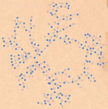
Biological Contagion

Principles of Complex Systems
CSYS/MATH 300, Fall, 2011

Prof. Peter Dodds

Department of Mathematics & Statistics | Center for Complex Systems | Vermont Advanced Computing Center | University of Vermont



Outline

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A confusion of contagions:

- Was Harry Potter some kind of virus?
- What about the Da Vinci Code?
- Did Sudoku spread like a disease?
- Language?
- Religion?
- Democracy...?

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Naturomorphisms

- "The feeling was contagious."
- "The news spread like wildfire."
- "Freedom is the most contagious virus known to man."
  —Hubert H. Humphrey, Johnson's vice president
- "Nothing is so contagious as enthusiasm."
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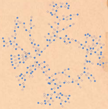

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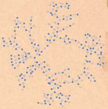

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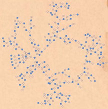

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The doctrine that everything is beautiful, including what is ugly, everything good, especially the bad, and everything right that is wrong. ...

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The doctrine that everything is beautiful, including what is ugly, everything good, especially the bad, and everything right that is wrong. ... It is hereditary, but fortunately not contagious.

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Eric Hoffer, 1902–1983

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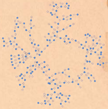

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The spread of fanaticism

Hoffer's acclaimed work: "The True Believer: Thoughts On The Nature Of Mass Movements" (1951) [3]

Quotes-aplenty:

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- "Where freedom is real, equality is the passion of the masses. Where equality is real, freedom is the passion of a small minority."

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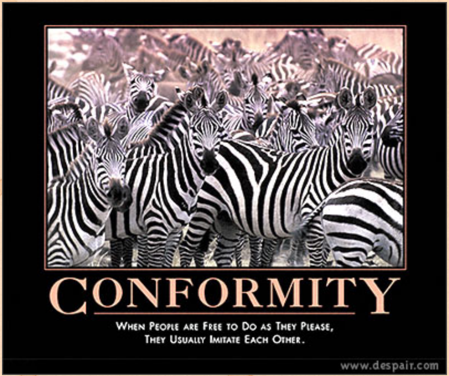
Imitation



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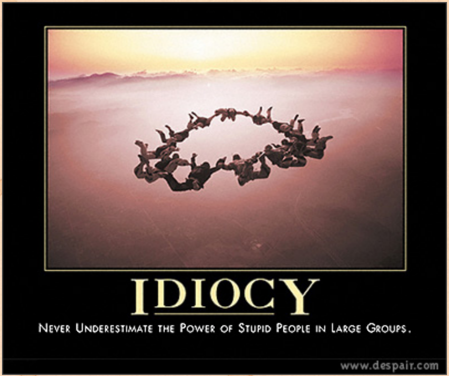
"When people are free to do as they please, they usually imitate each other."

—Eric Hoffer
"The Passionate State of Mind" [4]

The collective...



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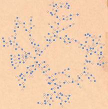
"Never Underestimate the Power of Stupid People in Large Groups."

Contagion

Definitions

- (1) The spreading of a quality or quantity between individuals in a population.
- (2) A disease itself:
  the plague, a blight, the dreaded lurgi, ...
- from Latin: *con* = 'together with' + *tangere* 'to touch.'
- Contagion has unpleasant overtones...
- Just Spreading might be a more neutral word
- But contagion is kind of exciting...



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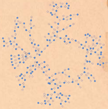

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The UNIVERSITY of VERMONT

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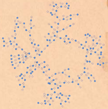

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Examples of non-disease spreading:

Interesting infections:

- Spreading of buildings in the US... (⊞)

- Viral get-out-the-vote video. (⊞)

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Contagions

Two main classes of contagion

1. Infectious diseases

2. Social contagion

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1. Infectious diseases:
   tuberculosis, HIV, ebola, SARS, influenza, ...

2. Social contagion

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Two main classes of contagion

1. Infectious diseases:
   tuberculosis, HIV, ebola, SARS, influenza, ...
2. Social contagion:
   fashion, word usage, rumors, riots, religion, ...

