Singular Value Decomposition

Matrixology (Linear Algebra)—Lecture 25/25
MATH 124, Fall, 2011

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Center for Complex Systems
Vermont Advanced Computing Center
University of Vermont

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Outline

The Fundamental Theorem of Linear Algebra

Approximating matrices with SVD

Fundamental Theorem of Linear Algebra

- Applies to any $m \times n$ matrix A.
- Symmetry of A and A^{T}.

Where $\vec{x}$ lives:

- Row space $C(A^{\mathrm{T}}) \subset R^n$.
- (Right) Nullspace $N(A) \subset R^n$.
- $\dim C(A^{\mathrm{T}}) + \dim N(A) = r + (n - r) = n$
- Orthogonality: $C(A^{\mathrm{T}}) \bigotimes N(A) = R^n$

Where $\vec{b}$ lives:

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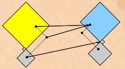

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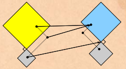

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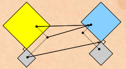

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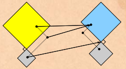

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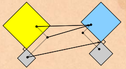

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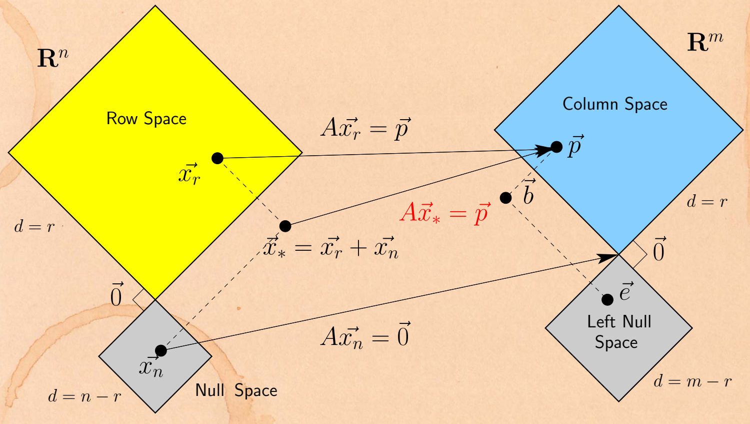
Best solution $\vec{x}_*$ when $\vec{b} = \vec{p} + \vec{e}$:



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Now we see:

- Each of the four fundamental subspaces has a 'best' orthonormal basis
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Happy bases

- $\{\hat{v}_1, \ldots, \hat{v}_r\}$ span Row space
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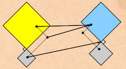

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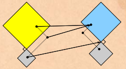

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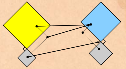

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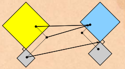

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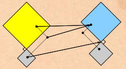

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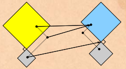

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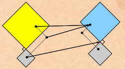

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How $A\vec{x}$ works:

- $$\boxed{A\hat{v}_i = \sigma_i \hat{u}_i} \text{ for } i = 1, \ldots, r.$$

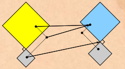
  and

  $$\boxed{A\hat{v}_i = \hat{0}} \text{ for } i = r+1, \ldots, n.$$

- Matrix version:

  $$\boxed{A = U\Sigma V^{\mathrm{T}}}$$

- A sends each $\hat{v}_i \in C(A^{\mathrm{T}})$ to its partner $\hat{u}_i \in C(A)$ with a positive stretch/shrink factor $\sigma_i > 0$.
- A is diagonal with respect to these bases.
- When viewed in the right way, every A is a diagonal matrix Σ.



