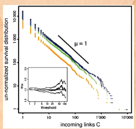

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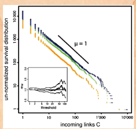

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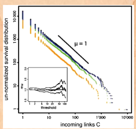

Random Competitive Replication

Example: Words in a text

- Consider words as they appear sequentially.
- With probability ρ, the next word has not previously appeared
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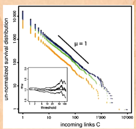

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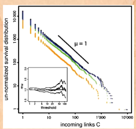

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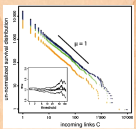

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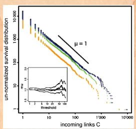

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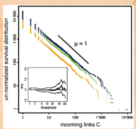

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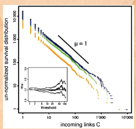
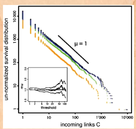


Random Competitive Replication

- Competition for replication between elements is random
- Competition for growth between groups is not random
- Selection on groups is biased by size
- Rich-gets-richer story
- Random selection is easy
- No great knowledge of system needed



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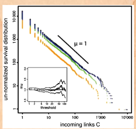

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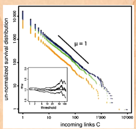

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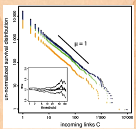

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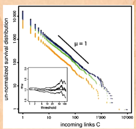

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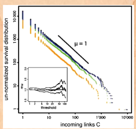

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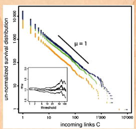
- Steady growth of system: +1 element per unit time.
- Steady growth of distinct flavors at rate ρ
- We can incorporate
  1. Element elimination
  2. Elements moving between groups
  3. Variable innovation rate ρ
  4. Different selection based on group size



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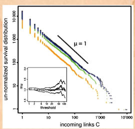

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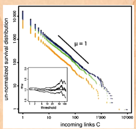

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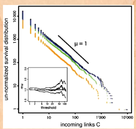

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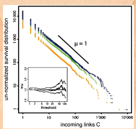

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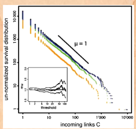
  2. Elements moving between groups
  3. Variable innovation rate ρ
  4. Different selection based on group size



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- Steady growth of system: +1 element per unit time.
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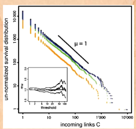

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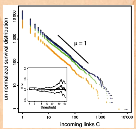
     (But mechanism for selection is not as simple...)



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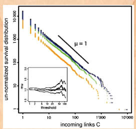
Definitions:

- k_i = size of a group i
- $N_k(t)$ = # groups containing k elements at time t.



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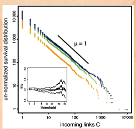

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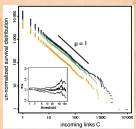

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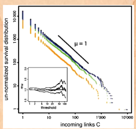
Basic question: How does $N_k(t)$ evolve with time?

First: $\sum_k kN_k(t) = t =$ number of elements at time t



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$P_k(t)$ = Probability of choosing an element that belongs to a group of size k:

- $N_k(t)$ size k groups
- $\Rightarrow kN_k(t)$ elements in size k groups
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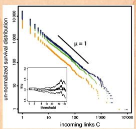

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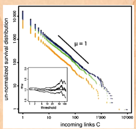

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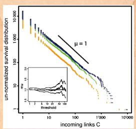

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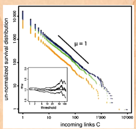

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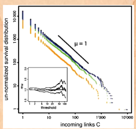

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$N_k(t)$, the number of groups with k elements, changes at time t if

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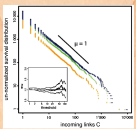

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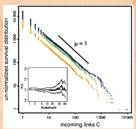

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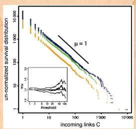

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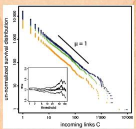

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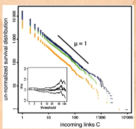

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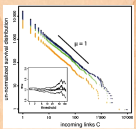

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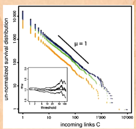

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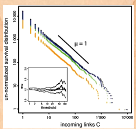
Special case for $N_1(t)$:

1. The new element is a new flavor:

2. A unique element is replicated.



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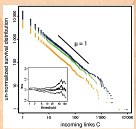

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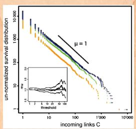

