

# Biological Contagion

## Principles of Complex Systems

### CSYS/MATH 300, Fall, 2011

Prof. Peter Dodds

Department of Mathematics & Statistics | Center for Complex Systems |  
Vermont Advanced Computing Center | University of Vermont

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References



Licensed under the *Creative Commons Attribution-NonCommercial-ShareAlike 3.0 License*.



# Outline

## Introduction

## Simple disease spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social catastrophe

## References

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References



## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

## A confusion of contagions:

- ▶ Was Harry Potter some kind of virus?
- ▶ What about the Da Vinci Code?
- ▶ Did Sudoku spread like a disease?
- ▶ Language?
- ▶ Religion?
- ▶ Democracy...?



## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

## Naturomorphisms

- ▶ “The feeling was contagious.”
- ▶ “The news spread like wildfire.”
- ▶ “Freedom is the most contagious virus known to man.”  
—Hubert H. Humphrey, Johnson’s vice president
- ▶ “Nothing is so contagious as enthusiasm.”  
—Samuel Taylor Coleridge



## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

## Optimism according to Ambrose Bierce: (田)

The doctrine that everything is beautiful, including what is ugly, everything good, especially the bad, and everything right that is wrong. ... **It is hereditary, but fortunately not contagious.**



## Eric Hoffer, 1902–1983

There is a grandeur in the uniformity of the mass. When a fashion, a dance, a song, a slogan or a joke sweeps like wildfire from one end of the continent to the other, and a hundred million people roar with laughter, sway their bodies in unison, hum one song or break forth in anger and denunciation, there is the overpowering feeling that in this country we have come nearer the brotherhood of man than ever before.

- ▶ Hoffer (田) was an interesting fellow...

### Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References



# The spread of fanaticism

## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

Hoffer's acclaimed work: "**The True Believer:**  
Thoughts On The Nature Of Mass Movements" (1951) [3]

## Quotes-aplenty:

- ▶ "We can be absolutely certain only about things we do not understand."
- ▶ "Mass movements can rise and spread without belief in a God, but never without belief in a devil."
- ▶ "Where freedom is real, equality is the passion of the masses. Where equality is real, freedom is the passion of a small minority."

## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References



## CONFORMITY

WHEN PEOPLE ARE FREE TO DO AS THEY PLEASE,  
THEY USUALLY IMITATE EACH OTHER.

[www.despair.com](http://www.despair.com)

[despair.com](http://despair.com)

“When people are free  
to do as they please,  
they usually imitate  
each other.”

—Eric Hoffer  
“The Passionate State  
of Mind” [4]





# The collective...

## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

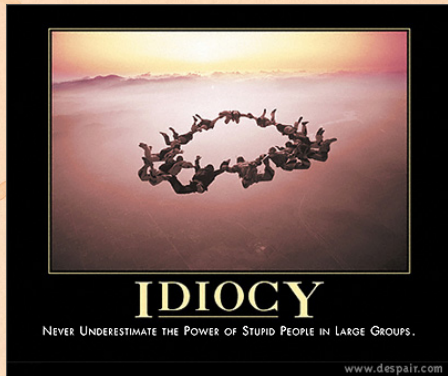
Model output

Conclusions

Predicting social  
catastrophe

References

“Never Underestimate  
the Power of Stupid  
People in Large  
Groups.”



despair.com



## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

## Definitions

- ▶ (1) The spreading of a quality or quantity between individuals in a population.
- ▶ (2) A disease itself:  
the plague, a blight, the dreaded lurgi, ...
- ▶ from Latin: *con* = 'together with' + *tangere* 'to touch.'
- ▶ Contagion has unpleasant overtones...
- ▶ Just **Spreading** might be a more neutral word
- ▶ But contagion is kind of exciting...