Outline

Mathematical Epidemiology

The standard SIR model [8]

- = basic model of disease contagion
- Three states:
  1. S = Susceptible
  2. I = Infective/Infectious
  3. R = Recovered or Removed or Refractory
- $S(t) + I(t) + R(t) = 1$
- Presumes random interactions (mass-action principle)
- Interactions are independent (no memory)
- Discrete and continuous time versions

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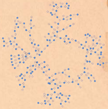

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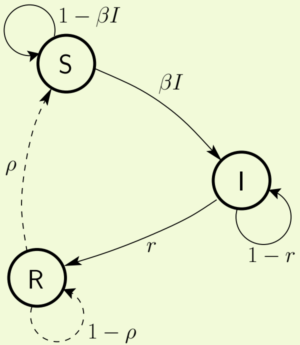

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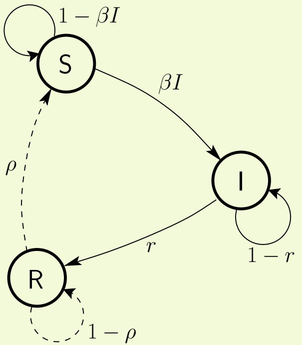

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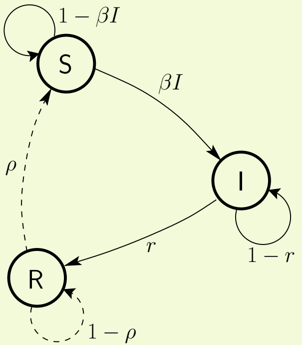

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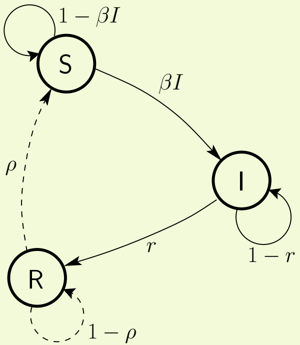

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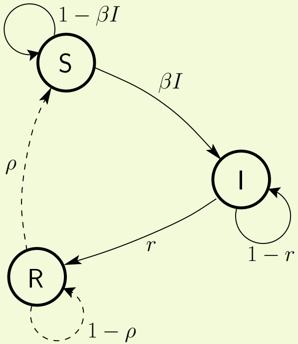

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ρ for loss of immunity

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Original models attributed to

- 1920's: Reed and Frost
- 1920's/1930's: Kermack and McKendrick [5, 7, 6]
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Independent Interaction models

Differential equations for continuous model

$$\frac{\mathrm{d}}{\mathrm{d}t} S = -\beta IS + \rho R$$

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$$\frac{\mathrm{d}}{\mathrm{d}t} R = rI - \rho R$$

β, r, and ρ are now rates.

Reproduction Number R_0:

- R_0 = expected number of infected individuals resulting from a single initial infective
- Epidemic threshold: If $R_0 > 1$, 'epidemic' occurs.

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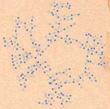

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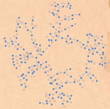

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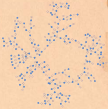

Reproduction Number R_0

Discrete version:

- Set up: One Infective in a randomly mixing population of Susceptibles
- At time $t = 0$, single infective random bumps into a Susceptible
- Probability of transmission = β
- At time $t = 1$, single Infective remains infected with probability $1 - r$
- At time $t = k$, single Infective remains infected with probability $(1 - r)^k$

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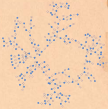

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Discrete version:

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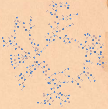

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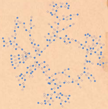

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For S_0 initial infectives ($1 - S_0 = R_0$ immune):

$$R_0 = S_0\beta/r$$

Independent Interaction models

For the continuous version

- Second equation:

$$\frac{\mathrm{d}}{\mathrm{d}t} I = \beta S I - r I$$

- Number of infectives grows initially if

$$\beta S(0) - r > 0$$

- Same story as for discrete model.

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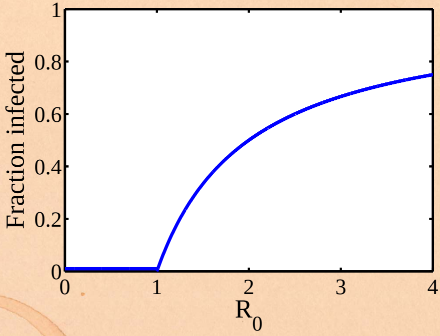

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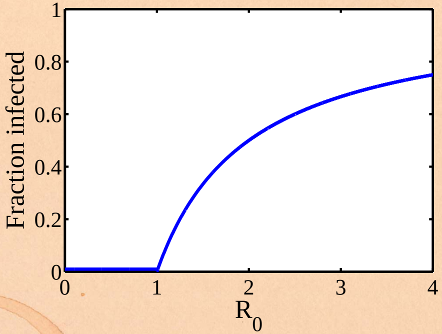

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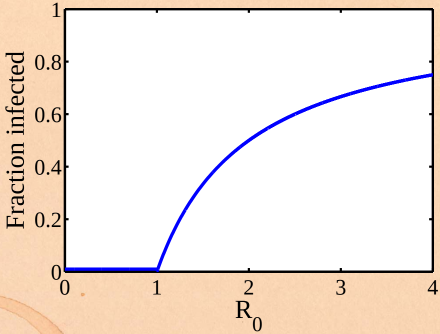

- Continuous phase transition.
- Fine idea from a simple model.