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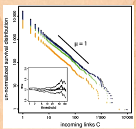

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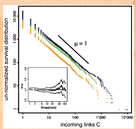

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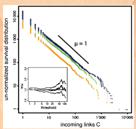

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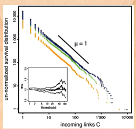
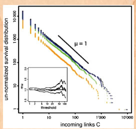


Random Competitive Replication

Put everything together:

For $k > 1$:

$$\langle N_k(t+1) - N_k(t) \rangle = (1-\rho)\left((k-1)\frac{N_{k-1}(t)}{t} - k\frac{N_k(t)}{t}\right)$$



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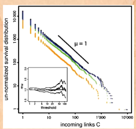
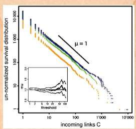


Random Competitive Replication

Assume distribution stabilizes: $N_k(t) = n_k t$

(Reasonable for t large)

- Drop expectations
- Numbers of elements now fractional
- Okay over large time scales
- n_k/ρ = the fraction of groups that have size k.



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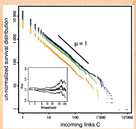

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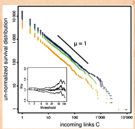

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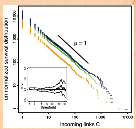

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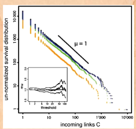

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Stochastic difference equation:

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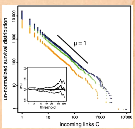

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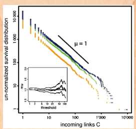

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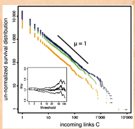

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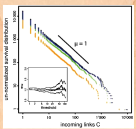

Random Competitive Replication

We have a simple recursion:

$$\frac{n_k}{n_{k-1}} = \frac{(k-1)(1-\rho)}{1+(1-\rho)k}$$

- Interested in k large (the tail of the distribution)
- Can be solved exactly.

  Insert question from assignment 4 (⊞)
- To get at tail: Expand as a series of powers of $1/k$

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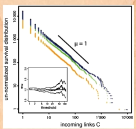

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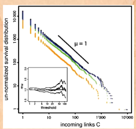

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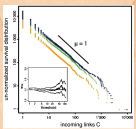

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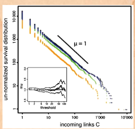

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- To get at tail: Expand as a series of powers of $1/k$

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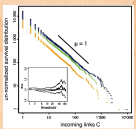


Random Competitive Replication

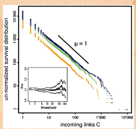
We have a simple recursion:

$$\frac{n_k}{n_{k-1}} = \frac{(k-1)(1-\rho)}{1+(1-\rho)k}$$

- Interested in k large (the tail of the distribution)
- Can be solved exactly.

  Insert question from assignment 4 (⊞)
- To get at tail: Expand as a series of powers of $1/k$

  Insert question from assignment 4 (⊞)



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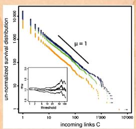
Random Competitive Replication

- We (okay, you) find

$$\frac{n_k}{n_{k-1}} \simeq (1 - \frac{1}{k})^{\frac{(2-\rho)}{(1-\rho)}}$$

- $$\frac{n_k}{n_{k-1}} \simeq \left(\frac{k-1}{k}\right)^{\frac{(2-\rho)}{(1-\rho)}}$$

- $$n_k \propto k^{-\frac{(2-\rho)}{(1-\rho)}} = k^{-\gamma}$$



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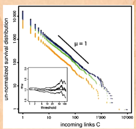

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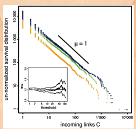

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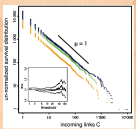

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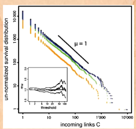
$$\boxed{\gamma = \frac{(2-\rho)}{(1-\rho)} = 1 + \frac{1}{(1-\rho)}}$$

- Micro to macros story with γ and ρ measurable.
- Observe $2 < \gamma < \infty$ as ρ varies.
- For $\rho \simeq 0$ (low innovation rate):

$$\gamma \simeq 2$$

- Recalls Zipf's law: $s_r \sim r^{-\alpha}$
  (s_r = size of the rth largest element)
- We found $\alpha = 1/(\gamma - 1)$
- $\gamma = 2$ corresponds to $\alpha = 1$