# Examples of non-disease spreading:

## Interesting infections:

- ▶ [Spreading of buildings in the US...](#) (田)



- ▶ [Viral get-out-the-vote video.](#) (田)

## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

## References



## Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

## Two main classes of contagion

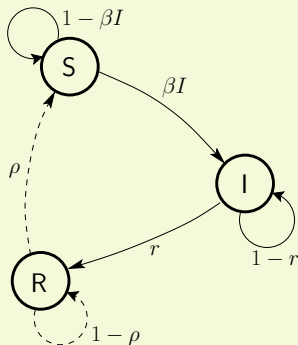
1. **Infectious diseases:**  
tuberculosis, HIV, ebola, SARS, influenza, ...
2. **Social contagion:**  
fashion, word usage, rumors, riots, religion, ...



## The standard SIR model <sup>[8]</sup>

- ▶ = basic model of disease contagion
- ▶ Three states:
  1. S = Susceptible
  2. I = Infective/Infectious
  3. R = Recovered or Removed or Refractory
- ▶  $S(t) + I(t) + R(t) = 1$
- ▶ Presumes random interactions (mass-action principle)
- ▶ Interactions are independent (no memory)
- ▶ Discrete and continuous time versions

## Discrete time automata example:



Transition Probabilities:

$\beta$  for being infected given

contact with infected

$r$  for recovery

$\rho$  for loss of immunity



Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

## Original models attributed to

- ▶ 1920's: Reed and Frost
- ▶ 1920's/1930's: Kermack and McKendrick [5, 7, 6]
- ▶ Coupled differential equations with a mass-action principle



## Differential equations for continuous model

$$\frac{d}{dt}S = -\beta IS + \rho R$$

$$\frac{d}{dt}I = \beta IS - rI$$

$$\frac{d}{dt}R = rI - \rho R$$

$\beta$ ,  $r$ , and  $\rho$  are now **rates**.

## Reproduction Number $R_0$ :

- ▶  $R_0$  = expected number of infected individuals resulting from a single initial infective
- ▶ Epidemic threshold: If  $R_0 > 1$ , 'epidemic' occurs.

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References





## Discrete version:

- ▶ Set up: One Infective in a randomly mixing population of Susceptibles
- ▶ At time  $t = 0$ , single infective random bumps into a Susceptible
- ▶ Probability of transmission =  $\beta$
- ▶ At time  $t = 1$ , single Infective remains infected with probability  $1 - r$
- ▶ At time  $t = k$ , single Infective remains infected with probability  $(1 - r)^k$



# Reproduction Number $R_0$

## Discrete version:

- ▶ Expected number infected by original Infective:

$$\begin{aligned}R_0 &= \beta + (1 - r)\beta + (1 - r)^2\beta + (1 - r)^3\beta + \dots \\&= \beta \left( 1 + (1 - r) + (1 - r)^2 + (1 - r)^3 + \dots \right) \\&= \beta \frac{1}{1 - (1 - r)} = \beta/r\end{aligned}$$

For  $S_0$  initial infectives ( $1 - S_0 = R_0$  immune):

$$R_0 = S_0\beta/r$$

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References



# Independent Interaction models

## For the continuous version

- ▶ Second equation:

$$\frac{d}{dt}I = \beta SI - rI$$

$$\frac{d}{dt}I = (\beta S - r)I$$

- ▶ Number of infectives grows initially if

$$\beta S(0) - r > 0 \Rightarrow \beta S(0) > r \Rightarrow \beta S(0)/r > 1$$

- ▶ Same story as for discrete model.

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

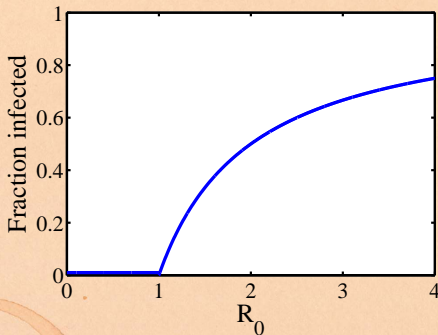
Predicting social  
catastrophe

References



# Independent Interaction models

Example of epidemic threshold:



- ▶ Continuous phase transition.
- ▶ Fine idea from a simple model.

