Complex Networks

Principles of Complex Systems
CSYS/MATH 300, Fall, 2011

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Department of Mathematics & Statistics | Center for Complex Systems | Vermont Advanced Computing Center | University of Vermont

Outline

Basic definitions

Examples of Complex Networks

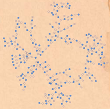
Properties of Complex Networks

Nutshell

Basic models of complex networks

- Generalized random networks
- Scale-free networks
- Small-world networks
- Generalized affiliation networks

References



net•work |ˈnetˌwərk|

noun

1 an arrangement of intersecting horizontal and vertical lines.

• a complex system of roads, railroads, or other transportation routes : *a network of railroads.*

2 a group or system of interconnected people or things : *a trade network.*

• a group of people who exchange information, contacts, and experience for professional or social purposes : *a support network.*

• a group of broadcasting stations that connect for the simultaneous broadcast of a program : *the introduction of a second TV network* | [as adj.] *network television.*

• a number of interconnected computers, machines, or operations : *specialized computers that manage multiple outside connections to a network* | *a local cellular phone network.*

• a system of connected electrical conductors.

verb [trans.]

connect as or operate with a network : *the stock exchanges have proven to be resourceful in networking these deals.*

• link (machines, esp. computers) to operate interactively : [as adj.] (**networked**) *networked workstations.*

• [intrans.] [often as n.] (**networking**) interact with other people to exchange information and develop contacts, esp. to further one's career : *the skills of networking, bargaining, and negotiation.*

Thesaurus deliciousness:

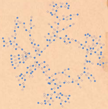
network

noun

1 *a network of arteries* WEB, lattice, net, matrix, mesh, crisscross, grid, reticulum, reticulation; Anatomy plexus.

2 *a network of lanes* MAZE, labyrinth, warren, tangle.

3 *a network of friends* SYSTEM, complex, nexus, web, webwork.

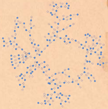


Ancestry:

From Keith Briggs's excellent etymological investigation: (⊞)

- Opus reticulatum:
- A Latin origin?

[http://serialconsign.com/2007/11/we-put-net-network]



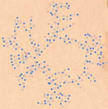
Ancestry:

First known use: Geneva Bible, 1560

'And thou shalt make unto it a grate like networke of brass (Exodus xxvii 4).'

From the OED via Briggs:

- 1658–: reticulate structures in animals
- 1839–: rivers and canals
- 1869–: railways
- 1883–: distribution network of electrical cables
- 1914–: wireless broadcasting networks



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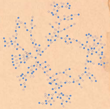

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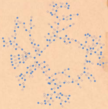

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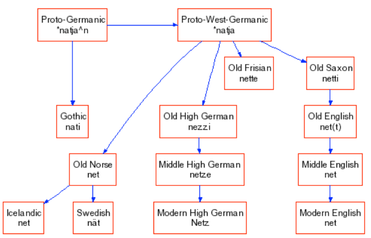
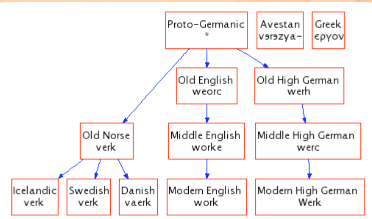
- 'Net' first used to mean spider web (King Ælfréd, 888).
- 'Work' appear to have long meant purposeful action.



The network of Germanic `net' words



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- c.f., ironwork, stonework, fretwork.

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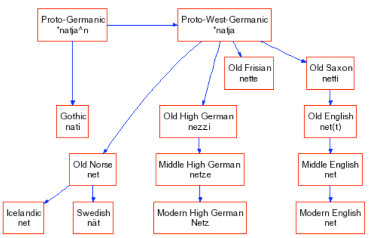

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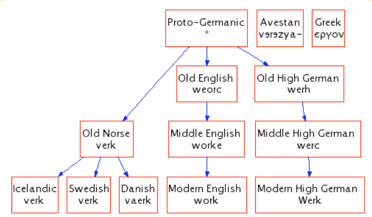

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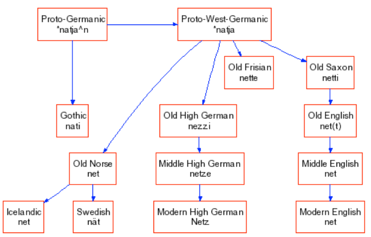

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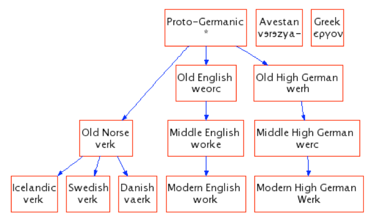

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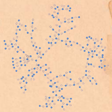
- Opens door to mathematical and numerical analysis.
- Dominant approach of last decade of a theoretical-physics/stat-mechish flavor.
- Mindboggling amount of work published on complex networks since 1998...
- ... largely due to your typical theoretical physicist:



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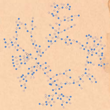

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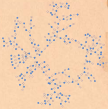

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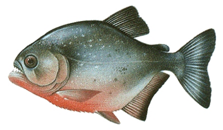

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- Hunt in packs.
- Feast on new and interesting ideas (see chaos, cellular automata, ...)

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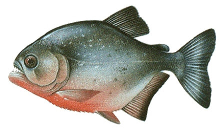

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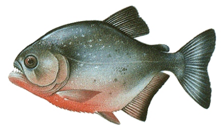

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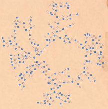


Popularity (according to ISI)

"Collective dynamics of 'small-world' networks" [31]

- Watts and Strogatz
  Nature, 1998
- Cited ≈ 4325 times (as of June 7, 2010)
- Over 1100 citations in 2008 alone.

"Emergence of scaling in random networks" [4]

- Barabási and Albert
  Science, 1999
- Cited ≈ 4769 times (as of June 7, 2010)
- Over 1100 citations in 2008 alone.

Popularity (according to ISI)

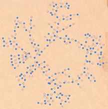
Review articles:

- S. Boccaletti et al.
  "Complex networks: structure and dynamics" [6]
  Times cited: 1,028 (as of June 7, 2010)
- M. Newman
  "The structure and function of complex networks" [21]
  Times cited: 2,559 (as of June 7, 2010)
- R. Albert and A.-L. Barabási
  "Statistical mechanics of complex networks" [2]
  Times cited: 3,995 (as of June 7, 2010)

Popularity according to textbooks:

Textbooks:

- Mark Newman (Physics, Michigan)
  "Networks: An Introduction" (⊞)
- David Easley and Jon Kleinberg (Economics and Computer Science, Cornell)
  "Networks, Crowds, and Markets: Reasoning About a Highly Connected World" (⊞)



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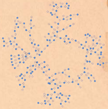

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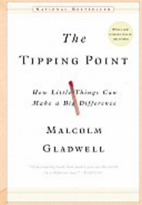
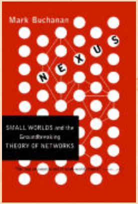
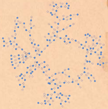


The Tipping Point: How Little Things can make a Big Difference—Malcolm Gladwell [14]



Nexus: Small Worlds and the Groundbreaking Science of Networks—Mark Buchanan



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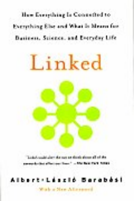
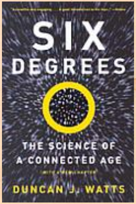


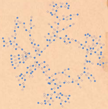
Linked: How Everything Is Connected to Everything Else and What It Means—Albert-Laszlo Barabási



Six Degrees: The Science of a Connected Age—Duncan Watts [29]

Numerous others:

- Complex Social Networks—F. Vega-Redondo [28]
- Fractal River Basins: Chance and Self-Organization—I. Rodríguez-Iturbe and A. Rinaldo [23]
- Random Graph Dynamics—R. Durette
- Scale-Free Networks—Guido Caldarelli
- Evolution and Structure of the Internet: A Statistical Physics Approach—Romu Pastor-Satorras and Alessandro Vespignani
- Complex Graphs and Networks—Fan Chung
- Social Network Analysis—Stanley Wasserman and Kathleen Faust
- Handbook of Graphs and Networks—Eds: Stefan Bornholdt and H. G. Schuster [8]
- Evolution of Networks—S. N. Dorogovtsev and J. F. F. Mendes [13]



More observations

- But surely networks aren't new...
- Graph theory is well established...
- Study of social networks started in the 1930's...
- So why all this 'new' research on networks?
- Answer: Oodles of Easily Accessible Data.
- We can now inform (alas) our theories with a much more measurable reality.*
- A worthy goal: establish mechanistic explanations.