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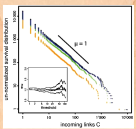

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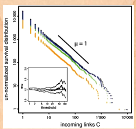

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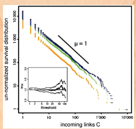

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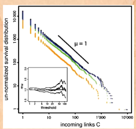

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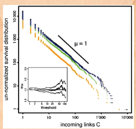
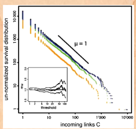


Random Competitive Replication

- We (roughly) see Zipfian exponent [25] of $\alpha = 1$ for many real systems: city sizes, word distributions, ...
- Corresponds to $\rho \rightarrow 0$ (Krugman doesn't like it) [9]
- But still other mechanisms are possible...
- Must look at the details to see if mechanism makes sense...



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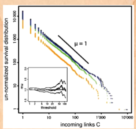

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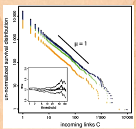

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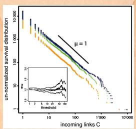

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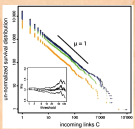
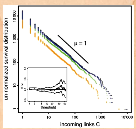


Random Competitive Replication

We had one other equation:

- $$\langle N_1(t+1) - N_1(t)\rangle = \rho - (1-\rho)1 \cdot \frac{N_1(t)}{t}$$
- As before, set $N_1(t) = n_1 t$ and drop expectations
- $$n_1(t+1) - n_1 t = \rho - (1-\rho)1 \cdot \frac{n_1 t}{t}$$
- $$n_1 = \rho - (1-\rho)n_1$$
- Rearrange:
$$n_1 + (1-\rho)n_1 = \rho$$
- $$\boxed{n_1 = \frac{\rho}{2-\rho}}$$



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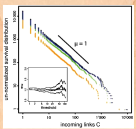

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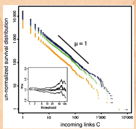

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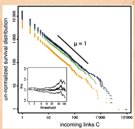

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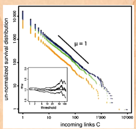


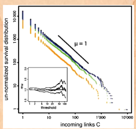
Random Competitive Replication

$$\text{So...} \qquad N_1(t) = n_1 t = \frac{\rho t}{2-\rho}$$

- Recall number of distinct elements = ρt.
- Fraction of distinct elements that are unique (belong to groups of size 1):

$$\frac{N_1(t)}{\rho t} = \frac{1}{2-\rho}$$

  (also = fraction of groups of size 1)
- For ρ small, fraction of unique elements $\sim 1/2$
- Roughly observed for real distributions
- ρ increases, fraction increases
- Can show fraction of groups with two elements $\sim 1/6$
- Model does well at both ends of the distribution



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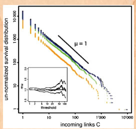

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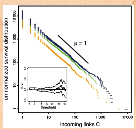

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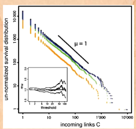

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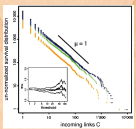

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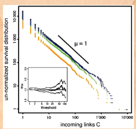

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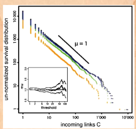

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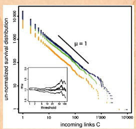
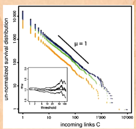


Outline

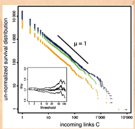


un-normalized survival distribution
μ = 1
threshold
incoming links C

Words

From Simon [19]:

Estimate $\rho_{\text{est}} = \#\text{ unique words}/\#\text{ all words}$



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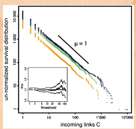
For Joyce's Ulysses: $\rho_{\text{est}} \simeq 0.115$



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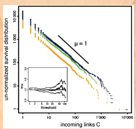
N_1 (real)	N_1 (est)	N_2 (real)	N_2 (est)
16,432	15,850	4,776	4,870



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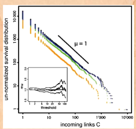

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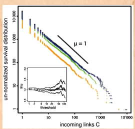

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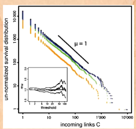

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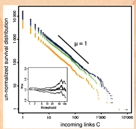

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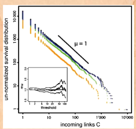

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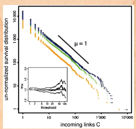