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

## Many variants of the SIR model:

- ▶ **SIS**: susceptible-infective-susceptible
- ▶ **SIRS**: susceptible-infective-recovered-susceptible
- ▶ compartment models (age or gender partitions)
- ▶ more categories such as 'exposed' (**SEIRS**)
- ▶ recruitment (migration, birth)



## For novel diseases:

1. Can we predict the size of an epidemic?
2. How important is the reproduction number  $R_0$ ?

## $R_0$ approximately same for all of the following:

- ▶ 1918-19 “Spanish Flu”  $\sim$  500,000 deaths in US
- ▶ 1957-58 “Asian Flu”  $\sim$  70,000 deaths in US
- ▶ 1968-69 “Hong Kong Flu”  $\sim$  34,000 deaths in US
- ▶ 2003 “SARS Epidemic”  $\sim$  800 deaths world-wide



## Size distributions are important elsewhere:

- ▶ earthquakes (Gutenberg-Richter law)
- ▶ city sizes, forest fires, war fatalities
- ▶ wealth distributions
- ▶ 'popularity' (books, music, websites, ideas)
- ▶ **Epidemics?**

Power laws distributions are common but not obligatory...

## Really, what about epidemics?

- ▶ Simply hasn't attracted much attention.
- ▶ Data not as clean as for other phenomena.

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

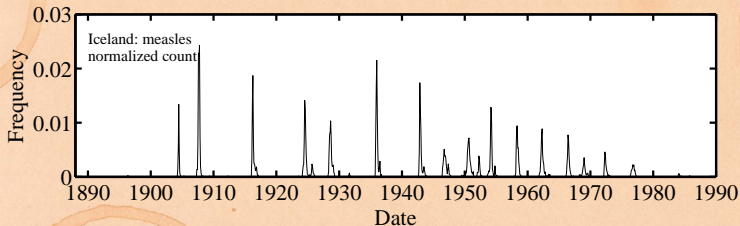
Predicting social  
catastrophe

References



# Feeling Ill in Iceland

Caseload recorded monthly for range of diseases in  
Iceland, 1888-1990



- ▶ Treat outbreaks separated in time as 'novel' diseases.

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

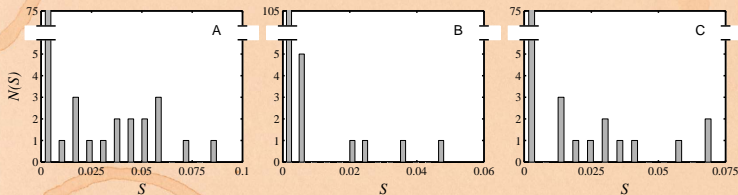
References





# Really not so good at all in Iceland

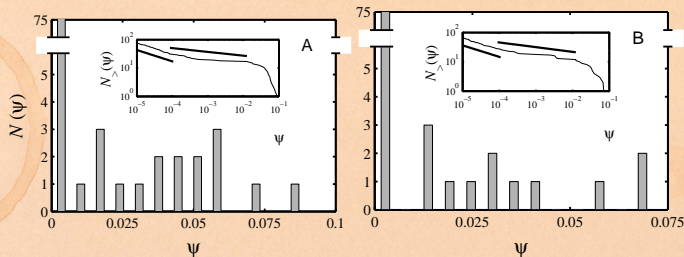
Epidemic size distributions  $N(S)$  for  
Measles, Rubella, and Whooping Cough.



Spike near  $S = 0$ , relatively flat otherwise.



# Measles & Pertussis



Insert plots:

Complementary cumulative frequency distributions:

$$N(\psi' > \psi) \propto \psi^{-\gamma+1}$$

Limited scaling with a possible break.

