Principles of Complex Systems, CSYS/MATH 300
University of Vermont, Fall 2011
Assignment 3

Dispersed: Friday, September 30, 2011.
Due: By start of lecture, 11:30 am, Thursday, October 6, 2011.
Some useful reminders:
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Course website: http://www.uvm.edu/∼pdodds/teaching/courses/2011-08UVM-300

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LATEX (or related TEX variant).

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1. Use a scaling argument to show that maximal rowing speed V increases as the number of oarspeople n as $V \propto N^{1/9}$.

   Assume the following:

   (a) Rowing shells are geometrically similar (isometric). The table below taken from McMahon and Bonner [3] shows that shell width is roughly proportional to shell length ℓ.

   Shell dimensions and performances.

No. of oarsmen	Modifying description	Length, l (m)	Beam, b (m)	l/b	Boat mass per oarsman (kg)	Time for 2000 m (min) I	II	III	IV
8	Heavyweight	18.28	0.610	30.0	14.7	5.87	5.92	5.82	5.73
8	Lightweight	18.28	0.598	30.6	14.7				
4	With coxswain	12.80	0.574	22.3	18.1				
4	Without coxswain	11.75	0.574	21.0	18.1	6.33	6.42	6.48	6.13
2	Double scull	9.76	0.381	25.6	13.6				
2	Pair-oared shell	9.76	0.356	27.4	13.6	6.87	6.92	6.95	6.77
1	Single scull	7.93	0.293	27.0	16.3	7.16	7.25	7.28	7.17

   (b) The resistance encountered by a shell is due largely to drag on its wetted surface.

   (c) Drag is proportional to the product of the square of the shell's speed (V^2) and the area of the wetted surface ($\propto \ell^2$ due to the shell isometry).

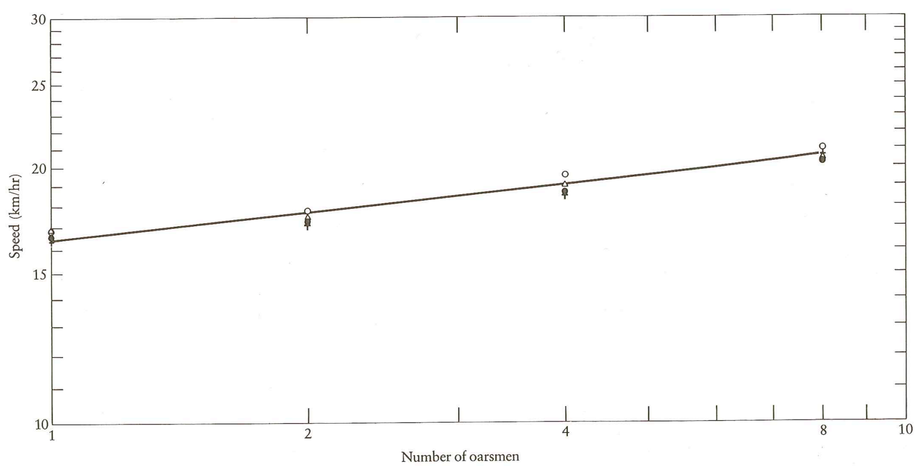
   (d) Power $\propto$ drag force $\times$ speed (in symbols: $P \propto D_f \times V$).

(e) Volume displacement of water by a shell is proportional to the number of oarspeople N (i.e., the team's combined weight).

(f) Assume the depth of water displacement by the shell grows isometrically with boat length ℓ.

(g) Power is proportional to the number of oarspeople N.

2. Find the modern day world record times for 2000 metre races and see if this scaling still holds up. Of course, our relationship is approximate as we have neglected numerous factors, the range is extremely small (1–8 oarspeople), and the scaling is very weak (1/9). But see what you can find. The figure below shows data from McMahon and Bonner.



3. Check current weight lifting records for the snatch, clean and jerk, and the total for scaling with body mass (three regressions).

   For weight classes, take the upper limit for the mass of the lifter.

   (a) Does 2/3 scaling hold up?

   (b) Normalized by the appropriate scaling, who holds the overall, rescaled world record?

4. Yes, even more on power law size distributions. It's good for you.

   For the probability distribution $P(x) = cx^{-\gamma}$, $0 < a \leq x \leq b$, compute the mean absolute displacement (MAD), which is given by $\langle |X - \langle X \rangle| \rangle$ where $\langle \cdot \rangle$ represents expected value. As always, simplify your expression as much as possible.

   MAD is a more reasonable estimate for the width of a distribution, but we like variance σ^2 because the calculations are much prettier. Really.

5. In the limit of $b \to \infty$, how does MAD behave as a function of γ? How does this compare with the behavior of the variance? (See the last question of Assignment 1.)

6. *Discrete random walks:*

   In class, we argued that the number of random walks returning to the origin for the first time after $2n$ time steps is given by

   $$N_{\text{first return}}(2n) = N_{\text{fr}}(2n) = N(1, 1, 2n-2) - N(-1, 1, 2n-2)$$

   where

   $$N(i, j, t) = \binom{t}{(t + j - i)/2}.$$

   Find the leading order term for $N_{\text{fr}}(2n)$ as $n \to \infty$.

   Hint: combine the terms and use Stirling's sterling approximation [1, 2].

   (If you enjoy this sort of thing, you may like to explore the same problem for random walks in higher dimensions. George Pólya is your man.)

   (And we are connecting to some good stuff in combinatorics; more to come in the solutions.)

References

[1] M. Abramowitz and I. A. Stegun, editors. *Handbook of Mathematical Functions*. Dover Publications, New York, 1974.

[2] I. Gradshteyn and I. Ryzhik. *Table of Integrals, Series, and Products*. Academic Press, San Diego, fifth edition, 1994.

[3] T. A. McMahon and J. T. Bonner. *On Size and Life*. Scientific American Library, New York, 1983.