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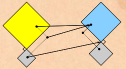

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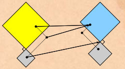

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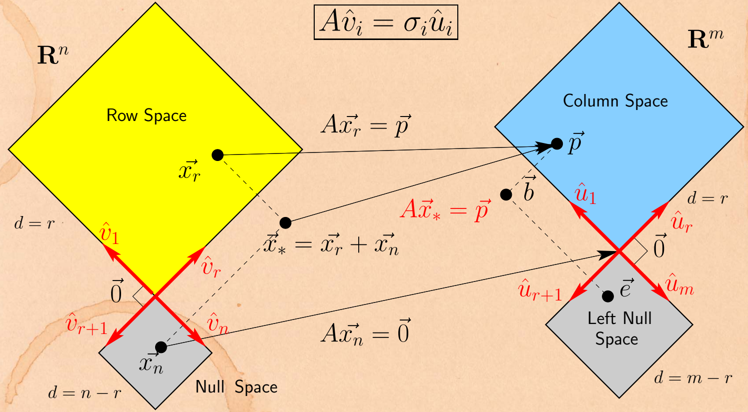


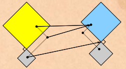
Image approximation (80x60)

Idea: use SVD to approximate images

- Interpret elements of matrix A as color values of an image.
- Truncate series SVD representation of A:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

- Use fact that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0$.
- Rank $r = \min(m, n)$.
- Rank $r =$ # of pixels on shortest side (usually).
- For color: approximate 3 matrices (RGB).



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- Use fact that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0$.
- Rank $r = \min(m, n)$.
- Rank $r =$ # of pixels on shortest side (usually).
- For color: approximate 3 matrices (RGB).

Image approximation (80x60)

Idea: use SVD to approximate images

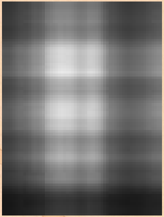
- Interpret elements of matrix A as color values of an image.
- Truncate series SVD representation of A:

$$A = U\Sigma V^{\mathrm{T}} = \sum_{i=1}^{r} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

- Use fact that $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_r > 0$.
- Rank $r = \min(m, n)$.
- Rank $r = \#$ of pixels on shortest side (usually).
- For color: approximate 3 matrices (RGB).

Image approximation (80x60)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



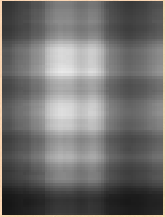
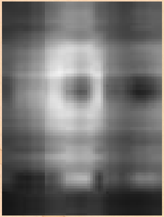


Image approximation (80x60)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



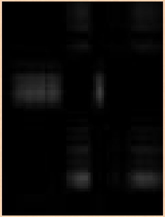
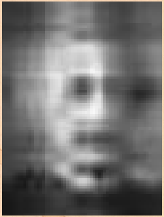
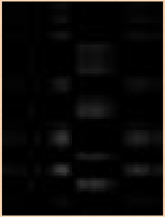


Image approximation (80x60)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





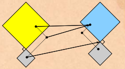


Image approximation (80x60)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

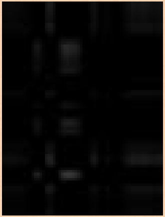


Image approximation (80x60)

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

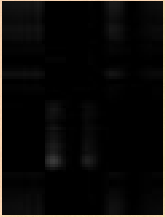


Image approximation (80x60)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

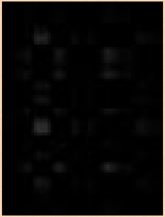


Image approximation (80x60)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

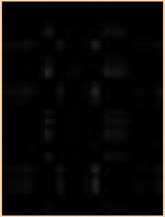


Image approximation (80x60)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

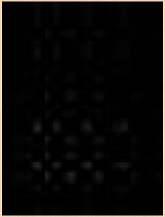


Image approximation (80x60)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

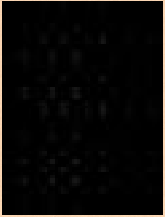


Image approximation (80x60)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

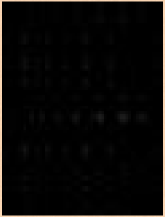
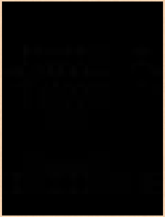


Image approximation (80x60)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



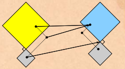


Image approximation (80x60)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

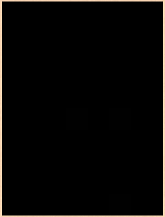


Image approximation (80x60)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

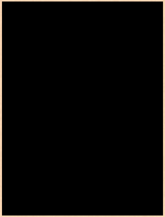
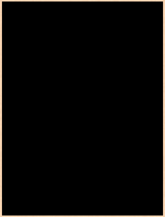