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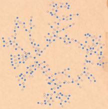

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The UNIVERSITY of VERMONT

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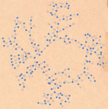

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* *If this is upsetting, maybe string theory is for you...*

More observations

- Web-scale data sets can be overly exciting.

Witness:

- The End of Theory: The Data Deluge Makes the Scientific Theory Obsolete (Anderson, Wired) (⊞)
- “The Unreasonable Effectiveness of Data,” Halevy et al.[15].

But:

- For scientists, description is only part of the battle.
- We still need to understand.

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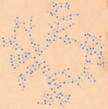


Super Basic definitions

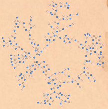
Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...

Links = Connections between nodes

- Links may be directed or undirected.
- Links may be binary or weighted.

Other spiffing words: vertices and edges.



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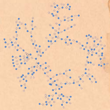

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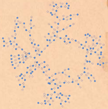

Super Basic definitions

Nodes = A collection of entities which have properties that are somehow related to each other

- e.g., people, forks in rivers, proteins, webpages, organisms,...

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- k_i = 0,1,2,. . . .
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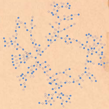

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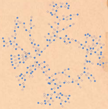

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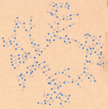

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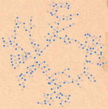

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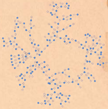

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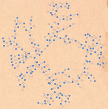

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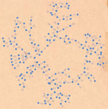

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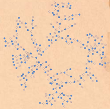

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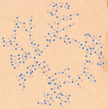

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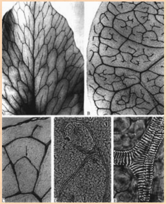

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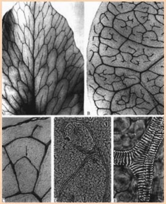

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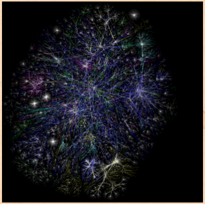

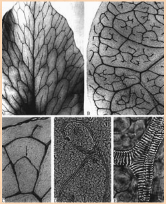

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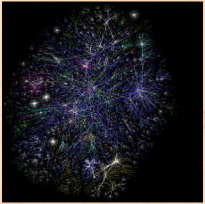

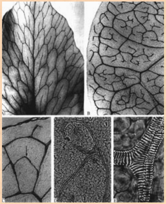

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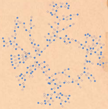

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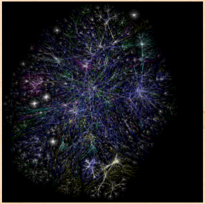

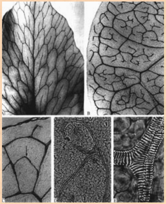

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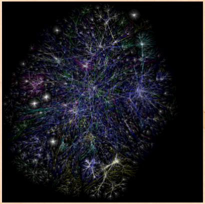

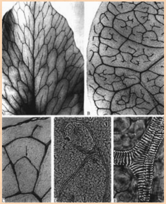

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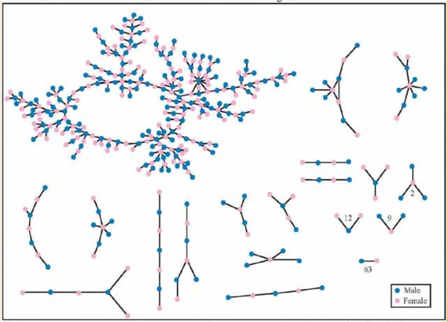

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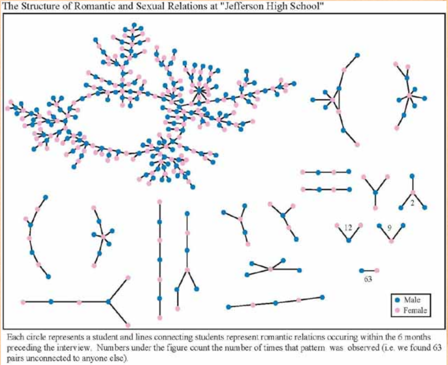

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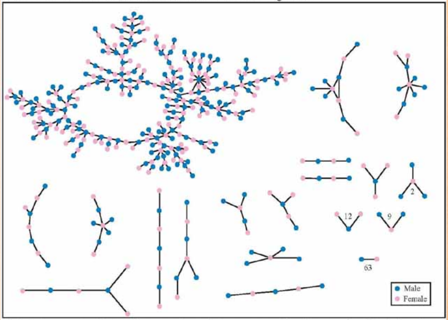

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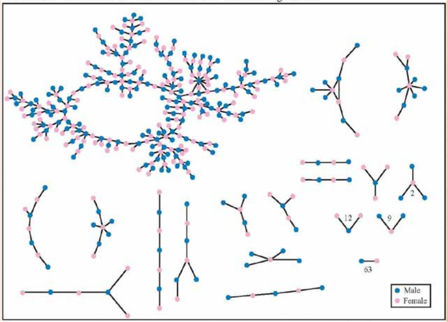

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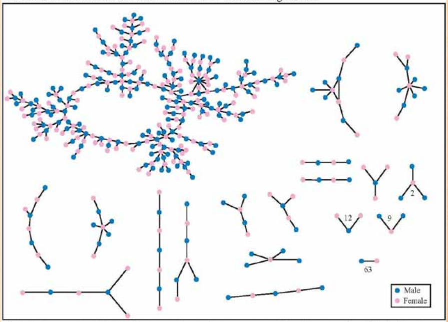

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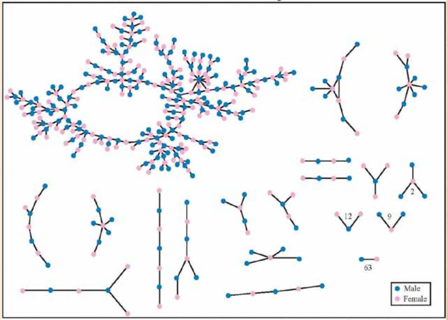

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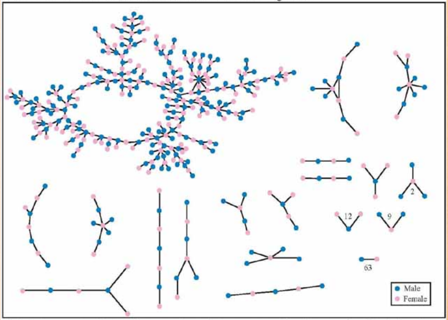

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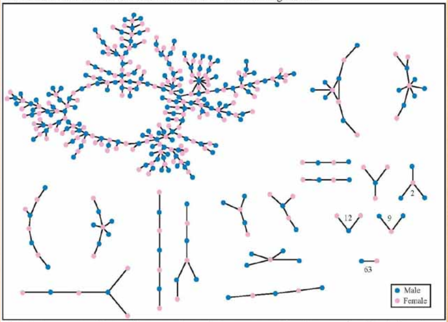

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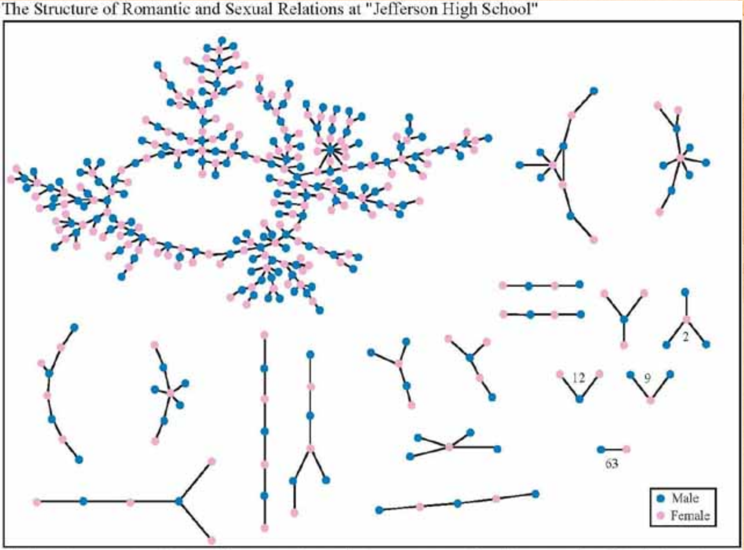