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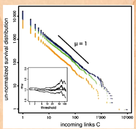

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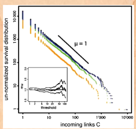

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More on Herbert Simon (1916–2001):

- Political scientist
- Involved in Cognitive Psychology, Computer Science, Public Administration, Economics, Management, Sociology
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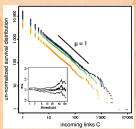

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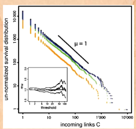

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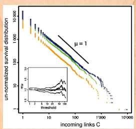

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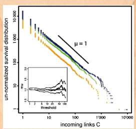

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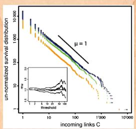

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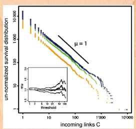

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- First to study network evolution with these kinds of models.
- Citation network of scientific papers
- Price's term: Cumulative Advantage
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- Directed network
- Two (surmountable) problems:
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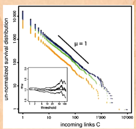

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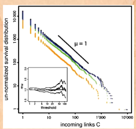

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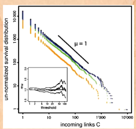

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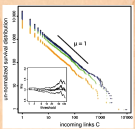

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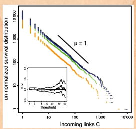

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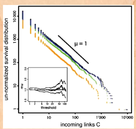

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- (Hath = suggested unit of purchasing power.)
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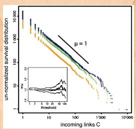

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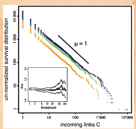

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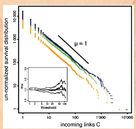

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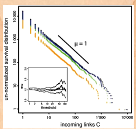

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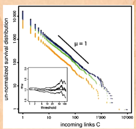

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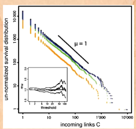

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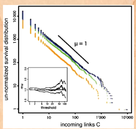

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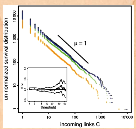
4. Focused interview → focus group



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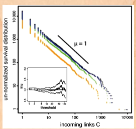

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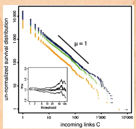

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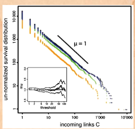

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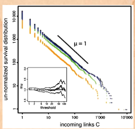

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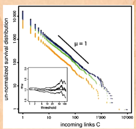

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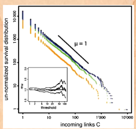

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- Barabasi and Albert [1]—thinking about the Web
- Independent reinvention of a version of Simon and Price's theory for networks
- Another term: "Preferential Attachment"
- Considered undirected networks (not realistic but avoids 0 citation problem)
- Still have selection problem based on size (non-random)
- Solution: Randomly connect to a node (easy) …
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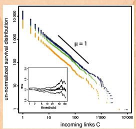
- Scale-free networks = food on the table for physicists



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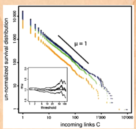

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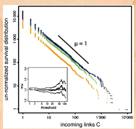

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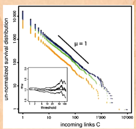

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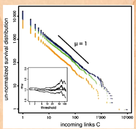

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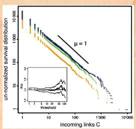

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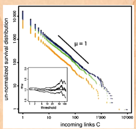

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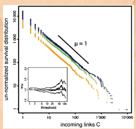

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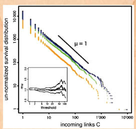
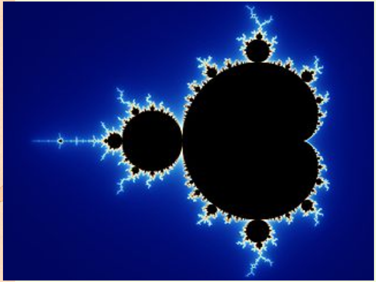


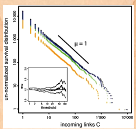
Benoît Mandelbrot (⊞)



Nassim Taleb's tribute:

> Benoit Mandelbrot, 1924-2010
>
> *A Greek among Romans*

- Mandelbrot = father of fractals
- Mandelbrot = almond bread
- Bonus Mandelbrot set action: here (⊞).



