Complex Networks, CSYS/MATH 303—Assignment 8
University of Vermont, Spring 2011

Dispersed: Sunday, April 10, 2011.
Due: By start of lecture, 2:30 pm, Thursday, April 21, 2011.
Some useful reminders:
Instructor: Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 3:45 pm to 4:15 pm Tuesday post class; 12:00 pm to 2:00 pm, Wednesday
Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2011-01UVM-303`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related variant).

---

1. Using Gleeson and Calahane's iterative equations below, derive the contagion condition for a vanishing seed by taking the limit $\phi_0 \to 0$ and $t \to \infty$.

$$\phi_{t+1} = \phi_0 + (1-\phi_0) \sum_{k=0}^{\infty} P_k \sum_{j=0}^{k} \binom{k}{j} \theta_t^j (1-\theta_t)^{k-j} B_{kj},$$

$$\theta_{t+1} = \phi_0 + (1-\phi_0) \sum_{k=1}^{\infty} \frac{kP_k}{\langle k \rangle} \sum_{j=0}^{k-1} \binom{k-1}{j} {\theta_t}^j (1-\theta_t)^{k-1-j} B_{kj},$$

where $\theta_0 = \phi_0$, and B_{kj} is the probability that a degree k node becomes active when j of its neighbors are active.

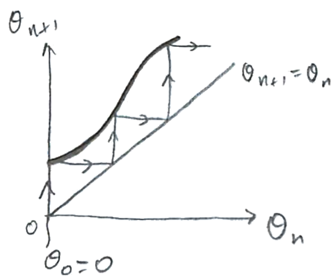
Recall that by contagion condition, we mean the requirements of a random network for macroscopic spreading to occur.

To connect the paper's model and notation to those of our lectures, given a specific response function F and a threshold model, the B_{kj} are given by $B_{kj} = F(j/k)$.

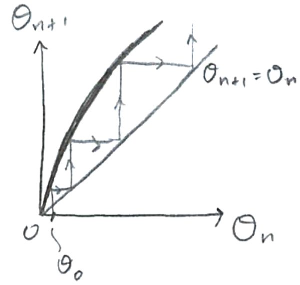
Allow B_{k0} to be arbitrary (i.e., not necessarily 0 as for simple threshold functions).

Here's a graphical hint for the three cases you need to consider as $\theta_0 \to 0$:

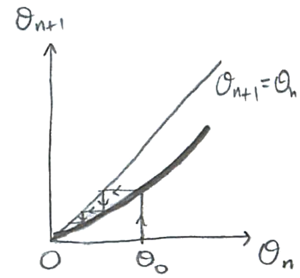
Success:



Sucesss:



Fail:



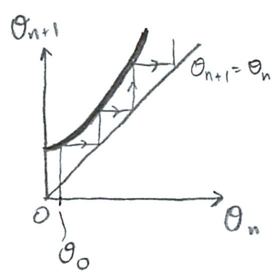
2. Derive equation 4 in Gleeson and Cahalane (2007) [1]:

$$C_\ell = \sum_{k=\ell+1}^{\infty} \sum_{j=0}^{\ell} \binom{k-1}{\ell} \binom{\ell}{j} (-1)^{\ell+j} \frac{kP_k}{\langle k \rangle} F\left(\frac{j}{k}\right).$$

3. (9 pts)

   (a) Derive equation 6 in Gleeson and Cahalane (2007), which is a second order approximation to the cascade condition for vanishing seeds.

   Here's an example of how this must work:

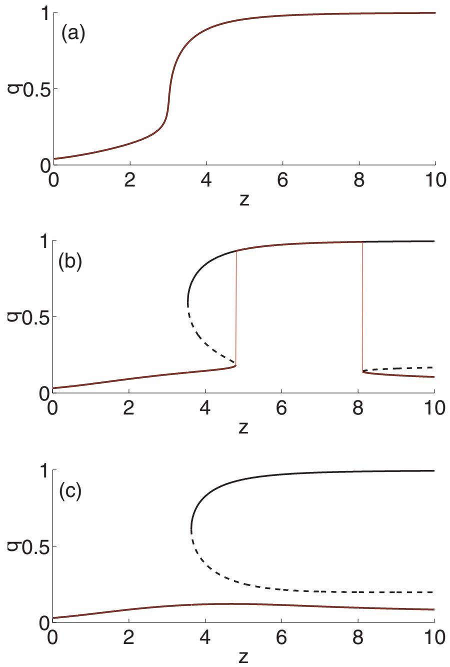


   (b) Hence reproduce the dashed analytic curve shown in Figure 1 of their paper.

   (c) Explain why there are jumps in the cascade window outline that do not occur at reciprocals of the integers.

4. (6 pts)

   (a) By solving for the fixed points of $\theta_{t+1} = G(\theta_t; 0)$, reproduce Figure 3 in Gleeson and Cahalane (2007):



(b) Also plot $G(\theta_t; 0)$ for an average threshold $\phi_*(= R)$ of 0.371 for $\langle k \rangle (= z) = 1, 2, 3, \ldots, 10$.

Add the cobweb diagram for a $\phi_0 = 0$ seed.

Do this by creating a recursive plotting script in matlab, for example.

You can use the following Matlab scripts/data as a basis, and most of the work is done. You'll need to improve the plots with some labels, and interpret them properly. The first function calls the other two.

`http://www.uvm.edu/~pdodds/share/matlab/Gfunction.m`
`http://www.uvm.edu/~pdodds/share/matlab/gleeson_fig3_02.mat`
`http://www.uvm.edu/~pdodds/share/matlab/cobweb3.m`

(c) Discuss how the stable points move with $\langle k \rangle$.

Note: $\phi_* = 0.371$ matches plot (b) in Figure 3 of [1].

References

[1] J. P. Gleeson and D. J. Cahalane. Seed size strongly affects cascades on random networks. *Phys. Rev. E*, 75:056103, 2007. pdf (⊞)