Complex Networks, CSYS/MATH 303—Assignment 9
University of Vermont, Spring 2011

Dispersed: Sunday, April 17, 2011.

Due: By start of lecture, 2:30 pm, Thursday, April 28, 2011.

Some useful reminders:

Instructor: Peter Dodds

Office: Farrell Hall, second floor, Trinity Campus

E-mail: peter.dodds@uvm.edu

Office hours: 3:45 pm to 4:15 pm Tuesday post class; 12:00 pm to 2:00 pm, Wednesday

Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2011-01UVM-303`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LATEX (or related variant).

---

1. (12 pts) Consider a family of undirected random networks with degree distribution

$$P_k = c\delta_{k1} + (1-c)\delta_{k3},$$

where δ_{ij} is the Kronecker delta function, and where c is a constant to be determined below. Also allow nodes to be correlated according to the following node-node mixing probabilities.

Conditional probability version, $P(k\,|k'\,)$:

$$P(1\,|\,1) = \frac{1}{2}(1+r),$$

$$P(3\,|\,1) = \frac{1}{2}(1-r),$$

$$P(1\,|\,3) = \frac{1}{2}(1-r),$$

$$\text{and } P(3\,|\,3) = \frac{1}{2}(1+r).$$

where $-1 \le r \le 1$ is the family's tunable parameter.

Newman's correlation probability version:

$$E = [e_{ij}] = \begin{bmatrix} e_{00} & e_{02} \\ e_{20} & e_{22} \end{bmatrix} = \frac{1}{4}\begin{bmatrix} (1+r) & (1-r) \\ (1-r) & (1+r) \end{bmatrix}$$

where e_{ij} is the probability that a randomly chosen edge connects a node of degree $i+1$ an a node of degree $j+1$, and only the non-zero values of E are shown.

For the following questions, you can use either formulation.

(a) Determine c so that purely disassortative networks are achievable if r is tuned to -1.

(b) Determine which networks in this family have a giant component. In other words, find the values of r for which a giant component exists.

Note which value (or values) of r mark a phase transition.

(c) Analytically determine the size of the giant component as a function of r.

(d) Determine the size of the largest component containing only degree 3 nodes as a function of r.

Hint: allow degree 3 nodes to be always vulnerable ($\beta_{3i} = 1$ for $i = 0$, 1, 2, and 3) and degree 1 nodes to be never vulnerable ($\beta_{1i} = 0$ for $i = 0$ and 1).

2. Spreading on assortative networks: Starting from

$$\theta_{j,t+1} = G_j(\vec{\theta_t}) = \phi_0 + (1 - \phi_0)\times$$

$$\sum_{k=1}^{\infty} \frac{e_{j-1,k-1}}{R_{j-1}} \sum_{i=0}^{k-1} \binom{k-1}{i} \theta_{k,t}^i (1 - \theta_{k,t})^{k-1-i} B_{ki}.$$

show the matrix for which we must have the largest eigenvalue greater than 1 for spreading to occur is

$$\frac{\partial G_j(\vec{0})}{\partial \theta_{k,t}} = \frac{e_{j-1,k-1}}{R_{j-1}}(k-1)(\beta_{k1} - \beta_{k0}).$$

3. Show that for uncorrelated networks, i.e, when $e_{jk} = R_j R_k$, the above condition collapses to the standard condition

$$\sum_{k=1}^{\infty}(k-1)\frac{kP_k}{\langle k \rangle}(\beta_{k1} - \beta_{k0}) > 1.$$