Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2011
Assignment 7

Dispersed: Monday, November 14, 2011.
Due: By start of lecture, 11:30 am, Thursday, December 1, 2011.
Some useful reminders:
Instructor: Peter Dodds
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Office hours: 12:50 pm to 3:50 pm, Wednesday
Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2011-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related TeX variant).

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1. Consider a modified version of the Barabàsi-Albert (BA) model [2] where two possible mechanisms are now in play. As in the original model, start with m_0 nodes at time $t = 0$. Let's make these initial guys connected such that each has degree 1. The two mechanisms are:

   M1: With probability p, a new node of degree 1 is added to the network. At time $t + 1$, a node connects to an existing node j with probability

   $$P(\text{connect to node } j) = \frac{k_j}{\sum_{i=1}^{N(t)} k_i} \qquad (1)$$

   where k_j is the degree of node j and $N(t)$ is the number of nodes in the system at time t.

   M2: With probability $q = 1 - p$, a randomly chosen node adds a new edge, connecting to node j with the same preferential attachment probability as above.

   Note that in the limit $q = 0$, we retrieve the original BA model (with the difference that we are adding one link at a time rather than m here).

   In the long time limit $t \to \infty$, what is the expected form of the degree distribution P_k?

   Do we move out of the original model's universality class?

   (3 points for set up, 3 for solving.)

2. Determine the clustering coefficient for toy model small-world networks [3] as a function of the rewiring probability p. Find C_1, the average local clustering coefficient:

$$C_1(p) = \left\langle \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2} \right\rangle_i = \frac{1}{N} \sum_{i=1}^{N} \frac{\sum_{j_1 j_2 \in \mathcal{N}_i} a_{j_1 j_2}}{k_i(k_i-1)/2}$$

where N is the number of nodes, $a_{ij} = 1$ if nodes i and j are connected, and $\mathcal{N}_i$ indicates the neighborhood of i.

As per the original model, assume a ring network with each node connected to a fixed, even number m local neighbors ($m/2$ on each side). Take the number of nodes to be $N \gg m$.

Start by finding $C_1(0)$ and argue for a $(1-p)^3$ correction factor to find an approximation of $C_1(p)$.

Hint 1: you can think of finding C_1 as averaging over the possibilities for a single node.

Hint 2: assume that the degree of individual nodes does not change with rewiring but rather stays fixed at m. In other words, take the average degree of individuals as the degree of a randomly selected individual.

For what value of p is $C_1 \simeq 1/2$?

(3 points for set up, 3 for solving.)

3. (Optional)

"Any good idea can be stated in fifty words or less."—Stanisław Ulam.[1]

Read through Anderson's seminal paper "More is different" [1] and generate three descriptions of complexification with exactly the following lengths:

(a) Three words,

(b) Six words,

(c) and Twelve words.

Things have sped up since Ulam made his claim. All three may contain one or more sentences.

References

[1] P. W. Anderson. More is different. *Science*, 177(4047):393–396, 1972.

[1] At the very least, Ulam's claim is self-consistent.

[2] A.-L. Barabási and R. Albert. Emergence of scaling in random networks. *Science*, 286:509–511, 1999.

[3] D. J. Watts and S. J. Strogatz. Collective dynamics of 'small-world' networks. *Nature*, 393:440–442, 1998.