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- Idea: Language is efficient
- Communicate as much information as possible for as little cost
- Need measures of information (H) and average cost (C)...
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- Equivalently: minimize C/H.
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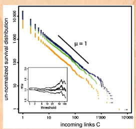

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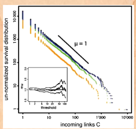

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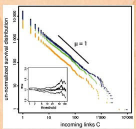

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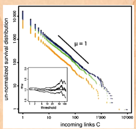

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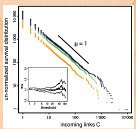

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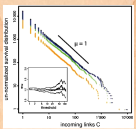

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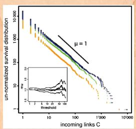

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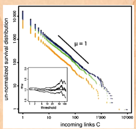

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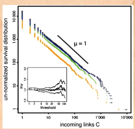

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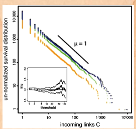

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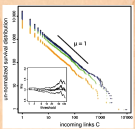

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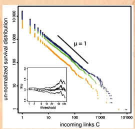

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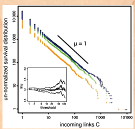

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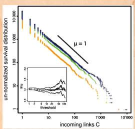

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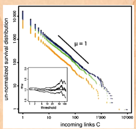

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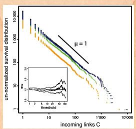

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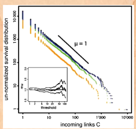

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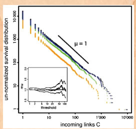

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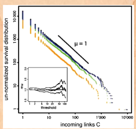

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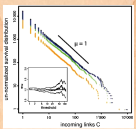

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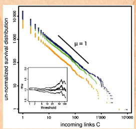

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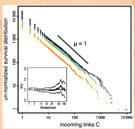

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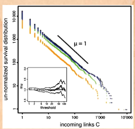

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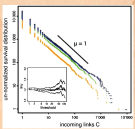

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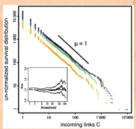
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Zipfarama via Optimization

Mandelbrot's Assumptions:

- Language contains n words: $w_1, w_2, \ldots, w_n$.
- ith word appears with probability p_i
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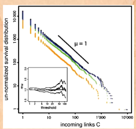
- Words = composition of letters is important
- Alphabet contains m letters
- Words are ordered by length (shortest first)



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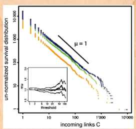

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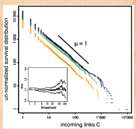

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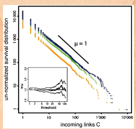

Zipfarama via Optimization

Mandelbrot's Assumptions:

- Language contains n words: $w_1, w_2, \ldots, w_n$.
- ith word appears with probability p_i
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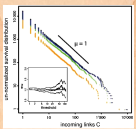

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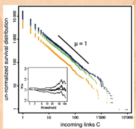

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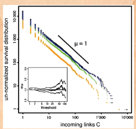

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Word Cost

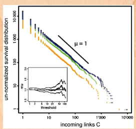
- Length of word (plus a space)
- Word length was irrelevant for Simon's method

Objection

- Real words don't use all letter sequences

Objections to Objection

- Maybe real words roughly follow this pattern (?)
- Words can be encoded this way
- Na na na-na naaaaa...



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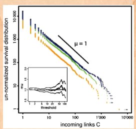

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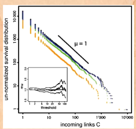

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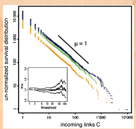

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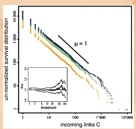

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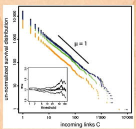

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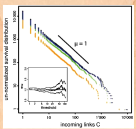
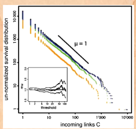


Zipfarama via Optimization

Binary alphabet plus a space symbol

i	1	2	3	4	5	6	7	8
word	1	10	11	100	101	110	111	1000
length	1	2	2	3	3	3	3	4
$1 + \ln_2 i$	1	2	2.58	3	3.32	3.58	3.81	4

- Word length of 2^kth word: $= k + 1$
- Word length of ith word $\simeq 1 + \log_2 i$
- For an alphabet with m letters, word length of ith word $\simeq 1 + \log_m i$.