Image approximation (80x60)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



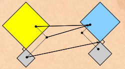
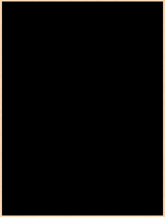


Image approximation (80x60)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



Decay of sigma values: Einstein

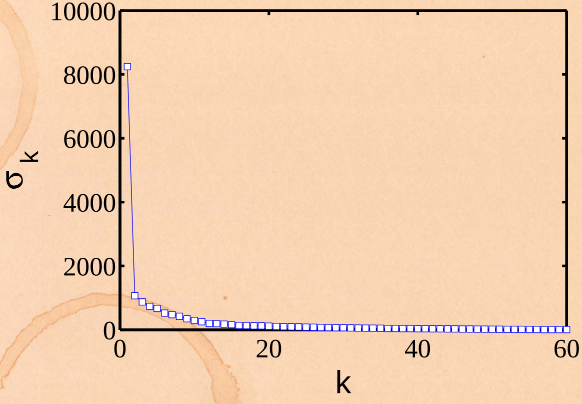


Image approximation (480x615)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

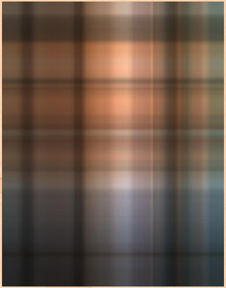
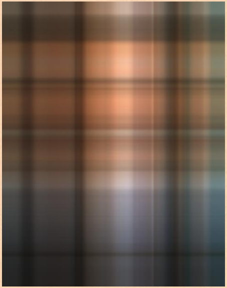
 

Image approximation (480x615)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

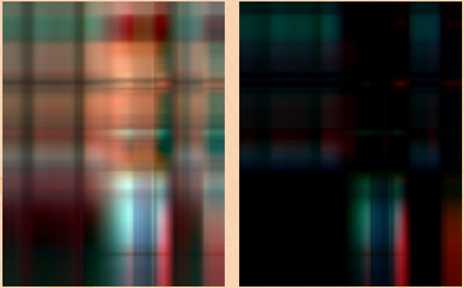
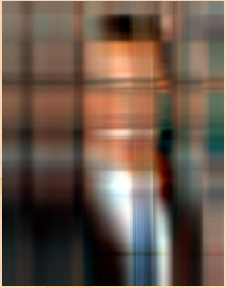


Image approximation (480x615)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



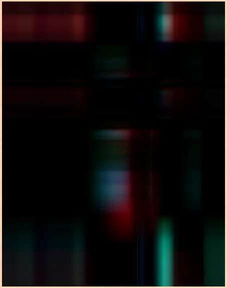


Image approximation (480x615)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

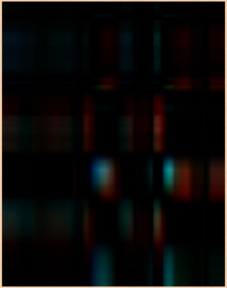


Image approximation (480x615)

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

Image approximation (480x615)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

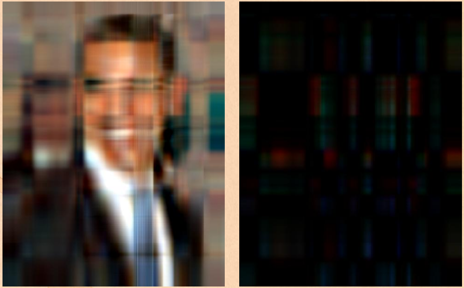


Image approximation (480x615)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

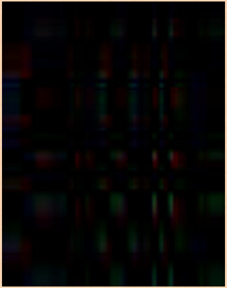


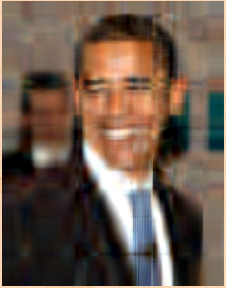
Image approximation (480x615)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