Independent Interaction models

Many variants of the SIR model:

- SIS: susceptible-infective-susceptible
- SIRS: susceptible-infective-recovered-susceptible
- compartment models (age or gender partitions)
- more categories such as 'exposed' (SEIRS)
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Disease spreading models

For novel diseases:

1. Can we predict the size of an epidemic?
2. How important is the reproduction number R_0?

R_0 approximately same for all of the following:

- 1918-19 “Spanish Flu” ∼ 500,000 deaths in US
- 1957-58 “Asian Flu” ∼ 70,000 deaths in US
- 1968-69 “Hong Kong Flu” ∼ 34,000 deaths in US
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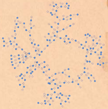

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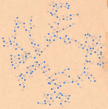

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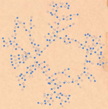
Size distributions

Size distributions are important elsewhere:

- earthquakes (Gutenberg-Richter law)
- city sizes, forest fires, war fatalities
- wealth distributions
- ‘popularity’ (books, music, websites, ideas)
- Epidemics?

Really, what about epidemics?

- Simply hasn’t attracted much attention.
- Data not as clean as for other phenomena.



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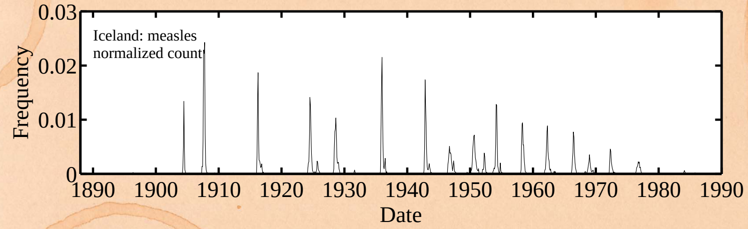
Feeling Ill in Iceland

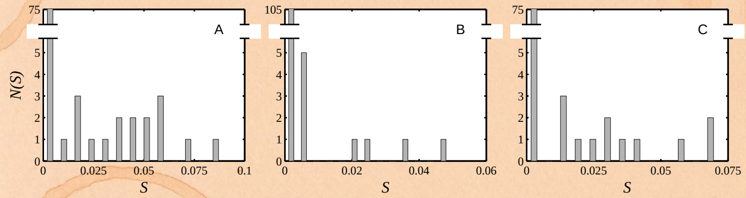
Caseload recorded monthly for range of diseases in Iceland, 1888-1990



- Treat outbreaks separated in time as 'novel' diseases.

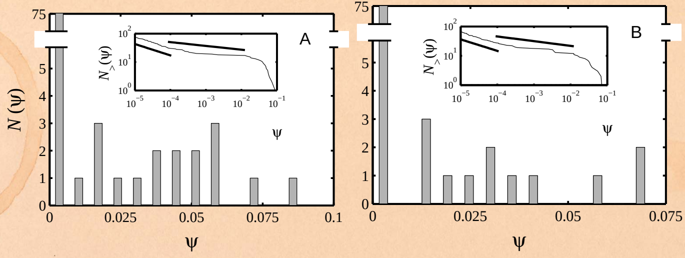
Really not so good at all in Iceland

Epidemic size distributions $N(S)$ for Measles, Rubella, and Whooping Cough.



Spike near $S = 0$, relatively flat otherwise.

Measles & Pertussis



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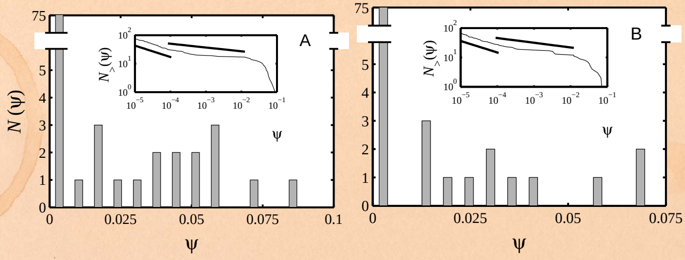


Insert plots:

Complementary cumulative frequency distributions:

$$N(\Psi' > \Psi) \propto \Psi^{-\gamma+1}$$

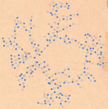
Limited scaling with a possible break.

Power law distributions

Measured values of γ:

- measles: 1.40 (low Ψ) and 1.13 (high Ψ)
- pertussis: 1.39 (low Ψ) and 1.16 (high Ψ)

- Expect $2 \leq \gamma < 3$ (finite mean, infinite variance)
- When $\gamma < 1$, can't normalize
- Distribution is quite flat.