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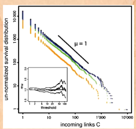

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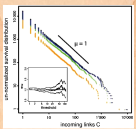

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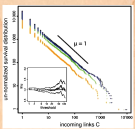

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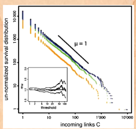

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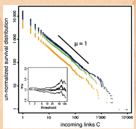
un-normalized survival distribution
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Zipfarama via Optimization

Total Cost C

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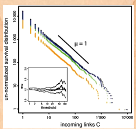
- Simplify base of logarithm:

$$C'_i \simeq \log_m(i+1) = \frac{\log_e(i+1)}{\log_e m}$$

- Total Cost:

$$C \sim \sum_{i=1}^{n} p_i C'_i \propto \sum_{i=1}^{n} p_i \ln(i+1)$$



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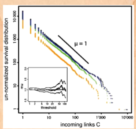

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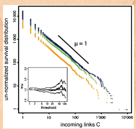

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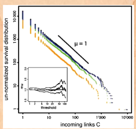

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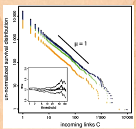

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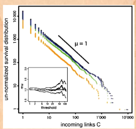


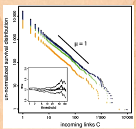
Zipfarama via Optimization

Information Measure

- Use Shannon's Entropy (or Uncertainty):

$$H = -\sum_{i=1}^{n} p_i \log_2 p_i$$

- (allegedly) von Neumann suggested 'entropy'...
- Proportional to average number of bits needed to encode each 'word' based on frequency of occurrence
- $-\log_2 p_i = \log_2 1/p_i$ = minimum number of bits needed to distinguish event i from all others
- If $p_i = 1/2$, need only 1 bit ($log_2 1/p_i = 1$)
- If $p_i = 1/64$, need 6 bits ($log_2 1/p_i = 6$)



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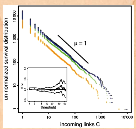

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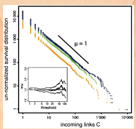

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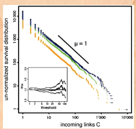

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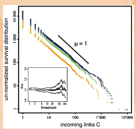

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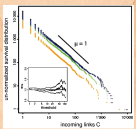

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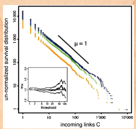
Information Measure

- Use a slightly simpler form:

$$H = -\sum_{i=1}^{n} p_i \log_e p_i / \log_e 2 = -g \sum_{i=1}^{n} p_i \ln p_i$$

where $g = 1/\ln 2$



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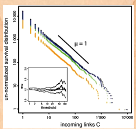
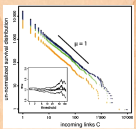


Zipfarama via Optimization

- Minimize
$$F(p_1, p_2, \ldots, p_n) = C/H$$
subject to constraint
$$\sum_{i=1}^{n} p_i = 1$$
- Tension:
(1) Shorter words are cheaper



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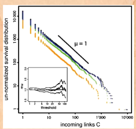

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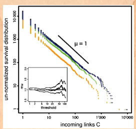
- Tension:
  (1) Shorter words are cheaper
  (2) Longer words are more informative (rarer)

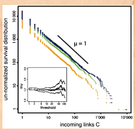


Outline

Growth Mechanisms
- Random Copying
- Words, Cities, and the Web

Optimization
- Minimal Cost
- Mandelbrot vs. Simon
- Assumptions
- Model
- **Analysis**
- Extra
- And the winner is...?

References



Zipfarama via Optimization

Time for Lagrange Multipliers:

- Minimize

$$\Psi(p_1, p_2, \ldots, p_n) =$$

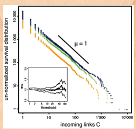
$$F(p_1, p_2, \ldots, p_n) + \lambda G(p_1, p_2, \ldots, p_n)$$

where

$$F(p_1, p_2, \ldots, p_n) = \frac{C}{H} = \frac{\sum_{i=1}^{n} p_i \ln(i+1)}{-g \sum_{i=1}^{n} p_i \ln p_i}$$

and the constraint function is

$$G(p_1, p_2, \ldots, p_n) = \sum_{i=1}^{n} p_i - 1 = 0$$



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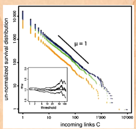

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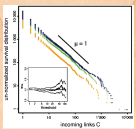
Insert question from assignment 5 (⊞)



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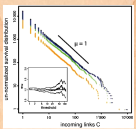
Some mild suffering leads to:

- $$p_j = e^{-1-\lambda H^2/gC}(j+1)^{-H/gC} \propto (j+1)^{-H/gC}$$
- A power law appears [applause]: $\boxed{\alpha = H/gC}$
- Next: sneakily deduce λ in terms of g, C, and H.
- Find
$$p_j = (j+1)^{-H/gC}$$



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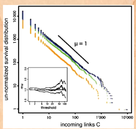

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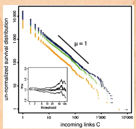

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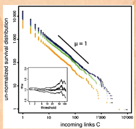

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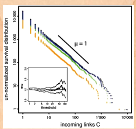


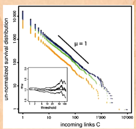
Zipfarama via Optimization

Finding the exponent

- Now use the normalization constraint:

$$1 = \sum_{j=1}^{n} p_j = \sum_{j=1}^{n} (j+1)^{-H/gC} = \sum_{j=1}^{n} (j+1)^{-\alpha}$$

- As $n \to \infty$, we end up with $\zeta(H/gC) = 2$ where ζ is the Riemann Zeta Function
- Gives $\alpha \simeq 1.73$ (> 1, too high)
- If cost function changes ($j+1 \to j+a$) then exponent is tunable
- Increase a, decrease α



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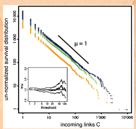

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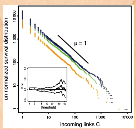

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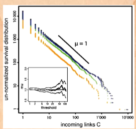

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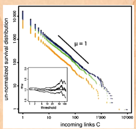

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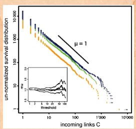

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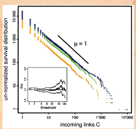
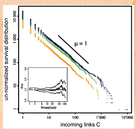


Zipfarama via Optimization

All told:

- Reasonable approach: Optimization is at work in evolutionary processes
- But optimization can involve many incommensurate elements: monetary cost, robustness, happiness,...
- Mandelbrot's argument is not super convincing
- Exponent depends too much on a loose definition of cost



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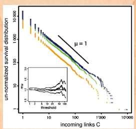

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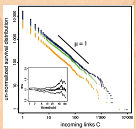

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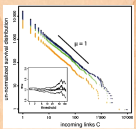

Reconciling Mandelbrot and Simon

- Mixture of local optimization and randomness
- Numerous efforts...

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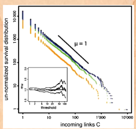

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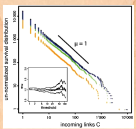

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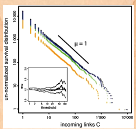

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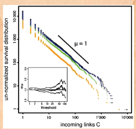

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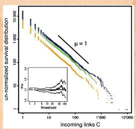

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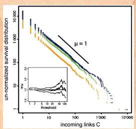

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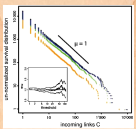

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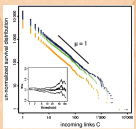

un-normalized survival distribution
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threshold
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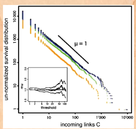

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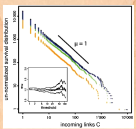

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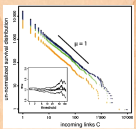

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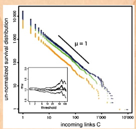

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From Berry [2]

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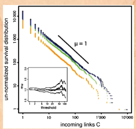

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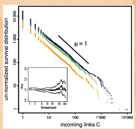

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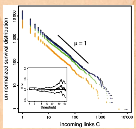

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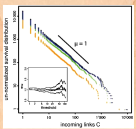

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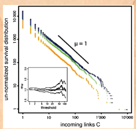
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So who's right?

Empirical Tests of Zipf's Law Mechanism in Open Source Linux Distribution

T. Maillart,[1] D. Sornette,[1] S. Spaeth,[2] and G. von Krogh[2]

[1]*Chair of Entrepreneurial Risks, Department of Management, Technology and Economics, ETH Zurich, CH-8001 Zurich, Switzerland*
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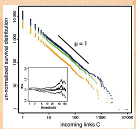
Zipf's power law is a ubiquitous empirical regularity found in many systems, thought to result from proportional growth. Here, we establish empirically the usually assumed ingredients of stochastic growth models that have been previously conjectured to be at the origin of Zipf's law. We use exceptionally detailed data on the evolution of open source software projects in Linux distributions, which offer a remarkable example of a growing complex self-organizing adaptive system, exhibiting Zipf's law over four full decades.