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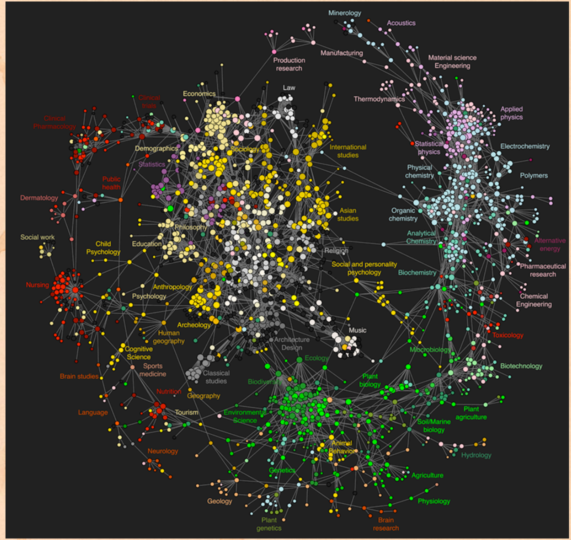
common tags cloud | list

community daily dictionary education encyclopedia english free imported info information internet knowledge learning news reference research resource resources search tools useful web web2.0 wiki wikipedia

Clickworthy Science:



Bollen et al. [7]

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- And even when renderings somehow look good:
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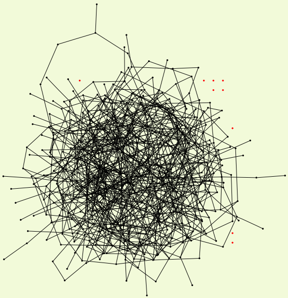

⇐ Typical hairball

- number of nodes $N = 500$
- number of edges $m = 1000$
- average degree $\langle k \rangle = 4$

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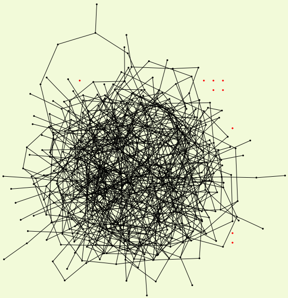

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- number of edges $m = 1000$
- average degree $\langle k \rangle = 4$

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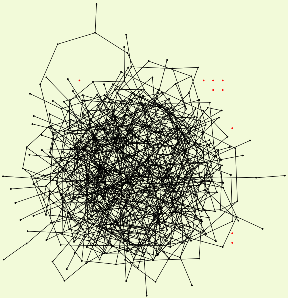

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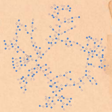

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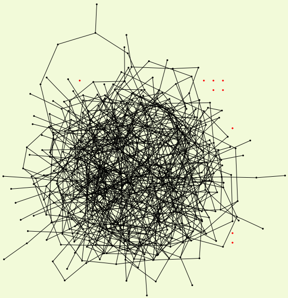

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Properties

Some key features of real complex networks:

- Degree distribution
- Assortativity
- Homophily
- Clustering
- Motifs
- Modularity
- Concurrency
- Hierarchical scaling
- Network distances
- Centrality
- Efficiency
- Robustness

- Coevolution of network structure and processes on networks.

Properties

1. Degree distribution P_k

- P_k is the probability that a randomly selected node has degree k
- Big deal: Form of P_k key to network's behavior
- ex 1: Erdős-Rényi random networks have a Poisson distribution:

$$P_k = e^{-\langle k \rangle} \langle k \rangle^k / k!$$

- ex 2: "Scale-free" networks: $P_k \propto k^{-\gamma} \Rightarrow$ 'hubs'
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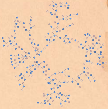

Properties

2. Assortativity/3. Homophily:

- Social networks: Homophily (⊞) = birds of a feather
- e.g., degree is standard property for sorting: measure degree-degree correlations.
- Assortative network: [20] similar degree nodes connecting to each other.
- Disassortative network: high degree nodes connecting to low degree nodes.

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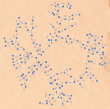

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  - *Often technological or biological: Internet, protein interactions, neural networks, food webs.*

Properties

4. Clustering:

- Your friends tend to know each other.
- Two measures:

$$C_1 = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i - 1)/2} \right\rangle_i \text{ due to Watts \& Strogatz [31]}$$

$$C_2 = \frac{3 \times \#\text{triangles}}{\#\text{triples}} \text{ due to Newman [21]}$$

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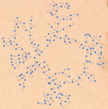

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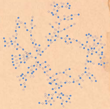

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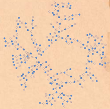

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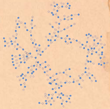

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- e.g., Feed Forward Loop:

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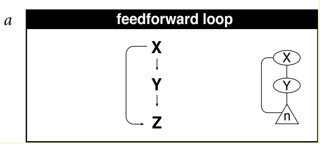

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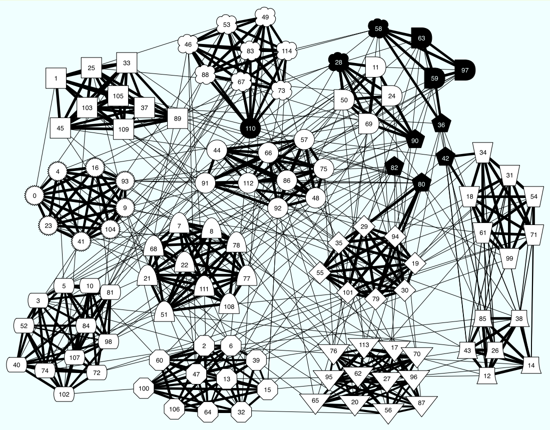

Clauset *et al.*, 2006 [10]: NCAA football

Properties

7. Concurrency:

- Transmission of a contagious element only occurs during contact [18]
- Rather obvious but easily missed in a simple model
- Dynamic property—static networks are not enough
- Knowledge of previous contacts crucial
- Beware cumulated network data!

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8. Horton-Strahler stream ordering:

- Metrics for branching networks:
  - Method for ordering streams hierarchically
  - Reveals fractal nature of natural branching networks
  - Hierarchy is not pure but mixed (Tokunaga). [26, 12]
  - Major examples: rivers and blood networks.

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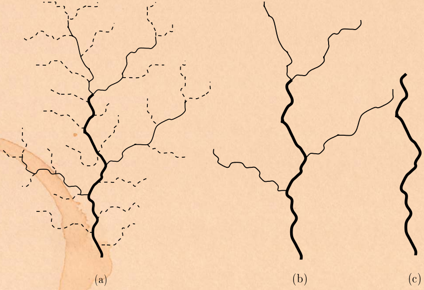

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- Fewest number of steps between nodes i and j.
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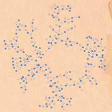

The UNIVERSITY of VERMONT

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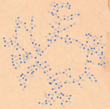

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(c) Network diameter $d_{\max}$:

- Maximum shortest path length in network.

(d) Closeness $d_{\mathrm{cl}} = [\sum_{ij} d_{ij}^{-1} / \binom{n}{2}]^{-1}$:

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- Closeness handles disconnected networks ($d_{ij} = \infty$)
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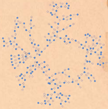

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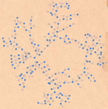

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- ex 2: Node i's betweenness
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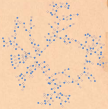

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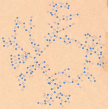

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- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
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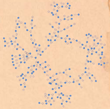

Nutshell:

Overview Key Points:

- The field of complex networks came into existence in the late 1990s.
- Explosion of papers and interest since 1998/99.
- Hardened up much thinking about complex systems.
- Specific focus on networks that are large-scale, sparse, natural or man-made, evolving and dynamic, and (crucially) measurable.
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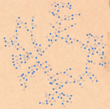

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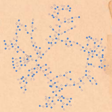

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Some important models:

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2. scale-free networks
3. small-world networks
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Outline

Basic definitions

Examples of Complex Networks

Properties of Complex Networks

Nutshell

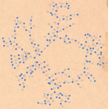
Basic models of complex networks
- Generalized random networks
- Scale-free networks
- Small-world networks
- Generalized affiliation networks

References

Models

Generalized random networks:

- Arbitrary degree distribution P_k.
- Create (unconnected) nodes with degrees sampled from P_k.
- Wire nodes together randomly.
- Create ensemble to test deviations from randomness.