The University of Vermont

Image approximation (480x615)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



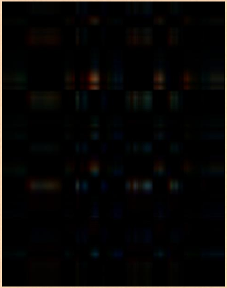
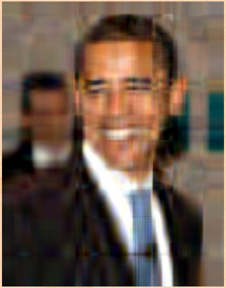


Image approximation (480x615)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



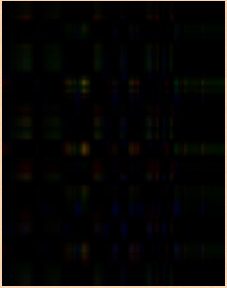


Image approximation (480x615)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

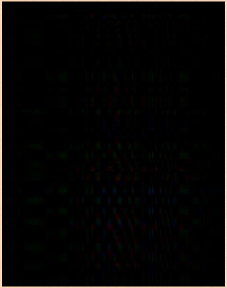


Image approximation (480x615)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

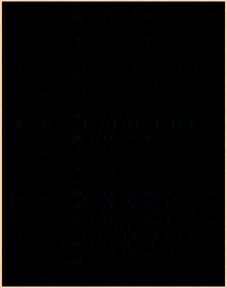
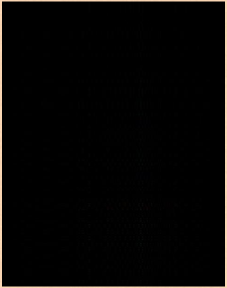


Image approximation (480x615)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



The Fundamental Theorem of Linear Algebra
Approximating matrices with SVD

Image approximation (480x615)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

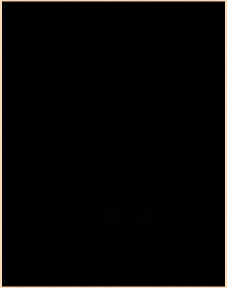


Image approximation (480x615)

$$A = \sum_{i=1}^{60} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

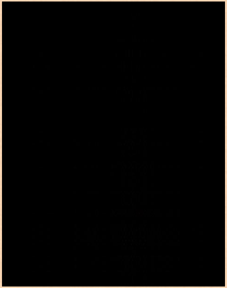


Image approximation (480x615)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

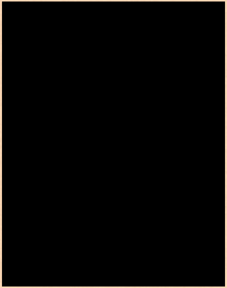


Image approximation (248x262)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

Image approximation (248x262)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

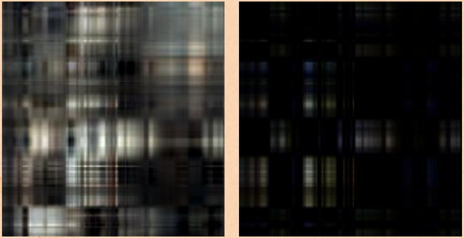
Image approximation (248x262)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

Image approximation (248x262)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



(wikipedia.org)

Image approximation (248x262)

The Fundamental Theorem of Linear Algebra
Approximating matrices with SVD

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

Image approximation (248x262)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

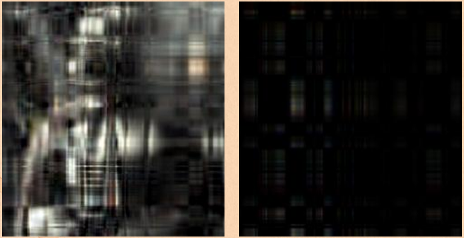
Image approximation (248x262)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

Image approximation (248x262)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



(wikipedia.org)

Image approximation (248x262)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

Image approximation (248x262)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

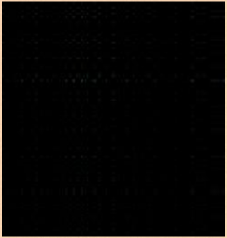
Image approximation (248x262)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