Introduction

Simple disease  
spreading models

Background

**Prediction**

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

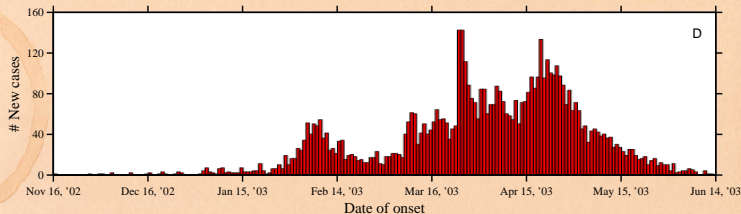


## Measured values of $\gamma$ :

- ▶ measles: **1.40** (low  $\Psi$ ) and **1.13** (high  $\Psi$ )
- ▶ pertussis: **1.39** (low  $\Psi$ ) and **1.16** (high  $\Psi$ )
  
- ▶ Expect  $2 \leq \gamma < 3$  (finite mean, infinite variance)
- ▶ When  $\gamma < 1$ , can't normalize
- ▶ Distribution is quite **flat**.



# Resurgence—example of SARS



- ▶ Epidemic slows...  
then an infective moves to a new context.
- ▶ Epidemic discovers new 'pools' of susceptibles:  
**Resurgence.**
- ▶ Importance of rare, stochastic events.

## Introduction

### Simple disease spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

## References



# The challenge

Introduction

Simple disease  
spreading models

Background

Prediction

**More models**

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

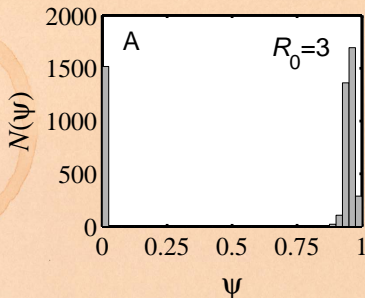
References

So... can a simple model produce

1. **broad epidemic distributions**  
and
2. **resurgence ?**



# Size distributions



Simple models typically produce **bimodal** or **unimodal** size distributions.

- ▶ This **includes** network models:  
random, small-world, scale-free, ...
- ▶ Exceptions:
  1. Forest fire models
  2. Sophisticated metapopulation models

Introduction

Simple disease spreading models

Background

Prediction

**More models**

Toy metapopulation models

Model output

Conclusions

Predicting social catastrophe

References



# Burning through the population

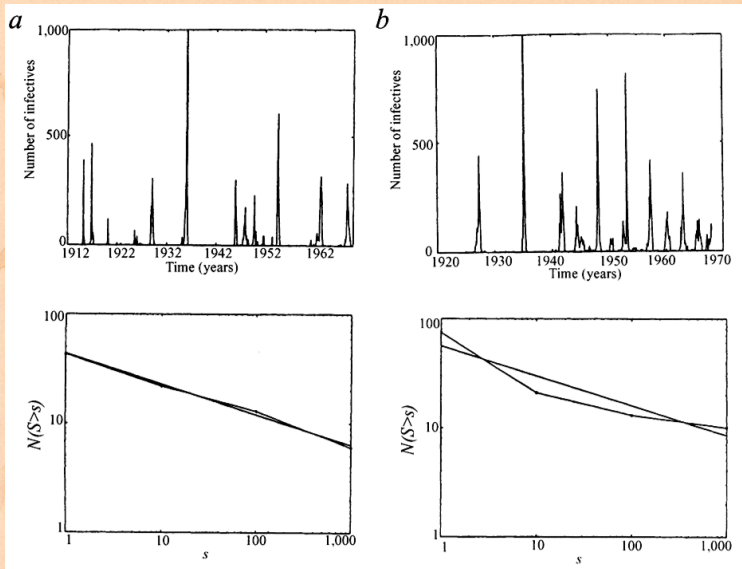
## Forest fire models: [9]

- ▶ Rhodes & Anderson, 1996
- ▶ The physicist's approach:  
"if it works for magnets, it'll work for people..."

## A bit of a stretch:

1. Epidemics  $\equiv$  forest fires spreading on 3-d and 5-d lattices.
2. Claim Iceland and Faroe Islands exhibit power law distributions for outbreaks.
3. Original forest fire model not completely understood.