The UNIVERSITY of VERMONT

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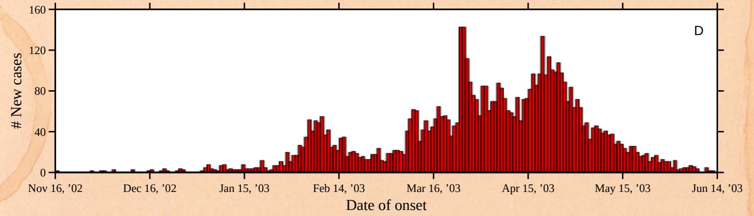

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- Epidemic discovers new 'pools' of susceptibles: Resurgence.
- Importance of rare, stochastic events.

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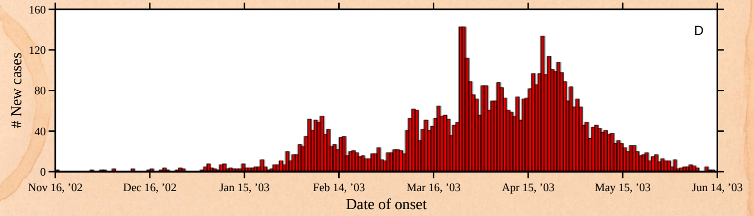

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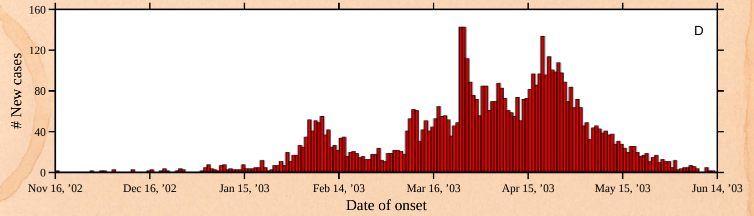

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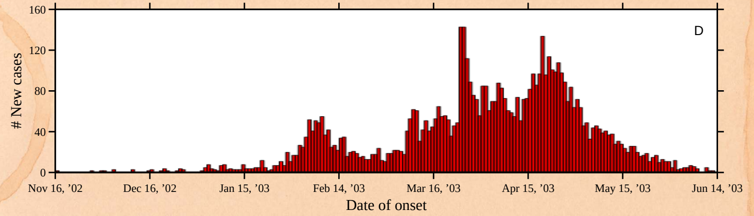

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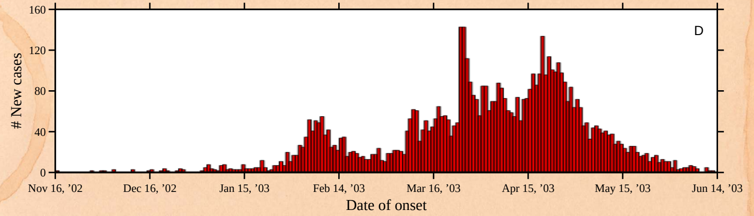

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The challenge

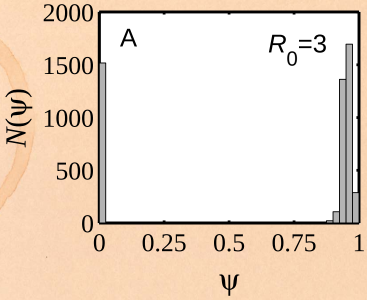
So... can a simple model produce

1. broad epidemic distributions

   and

2. resurgence ?

Size distributions



Simple models typically produce bimodal or unimodal size distributions.

- This includes network models: random, small-world, scale-free, ...
- Exceptions:
  1. Forest fire models
  2. Sophisticated metapopulation models

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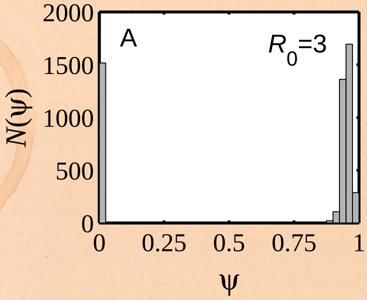

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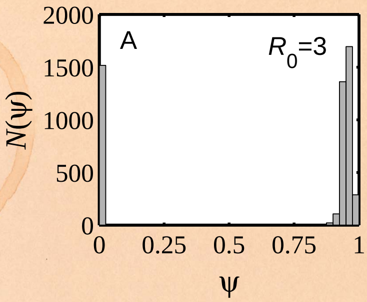

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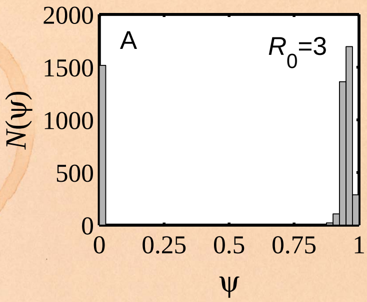

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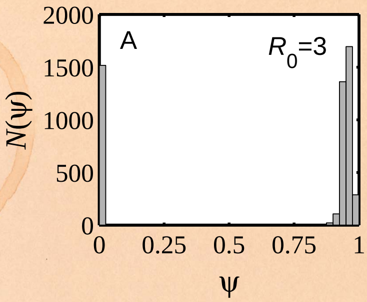

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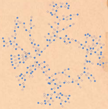
Burning through the population

Forest fire models: [9]

- Rhodes & Anderson, 1996
- The physicist's approach:
  "if it works for magnets, it'll work for people..."