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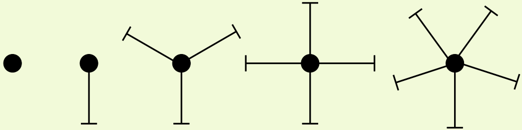
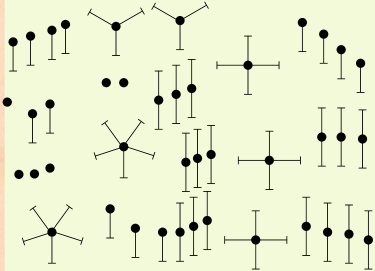
Building random networks: Stubs

Phase 1:

- Idea: start with a soup of unconnected nodes with stubs (half-edges):





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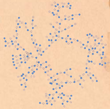

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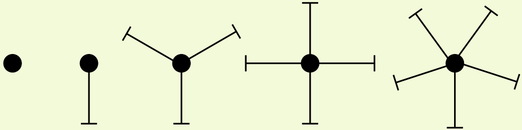

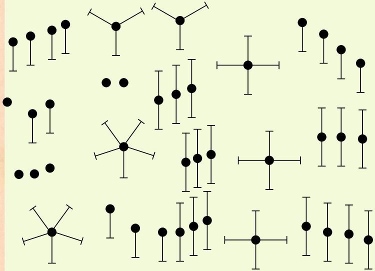

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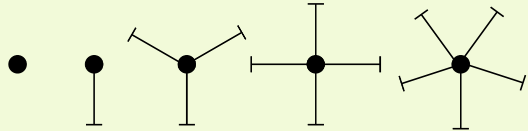

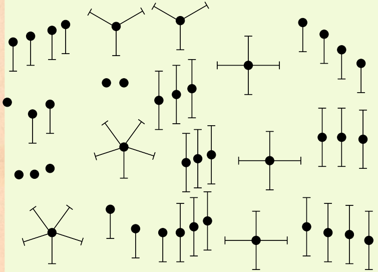

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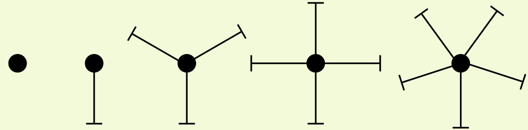

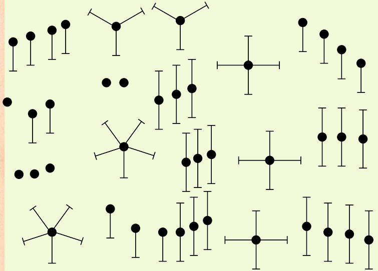

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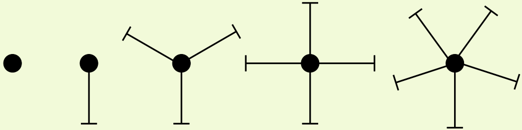

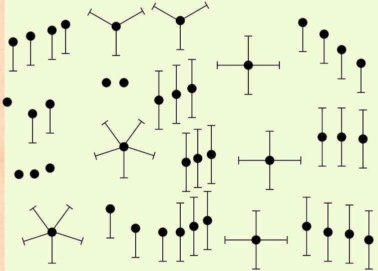

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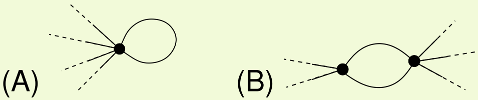
- Now find any (A) self-loops and (B) repeat edges and randomly rewire them.



- Being careful: we can't change the degree of any node, so we can't simply move links around.
- Simplest solution: randomly rewire two edges at a time.

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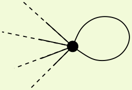

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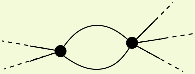

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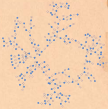

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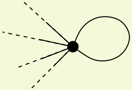
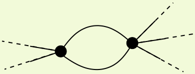
(A)  (B) 

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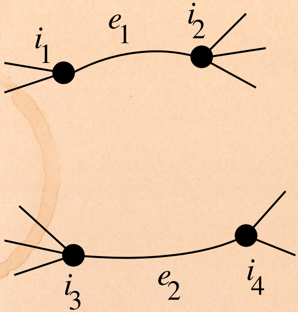

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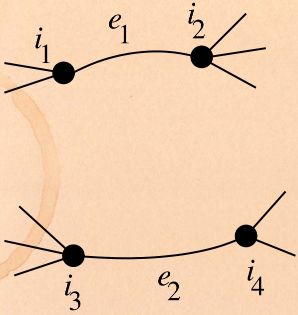

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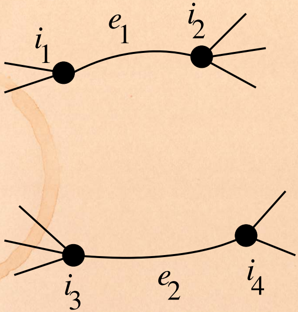

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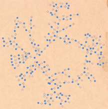

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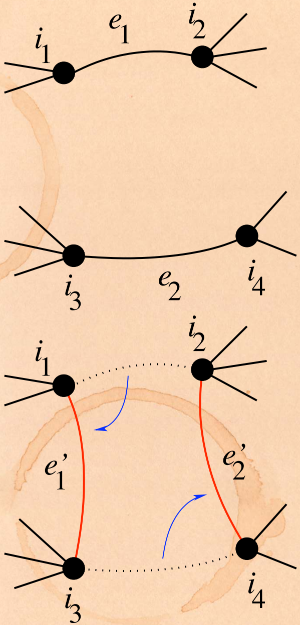

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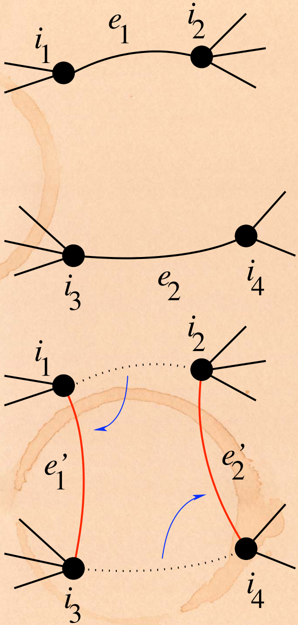

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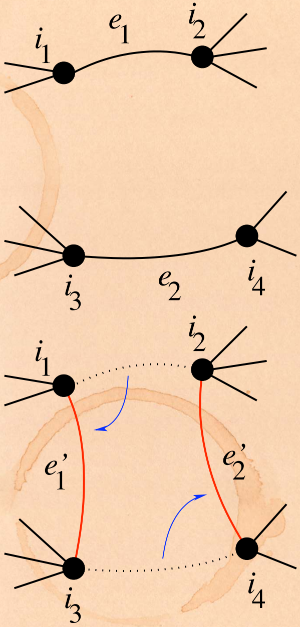

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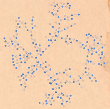

Sampling random networks

Phase 2:

- Use rewiring algorithm to remove all self and repeat loops.

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- Randomize network wiring by applying rewiring algorithm liberally.
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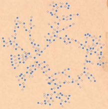

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- Scale-free refers specifically to the degree distribution having a power-law decay in its tail:

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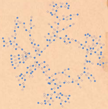

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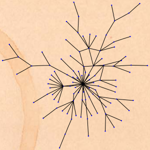
Random networks: largest components



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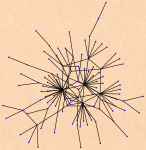
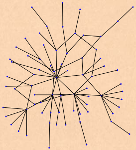
$\langle k \rangle = 1.8$



$\gamma = 2.5$
$\langle k \rangle = 2.05333$



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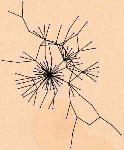

$\gamma = 2.5$
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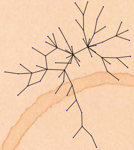

$\gamma = 2.5$
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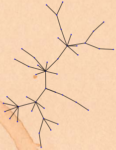

$\gamma = 2.5$
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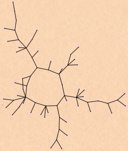

$\gamma = 2.5$
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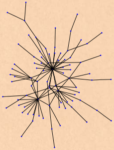

$\gamma = 2.5$
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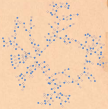

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Scale-free networks

The big deal:

- We move beyond describing networks to finding mechanisms for why certain networks are the way they are.