# Size distributions



From Rhodes and Anderson, 1996.

Introduction

Simple disease  
spreading models

Background

Prediction

**More models**

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References





# Sophisticated metapopulation models

- ▶ Community based mixing: Longini (two scales).
- ▶ Eubank et al.'s EpiSims/TRANSIMS—city simulations.
- ▶ Spreading through countries—Airlines: Germann et al., Corlizza et al.
- ▶ Vital work but perhaps hard to generalize from...
- ▶ ⇒ Create a simple model involving multiscale travel
- ▶ Multiscale models suggested by others but not formalized (Bailey, Cliff and Haggett, Ferguson et al.)



# Size distributions

Introduction

Simple disease  
spreading models

Background

Prediction

**More models**

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

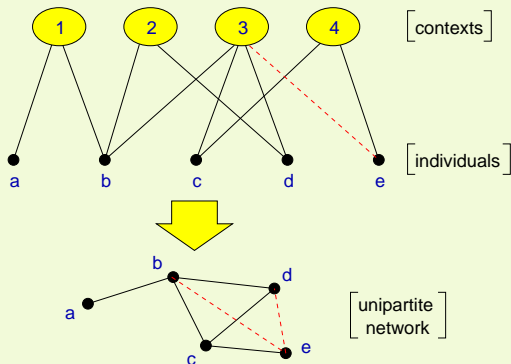
References

- ▶ Very big question: **What is  $N$ ?**
- ▶ Should we model SARS in Hong Kong as spreading in a neighborhood, in Hong Kong, Asia, or the world?
- ▶ For simple models, we need to know the final size beforehand...



# Improving simple models

## Contexts and Identities—Bipartite networks



- ▶ boards of directors
- ▶ movies
- ▶ transportation modes (subway)

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References



# Improving simple models

Idea for social networks: incorporate **identity**.

Identity is formed from attributes such as:

- ▶ Geographic location
- ▶ Type of employment
- ▶ Age
- ▶ Recreational activities

Groups are crucial...

- ▶ formed by people with at least one similar attribute
- ▶ Attributes  $\Leftrightarrow$  Contexts  $\Leftrightarrow$  Interactions  $\Leftrightarrow$  Networks. <sup>[11]</sup>

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

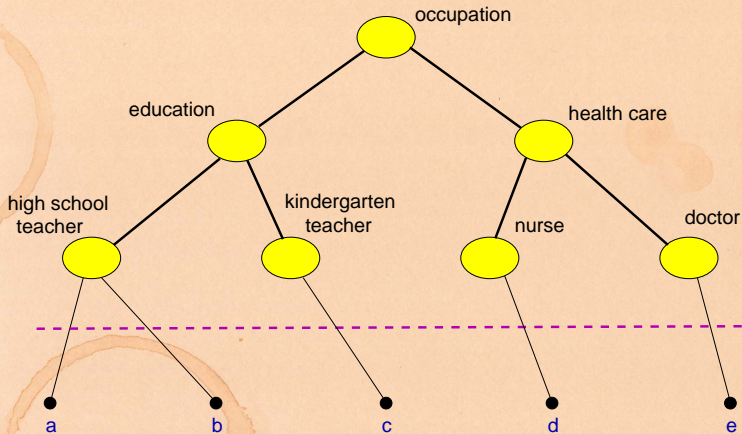
Conclusions

Predicting social  
catastrophe

References

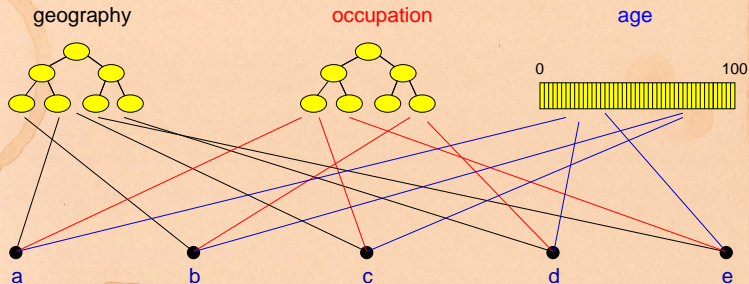


# Infer interactions/network from identities



Distance makes sense in identity/context space.