A bit of a stretch:

1. Epidemics ≡ forest fires spreading on 3-d and 5-d lattices.
2. Claim Iceland and Faroe Islands exhibit power law distributions for outbreaks.
3. Original forest fire model not completely understood.



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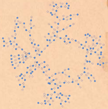

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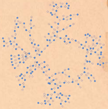

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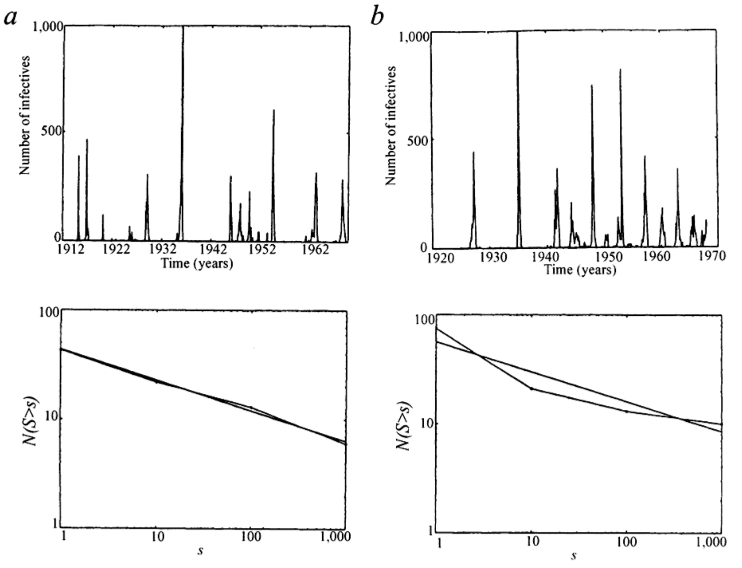
Size distributions



From Rhodes and Anderson, 1996.

Sophisticated metapopulation models

- Community based mixing: Longini (two scales).
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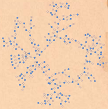

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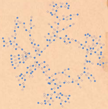

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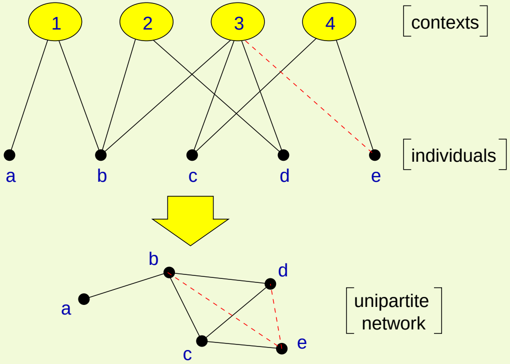
Outline

Improving simple models

Contexts and Identities—Bipartite networks



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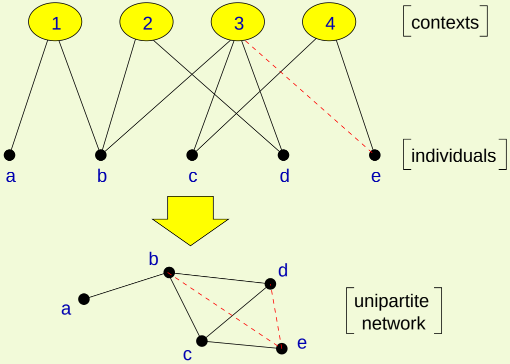

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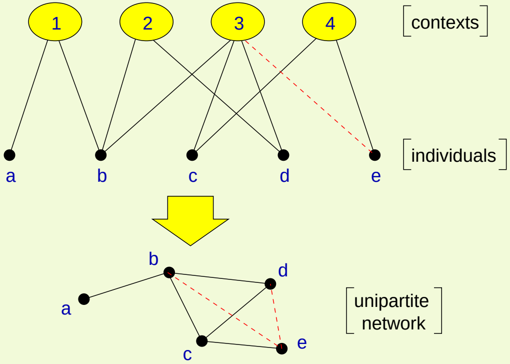

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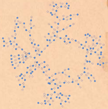

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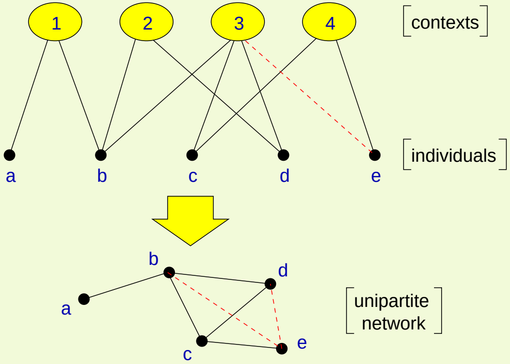

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Idea for social networks: incorporate identity.