Image approximation (248x262)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



(wikipedia.org)

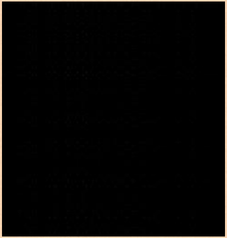
Image approximation (248x262)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$


(wikipedia.org)

Image approximation (248x262)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



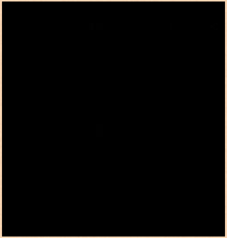
(wikipedia.org)

Image approximation (248x262)

The Fundamental Theorem of Linear Algebra

Approximating matrices with SVD

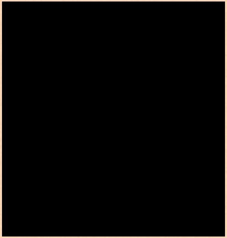
$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



(wikipedia.org)

Image approximation (248x262)

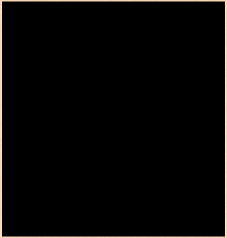
$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



(wikipedia.org)

Image approximation (248x262)

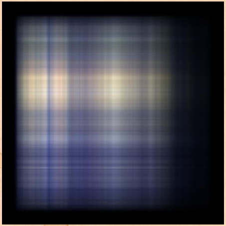
$$A = \sum_{i=1}^{240} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

(wikipedia.org)

Image approximation (540x540)

$$A = \sum_{i=1}^{1} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



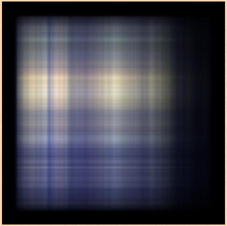
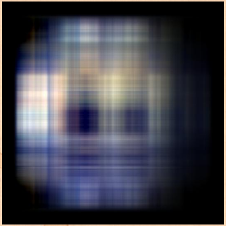


Image approximation (540x540)

$$A = \sum_{i=1}^{2} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



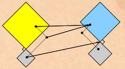
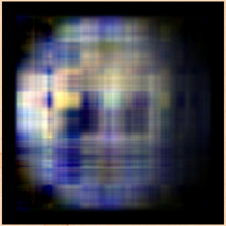


Image approximation (540x540)

$$A = \sum_{i=1}^{3} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



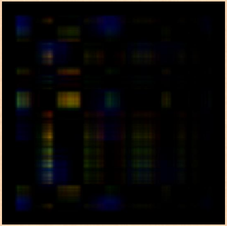
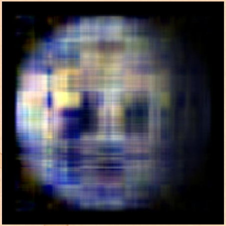


Image approximation (540x540)

$$A = \sum_{i=1}^{4} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



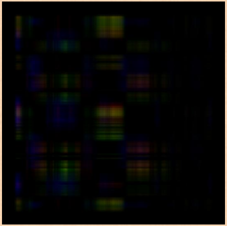


Image approximation (540x540)

$$A = \sum_{i=1}^{5} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

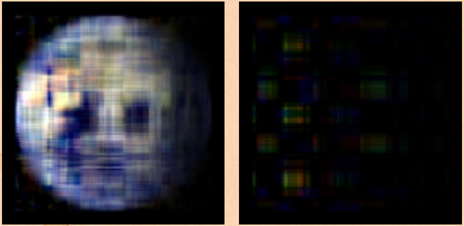


Image approximation (540x540)

$$A = \sum_{i=1}^{6} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

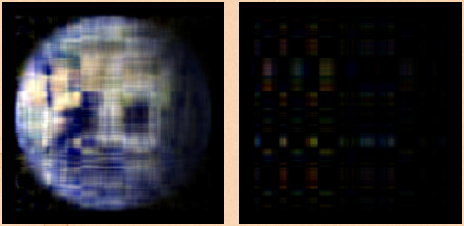


Image approximation (540x540)