A big deal for scale-free networks:

- How does the exponent γ depend on the mechanism?
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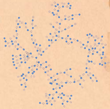

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- Barabási-Albert model = BA model.
- Key ingredients:
  Growth and Preferential Attachment (PA).
- Step 1: start with m_0 disconnected nodes.
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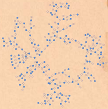

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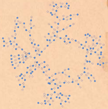

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- Definition: A_k is the attachment kernel for a node with degree k.
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- Definition: $P_{\mathrm{attach}}(k, t)$ is the attachment probability.
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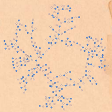

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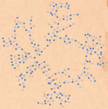

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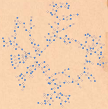

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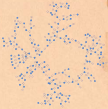

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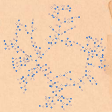

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Approximate analysis

- When $(N+1)$th node is added, the expected increase in the degree of node i is

$$E(k_{i,N+1} - k_{i,N}) \simeq m \frac{k_{i,N}}{\sum_{j=1}^{N(t)} k_j(t)}.$$

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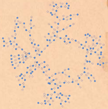

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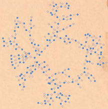

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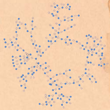

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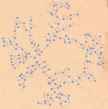

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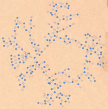

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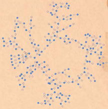

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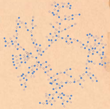

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- So for $i > m_0$ (exclude initial nodes), we must have

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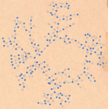

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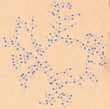

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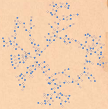
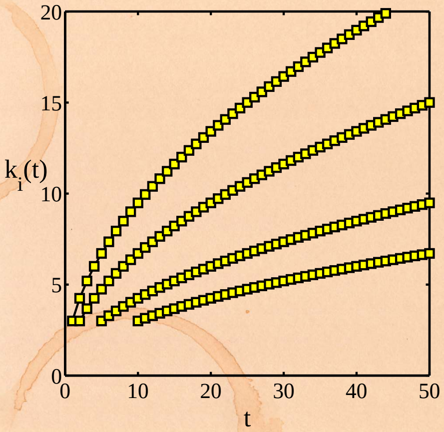
$$k_i(t) = m \left(\frac{t}{t_{i,\mathrm{start}}} \right)^{1/2} \quad \text{for } t \geq t_{i,\mathrm{start}}.$$

- All node degrees grow as $t^{1/2}$ but later nodes have larger $t_{i,\mathrm{start}}$ which flattens out growth curve.
- Early nodes do best (First-mover advantage).



Approximate analysis



- $m = 3$
- $t_{i,\text{start}} =$ 1, 2, 5, and 10.

Degree distribution

- So what's the degree distribution at time t?
- Use fact that birth time for added nodes is distributed uniformly:

$$\mathbf{Pr}(t_{i,\mathrm{start}})\mathrm{d}t_{i,\mathrm{start}} \simeq \frac{\mathrm{d}t_{i,\mathrm{start}}}{t}$$

- Also use

$$k_i(t) = m\left(\frac{t}{t_{i,\mathrm{start}}}\right)^{1/2} \Rightarrow t_{i,\mathrm{start}} = \frac{m^2 t}{k_i(t)^2}.$$

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$$\frac{\mathrm{d}t_{i,\mathrm{start}}}{\mathrm{d}k_i} = -2\frac{m^2 t}{k_i(t)^3}.$$

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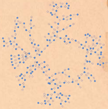

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- Typical for real networks: $2 < \gamma < 3$.
- Range true more generally for events with size distributions that have power-law tails.
- $2 < \gamma < 3$: finite mean and 'infinite' variance (wild)
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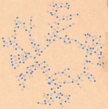

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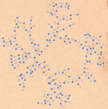

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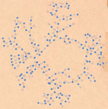

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Examples

WWW	$\gamma \simeq 2.1$ for in-degree
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Movie actors	$\gamma \simeq 2.3$
Words (synonyms)	$\gamma \simeq 2.8$

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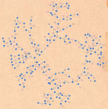

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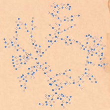

Real data

From Barabási and Albert's original paper [4]:

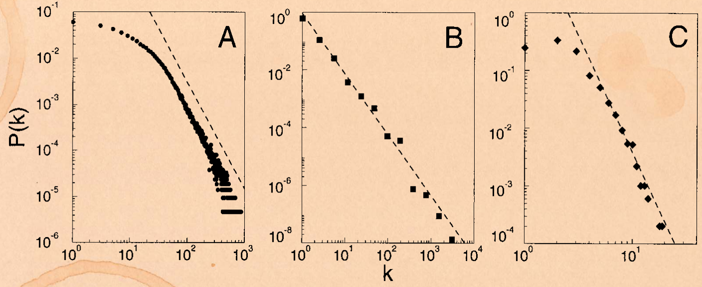

Fig. 1. The distribution function of connectivities for various large networks. (**A**) Actor collaboration graph with N = 212,250 vertices and average connectivity $\langle k \rangle$ = 28.78. (**B**) WWW, N = 325,729, $\langle k \rangle$ = 5.46 (6). (**C**) Power grid data, N = 4941, $\langle k \rangle$ = 2.67. The dashed lines have slopes (A) γ_{actor} = 2.3, (B) γ_{www} = 2.1 and (C) γ_{power} = 4.

Things to do and questions

- Vary attachment kernel.
- Vary mechanisms:
  1. Add edge deletion
  2. Add node deletion
  3. Add edge rewiring
- Deal with directed versus undirected networks.
- Important Q.: Are there distinct universality classes for these networks?
- Q.: How does changing the model affect γ?
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- The answer is (surprisingly) yes. See Simon's model of Zipf.

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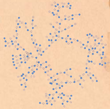

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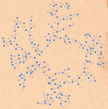

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Preferential attachment

- Let's look at preferential attachment (PA) a little more closely.
- PA implies arriving nodes have complete knowledge of the existing network's degree distribution.
- For example: If $P_{\text{attach}}(k) \propto k$, we need to determine the constant of proportionality.
- We need to know what everyone's degree is...
- PA is $\therefore$ an outrageous assumption of node capability.
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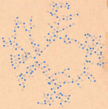

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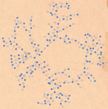

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Preferential attachment through randomness

- Instead of attaching preferentially, allow new nodes to attach randomly.
- Now add an extra step: new nodes then connect to some of their friends' friends.
- Can also do this at random.
- Assuming the existing network is random, we know probability of a random friend having degree k is

$$Q_k \propto kP_k$$

- So rich-gets-richer scheme can now be seen to work in a natural way.

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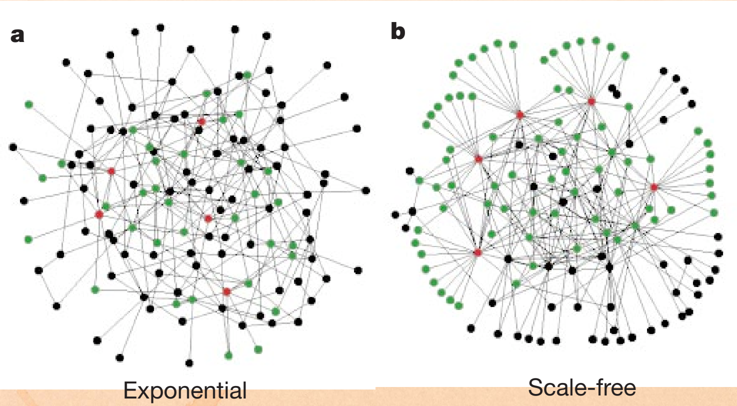
Robustness

- Albert et al., Nature, 2000:
  "Error and attack tolerance of complex networks" [3]
- Standard random networks (Erdős-Rényi) versus Scale-free networks:



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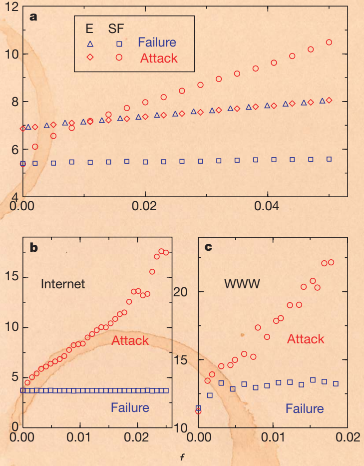

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- Plots of network diameter as a function of fraction of nodes removed
- Erdős-Rényi versus scale-free networks
- blue symbols = random removal
- red symbols = targeted removal (most connected first)

Robustness

- Scale-free networks are thus robust to random failures yet fragile to targeted ones.
- All very reasonable: Hubs are a big deal.
- But: next issue is whether hubs are vulnerable or not.
- Representing all webpages as the same size node is obviously a stretch (e.g., google vs. a random person's webpage)
- Most connected nodes are either:
  1. Physically larger nodes that may be harder to 'target'
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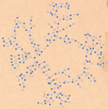
- Need to explore cost of various targeting schemes.