Identity is formed from attributes such as:

- Geographic location
- Type of employment
- Age
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Groups are crucial...

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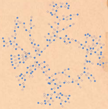

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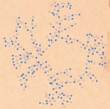

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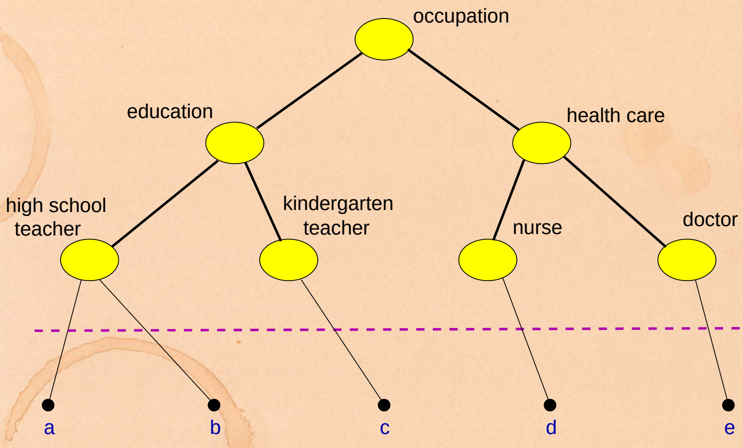
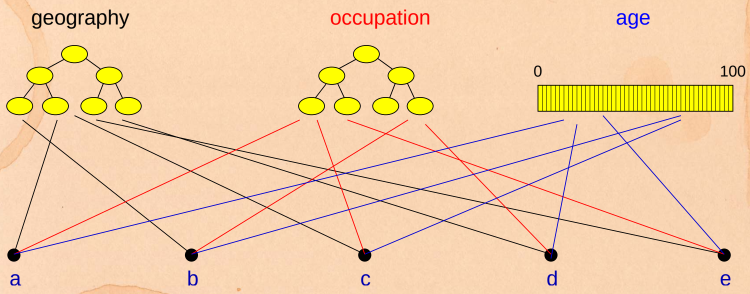
Infer interactions/network from identities



Distance makes sense in identity/context space.

Generalized context space



(Blau & Schwartz [1], Simmel [10], Breiger [2])

A toy agent-based model

Geography—allow people to move between contexts:

- Locally: standard SIR model with random mixing
- discrete time simulation
- β = infection probability
- γ = recovery probability
- P = probability of travel
- Movement distance: $\Pr(d) \propto \exp(-d/\xi)$
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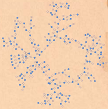

The UNIVERSITY of VERMONT

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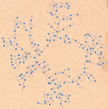

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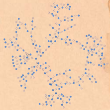

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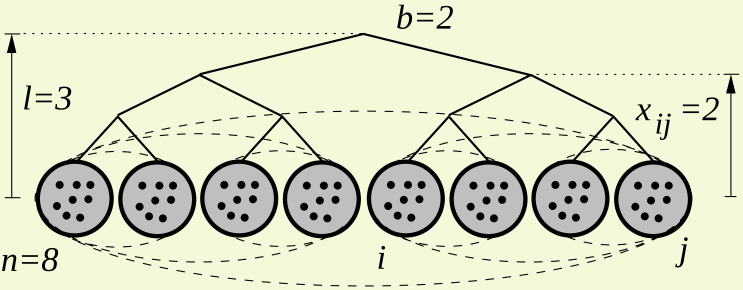
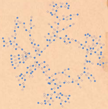
Schematic:



Outline

Model output

- Define P_0 = Expected number of infected individuals leaving initially infected context.
- Need $P_0 > 1$ for disease to spread (independent of R_0).
- Limit epidemic size by restricting frequency of travel and/or range.