# Generalized context space



(Blau & Schwartz <sup>[1]</sup>, Simmel <sup>[10]</sup>, Breiger <sup>[2]</sup>)

# A toy agent-based model

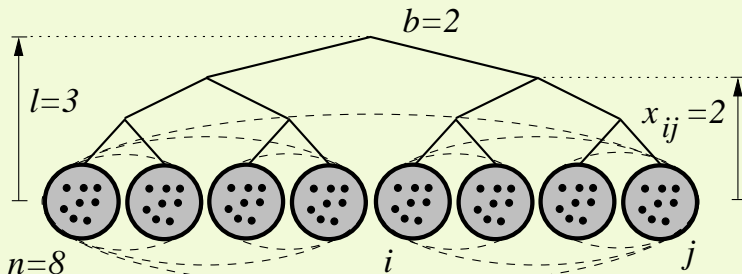
## Geography—allow people to move between contexts:

- ▶ Locally: standard SIR model with random mixing
- ▶ discrete time simulation
- ▶  $\beta$  = infection probability
- ▶  $\gamma$  = recovery probability
- ▶  $P$  = probability of travel
- ▶ **Movement distance:**  $\Pr(d) \propto \exp(-d/\xi)$
- ▶  $\xi$  = typical travel distance



# A toy agent-based model

## Schematic:

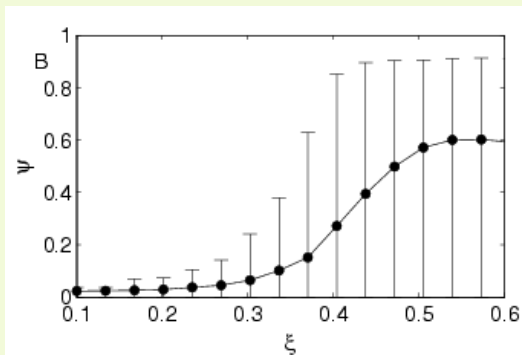




- ▶ Define  $P_0$  = Expected number of infected individuals **leaving** initially infected context.
- ▶ Need  $P_0 > 1$  for disease to spread (independent of  $R_0$ ).
- ▶ Limit epidemic size by **restricting frequency of travel and/or range**



## Varying $\xi$ :



- ▶ Transition in expected final size based on typical movement distance (**sensible**)

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

**Model output**

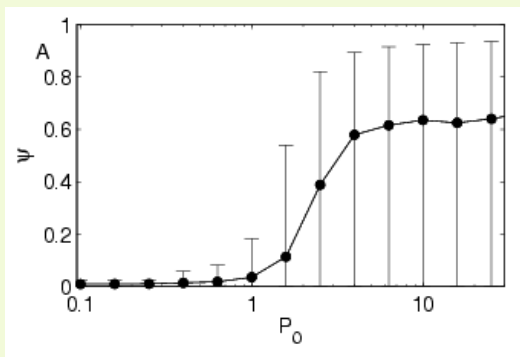
Conclusions

Predicting social  
catastrophe

References



## Varying $P_0$ :



- ▶ Transition in expected final size based on typical number of infectives leaving first group (also sensible)
- ▶ Travel advisories:  $\xi$  has larger effect than  $P_0$ .