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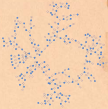

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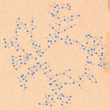

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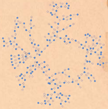

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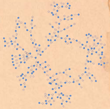

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- Scale-free networks are thus robust to random failures yet fragile to targeted ones.
- All very reasonable: Hubs are a big deal.
- But: next issue is whether hubs are vulnerable or not.
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Outline

People thinking about people:

How are social networks structured?

- How do we define and measure connections?
- Methods/issues of self-report and remote sensing.

What about the dynamics of social networks?

- How do social networks/movements begin & evolve?
- How does collective problem solving work?
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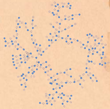
Sociotechnical phenomena and algorithms:

- What can people and computers do together? (google)
- Use Play + Crunch to solve problems. Which problems?



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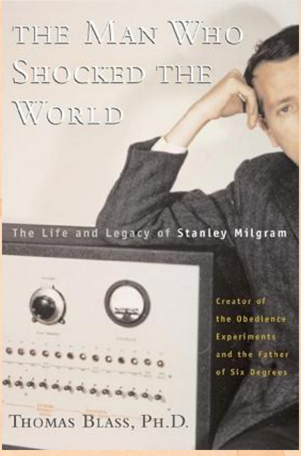
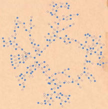


http://www.stanleymilgram.com

- Target person = Boston stockbroker.
- 296 senders from Boston and Omaha.
- 20% of senders reached target.
- chain length $\simeq 6.5$.

Popular terms:

- The Small World Phenomenon;
- "Six Degrees of Separation."



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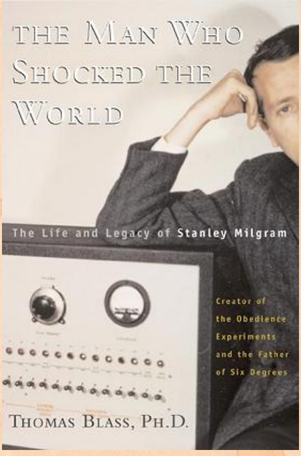

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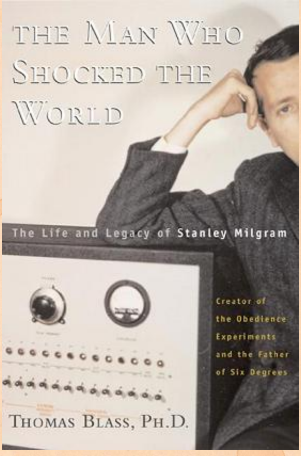

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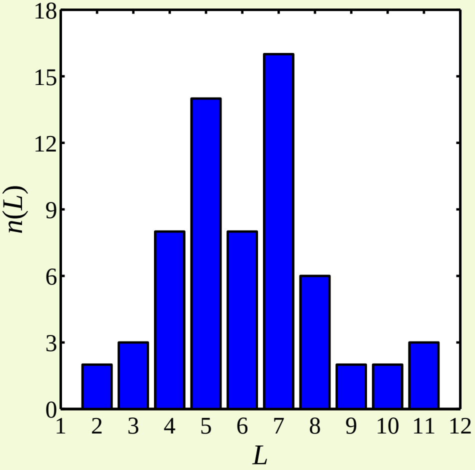

From Travers and Milgram (1969) in Sociometry: [27]
"An Experimental Study of the Small World Problem."

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1. Short paths exist

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Milgram's small world experiment with email:

"An Experimental study of Search in Global Social Networks"
P. S. Dodds, R. Muhamad, and D. J. Watts,
Science, Vol. 301, pp. 827–829, 2003. [11]

Social search—the Columbia experiment

- 60,000+ participants in 166 countries
- 18 targets in 13 countries including
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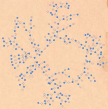

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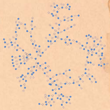

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- Probability of a chain of length 10 getting through:

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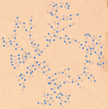

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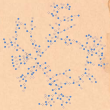

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- weak ties (Granovetter)
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- target's work (65% vs. 40%)

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- hubs (8% vs. 1%) (+ no evidence of funnels)
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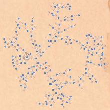

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Senders of successful messages showed little absolute dependency on

- age, gender
- country of residence
- income
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Range of completion rates for subpopulations:

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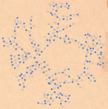

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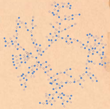

Nevertheless, some weak discrepencies do exist...

An above average connector:

Norwegian, secular male, aged 30-39, earning over $100K, with graduate level education working in mass media or science, who uses relatively weak ties to people they met in college or at work.

A below average connector:

Italian, Islamic or Christian female earning less than $2K, with elementary school education and retired, who uses strong ties to family members.

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Mildly bad for continuing chain:

choosing recipients because "they have lots of friends" or because they will "likely continue the chain."

Why:

- Specificity important
- Successful links used relevant information. (e.g. connecting to someone who shares same profession as target.)

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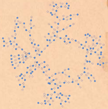

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Basic results:

- $\langle L \rangle = 4.05$ for all completed chains
- L_* = Estimated 'true' median chain length (zero attrition)
- Intra-country chains: $L_* = 5$
- Inter-country chains: $L_* = 7$
- All chains: $L_* = 7$
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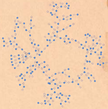

Usefulness:

Harnessing social search:

- Can distributed social search be used for something big/good?
- What about something evil? (Good idea to check.)
- What about socio-inspired algorithms for information search? (More later.)
- For real social search, we have an incentives problem.
- Which kind of influence mechanisms/algorithms would help propagate search?
- Fun, money, prestige, ... ?
- Must be ‘non-gameable.’

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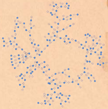

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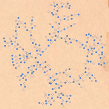
Red balloons:

A Grand Challenge:

- 1969: The Internet is born (⊞) (the ARPANET (⊞)—four nodes!).
- Originally funded by DARPA who created a grand Network Challenge (⊞) for the 40th anniversary.
- Saturday December 5, 2009: DARPA puts 10 red weather balloons up during the day.
- Each 8 foot diameter balloon is anchored to the ground somewhere in the United States.
- Challenge: Find the latitude and longitude of each balloon.
- Prize: $40,000.

*DARPA = Defense Advanced Research Projects Agency (⊞).



Red balloons:

A Grand Challenge:

- 1969: The Internet is born (⊞) (the ARPANET (⊞)—four nodes!).
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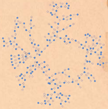

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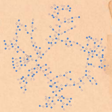

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Finding red balloons:

The winning team and strategy:

- MIT's Media Lab (⊞) won in less that 9 hours. [22]
- Pickard et al. "Time-Critical Social Mobilization," [22] Science Magazine, 2011.
- People were virally recruited online to help out.
- Idea: Want people to both (1) find the balloons and (2) involve more people.
- Recursive incentive structure with exponentially decaying payout:
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The UNIVERSITY of VERMONT

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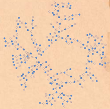

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Finding balloons:

Clever scheme:

- Max payout = \$4000 per balloon.
- Individuals have clear incentives to both
  1. involve/source more people (spread), and
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- Gameable?
- Limit to how much money a set of bad actors can extract.

Extra notes:

- MIT's brand helped greatly.
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Examples of Complex Networks
Properties of Complex Networks
Nutshell
Basic models of complex networks
Generalized random networks
Scale-free networks
Small-world networks
Generalized affiliation networks
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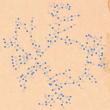

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The social world appears to be small... why?