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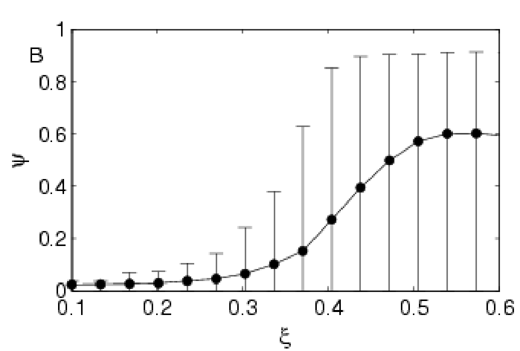
Varying ξ:



- Transition in expected final size based on typical movement distance

Model output

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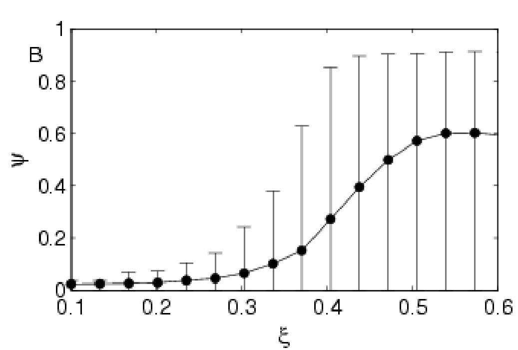
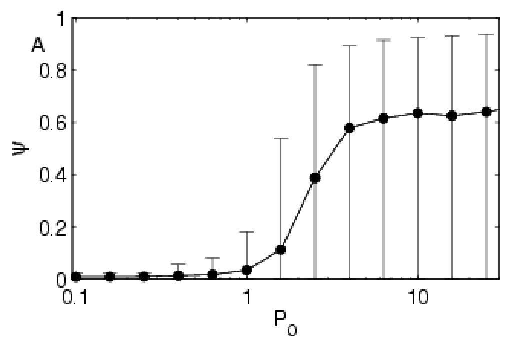


- Transition in expected final size based on typical movement distance (sensible)

Model output

Varying P_0:



- Transition in expected final size based on typical number of infectives leaving first group
- Travel advisories: ξ has larger effect than P_0.

Model output

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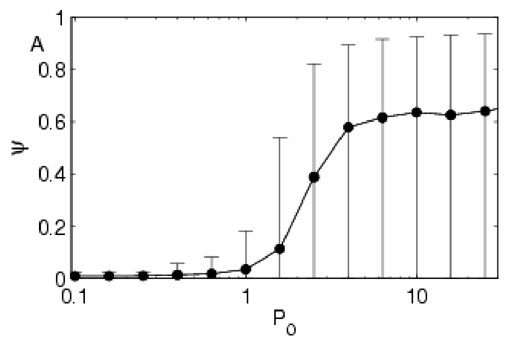

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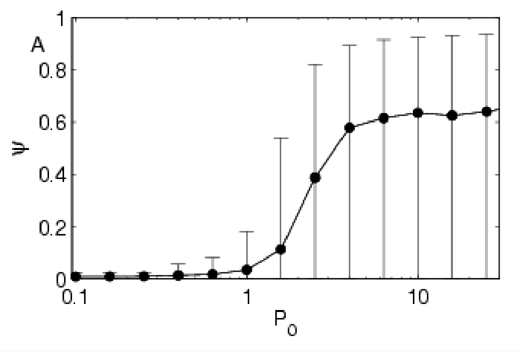

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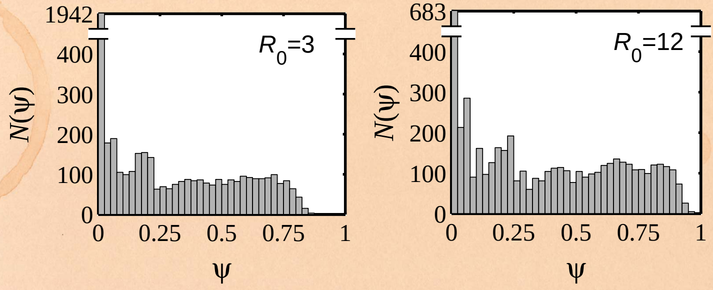

- Flat distributions are possible for certain ξ and P.
- Different R_0's may produce similar distributions
- Same epidemic sizes may arise from different R_0's

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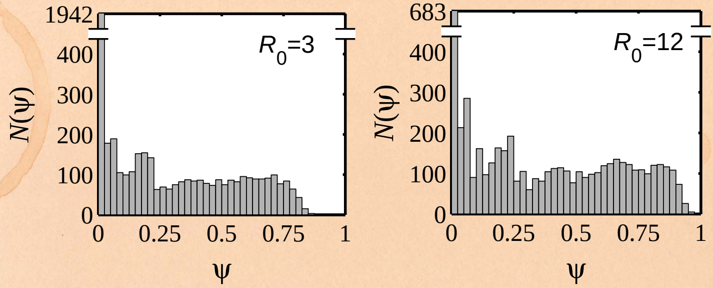