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

**Model output**

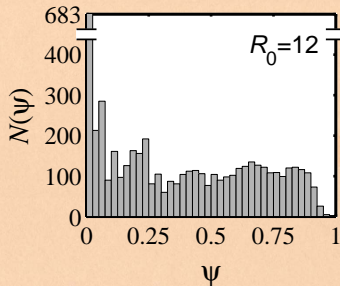
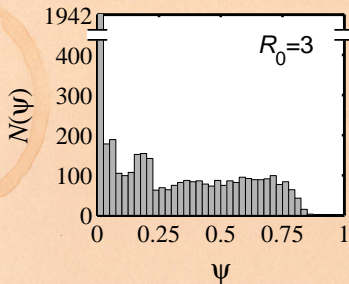
Conclusions

Predicting social  
catastrophe

References



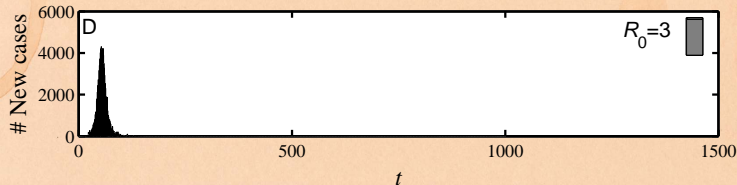
# Example model output: size distributions



- ▶ Flat distributions are possible for certain  $\xi$  and  $P$ .
- ▶ Different  $R_0$ 's may produce similar distributions
- ▶ Same epidemic sizes may arise from different  $R_0$ 's

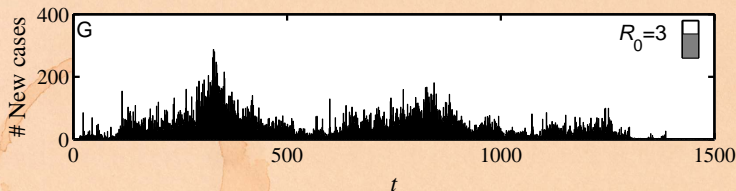
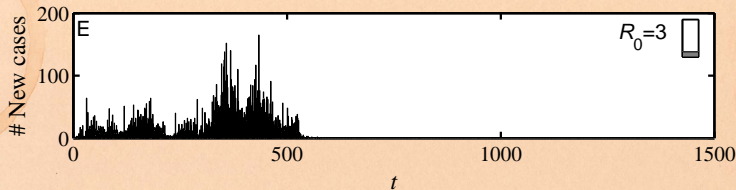
# Model output—resurgence

## Standard model:



# Model output—resurgence

## Standard model with transport:



# The upshot

Simple multiscale population structure  
+  
stochasticity

leads to

resurgence

+  
broad epidemic size distributions

## Biological Contagion

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

**Model output**

Conclusions

Predicting social  
catastrophe

References



# Conclusions

- ▶ For this model, epidemic size is highly unpredictable
- ▶ Model is more complicated than SIR but still simple
- ▶ We haven't even included normal social responses such as travel bans and self-quarantine.
- ▶ The reproduction number  $R_0$  is not terribly useful.
- ▶  $R_0$ , however measured, is not informative about
  1. how likely the observed epidemic size was,
  2. and how likely future epidemics will be.
- ▶ Problem:  $R_0$  summarises **one** epidemic after the fact and enfolds movement, the price of bananas, everything.





# Conclusions

- ▶ Disease spread highly sensitive to population structure
- ▶ Rare events may matter enormously (e.g., an infected individual taking an international flight)
- ▶ More support for controlling population movement (e.g., travel advisories, quarantine)



## What to do:

- ▶ Need to separate movement from disease
- ▶  $R_0$  needs a friend or two.
- ▶ Need  $R_0 > 1$  and  $P_0 > 1$  and  $\xi$  sufficiently large for disease to have a chance of spreading

## More wondering:

- ▶ Exactly how important are rare events in disease spreading?
- ▶ Again, what is  $N$ ?

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

## Valiant attempts to use SIR and co. elsewhere:

- ▶ Adoption of ideas/beliefs (Goffman & Newell, 1964)
- ▶ Spread of rumors (Daley & Kendall, 1965)
- ▶ Diffusion of innovations (Bass, 1969)
- ▶ Spread of fanatical behavior (Castillo-Chávez & Song, 2003)
- ▶ Spread of Feynmann diagrams (Bettencourt et al., 2006)



# Predicting social catastrophe isn't easy...

## “Greenspan Concedes Error on Regulation”

- ▶ ... humbled Mr. Greenspan admitted that he had put too much faith in the self-correcting power of free markets ...
- ▶ “Those of us who have looked to the self-interest of lending institutions to protect shareholders' equity, myself included, are in a state of shocked disbelief”
- ▶ Rep. Henry A. Waxman: “Do you feel that your ideology pushed you to make decisions that you wish you had not made?”
- ▶ Mr. Greenspan conceded: “Yes, I've found a flaw. I don't know how significant or permanent it is. But I've been very distressed by that fact.”