Theory: how do we understand the small world property?

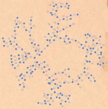
- Connected random networks have short average path lengths:

$$\langle d_{AB} \rangle \sim \log(N)$$

  N = population size,

  d_{AB} = distance between nodes A and B.

- But: social networks aren't random...



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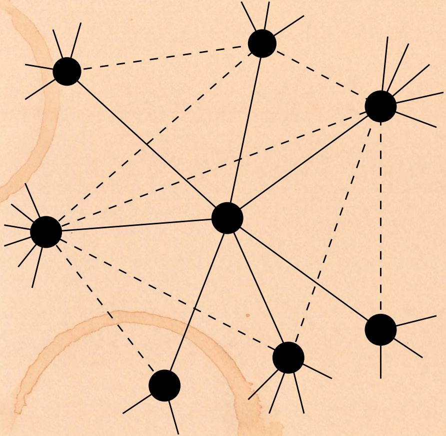
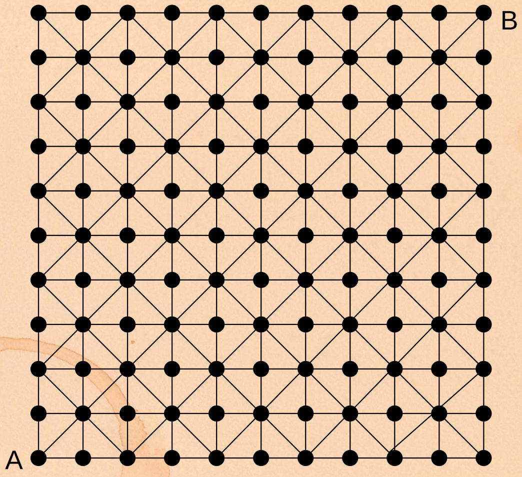
Simple socialness in a network:



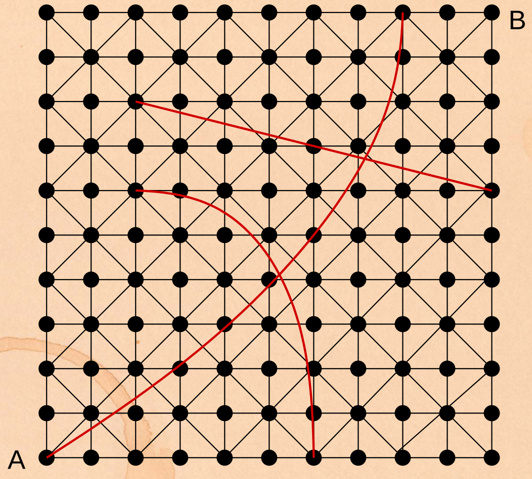
Need "clustering" (your friends are likely to know each other):

Non-randomness gives clustering:



$d_{AB} = 10 \rightarrow$ too many long paths.

Randomness + regularity



Now have $d_{AB} = 3$

$\langle d \rangle$ decreases overall

Small-world networks

Introduced by Watts and Strogatz (Nature, 1998) [31]
"Collective dynamics of 'small-world' networks."

Small-world networks were found everywhere:

- neural network of C. elegans,
- semantic networks of languages,
- actor collaboration graph,
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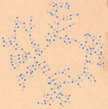

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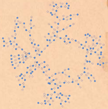

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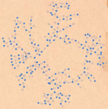

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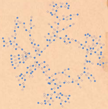

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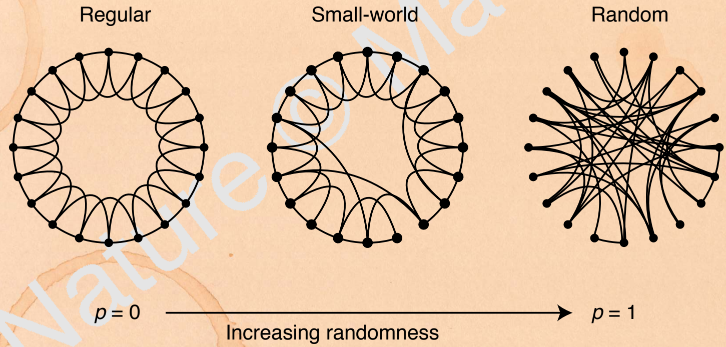
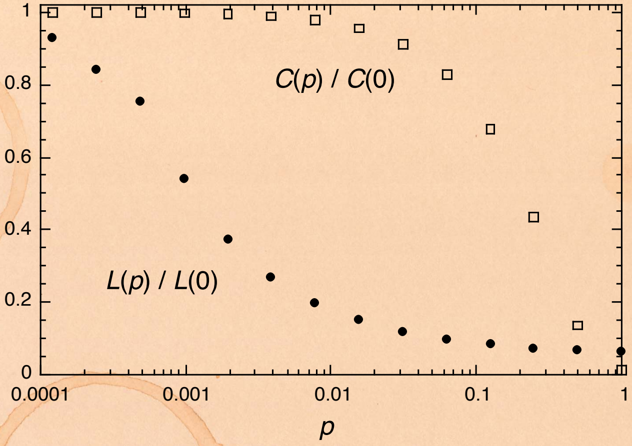
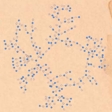
Toy model:



The structural small-world property:



- $L(p)$ = average shortest path length as a function of p
- $C(p)$ = average clustring as a function of p



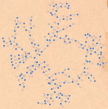
Previous work—finding short paths

But are these short cuts findable?

Nope.

Nodes cannot find each other quickly with any local search method.

Need a more sophisticated model...



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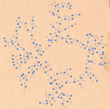

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- Target's identity
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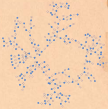

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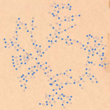

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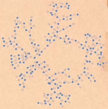
Kleinberg's Network:

1. Start with regular d-dimensional cubic lattice.
2. Add local links so nodes know all nodes within a distance q.
3. Add m short cuts per node.
4. Connect i to j with probability

$$p_{ij} \propto x_{ij}{}^{-\alpha}.$$

- $\alpha = 0$: random connections.
- α large: reinforce local connections.
- $\alpha = d$: connections grow logarithmically in space.



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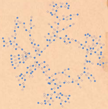

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- $\alpha = d$: connections grow logarithmically in space.

Previous work—finding short paths

Theoretical optimal search:

- “Greedy” algorithm.
- Number of connections grow logarithmically (slowly) in space: $\alpha = d$.
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But: social networks aren’t lattices plus links.

Previous work—finding short paths

- If networks have hubs can also search well: Adamic et al. (2001) [1]

$$P(k_i) \propto k_i^{-\gamma}$$

where k = degree of node i (number of friends).

- Basic idea: get to hubs first (airline networks).
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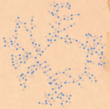

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Outline

Basic definitions

Examples of Complex Networks

Properties of Complex Networks

Nutshell

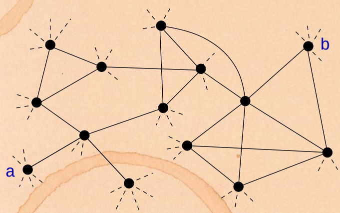
Basic models of complex networks

- Generalized random networks
- Scale-free networks
- Small-world networks
- Generalized affiliation networks

References

The problem

If there are no hubs and no underlying lattice, how can search be efficient?



Which friend of a is closest to the target b?

What does 'closest' mean?

What is 'social distance'?

Models

One approach: incorporate identity.

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Identity is formed from attributes such as:

- Geographic location
- Type of employment
- Religious beliefs
- Recreational activities.