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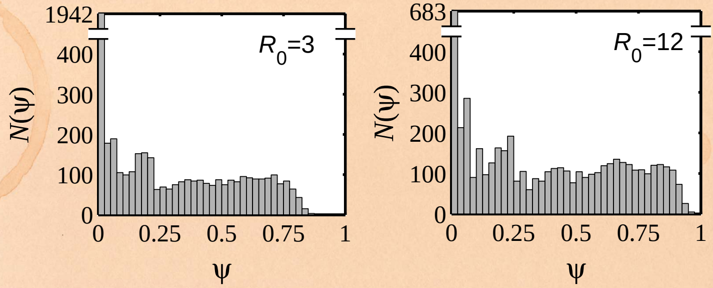

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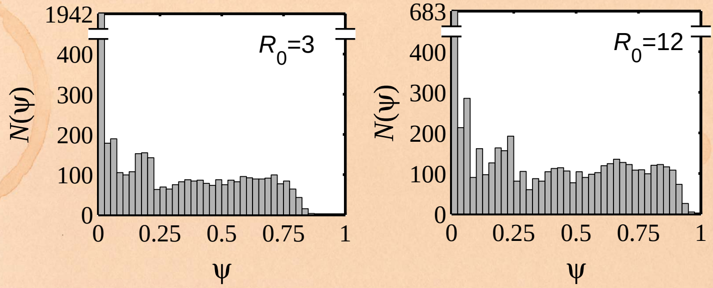

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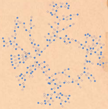

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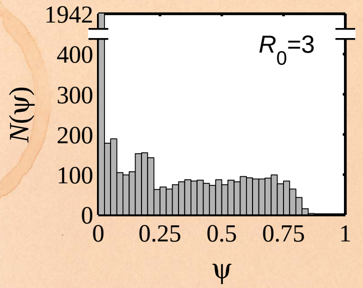

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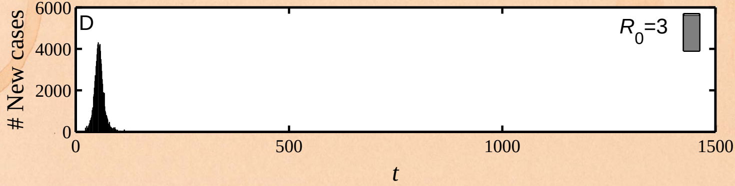
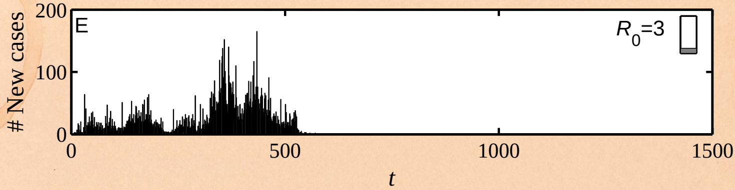
Model output—resurgence

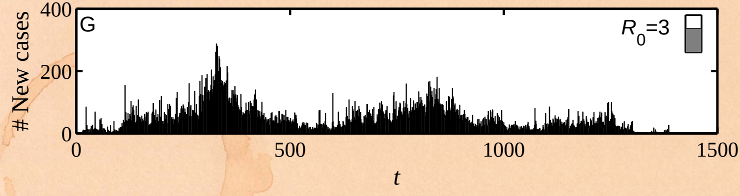
Standard model:



Model output—resurgence

Standard model with transport:





The upshot

Simple multiscale population structure

The upshot

Simple multiscale population structure

+

stochasticity

The upshot

Simple multiscale population structure

+

stochasticity

leads to

resurgence

+

broad epidemic size distributions

Outline

Introduction

Simple disease spreading models

- Background
- Prediction
- More models
- Toy metapopulation models
- Model output
- **Conclusions**
- Predicting social catastrophe

References

Conclusions

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- Model is more complicated than SIR but still simple
- We haven't even included normal social responses such as travel bans and self-quarantine.
- The reproduction number R_0 is not terribly useful.
- R_0, however measured, is not informative about
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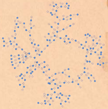
- Problem: R_0 summarises one epidemic after the fact and enfolds movement, the price of bananas, everything.



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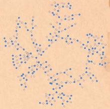

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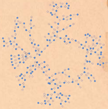

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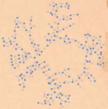

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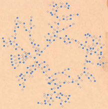

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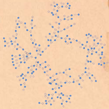

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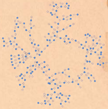

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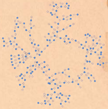

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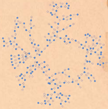

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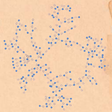

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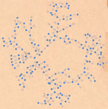

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wildbluffmedia.com

- From the Daily Show (⊞) (September 18, 2007)
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