$$A = \sum_{i=1}^{7} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

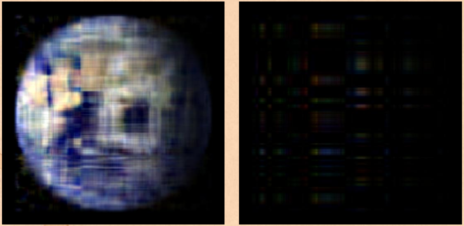
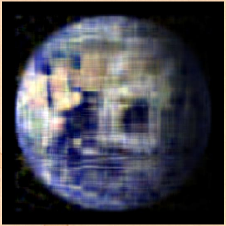


Image approximation (540x540)

$$A = \sum_{i=1}^{8} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



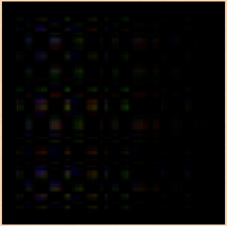
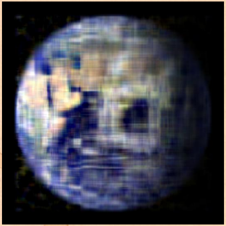


Image approximation (540x540)

$$A = \sum_{i=1}^{9} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$



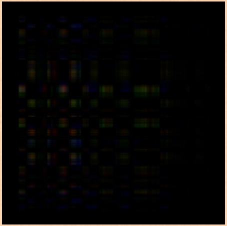
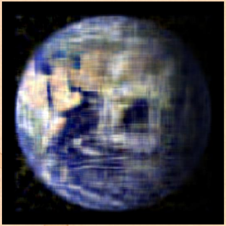
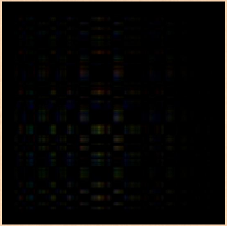


Image approximation (540x540)

$$A = \sum_{i=1}^{10} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$





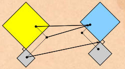


Image approximation (540x540)

$$A = \sum_{i=1}^{20} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

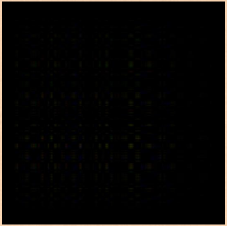


Image approximation (540x540)

$$A = \sum_{i=1}^{30} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

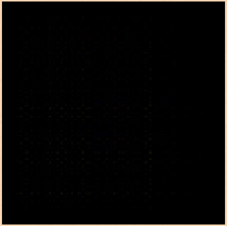
 

Image approximation (540x540)

$$A = \sum_{i=1}^{40} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

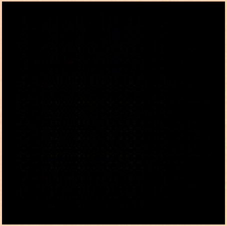


Image approximation (540x540)

$$A = \sum_{i=1}^{50} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

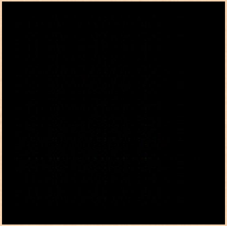


Image approximation (540x540)

$$A = \sum_{i=1}^{100} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

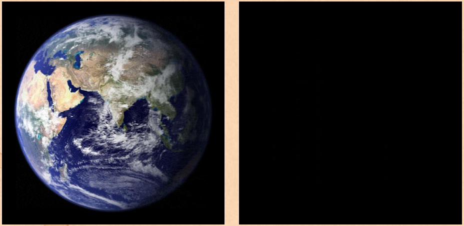


Image approximation (540x540)

$$A = \sum_{i=1}^{200} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$

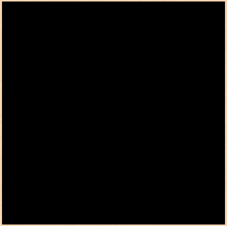


Image approximation (540x540)

$$A = \sum_{i=1}^{480} \sigma_i \hat{u}_i \hat{v}_i^{\mathrm{T}}$$