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

New York Times, October 23, 2008 (田)

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

Alan Greenspan (September 18, 2007):

“I’ve been dealing with these big mathematical models of forecasting the economy ...

If I could figure out a way to determine whether or not people are more fearful or changing to more euphoric,

I don’t need any of this other stuff.

I could forecast the economy better than any way I know.”



<http://wikipedia.org>

# Economics, Schmeconomics

## Greenspan continues:

“The trouble is that we can’t figure that out. I’ve been in the forecasting business for 50 years. **I’m no better than I ever was, and nobody else is.** Forecasting 50 years ago was as good or as bad as it is today. **And the reason is that human nature hasn’t changed.** We can’t improve ourselves.”

## Jon Stewart:

“You just bummed the @\*!# out of me.”



wildbluffmedia.com

- ▶ From the Daily Show (田) (September 18, 2007)
- ▶ The full interview is here (田).

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References



James K. Galbraith:

NYT But there are at least 15,000 professional economists in this country, and you're saying only two or three of them foresaw the mortgage crisis? [JKG] Ten or 12 would be closer than two or three.

NYT What does that say about the field of economics, which claims to be a science? [JKG] It's an enormous blot on the reputation of the profession. There are thousands of economists. Most of them teach. And most of them teach a theoretical framework that has been shown to be fundamentally useless.

From the New York Times, 11/02/2008 (田)

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References

# References I

- [1] P. M. Blau and J. E. Schwartz.  
Crosscutting Social Circles.  
Academic Press, Orlando, FL, 1984.
- [2] R. L. Breiger.  
The duality of persons and groups.  
Social Forces, 53(2):181–190, 1974. [pdf](#) (田)
- [3] E. Hoffer.  
The True Believer: On The Nature Of Mass Movements.  
Harper and Row, New York, 1951.
- [4] E. Hoffer.  
The Passionate State of Mind: And Other Aphorisms.  
Buccaneer Books, 1954.

Introduction

Simple disease  
spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social  
catastrophe

References





# References II

- [5] W. O. Kermack and A. G. McKendrick.  
A contribution to the mathematical theory of epidemics.  
[Proc. R. Soc. Lond. A, 115:700–721, 1927. pdf](#) (⊞)
- [6] W. O. Kermack and A. G. McKendrick.  
A contribution to the mathematical theory of epidemics. III. Further studies of the problem of endemicity.  
[Proc. R. Soc. Lond. A, 141\(843\):94–122, 1927. pdf](#) (⊞)
- [7] W. O. Kermack and A. G. McKendrick.  
Contributions to the mathematical theory of epidemics. II. The problem of endemicity.  
[Proc. R. Soc. Lond. A, 138\(834\):55–83, 1927. pdf](#) (⊞)

Introduction

Simple disease spreading models

Background

Prediction

More models

Toy metapopulation models

Model output

Conclusions

Predicting social catastrophe

References



# References III

- [8] J. D. Murray.  
Mathematical Biology.  
Springer, New York, Third edition, 2002.
- [9] C. J. Rhodes and R. M. Anderson.  
Power laws governing epidemics in isolated populations.  
Nature, 381:600–602, 1996. pdf (田)
- [10] G. Simmel.  
The number of members as determining the sociological form of the group. I.  
American Journal of Sociology, 8:1–46, 1902.
- [11] D. J. Watts, P. S. Dodds, and M. E. J. Newman.  
Identity and search in social networks.  
Science, 296:1302–1305, 2002. pdf (田)

