Complex Networks, CSYS/MATH 303—Assignment 5
University of Vermont, Spring 2011

Dispersed: Thursday, March 10, 2011.
Due: By start of lecture, 2:30 pm, Thursday, March 24, 2011.
Some useful reminders:
Instructor: Peter Dodds
Office: Farrell Hall, second floor, Trinity Campus
E-mail: peter.dodds@uvm.edu
Office hours: 3:45 pm to 4:15 pm Tuesday post class; 1:00 pm to 3:00 pm, Wednesday
Course website: `http://www.uvm.edu/~pdodds/teaching/courses/2011-01UVM-303`

All parts are worth 3 points unless marked otherwise. Please show all your working clearly and list the names of others with whom you collaborated.

Graduate students are requested to use LaTeX (or related variant).

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1. Given N labelled nodes and allowing for all possible number of edges m, what's the total number of undirected, unweighted networks we can construct?

   How does this number scale with N?

2. Given N labelled nodes and a variable number of m edges, for what value of m do we obtain the largest diversity of networks? And for this m, how does the number of networks scale with N?

3. We've seen that large random networks have essentially no clustering, meaning that locally, random networks are pure branching networks. Nevertheless, a finite, non-zero number of triangles will be present.

   For pure random networks, with connection probability $p = \langle k \rangle/(N-1)$, what is the expected total number of triangles as $N \rightarrow \infty$?

4. Repeat the preceding calculation for cycles of length 4 and 5 (triangles are cycles of length 3).

5. We've figured out in class that for large enough N (and $\langle k \rangle$ fixed), a random network always has a Poisson degree distribution. And as we've discussed, we don't find these networks in the real world (they don't arise due to simple mechanisms). Let's investigate this oddness a little further.

   (a) Compute the expected size of the largest degree in a random network given $\langle k \rangle$ and as a function of increasing N. In other words, in selecting (with replacement) N degrees from a Poisson distribution with mean $\langle k \rangle$, what's the expected value of the largest degree $k_{\max}$?

A good way to compute $k_{\max}$ is to equate it to the value for which we expect $1/N$ of our random selections to exceed. (We had a question in 300 along these lines for power-law distributions.)

(b) Now let's flip the question: How likely is it to find a very high degree node in a pure random network?

Compute the probability that a randomly selected node in a randomly selected network (specified again by N and $\langle k \rangle$) has a degree exceeding $\simeq 1 \cdot N^{\alpha}$ where $\alpha \leq 1$.

Start with the case $\alpha = 1$ which is to be interpreted as a node being connected to all other $N - 1$ nodes.