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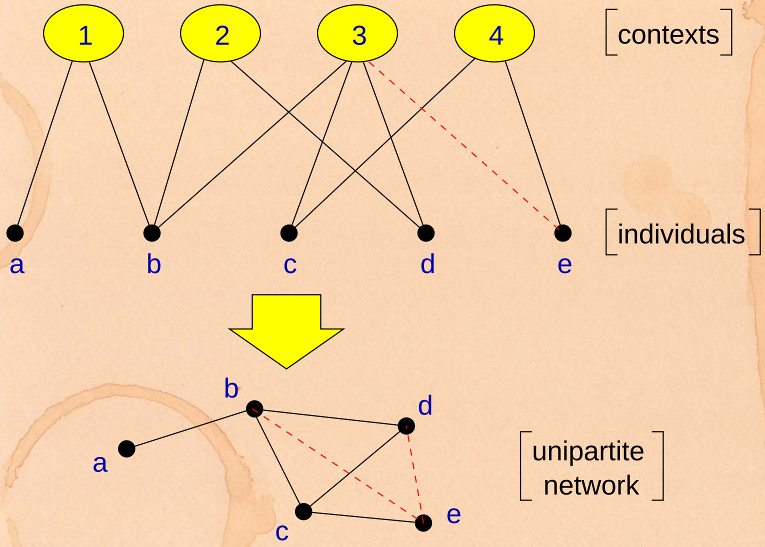
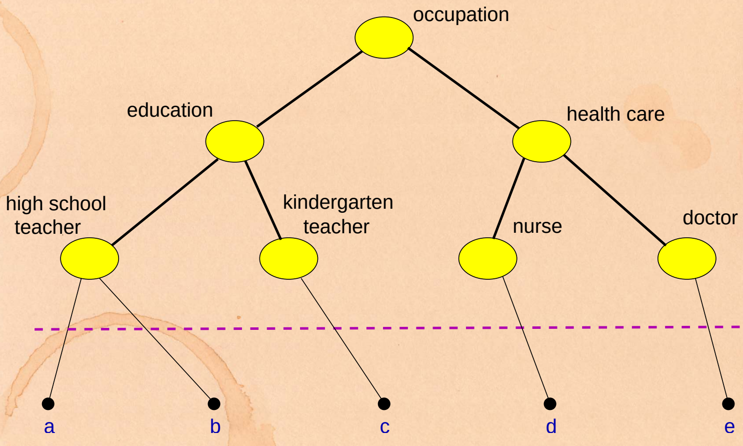
Attributes $\Leftrightarrow$ Contexts $\Leftrightarrow$ Interactions $\Leftrightarrow$ Networks.

Social distance—Bipartite affiliation networks



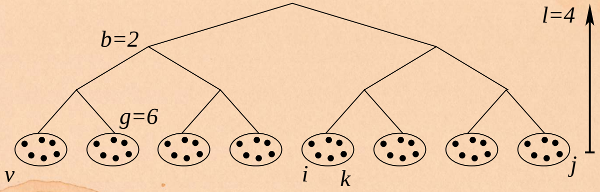
Bipartite affiliation networks: boards and directors, movies and actors.

Social distance—Context distance



Models

Distance between two individuals x_{ij} is the height of lowest common ancestor.



$x_{ij} = 3$, $x_{ik} = 1$, $x_{iv} = 4$.

Models

- Individuals are more likely to know each other the closer they are within a hierarchy.
- Construct z connections for each node using

$$p_{ij} = c \exp\{-\alpha x_{ij}\}.$$

- $\alpha = 0$: random connections.
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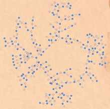
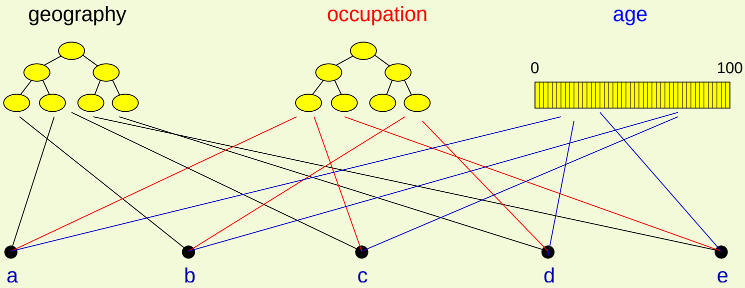


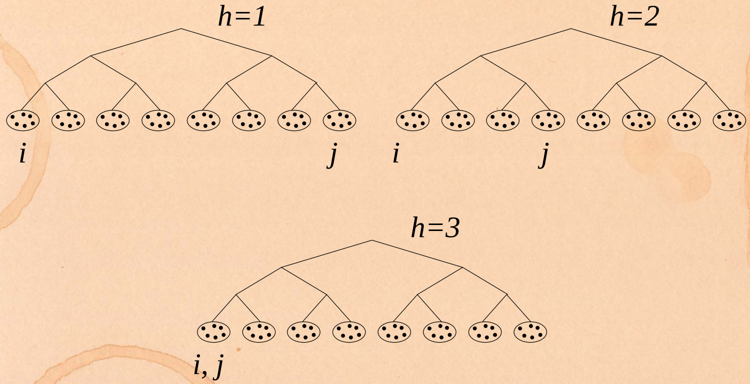
Models

Generalized affiliation networks



- Blau & Schwartz [5], Simmel [25], Breiger [9], Watts *et al.* [30]

The model



$\vec{v}_i = [1\ 1\ 1]^T$, $\vec{v}_j = [8\ 4\ 1]^T$

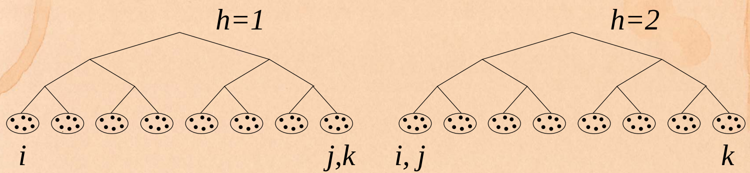
$x_{ij}^1 = 4,\ \ x_{ij}^2 = 3,\ \ x_{ij}^3 = 1.$

Social distance:

$$\boxed{y_{ij} = \min_h x_{ij}^h.}$$

The model

Triangle inequality doesn't hold:



$$y_{ik} = 4 > y_{ij} + y_{jk} = 1 + 1 = 2.$$

The model

- Individuals know the identity vectors of
  1. themselves,
  2. their friends,
     and
  3. the target.
- Individuals can estimate the social distance between their friends and the target.
- Use a greedy algorithm + allow searches to fail randomly.

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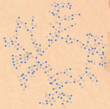

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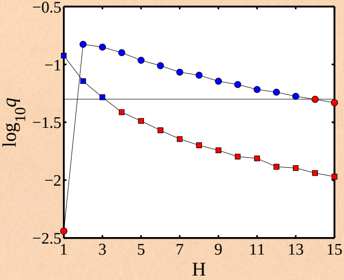
The model-results—searchable networks

$\alpha = 0$ versus $\alpha = 2$ for $N \simeq 10^5$:



$q \geq r$

$q < r$

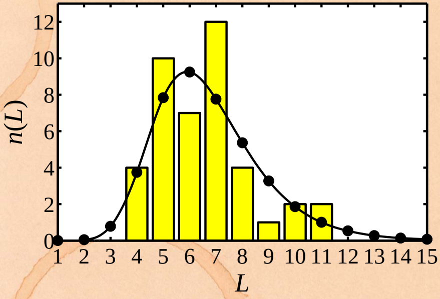
$r = 0.05$

q = probability an arbitrary message chain reaches a target.

- A few dimensions help.
- Searchability decreases as population increases.
- Precise form of hierarchy largely doesn't matter.

The model-results

Milgram's Nebraska-Boston data:



Model parameters:

- $N = 10^8$,
- $z = 300$, $g = 100$,
- $b = 10$,
- $\alpha = 1$, $H = 2$;

- $\langle L_{\mathrm{model}} \rangle \simeq 6.7$
- $L_{\mathrm{data}} \simeq 6.5$

Social search—Data

Adamic and Adar (2003)

- For HP Labs, found probability of connection as function of organization distance well fit by exponential distribution.
- Probability of connection as function of real distance $\propto 1/r$.

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Social Search—Real world uses

- Tags create identities for objects
- Website tagging: `http://www.del.icio.us`
- (e.g., Wikipedia)
- Photo tagging: `http://www.flickr.com`
- Dynamic creation of metadata plus links between information objects.
- Folksonomy: collaborative creation of metadata

Social Search—Real world uses

Recommender systems:

- Amazon uses people's actions to build effective connections between books.
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Nutshell for Small-World Networks:

- Bare networks are typically unsearchable.
- Paths are findable if nodes understand how network is formed.
- Importance of identity (interaction contexts).
- Improved social network models.
- Construction of peer-to-peer networks.
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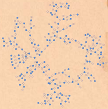

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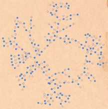

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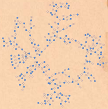

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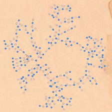

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