So who's right?

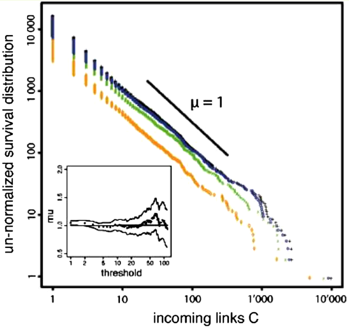
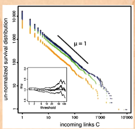


FIG. 1 (color online). (Color Online) Log-log plot of the number of packages in four Debian Linux Distributions with more than C in-directed links. The four Debian Linux Distributions are Woody (19.07.2002) (orange diamonds), Sarge (06.06.2005) (green crosses), Etch (15.08.2007) (blue circles), Lenny (15.12.2007) (black+'s). The inset shows the maximum likelihood estimate (MLE) of the exponent μ together with two boundaries defining its 95% confidence interval (approximately given by $1 \pm 2/\sqrt{n}$, where n is the number of data points using in the MLE), as a function of the lower threshold. The MLE has been modified from the standard Hill estimator to take into account the discreteness of C.

Maillart et al., PRL, 2008:
"Empirical Tests of Zipf's Law Mechanism in Open Source Linux Distribution" [11]



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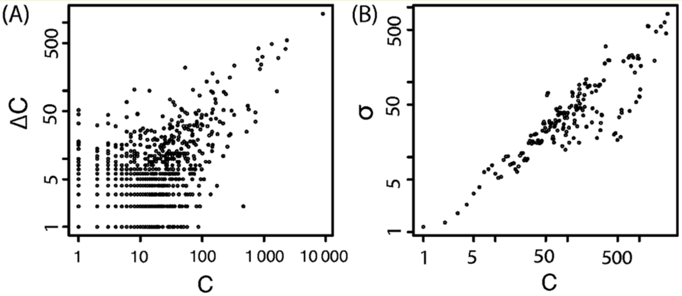
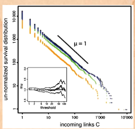


FIG. 2. Left panel: Plots of ΔC versus C from the Etch release (15.08.2007) to the latest Lenny version (05.05.2008) in double logarithmic scale. Only positive values are displayed. The linear regression $\Delta C = R \times C + C_0$ is significant at the 95% confidence level, with a small value $C_0 = 0.3$ at the origin and $R = 0.09$. Right panel: same as left panel for the standard deviation of ΔC.

- Rough, approximately linear relationship between C



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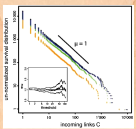
Bornholdt and Ebel (PRE), 2001:
"World Wide Web scaling exponent from Simon's 1955 model" [3].

- Show Simon's model fares well.
- Recall ρ = probability new flavor appears.
- Alta Vista (⊞) crawls in approximately 6 month period in 1999 give $\rho \simeq 0.10$
- Leads to $\gamma = 1 + \frac{1}{1-\rho} \simeq 2.1$ for in-link distribution.
- Cite direct measurement of γ at the time: 2.1 ± 0.1 and 2.09 in two studies.



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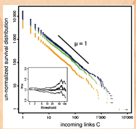

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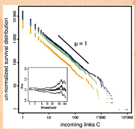

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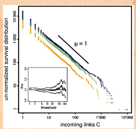

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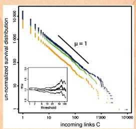

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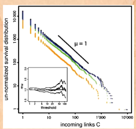

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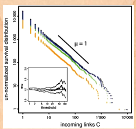
Nutshell:

- Simonish random 'rich-get-richer' models agree in detail with empirical observations.
- Power-lawfulness: Mandelbrot's optimality is still apparent.
- Optimality arises for free in Random Competitive Replication models.



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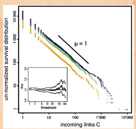

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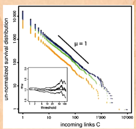

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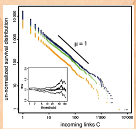

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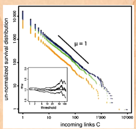

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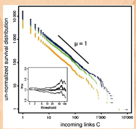

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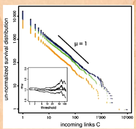

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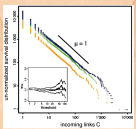

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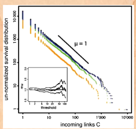

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